Fire Protection, Services and Maintenance Management of Building
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Lecture - 17
Design of Lift systems: expected stops and floor of reversal

Welcome back. We will do the lift again, I mean you know we will continue with the lift from wherever we have stopped in the last class, you know last half an hour slot. So, we follow it up, right.

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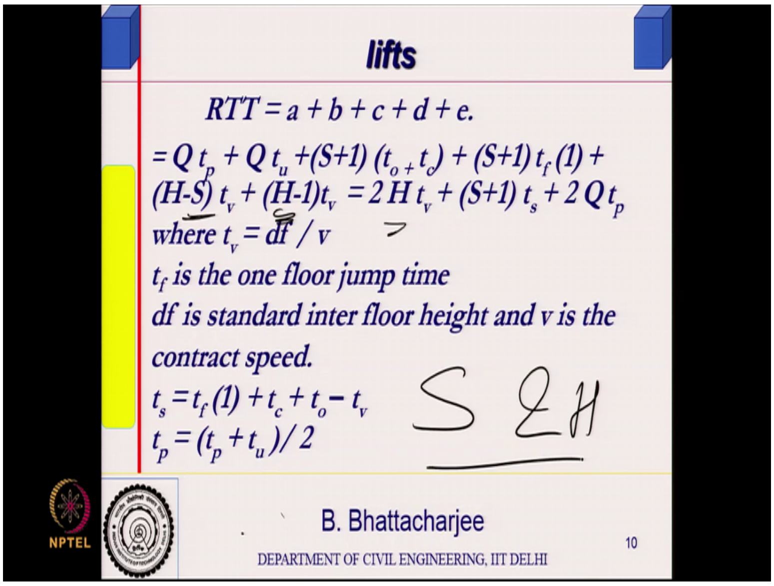

So, lifts that is what we were doing last time and then we if we recall that we talked of round trip time as sum of a, b, c, d, e 5 components. I will not repeat this too much now, but only thing that I want to highlight here today because we have done it last class itself, highlight today is that we got to know two things S and H.

What is S? S is the probable number of stops, and H is the probable height of reversal, expected height of reversal that is what we said in the last class right. And this we can determine basically heuristically that means, by just simple you know logical thinking or from the concepts of assuming you know process of people boarding in the lift as a and boarding in the lift is an waiting situation with persons process, assuming it to be a person's process. Waiting for the lift and then lift serves the people you know.

So, as if people are the customer lift is the service provider, then you know the person enters and disembarks somewhere. So, this can be thought of as a waiting situation waiting line situation but then we are not going to do that today. We will look into how do we determine S and H heuristically that means, by thinking process. Of course the probability comes into picture. So, that is what you can do.

So, to do that let us now look at first of all let me first to you know try to find out the probable, probable number of stops, probable number of stops that is S, right. Let me find out probable number of stops that is S, right.

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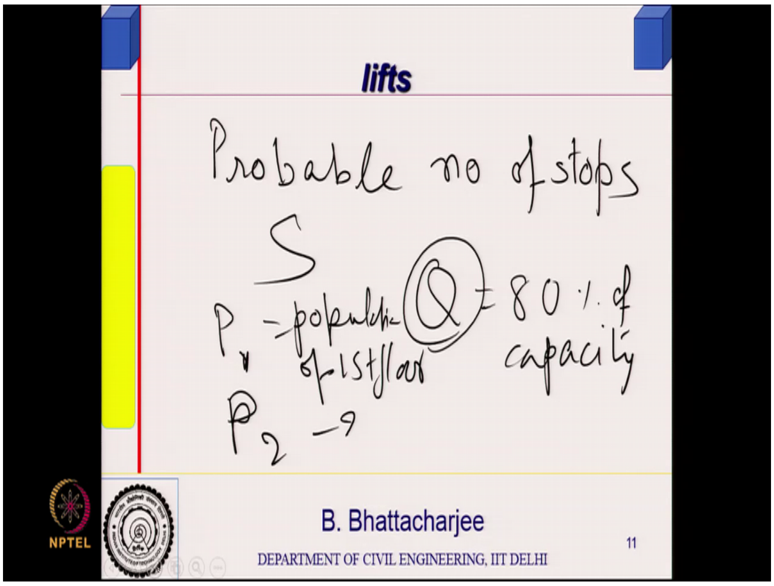

Now, how do I find out S what is what actually it would be people because we said this is a one way traffic up going traffic only we are designing the system for up going traffic only. And we have Q number of people boarding right, Q which is actually 80 percent of the capacity, which is actually 80 percent of you know 80 percent of capacity lift capacity.

