Sustainable Materials and Green Buildings
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Lecture No. 23
Operational Energy: Estimation of Thermal Conductivity

So we will continue from where we were doing last class we are trying to look estimate k thermal conductivity.

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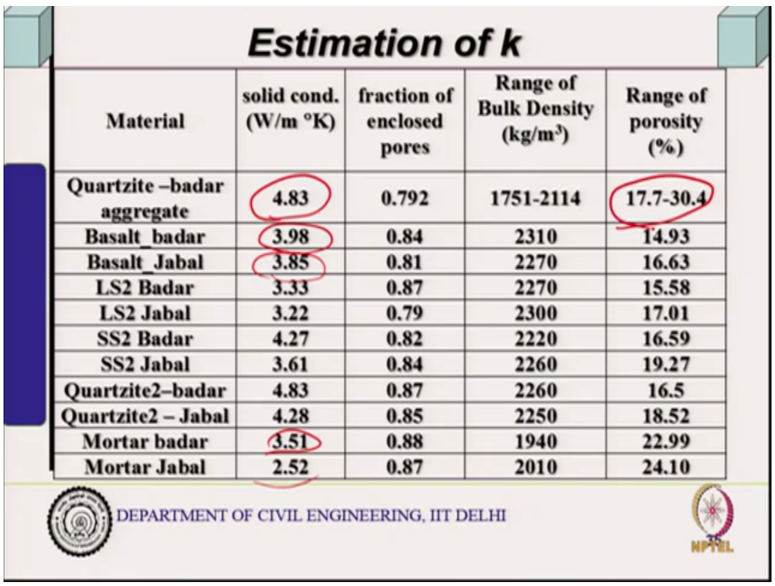

Material	solid cond. (W/m °K)	fraction of enclosed pores	Range of Bulk Density (kg/m^3)	Range of porosity (%)
Quartzite –badar aggregate	4.83	0.792	1751-2114	17.7-30.4
Basalt_badar	3.98	0.84	2310	14.93
Basalt_Jabal	3.85	0.81	2270	16.63
LS2 Badar	3.33	0.87	2270	15.58
LS2 Jabal	3.22	0.79	2300	17.01
SS2 Badar	4.27	0.82	2220	16.59
SS2 Jabal	3.61	0.84	2260	19.27
Quartzite2–badar	4.83	0.87	2260	16.5
Quartzite2 – Jabal	4.28	0.85	2250	18.52
Mortar badar	3.51	0.88	1940	22.99
Mortar Jabal	2.52	0.87	2010	24.10

And I think we are talking about some of those you know like solid estimated solid conductivity, estimated solid conductivity, from for various kinds of concrete infact, this is concrete which concrete this is different types of aggregates**.** So one can actually estimate them for mortar and concrete, what is this solid conductivity. Now, we are trying to look at their porosities also one can also determine through water permeable porosity test and things like that. Because this water permeable porosity can be very easily measured.

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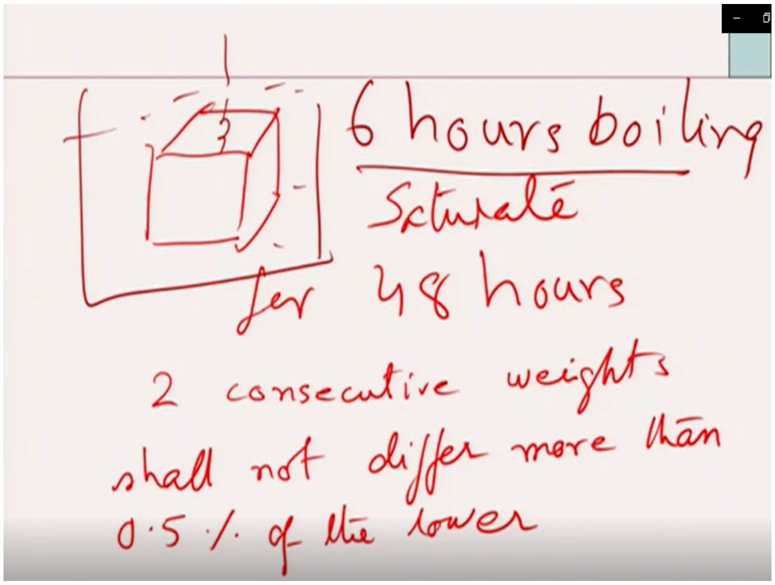

First of all, take a cubic sample or you know solid sample, saturated for 48 hours and for confirmation another 24 hours. 2 weights should not be different by 0.5 percent. So, two consecutive weight, 2 consecutive weight, 2 consecutive weights shall be shall not differ more than 0.5 percent of the lower, so then it is fully saturated.

Then boil for 6 hours, then boil for 6 hours, 6 hours boiling, 6 hours boiling and the difference in weights of the dry and after boiling that is full saturation gives you the mass of moisture absorbed and its volume you can find out by suspending it in you know by taking a suspended weight through a spring balance in a immersed condition, fully immersed condition.

So, you know the mass loss is equal to the volume of water displaced and so on you can find out. So, therefore, overall volume can be found out and volume of portion we represented by the mass of water that is gone in two different weights so one can estimate them, STM procedure is there for this. So permeable porosity one can determine. So this permeable porosity.

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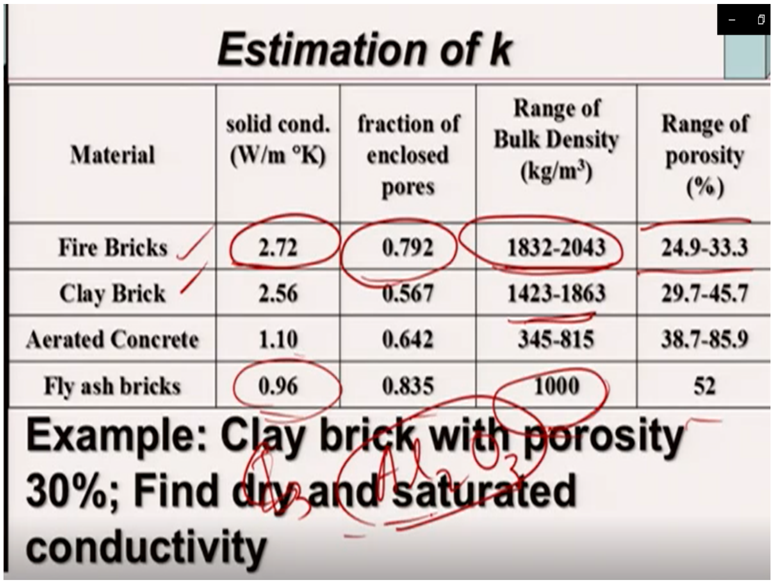

Material	solid cond. (W/m °K)	fraction of enclosed pores	Range of Bulk Density (kg/m³)	Range of porosity (%)
Fire Bricks	2.72	0.792	1832-2043	24.9-33.3
Clay Brick	2.56	0.567	1423-1863	29.7-45.7
Aerated Concrete	1.10	0.642	345-815	38.7-85.9
Fly ash bricks	0.96	0.835	1000	52

This permeable porosity, this permeable porosity, so this is permeable porosity, so for various bricks for example, fire bricks are those bricks which are used in the factories. So generally they have high, I mean sorry, now high Al2O3, high alumina content in this type of fire bricks. So obtained from surface clay bricks are obtained from, clay bricks are obtained, first you remove the organic soil, then go to one level below the clay that you get it is called the surface clay and that bricks are our ordinary clay bricks.

