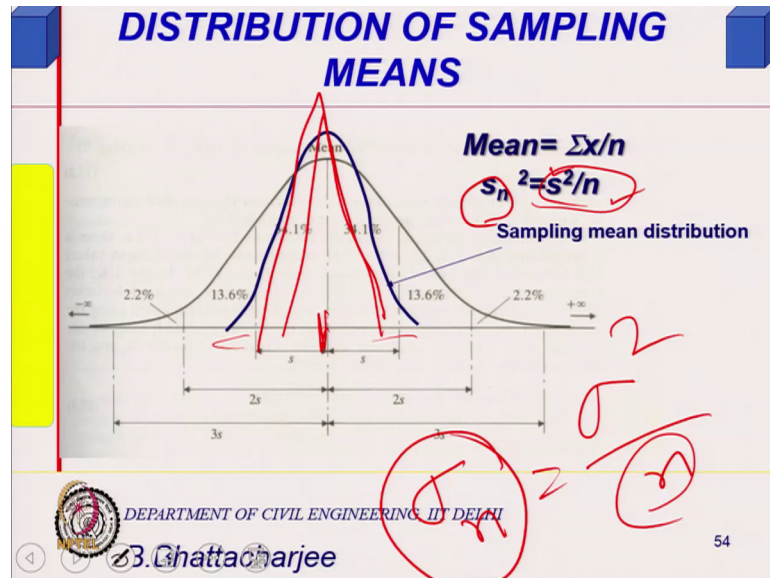


Fire Protection, Services and Maintenance Management of Building
Prof. B. Bhattacharjee
Department of Civil Engineering
Indian Institute of Technology, Delhi

Lecture - 48
Sampling and choice of test location

(Refer Slide Time: 00:20)



So, following from there distribution of sampling mean right, distribution of sampling mean as I said now with an algebra we can understand. Simply we can understand the algebra easily.

(Refer Slide Time: 00:31)

Handwritten equations on a whiteboard:

$$\bar{X}_1 = \frac{x_{11} + x_{12} + x_{13} + \dots + x_{1n}}{n}$$

$$\bar{X}_2 = \frac{x_{21} + x_{22} + x_{23} + \dots + x_{2n}}{n}$$

At the bottom of the whiteboard, there is a logo of IIT Delhi and the text "DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI".

It is like this you know like supposing I have \bar{X}_1 bar I said it is a mean of n samples. Which will be let me call it as x_{11} plus x_{12} plus x_{13} . These are the samples up to x_{1n} divided by n , \bar{X}_2 bar would be simply x_{21} plus x_{22} plus x_{23} etcetera x_{2n} divided by again n .

(Refer Slide Time: 01:11)

Handwritten equations on a whiteboard:

$$\bar{X} = \frac{\bar{X}_1 + \bar{X}_2 + \bar{X}_3 + \dots + \bar{X}_m}{m}$$

$$\bar{X} = \frac{x_{11} + x_{12} + \dots + x_{1n} + x_{21} + x_{22} + \dots + x_{2n} + \dots + x_{m1} + x_{m2} + \dots + x_{mn}}{mn}$$

At the bottom of the whiteboard, there is a logo of IIT Delhi and the text "DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI".

And if I sum all of them up that is sum up you know sum up \bar{X}_1 bar \bar{X}_2 bar plus \bar{X}_3 bar etcetera etcetera and divided by m of them \bar{X}_m because I said m number of times I have repeated this experiment. So, this will be my overall \bar{X} bar now,

this can be expanded and what it would be? It would be simply $x_1^2 + x_1 + x_1^2 + \dots$ etcetera etcetera and this will be then divided by n into m because each one of them is this will be $x_2^2 + x_2 + x_2^2 + \dots$ etcetera etcetera plus $x_2 + n$ divided by n .

So, this will be sum of all these $x_1^2 + x_2^2 + \dots + x_m^2$ plus $x_1 + x_2 + \dots + x_m$ etcetera etcetera divided by m . That means, large number of them divided by total number of my observation, which will be actually it is a simple yeah n into m and I have got total n into n into m number of data divided by n into n into m ; that means, it will be the true mean. So, mean of the sampling mean is same as the mean of the population, sampling mean mean of sampling mean, you know mean of the means is population mean that is one thing we understand right.