So, it runs at Q number of people boarding, the lift car, and then they get their disembark in between wherever they are supposed to go. So, we are looking at the up going traffic only, because as you remember in the round trip time and the design of the lift system we take pick 5 minutes time for single purpose, you know single purpose office which is a most crucial and that is how we do the lift design. This concept can be extended to anywhere else, right so because this is most crucial. So, we said that Q people would be boarding the lift and they will disembark.

So, if I you know my, if I find if I know the population of each flow. Let us say population of first floor is P 1 right population of first floor, P 2 is a population of second floor and so on. And P i is a population of ith floor, P i is a population of ith floor, P i is a population of ith floor, right; P i population of ith floor. I mean let me use keep it capital P only population of ith floor.

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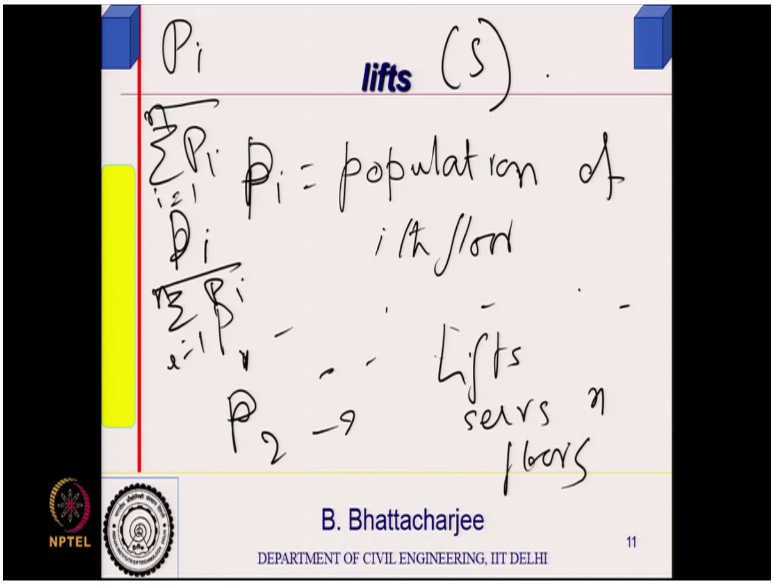

We are trying to find out S then probability of finding a person belonging to ith floor will be given by P i divided by sigma P i I going from 1 to n, where n is a lift serves lift serves, n number of floors. So, lift serves n floors.

So, probability of finding a person and population in first floor is P 1, second floor is P 2 third floor is P 3 etcetera etcetera, and P i is the population in ith floor. If I sum up for all the floors that will give me the total population to be served for nth floor. So, probability of person boarding the lift and belonging to ith floor is P i by, P i by sum of all P i's, i going from 1 to n, right.

And if that you know if, so therefore, probability of finding a person in ith floor is given by this, and I am interested in finding out probability of finding at least one person in the ith floor, out of all the Q people who have boarded; out of all the Q people who have boarded. So, at least one person right, not all the persons at least one person; must be in ith floor, then ith floor will be the stop.

So, I am interested in finding out at least one person. So, P i divided by sigma P i 1 minus that is probability of not finding a person in ith floor ok, probability of not finding a person in ith floor. So, let us let us look at it again, let us look at it again right, probability of this is what I am looking at; this is what I am looking at. Probability of finding S is what I am looking.

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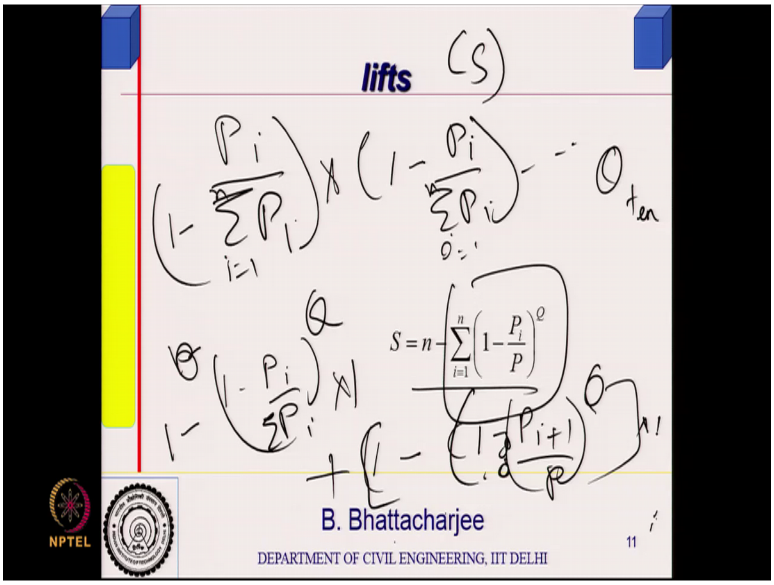

So, we have seen P i divided by sigma P i is the probability of finding I going from 1 to n, probability of finding one person you know probability of finding a person out of Q in out of Q belonging to ith floor. 1 minus that is probability of not finding a person, probability of not finding a person right in ith floor, and 1 minus that is the probability of finding at least one person.