Normally they will have less of this one, this is fire bricks which are produced at a you know it comes down below a lower level and can sustain much higher temperature. So that is why they are used in the factories but coming back to this. So, their solid conductivity estimated is something like this, the fraction of enclosed force is something like this and bulk density would be somewhat higher you can see and then aerated concrete there was a something of this kind, fly ash bricks is of similar kind density is much less because they would have actually done some autoclaving in this particular types.

Fly ash, some local sand and part of the lime or cement and then heat it up, heat it up to get the autoclave to cure it and then that is what it is. So, this solid conductivity is less in this particular one. So, supposing you want to find out the thermal conductivity of bricks with porosity let us say 30 percent, you know this range is from 29 to 45.7.

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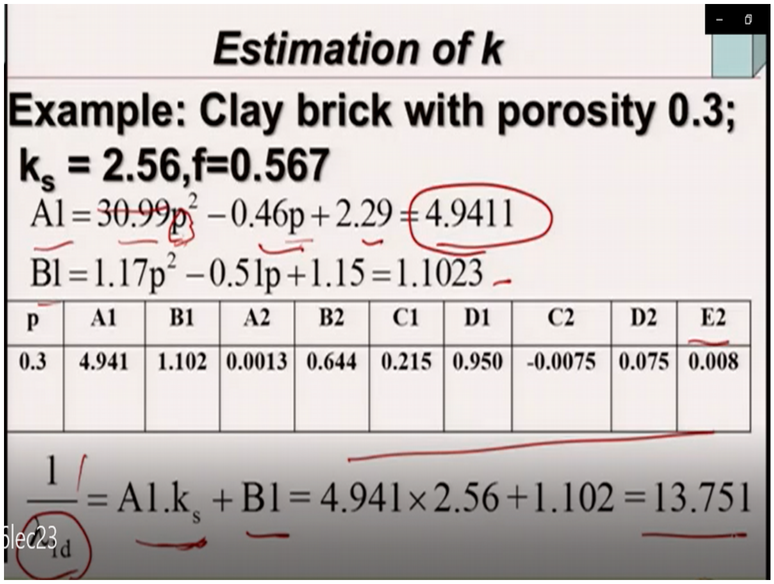

So first is clay brick with porosity 30 percent. Now, we have seen this conductivity of solid is 2.56, so and fraction of enclosed pores is 0.567. So, our equation was something like this, this constants A1, B1, etcetera. etcetera. up to E2 one can estimate putting this p equal to 0.3. So, 0.3 means, 0.09 since 10 percent here, something like this so you know if you find out the value of A1 which is this, so put p equals to 0.3 and this you calculate out this comes out to 4.9411.

Similarly, B1 using the same equation which I have shown you earlier and all the constants work out to be like this. Now, remember then we expanded this and use this to find out the value of some equivalent conductivity all in dry state at the moment, so A1 into solid conductivity plus B1 it comes out to be 13.751.

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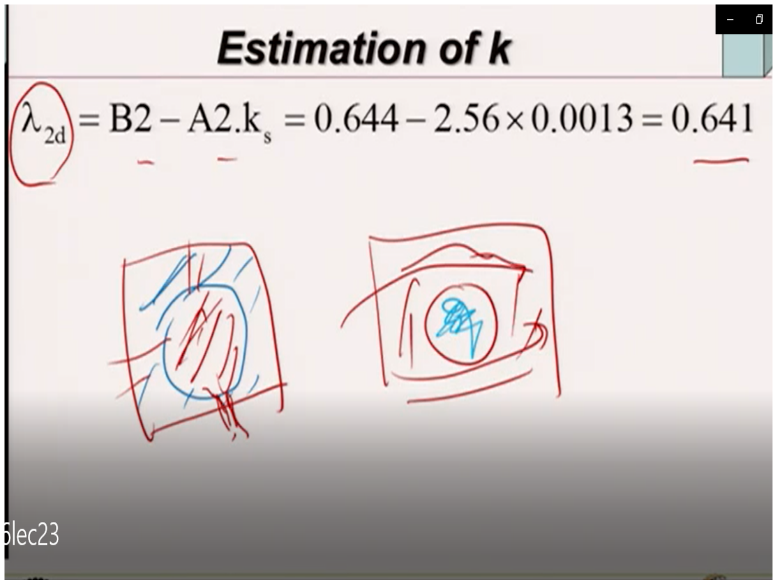

1 by Lambda 1d we have find out, Lambda 2d similarly one can find out, it is 0.641. So, one of them were, were, you can see that conductivity is very less here which means that actually pores are enclosing the solid, so they are enclosing pores, Lambda 1 d, because 1 by Lambda d so 1 by Lambda Lambda 1d will be 1 by 13.751 which will be small and this is pore is enclosed so you have got solid part all around the pore so this is your pore, this let us say is our pore, this let us say is our pore, so this is the pore.

Now, solid will have path, heat flow path would be like this, heat flow path, solids can go like this, heat flow path can go like this but when you have the other way around the case where you have, this is your pores essentially, majority of it is pore, this is pore, and the solid is inside, the solid is inside, solid is inside, solid is inside, solid is inside in that case conductivity will be much less because you know the path of solid path is just connected through some interconnectivity. This is pores, inside the solid so solid is connected to some interconnection, so therefore this is much less. Here the pore, these are all solid so heat flow path would be much less here because there is an (inner) around the pore there is an interconnection. So, these are largely open types of pores, these are closed type of pores and therefore these conductivity values comes something like this.