Standard deviation one can derive and show standard deviation; one can derive and show right standard deviation; one can derive and show that standard deviation is actually given by this formula which I have just written straight away. For example, you determine for n is the sample size, s is the population estimated population standard deviation. If it is for if it is known well known then we write it like σ^2 by n and this is σ of n where this stand for standard deviation of the sampling mean with n sample size.

So, you can see that supposing I have n equals to 4, my sampling mean standard deviation of the sampling mean will be population standard deviation divided by 2. If I take 3 it will be under root 3 see the if I take 3 and take their mean the standard deviation of such means will be population standard deviation divided by under root 3 because, I am squeezing down reducing down. So, this is this you can again prove same manner I have done it from mean very quickly you see any statistics books you will find it out, but you can prove it also.

So, more number of samples you take you are likely to be closer to this value supposing I take still larger number of sample, very large number of sample, infinite number of samples and find out their mean. Standard deviation will be 0 because, every time the mean will be here, supposing I take 100 or 1000 and find out the mean. So, the value will be very close to this. Next time I do again I will find value close to very this. So, if it is theoretically infinite, it will be standard deviation will be 0.

So, that is why inversely proportional to n if n equals to 1, then it will be same as the population itself. So, do you understand that so, if you increase the number of sample size in situ testing also we can adopt this kind of principles. This is called you know strains to stress formula when I am doing it from small samples.

(Refer Slide Time: 04:40)

ACCEPTANCE

$$f_m = f_{ck} + 1.65\sigma$$

Mean of group of 4 non-overlapping consecutive samples

f_m (for 4 samples) \geq
Max ($f_{ck} + 0.825\sigma$, $f_{ck} + 3$) for M15 grade

$f_{ck} + 1.65\sigma$
 $= 30 + 1.65 \times 10 = 46.5$

DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

So, the number of samples I can do it that way now just to go to example that this is what we use in concrete technology or concrete practices, it says that acceptance criteria of concrete is it says that 4 non overlapping consecutive samples. And their mean should be greater than f_{ck} plus 0.825 sigma, f_{ck} plus 1.65 sigma is the mean.

And mean plus you know if this mean plus maximum variation will be mean plus you know mean minus sorry on the lower side 3 sigma 99.9 percent of least value. So, this is if my mean is these standard deviation is this, my no cube strength, sample strength or cube strength should go below this. If I am taking mean of 4 samples standard deviation becomes this under root 4 and this also becomes under root 4 right.

And if instead of taking 99.9 percent of the time if I take it to be 95 percent of this time my strength should not be falling below this. My strength you know the point is 99 percent 95 percent of the time, my mean is this, the least value 95 percent of the time.,

(Refer Slide Time: 06:07)

ACCEPTANCE

$$f_m = f_{ck} + 1.65\sigma$$

95% $\left\{ \begin{array}{l} f_{ck} + 1.65\sigma \\ \sigma \end{array} \right. \begin{array}{l} T_4 \\ T_4 \end{array}$

$\sigma / \sqrt{4}$

$f_{ck} + 1.65\sigma$ $\begin{array}{l} \cancel{1.65\sigma} \\ 0.825\sigma \\ T_4 \end{array}$

DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

My cube strength should be greater than $f_{ck} + 1.65\sigma$ minus 1.65σ 95 percent of the time right. Now, supposing I am taking mean of four samples, then this is σ under root 4 and this is σ under root 4. So, 95 percent of the time the 399.9 percent, but 95 percent I am taking then, 1.65σ . So, what I am saying 95 percent of the time mean of four samples should not be should be greater than thing 95 percent of the time, 95 percent corresponds to 1.65 99 percent corresponds to 3σ 99.9.

So, if you calculate this out this comes out to be 0.825σ . Because, half of this root 2 divided by 2 and this is also you know it will come out to be. So, if you know this is what it should be that is why is my you know acceptance criteria as given like this, I can you can you can also derive yourself.

(Refer Slide Time: 07:15)

ACCEPTANCE

$$f_m = f_{ck} + 1.65\sigma$$

Mean of group of 4 non-overlapping consecutive samples

f_m (for 4 samples) \geq
Max ($f_{ck} + 0.825\sigma$, $f_{ck} + 3$) for M15 grade

$f_{ck} + 1.65\sigma - \frac{1.65\sigma}{4}$

DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

The same thing just look at this, mean of 4 sample should be greater than this f_{ck} plus 0.825σ because you know f_{ck} plus mean is sorry mean is f_{ck} plus 1.65σ minus 1.65σ divided by 4. Because, this is mean, mean minus so, 95 percent of the time this should be greater than this value. So, that is why this 0.825σ comes in ok, forget it this part I am not interested in telling you in this class right. So, this was earlier starting point of the code, now they have modified it, but that is not I am not interested in this.

So, all that I am trying to say is this concept is used in acceptance criteria also example I have just shown you. So, this if you see it you see in the code it gives you that mean of four sample should be greater than this or something f_{ck} plus something that is you know been put in arbitrarily because, there may be controversy related to sigma. So, code has to do with the legal issues also so, they have put in some value f_{ck} plus 4 or something 3, it should be 90 you know about f_{ck} plus 3 for acceptance.

(Refer Slide Time: 08:33)

ACCEPTANCE

Individual test results shall be greater than
 $f_c \geq f_{ck} - 3 \text{ MPa for M15 grade}$

DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

So, if you see this concept is applied in acceptance also alright so, I think that is it. So, this also says individual strength result should be greater than f_{ck} minus 3, you can be less than f_{ck} also. Because, you know m 15 does not exist for m 20 f_{ck} whatever it is this I am not interested.

(Refer Slide Time: 08:50)

ACCEPTANCE

Probability of acceptance per cent

True percentage of defective concrete with respect to characteristic strength

Average of any four test results for a standard deviation of 4 MPa (600 psi)

Ideal curve

❖ OC Curve for RMC

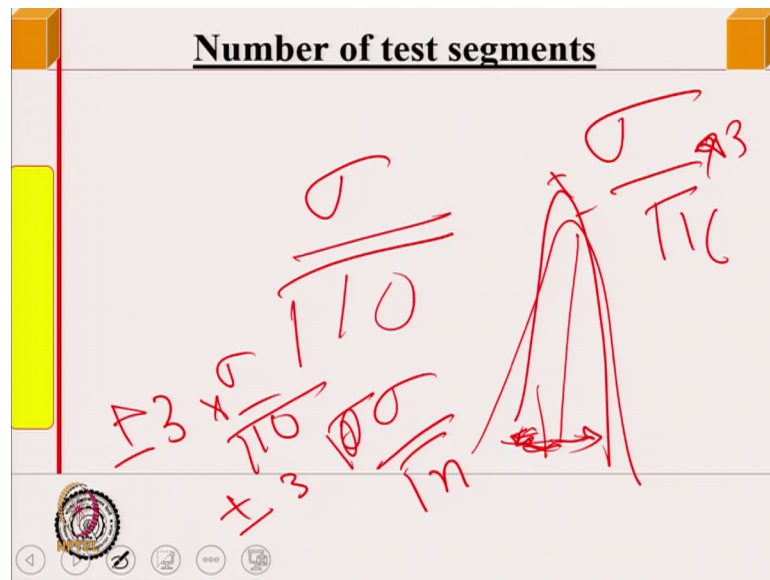
$$p = \frac{\binom{Nd}{0} \binom{N(1-d)}{n} + \dots + \binom{Nd}{x} \binom{N(1-d)}{x}}{\binom{N}{n}}$$

$$f_m = f_{ck} + 2\sigma$$

DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

So, this part I am not interested I will not solve the problem either, but I think I have given you the concepts.

(Refer Slide Time: 08:55)



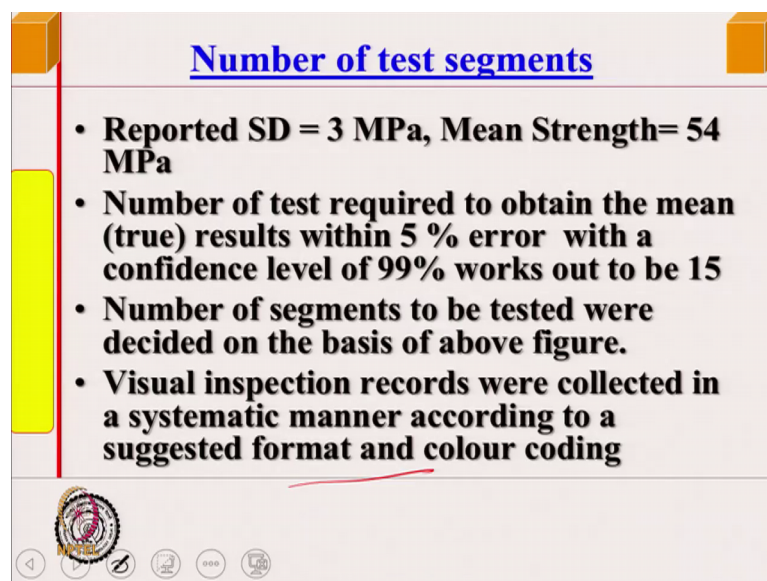
So, number of segment therefore, or number of samples is related to that. Supposing I test 10 of them, then my standard deviation would be sigma under root 10, it means that I will be closer to this. Now if I test 16 of them I will be still closer now I will be closer to the them; that means, my error maximum error if I assume 99.9 percent of the time will be plus minus you know 3 of this. Maximum error will be plus minus 3 times sigma under root 10 or plus minus 399.9 percent of the time sigma under root n.