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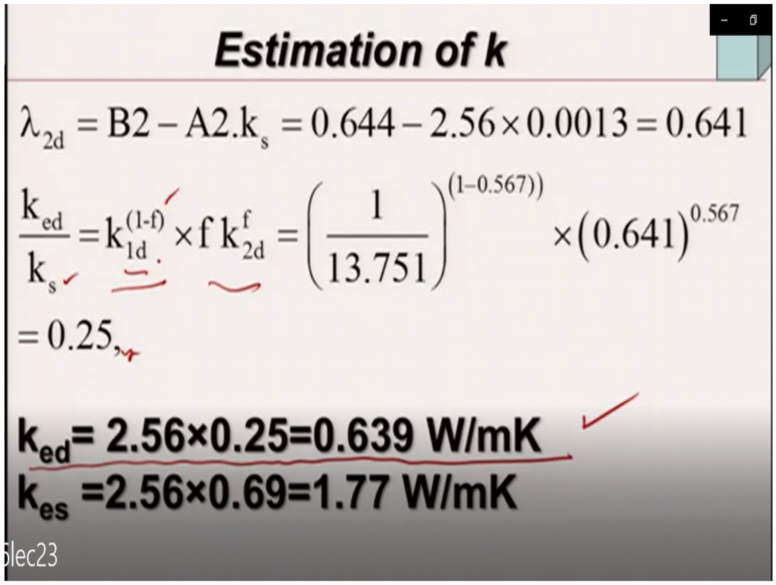

Then similarly one can find out, then one can find out Ked by Ks using that formula which we which we gave earlier k 1d to the power 1 by this fraction of pores, and this is the fraction of this is the other type, so fraction of so putting this into the equation one gets Ked by Ks equals to 0.25. So, therefore, conductivity will be 2.56 into (0.63) 0.639-watt meter Kelvin because Ked by Ks is equal to 0.25, Ks we have 2.56 just we have seen that from this table we have taken so we have taken from this table 2.56. So, if you know the solid conductivity you know you know you can know the solid conductivity, you can actually estimate in this manner and if you can find out in the similar manner Kes.

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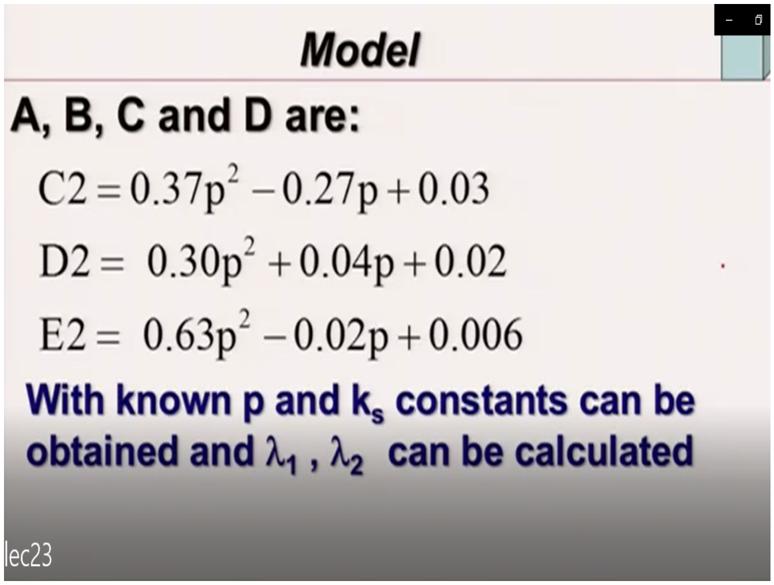

To find out Kes what you will do instead of finding 1 by Lambda d, you will find out 1 by, you know you will find out 1 by Lambda 1s and then Lambda 2s in same equation. So if you want to find out Lambda 1s and 2s, the formula is here, formula you are here A, B, C, D, A, B, C, D, E you know so this formula was there.

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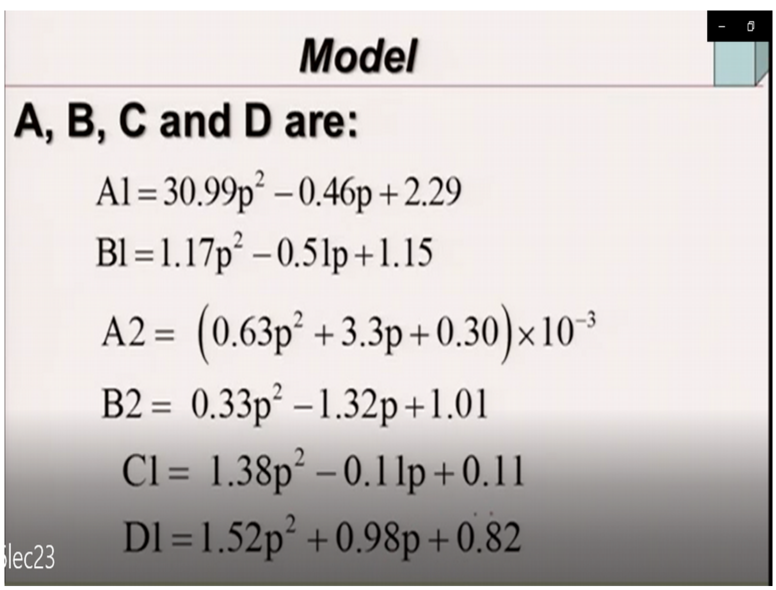