If I you know sigma very well, which means that if I increase number of samples then, I will be my error will be less. Error is plus minus you know you always whenever you do experiments you say mean or results plus minus some value. So, this error is given by this formula 3, so, if I have more n error will be less that is the idea. So, then this particular case of that you know segments that I was talking about the tunnel lining segments that I was talking about.

be obtained from table depending upon how many what is the size of your n to for determination of s and similarly you can find out n.

So, number of test is determined based on this depending upon how much error you need, but there is another point more the number of test cost of testing will be more. So, you have to have a compromise on the cost and accuracy, you want higher accuracy you have to do more test. You want higher accuracy then, you have to spend money, but if your budget is limited then, obviously, you know you compromise.

(Refer Slide Time: 13:07)



Number of test segments

- **Reported SD = 3 MPa, Mean Strength= 54 MPa**
- **Number of test required to obtain the mean (true) results within 5 % error with a confidence level of 99% works out to be 15**
- **Number of segments to be tested were decided on the basis of above figure.**
- **Visual inspection records were collected in a systematic manner according to a suggested format and colour coding**

So, number of test segments one can determine for example, you found out standard deviation of 3 MPa mean strength 54 MPa you know we determine. So, number of test required to obtain mean true result within 5 percent error with a confidence levels of 99 percent in this particular case of, the segments I was talking about the tunnel lining segment worked out to be 15 in this particular. So, 15 segments were tested for randomly for various kind of test you know to wherever it was is there or similar kind of concepts you can apply elsewhere to order to you know see your accuracy.

So, here accuracy was more important cost was not affected, some cases cost would be a factor if there you have to take it right. So, that is what it is. So, we collected visual inspection records in a systematic manner according to a suggested format and colour coding which I told you in the last class that we looked at the crack, mark them double

red to green colour, double red was maximum size crack, green was the minimum size crack and so on. So, that is how one can determine this right ok.

(Refer Slide Time: 14:17)

The slide is titled "Number of test locations" and contains the following content:

- Cost of test includes three components
 - Actual Cost of test
 - Cost of damage to the structure
 - Cost of disruption
- Total cost of one i th type of test :
 - $C_i = C_{T_i} + C_{D_i} + C_{I_i}$
- For all N type of test
 - Total Cost = $\sum C_i n_i$
- This total cost shall be minimized or within a budget B

At the bottom of the slide, there is a logo of IIT Delhi, the text "DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI", the name "Bhattacharya", and the page number "-62".

So number of test locations let us look at it now. So, in case of general structure this was a case and I should you an example how you find out the number of test required? It is not arbitrary you go to a consultant who does testing they would suggest as many number of tests possible, that should not be the case. It should be done depending upon the requirement, I think I must be having another slide related to the number of test, let us see if I have; yeah this first I will talk about this one, then I will go back again the location part of it.

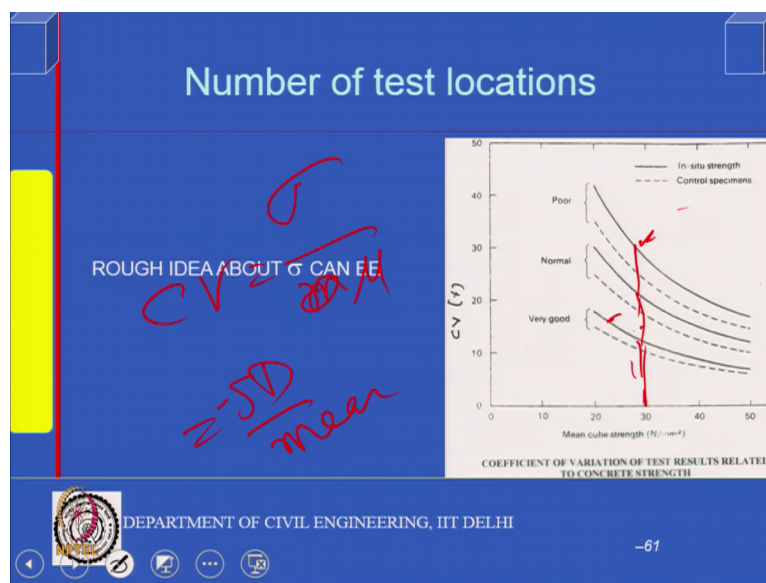
Cost of test will include actual cost of test, because you have instrument cost of the you know like they would like to recover the cost of the instrument the whoever is doing it the labour cost setting up cost etcetera etcetera. The cost of damage to the structure if there is any for example; you do a code test you have to fill it back. So, more number of test you do, more damage to the structure so, some cases there is damages. So, where there is no damages this cost is 0, but if there is a damage then that will.

Then cost of disruption, if you are stopping the activity not allowing traffic to pass through the bridge or not allowing the class to occur in the classroom there is a disruption cost involved. So, therefore, the total cost of i th type of test for any test this time would be different cost would so, the total cost will be this and for all n type of test

the sum total will be C_i into n_i right. The total cost should be minimized for a given budget so, you keep that B , then you decide on accuracy and say compromise between the two right, compromise between the two.

Because accuracy has to be maximized you know it is a kind of a goal programming situation. You have cost. Cost has to be minimized and accuracy has to be maximized right, more the number of test cost goes up, but accuracy also improves error reduces. So, that is how we decide number of test location right ok.

(Refer Slide Time: 16:29)



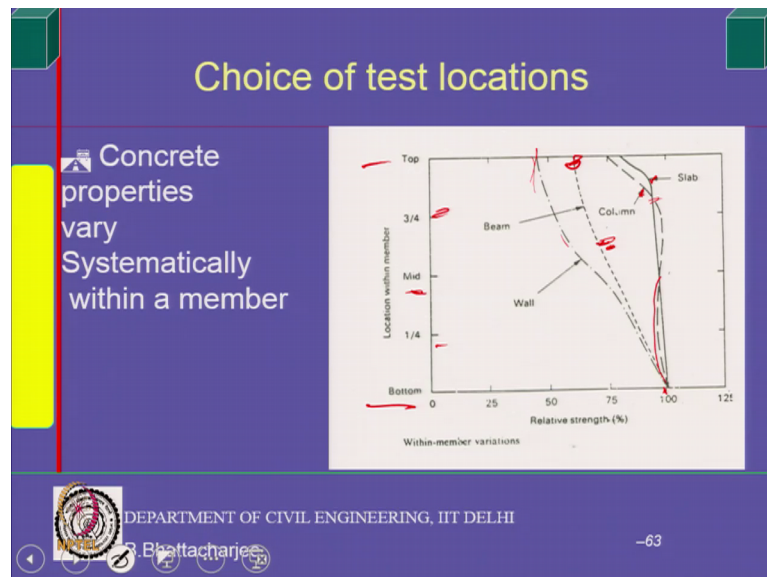
So, now, we come to choice of test location you know number of test location we have already talked about. Now then, we can find out the standard deviation from CV values from literature if I do not have any idea. For example, coefficient of variation is nothing, but standard deviation divided by mean. So, it is given for very good concrete the dark line is in situ concrete where CV will be higher and dotted ones are standard I mean laboratory samples.

Poor concrete so, if you know the grade or mean strength mean strength then, you can find out these values for very good concrete for normal and for poor concrete. Even if you do not have idea about the standard deviation because CV is nothing, but CV is nothing, but standard deviation divided by μ or mean you know. So, SD divided by mean. So, percentage is given so, you can get an estimate this from a book by bangi and

milod you know they would have done it for in UK. They would have done it so, from that it is it is that way right.

So, the CV is the one can find out CV this manner. So, standard deviation mean 1 can find out right. So, rough idea you can get it, but then you can do a small number of test and then establish the standard deviation yourself and use t test you know that other formula strain to stress formula which I have given you with t that you can utilize so, that is what it is. Now, this is the cost aspect that I was talking of so, total cost should be within the budget.