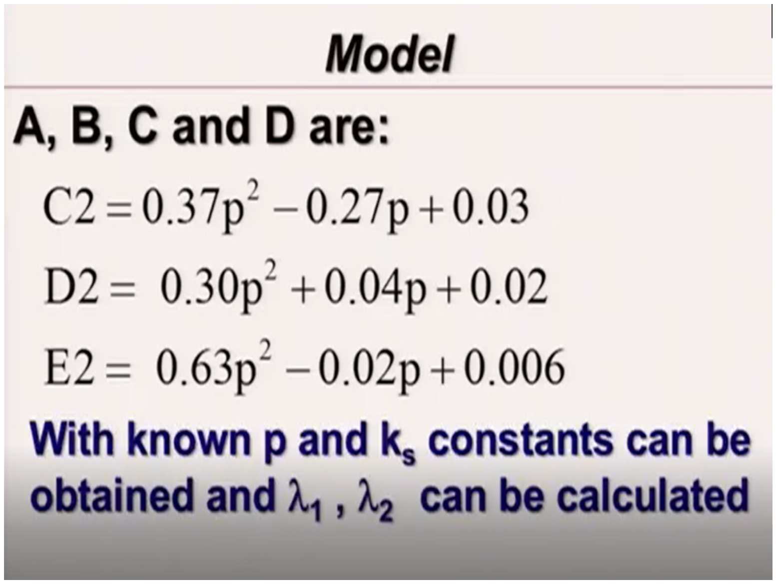

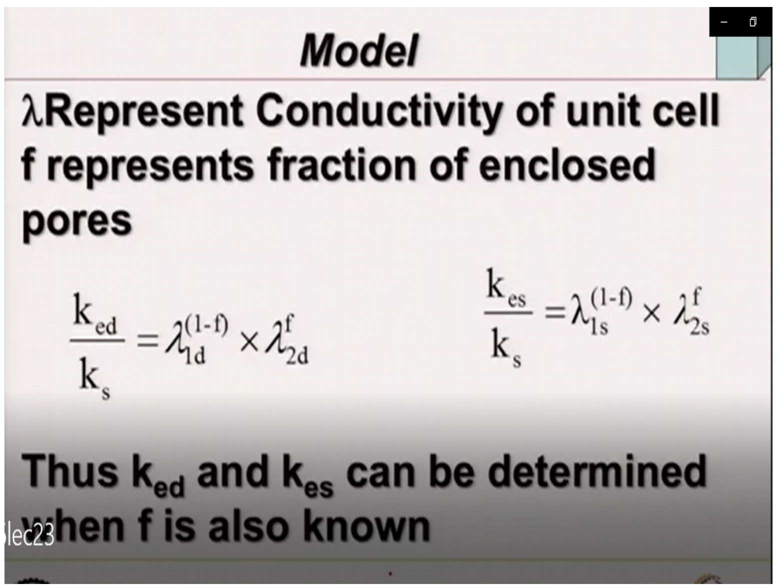

So, for example, it was you know this formula one can use and find out the values for, we just use write down this A1 and B1 and A2 and B2 and then C1, D1, C2, D2, E2, this three one can utilize to find out the value of the constants and once you have found out the value of the constants you can use to find out Kes is given by this formula, Kes is given by this formula, Kes by Ks, this is 2.56, this value you have found out, this value you have found out, f is 0.657 whatever it is that you looked into so, then you can estimate the Kes also.

So, estimated Kes comes out to be 0.69 multiplied by 2.56, 1.77. So, using this tables and charts, you can actually estimate the conductivity like this example is there. Just you got to check your calculation yourself, you can check them. So, but, this is you know saturated. What about the intermediate moisture content? The empirical one was Jacob's factor which is a multiply factor I mentioned sometime there is a factor, but you want to do it a little bit more accurately or somewhat better.

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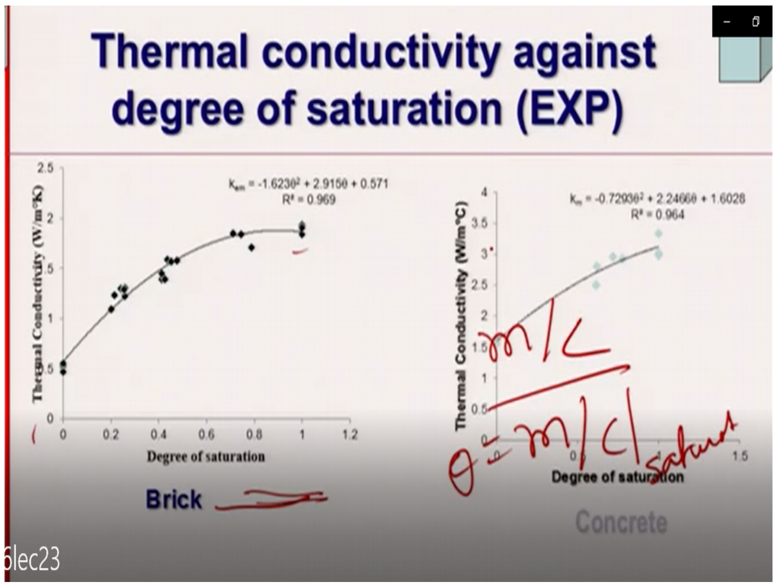

Generally, one can observe this is your degree of saturation, degree of saturation for example brick. What is degree of saturation? The moisture content divided by the moisture content at saturation, at saturation. So, this is what we call and generally denote sometimes by Theta is degree of saturation. So, degree of saturation is the mass of the moisture or moisture content at a particular moisture level divided by the saturation moisture content.

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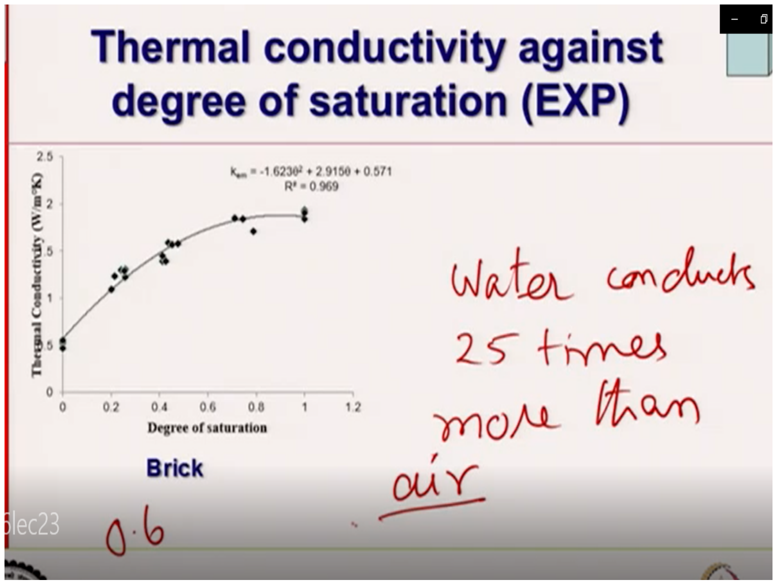

Now degree of saturation, if you if you plot against thermal conductivity you get something like this. It tends to increase with degree of saturation. As the moisture content increases, that we have seen earlier and then becomes a steady value, at the saturation near saturation level. What is the reason behind this? What is the reason behind this?

Actually, water conducts 25 times more than that of air that is the first thing, so water conducts 25 times more than that of air, this was one issue more than air so if it is moisture comes into such porous material then thermal conductivity will increase and as you have seen the last example what you have seen. The dry conductivity of something like 0.65 or something if you are seen last time 0.639 and 1. 77, so it can be few folds, it can be few folds.

So the insulation quality can reduce significantly in presence of moisture, in presence of moisture actually three times, nearly three times, so this can change significantly, this can change significantly, so water content is 25 times more than air and also there can be a phenomenon like evaporation, condensation. What is evaporation, condensation?

The moisture at the hot phase evaporates and when it goes to the colder region it again becomes water, from vapor to the water, so, therefore, it is just converts, latent heat is taken and absorbed first and discharged or dissipated on the cold zone, so hot to cold zone there can be heat transfer by evaporation and condensation. Now, one can actually model that but in the real material, with the types of pores their interconnectivity becomes difficult but empirically one can see that the Kem is the thermal conductivity with the moisture. It is a function of degree of saturation in a empirical equation could be in a quadratic form because it increases and therefore it stabilizes. So this is for brick let us say.