(Refer Slide Time: 18:07)



Then test locations now, I said concrete strength varies systematically in concrete. For example, if you see typically for a column strength at the bottom is if I assume it to be 100 percent 1 percent variation is much less than the top. So this is top, this is bottom, this is mid height three fourth height one fourth height etcetera etcetera.

So, it is because the cement you know and also with the way we construct and the reinforcement there right. So, essentially what we will do? I will pour concrete from the bottom may be in two lifts right, but the water cement ratio is minimal at the bottom cement will go most water will come up.

And reinforcement depending upon because I may not be you know like part of the shuttering I might have put in side shuttering then I have done the casting. As it becomes

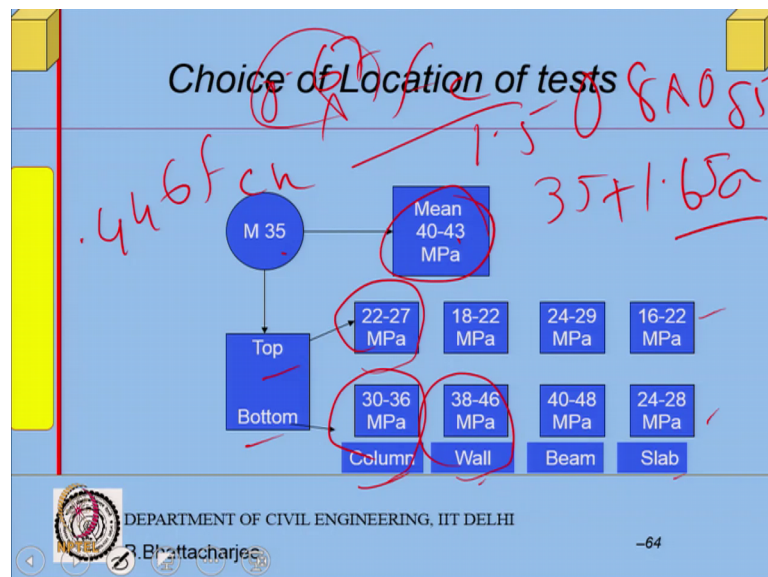
deeper tendency to compact it also you know you know the possibility of compacting it becomes also somewhat less.

So, this is from experience wall something like this, the slab something like this and beam something of column is here this is for beam, this is for wall because it is also depends upon the reinforcement you know congestion and all that. So, this is for you know beam, this is for column top is bad double layer so, you can see that one lift maybe something of this kind you know so, typically this is what is it.

So, the point that I am I am trying to make is strength of concrete is not same at all heights. So, while doing sampling you do not do it from same height at all locations you randomize it, one third from the bottom, one third from the middle, may be one third from the top.

If you take everything from the top you are likely to bias your results with poor concrete, if you take everything from the bottom of a column bottom of all the columns, you are likely bias it towards the positive side. So, this has to be understood normally these people may not understand you know take care of this and just do it as it is.

(Refer Slide Time: 20:20)



Then you have got this is being also observed typically if you have you know supposing M 35 grade of concrete. So, mean strength will be 35 plus 1.65 sigma could be

something of this order depending upon the standard deviation, depending upon standard deviation.

But then, top and bottom if you see in case of column you might find 20-27 while bottom 30-36 wall this is in situ I am talking of cube strength is 40-43 mean cube strength right. Mean cube strength you know this is the cube this is the grade this is the mean cube strength and then wall is something like, this slab something like this. So, you can see the strength in structure is much lower; strength in structure is much much lower than cube strength because, cube is a standard cube strength strength is much much lower in actual structure.

Besides that this is taken care of in design nobody has to really worry about it because, you already multiplied or divided by you know multiplied by 0.67 in order to take care of this divide it by 1.5 you can say, but then there is a partial factor of safety that is also 1.5. So, first if you remember you take column you know f_{ck} you take f_{ck} for compressive strength of concrete that is because, f_{ck} divided into 0.67 divided by 1.5 as a partial factor of safety of material.

So, basically this ϕ takes care of this and something else adverse effect of loading as well you know. So, all this factor put together it has been seen that in situ strength that one can realize, because of the nature of cube is tested in a standard manner, while structural loading could be different, also the strength variation occurs in structure. So, therefore, it has been observed actually it is about 80 percent of the 85 percent of the cylinder strength American code. So, cylinder strength itself is 80 percent of the cube strength.

So, 0.8 multiplied by 0.85, if you look at it how much it will come to so, 0.68 or something so, a British code or Indian code they took it as 0.67 of the cube strength right. So, that is taken care of, but one must realise that strength that you get inside would be lower than cube strength or grade strength, you have to interpretation has to be done then you go to do it that way ok.

So, this is this will be handy coming later on, but my point here more important point here that I am talking about that you select the locations, select the locations randomly at all height level. And do not select the location which is vulnerable; do not cut a core to

the inlines of the slab because you are making it vulnerable, avoid cutting possible reinforcement bars. So, this is what it is this is what it is right.

(Refer Slide Time: 23:30)

Choice of Location of tests

★ Concrete properties varies systematically within a member

ILLUSTRATION OF APPROXIMATE RELATIONSHIP OF COMPRESSIVE STRENGTHS

Test Type	Approximate Relationship
Top of member	0.85
Bottom of member	1.15
Left side of member	0.85
Right side of member	1.15
Top of slab	0.85
Bottom of slab	1.15
Left side of slab	0.85
Right side of slab	1.15

DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

Bhattacharya

-65

So, choice on choice of location of tests as I said concrete properties varies systematically within member and this is what again another graph shows it is from another British codes earlier British code right. So, actual strength in the structure top would be lower bottom will be higher, cube strengths will be equivalent, cube strengths you can think of that will be a lower here, higher here right cylinder strength and so on. And standard cube strength is here. So, this is this same idea is given again here. So, if you want to compare particularly then, randomization is not necessary, but, take it from such place supposing I want to compare the strength of column.

(Refer Slide Time: 24:15)

Location of tests

- While comparing properties of elements say columns, to test the hypothesis that one set of column is as good as others same, relative locations need to be chosen in all the elements.
- When it is necessary to obtain overall properties in a structure, random selection at top, middle and bottom, keeping in view of the within member variation is desirable

DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

—66

This has happened to me you know this is a this practical cases in many places, practical cases in many places; it happens that you would like to compare. You know supposing you do not do somebody does not want you to do actually extensive testing; just you can do non destructive testing. And tell me this concrete is acceptable in this set of columns, this has happened you know some cases it has happened.

In fact, one of the flyovers here also has a similar case, some portion was acceptable, some other portions they were doubt so, what you do? And they say no you do not take too many codes already traffic is going on or the column office is on. So, you can I can give you only short period of time, you do non destructive testing and find out. So, what you can do you can compare and if this is good and this is also belong to the same population statistically you can say this acceptable. If it does not belong to that population actually showing lower properties you say the no this is not good you need further testing.

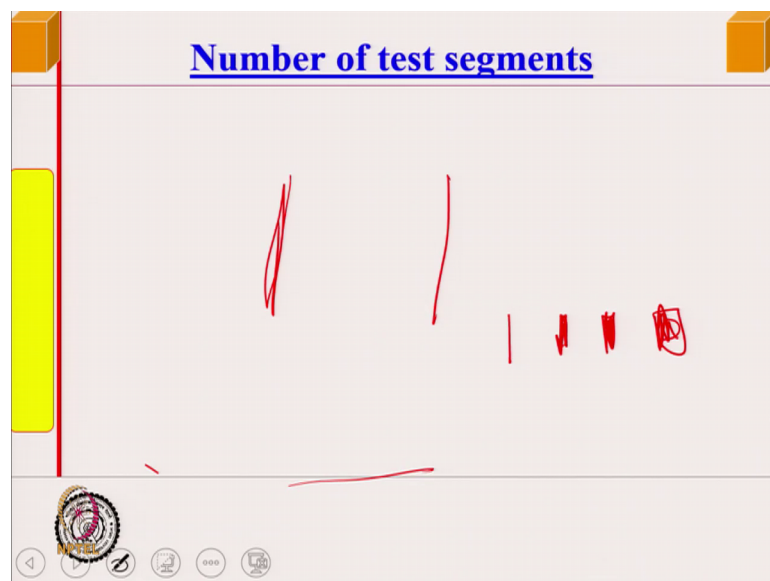
So, comparison if you want to do then test at the same locations both places. So, set of columns suppose you are given these are good columns, these are bad columns. So, you should be testing mid height for all the column this the height and this height, do not take from top height, bottom height and one can bias that is also that is as biasing should not be there. So, while comparing elements say columns are test hypothesis that one set of

column is as good as other one relative location need to be chosen in relatively same location, same relative location should be chosen in both the set of columns right.