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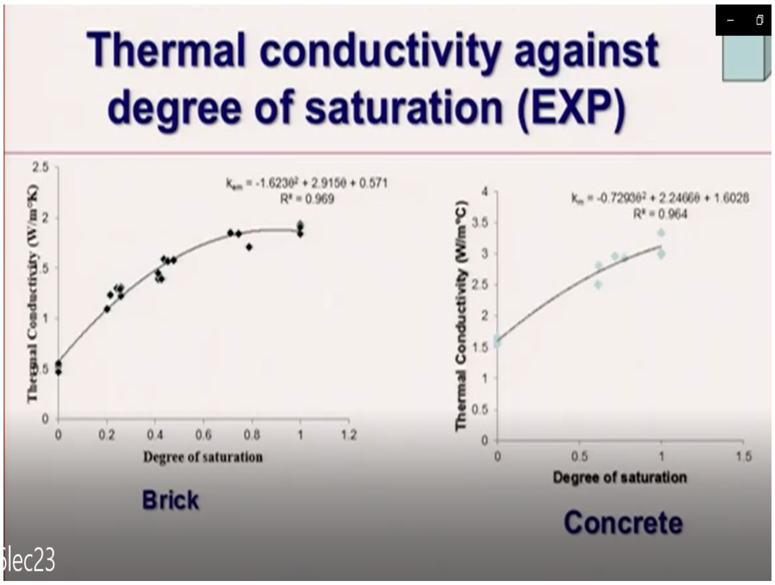

For other porous material, it would be similar, for concrete also, it is similar, degree of saturation to conductivity. So this is something of the kind. So, therefore it is a general equation of this form.

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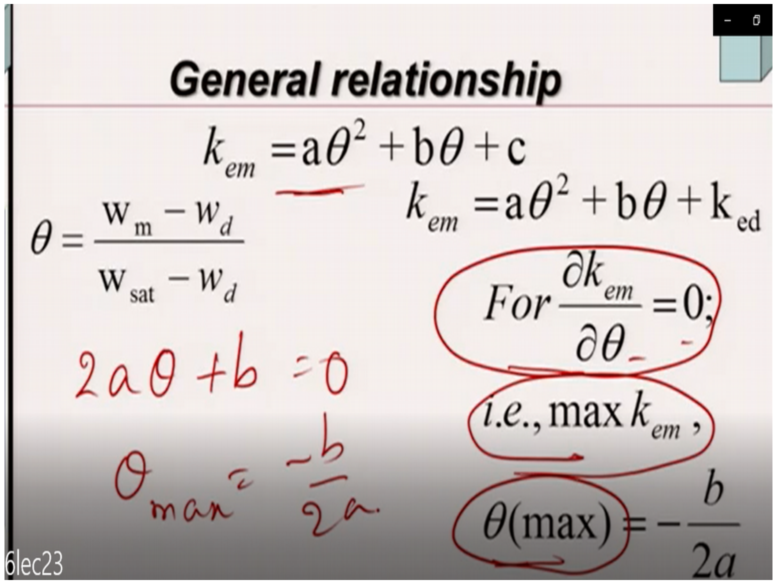

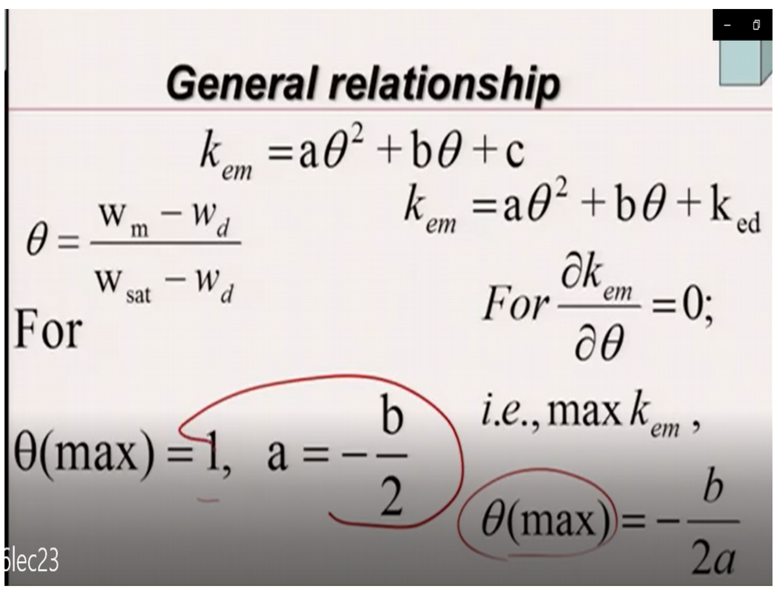

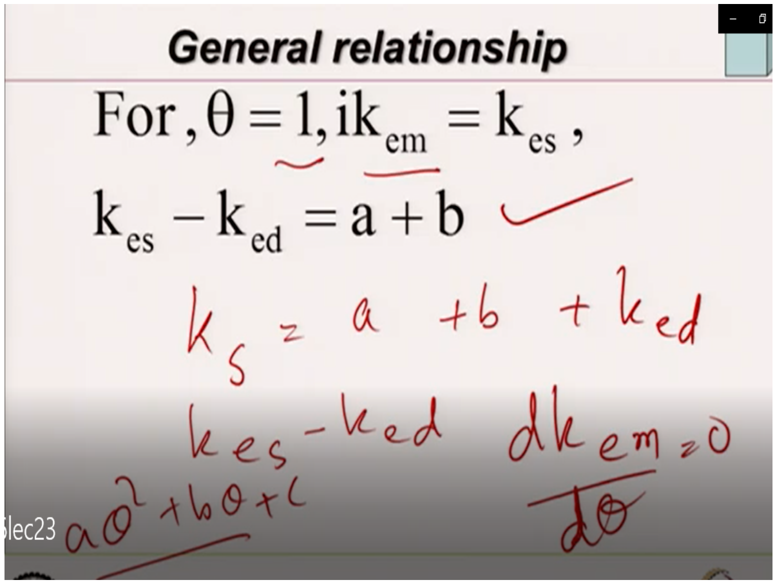
General relationship
$\text{For}, \theta = 1, ik_{em} = k_{es},$
$k_{es} - k_{ed} = a + b$
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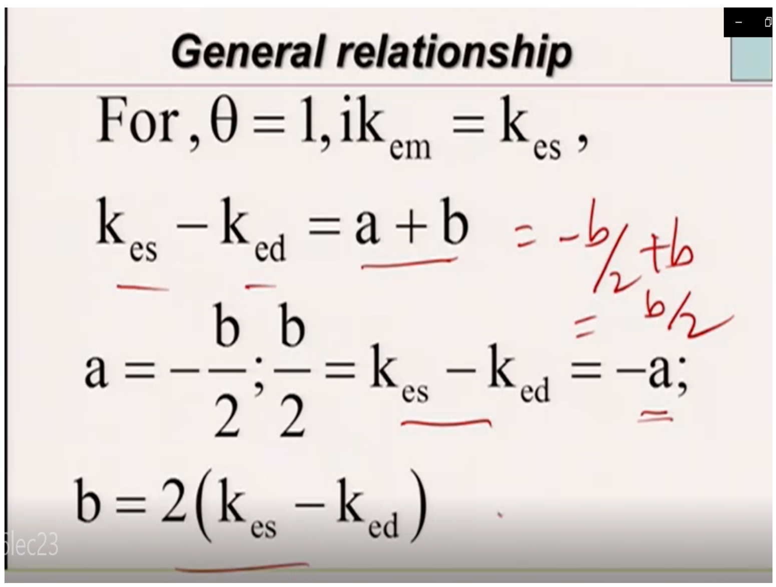
General relationship
$\text{For}, \theta = 1, ik_{em} = k_{es},$
$k_{es} - k_{ed} = a + b$
$a = -\frac{b}{2}; \frac{b}{2} = k_{es} - k_{ed} = -a;$
$b = 2(k_{es} - k_{ed})$
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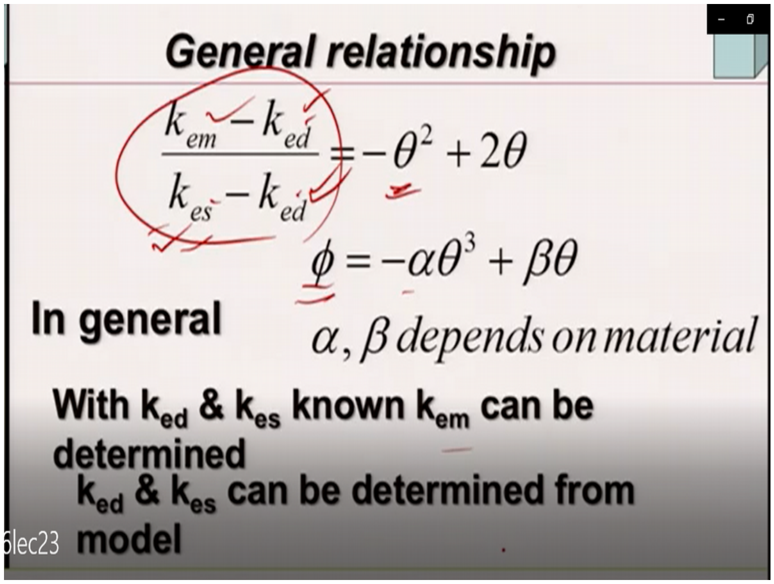
General relationship

$$k_{em} = -(k_{es} - k_d)\theta^2 + 2(k_{es} - k_d)\theta + k_{ed}$$

$$k_{em} = k_{ed}$$

$$k_{em} = -(k_{es} - k_{ed})\theta^2 + 2(k_{es} - k_{ed})\theta + k_{ed}$$



Kem the moist conductivity, conductivity at some moisture content or at Theta degree of saturation is given by something of this form. And I can then you know Theta Theta is defined in this manner. Weight of the sample, moist sample minus the dry weight divided by saturated minus dry weight. So that is how we define degree of saturation.

So, c is when Theta is equal to 0, degree of saturation is equal to 0 is the dry conductivity. So c is nothing but you put Theta is equal to 0 Kem becomes Ked. So, this is Ked and one thing we have seen that since it is parabolic if I differentiate it, you know it is a curve of this form, so either at the peak it will be maximum or near peak it will be maximum and there may be sometime some deduction also, so one can find out the value, you know one can use this kind of relationship, for example, differentiate this with respect to Theta, that will be 0, there will be maximum Kem.

Differentiate this with respect to Theta. One can then get the degree of saturation corresponding to maximum moist conductivity from this one, simply differentiate this, you will get 2a Theta plus b equals to 0. So Theta max can be simply Theta max at maximum thermal conductivity is minus b by 2a, that is what all you get.

So once therefore using this 2, if Theta max is equal to 1 which is a peak because the saturation is the maximum, then simply this is equal to 1, so a is equal to minus b by 2. So, usually the curve you have seen, the curve is usually 1, it is maximum at peak at 1, this is maximum at peak at 1. So, using this condition one gets a equals to minus b by 2..

So, try this generalize, do some generalization, for Theta is equals to 1 Kem is equal to Kes saturated, Theta equals to 0, moist conductivity is equal to Kdry and when Theta is equals to 1 Kem is equal to Kes.

You know this was the equation so, this degree of saturation when this is equal to 0, Kem is equal to simply c is equal to Ked when Theta is equal to 1 this is nothing but Kem Kem equals to K saturated. So, use this, use this relationship, so, therefore, I can write Ksaturated is equal to a Theta square plus B Theta plus Ked because when Theta is equal to 1, you know it is saturated and Theta equals to 1 sorry a plus b, this si Theta is equals to 1 this can be written as Ks is equal to, let me use the regular eraser and erase this out.

When Theta is equal to 1, for Theta is equal to 1. Kem is equal to Kes so I can write it like this. In other words, in other words, Kes minus Ked is equal to this because maximum value you get at saturation. Generally, maximum value you get at saturation so you can what you can do is, you can take Delta Kem divided by dTheta equals to 0 of that equation which was of this form, you know (a square plus) a Theta square plus b Theta (minus) plus c and you can differentiate this is equal to 0 and generally this is at Theta max corresponding Theta max corresponds to Theta equals to 1. At maximum saturation, you will get 1. So this is the other equation that you get, this is the other equation you get, this is the other equation you get.

So, we have seen that a is equal to minus b by 2, a is equal to minus b by 2, so (k) b by 2 is equal to Kes , b by 2 is equal to Kes minus Ked, you know you put them in equation, this equation, so you will get b by 2 is equal to Kes because this is equal to a plus b is equal to, to a plus b is equal to Kes minus Ked, put a equal to minus b by 2, you know put a equal to minus b by 2 so put minus b by 2 plus b which is equal to b by 2. So, Kes minus Ked is equal to either a or you know it is b by 2 or you put b is equal to 2a, minus 2a either way so you get this expression you get. So, just I put them in our earlier equation Kem by, I just put them in the equation straightway, b is equal to 2 Kes minus Ked, a is equal to Kes minus Ked

So my equation therefore would be equals to Kem is equals to a that would be minus you know that was minus Kes minus Kd, Kes minus Kd and the second one is b, b the expression comes out to be 2 Kes minus Kd, so 2 Kes minus Kd into Theta square and this Theta minus Ked. So if I simplify this a little bit you know or take Kes Kd on Kes Kd on one side I will get, I will get, now I will get an expression of this kind, Kes Ked I can actually get Kem minus Ked, ao take this as I said Kem was equals to first of all what was it minus a which was Kes minus Ked into Theta square plus 2 Kes minus Ked Theta plus Ked, so I can take this on the other side, I will get Kem minus Ked Ked divided by this two-term is common I can take it out and I get this kind of expression, so what type of expression I am getting, I am getting something like this.