While comparing properties of elements say columns test hypothesis ok, when it is necessary to obtain overall properties then, random selection at the top, bottom and middle you should be doing as much as possible satisfied it one third, one third, one one third something. So, you know that would be desirable so, that is what that is what that is what how we choose location of test so, first part was visual survey, second part was choosing the test and third is location and number. So, I think we have covered that part.

Yes, last class I was talking to you related to crack depth measurement. Now this can be done through let me see if I can show you how it just this is a simple device in my hand, I just show you if I can. So, this is a simple device right.

(Refer Slide Time: 26:50)



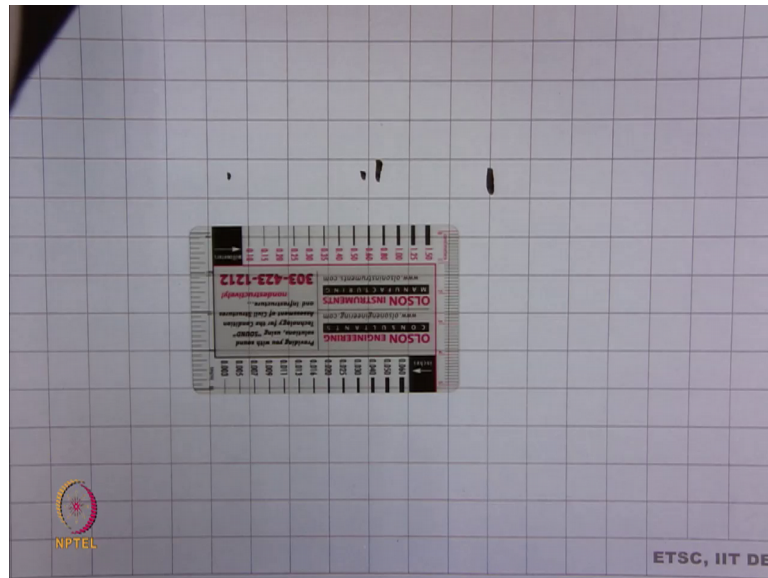
It will not come in it will not come in does it come in here, no it will not come in may come in somewhere there it is a simple device maybe I will have it again somewhere supposing this my crack thickness. So, this is a device it has got a graduation it is got a graduation.

Student: (Refer Time: 27:11).

Ah (Refer Time: 27:13) just quickly (Refer Time: 27:17). So, you know these are device for example, this device it has got a scale right and thick lines like this, you know some

you will have one line then one line slightly thicker then, another line slightly thicker, another line slightly thicker. So, you place the, place the, place the scale on that right place the scale on that then you can find out the (Refer Time: 27:45) simply this is a simple scale there are more complicated ones ok.

(Refer Slide Time: 27:59)



So, for example, for example, you know I want find out if you see this scale, this scale, now, this line I just match with this particular one. So, it is about 3.35 mm right. Now, supposing yeah zoom zoom zoom zoom zoom zoom yeah as for (Refer Time: 28:14) what can I use for writing here this pen right.

Now, you know this is a simple it shows how much thickness it shows 0.35 mm you know this scale you can read. Now supposing the thickness was something like this, then I will match it this particular one right this particular one. You see these thicknesses are different so, this is different so, this is 1.5 mm this is 1.5 mm.

So, I place the crack I mean place this scale over the crack, supposing the crack is something of this dimension place it like this then, I see the crack width is 1.5 mm. And if it is very fine finer than even this line, this line is 0.35 mm as I have seen or something like that there will be slight error. So, this is how we can find out the crack both in inches and this is a simple scale there are better ones of course, but this is a simple scale which one can use to find out the crack width, find out the crack width right.

So, this is still zoom it can be zoomed further yeah . Now this now zoom zoom zoom plus plus plus plus plus plus right yeah this you can see now properly. So, this is this is a simple scale and as I said you know like this thickness is 1.5 mm or 1.5 mm, accuracy would be of course, depending upon its not very easy to make it accurate, but maybe if you put a magnifying glass, then you will see it more accurately you can measure. For example, this is coming out be 0.40 at the moment as I am able to see this particular line here is line here is 0.40 or 0.35 this line or may be 0.35.

But if I magnify it further with a magnifying glass putting here I can see it match it better, match it better. So, this is how one can measure the cracks. So, we have seen now crack measurements maybe next class we will look into some of the test measurement right.