So, Kem by you know so just 2 Theta, so this I define as you know this was degree of saturation which is nothing but relative moisture content, this is nothing but if you can see this is the saturation, this is dry, this is again dry and this is under the moist condition. So this degree of saturation, how do we define? We define like this weight moist minus Wd and W saturation minus Wd that is equal to Theta, so there is a similarity here, there is a similarity here and we define this as, we define this as fi, fi is nothing but, fi is nothing but relative thermal conductivity.

Relative thermal conductivity is degree of saturation, so fi is Alpha Theta, there can be a general equation of this form, but usually, you know you will get the relative moisture, the relative conductivity is a function of Theta and Theta square in this form only. So, these are known from, these are known, if this is known one can obtain Kem and we have seen that we can using the model find out Kes, I can find out Ked and at any moisture content, at any degree of saturation therefore I can determine Kem, so moist conductivity I can.

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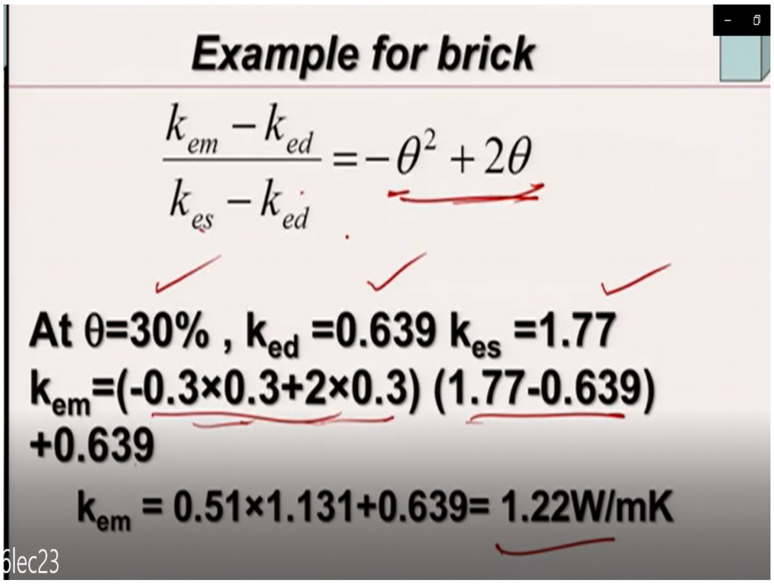

So, Kes, Kd can be determined from the model as we have done earlier. So, now earlier case at 30 percent porosity Ked was 0.639, Kes was 1.77, so Kem at let us say some moisture content right 30 percent degree of saturation this value will come out to be 0.3 square, 0.3 square plus 2 into 0.3 and if you, this is this term and divide this by the difference between this two plus 8.639 So 30 percent degree of saturation you get conductivity is equals to 1.22. So one can estimate the moist conductivity it is slightly more realistically using this equation, so what you see is that the relative thermal conductivity is a function of degree of saturation which is nothing but relative moisture content. So one can, one can estimate this conductivity.

Now one can look into evaporation and condensation for modeling purpose if one wants to look into but as I said this is not easy to incorporate into a porous material thing because a porous interconnectivity, the condensation occurring from, you know the evaporation is occurring from which front and their connectivity, there is a little bit of a problem although one can possibly extend this model because we know the pores sizes at which capillary condensation occurs, you know that one can find out from Kelvin's equation, but I think I will not go into that .

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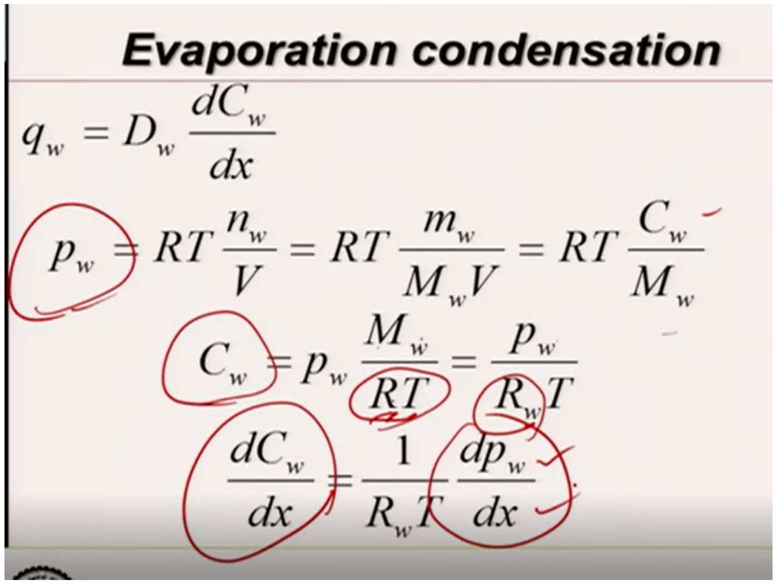

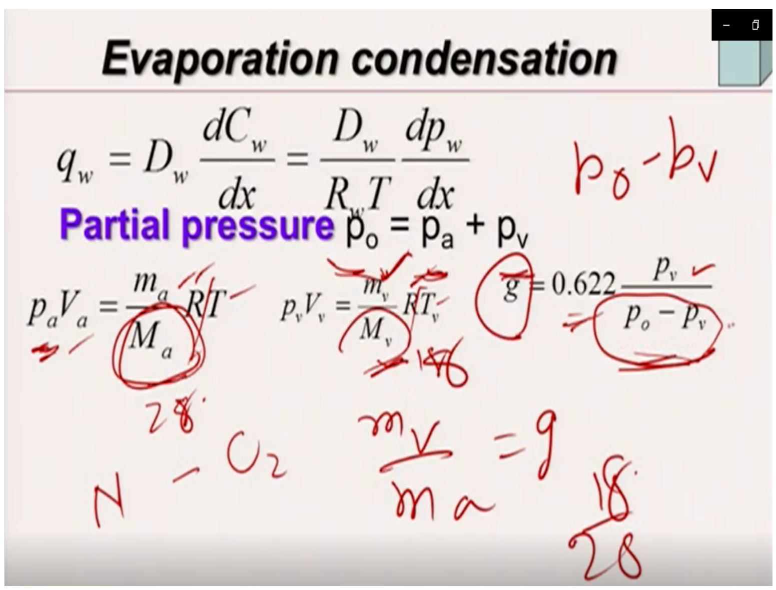

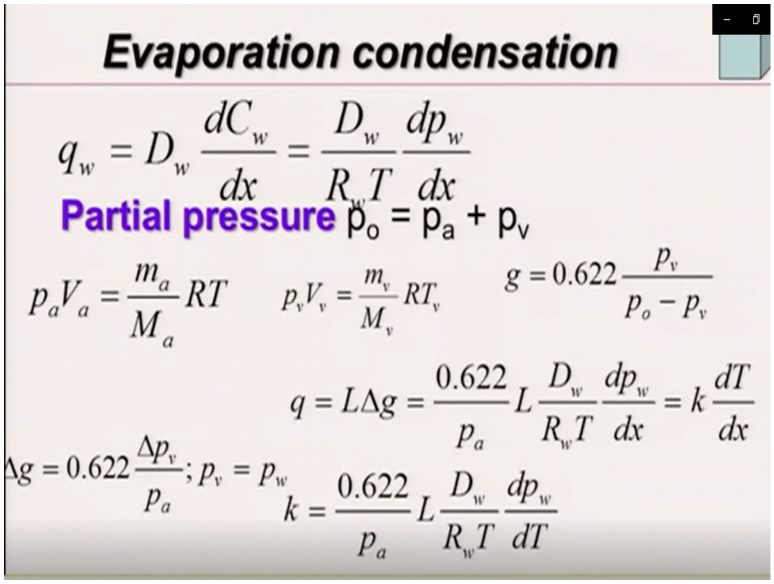

But, just try to explain the phenomenon of evaporation condensation. Fick's law again. So the amount of moisture vapor that would evaporate is a function of we know the concentration, so the function of concentration gradient. So moisture flux, now assuming this as a ideal gas, moisture vapor as an ideal gas, so you can write pressure because it is basically it is a vapor pressure difference which causes the flow, vapor to flow.

Now, more moisture content, you know vapor pressure is the function of moisture content, itself. So this is ideal gas law and for moisture vapor, you can put it this way , so what is this? V by you know M basically see V by Mw is nothing but V by or Mw by V sorry V by M small mw by small v is concentration is defined as mass per unit volume, so this is the mass, this is volume so this is nothing but Cw.

So one can then find out the, one can, one can Cw can be written in terms of Pw, so this is Pw divided by you know Pw into Mw divided by RT, and you know Mw by RT or RT by Mw can be written as Rw which is specific gas constant for moisture vapor, you know this universal gas constant divided by the molecular weight that gives you the specific gas constant of the moisture vapor.

So Pw by T, so, therefore, dc dx one can write like dp dx because it can be expressed in terms of vapor pressure and if you have done earlier the course, those who have done the course on building science will actually derive relative humidity versus relative humidity which is (())

(26:21) to be a moisture content relationship there it would have come and the same thing, nothing, the same thing. So you using ideal gas law one can show that concentration, vapor concentration is a function of gradient is a function of vapor pressure gradient in this manner.

So, therefore this can be written as q flow can be written as dp d dpw dx in this manner. Partial pressure is the sum of pressure of air plus pressure of you know total pressure, atmospheric pressure is the sum of total partial pressure of air dry air, plus partial vapor pressure on (()) (26:56) vapor pressure. So pa, they will all follow ideal gas law. So, paVa is equals to I can write for air, dry air, similarly I can write for vapor and g is the moisture content from this it follows because moisture content is nothing but mass of the vapor divided by mass of dry air and molecular weights of air is around 28 because nitrogen you know it dominates so majority of it (()) (27:27) .78 percent is nitrogen, around 20 percent is oxygen so one can calculate out molecular weight of nitrogen, oxygen, etcetera.

So molecular weight of air is around 28 point something, this is 18, molecular weight of water is 18 so this ratio comes out to be 18 divided, so moisture content can be written in this manner, moisture content can be written in this manner, moisture content can be written in this manner because mv divided by ma is the g moisture content and from this equation it will follow, this value is around 18 point something, this is around 28 point something, the R will cancel out T etcetera. will cancel out because at the same temperature so pa pressure of pa, pa pa can be written as p0 minus pv.

So that is how it is so. The ratio of this two 18 divided by 28 and this 0.62 and you know 18 point something and this and rest all this cancels out, this cancels out volume is same so, pv divided by you know g moisture content is related to pressure of air which is p0 minus pv into this. So one can obtain this sort of relationship for moisture content in the air.

So, Delta g, change in moisture content, causes change in vapor pressure, change in moisture content, so Delta g if my moisture content changes so that would cause vapor pressure change as Delta pv. So then q can be written, q was dw this is dpd, you know it can be written in terms of you know what we did, so this can be written concentration I am just changing so if if delta g L delta g I can write.

Latent heat of evaporation, so heat, if this is the amount of heat that is going in that will be multiplied by amount of moisture vapor transporting, multiplied by the latent heat of evaporation. So Delta g is this, therefore if I write it in terms of equivalent conductivity dpd you know equivalent conductivity will be per unit area, this is also per unit area, thus 622 everything remains same. What I will do is I will write this as dpw, dpw dt can be written as dp dx and dt dx, so this can this will just make in dpw dx into 1 by dt dx or whatever it is. So you can get an equivalent conduction term, equivalent conduction term, you can get it.

You can get an equivalent conduction term. You can get an equivalent conduction term from here because this dt dx this k is dt dx, k dt dx so from this one can get equivalent conduction term. Now if you put this into the equation of k, you know moisture equation into the, put it into the equation of Kem and try to estimate Kem, one can actually do that but I think at the moment it is not easy to do that. So the phenomenon of evaporation condensation you understand what is happening.

Basically, there is a vapor pressure difference due to moisture content itself. Where there is high moisture content, vapor pressure is higher, where there is a low moisture content, vapor pressure is lower and due to that vapor tends to flow and at certain temperature there will be condensation occurring, at dew point condensation will be occurring.

So if you are in a phase or the cold phase is lower than dew point condensation will occur and the amount of heat that it gone, so you can derive some kind of equivalent conductivity but relating it to the porous material at the moment is difficult. So the concept of evaporation condensation is something like this. But perhaps we will restrict ourselves to the empirical equation of this kind that we looked into and one can calculate it. So that is related to effect of moisture and this is very important one has to take that.

So if there is moisture particularly post rainy season one we meet scenario, the moisture content of the bricks could be much higher, walls could be much higher, so thermal conductivity can change significantly, insulation quality may be reduced. Although people do not estimate it in that way but I mean if you are looking at, looking at a building then you might look into this. Then we will look into thermal diffusability. We will break for a moment and then come back to this.