Now, what I can do is this is since I am dealing with Q persons, this is the probability of not finding a person in ith floor. Probability of not finding the second person two persons will be simply multiplied because this is joint probability and they are independent persons, this is independent event they being there one person being there second person being there. So, it is simply joint probability 1 by P i divided by sigma P i again i going from 1 to n.

So, I will not write that again and this for the Q terms, Q terms, that is joint probability of not finding any one of those Q persons; joint probability of not finding any one of these Q persons belonging to any of these Q persons belonging to joint probability of not finding any one of these Q persons belonging to ith floor you know this product. So, this is the term actually.

If you look at it this is the term this is the term right, this is the term, this term. This term 1 minus P i, I mean only I am just writing 1 minus P i divided by P to the power Q is the joint probability of not finding anyone of this Q person belonging to ith floor. And 1 minus that, 1 that gives me probability of finding at least one person. You see probability of not finding any person, probability of not finding any person and probably 1 minus that gives you probability of finding at least one person, probability of finding at least one person right, which means that ith floor will be the floor of reversal.

So, probability of finding a person in ith floor 1 minus that gives me probability or not finding a person in ith floor to the power cube gives you all the Q people who have boarded not finding any one of them in ith floor. 1 minus that gives me probability of finding at least one person, right. 1 that gives me at least one person will belong to out of those Q will belong to ith floor.

This multiplied by 1 is the expected ith floor being the being a stop, this is multiplied by 1 is you know. So, expected value of ith floor being the stops and this I repeat for all the floors that means, I sum up you know sum up for next floor.

1 minus you know 1 minus 1 minus P i plus 1 divided by P, this to the power whole thing to the power Q whole thing to the power Q not this bracket is not there. This is the probability of two floors being expected number of rows. And if I repeat this for all n floors that gives me the value of S; that gives me the value of S. You know if I repeat this for all floors that gives me the value of S.

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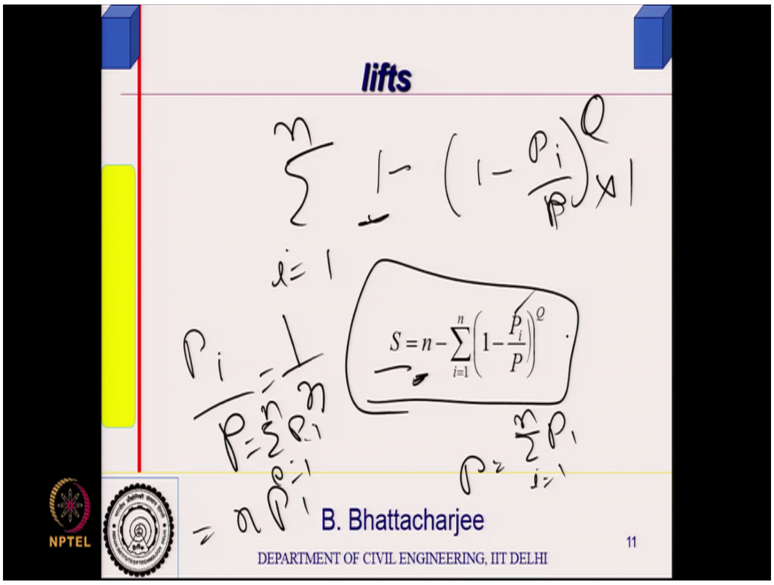

So, let me just explain again. This is the probability of finding a person in, P is the total population which is P is equals to sigma P i I going from one to n for all floors. So, probably of finding a person in ith rho, 1 minus that is probability of not finding one person in ith floor. Probability of all Q persons not belonging to ith floor is this.

And this you know 1 minus this, 1 minus this term alone not the sigma 1 minus this, 1 minus this, right, 1 minus, 1 minus P ith by P to the power Q that is the probability of finding at least one person in a given floor. When I sum it up for all n floors multiplied by 1, I get n here, because summing up and the n sum up comes here. So, that is the probable number of stops, that is the probable number of stops, that is the probable number of stops, that is the probable number of stops, right.

Now, this is when populations are different. If populations are same in all the floors, in population in all the floors are same then P i by P will be simply 1 by n, because this would be simply sigma P i, i going from 1 to n and since this value is constant equal population. So, let me you know whatever the value is it will be simply n into P i or you know. So, P i's will cancel out. So, when population is same the formula becomes a little bit simpler, formula becomes a little bit simpler.

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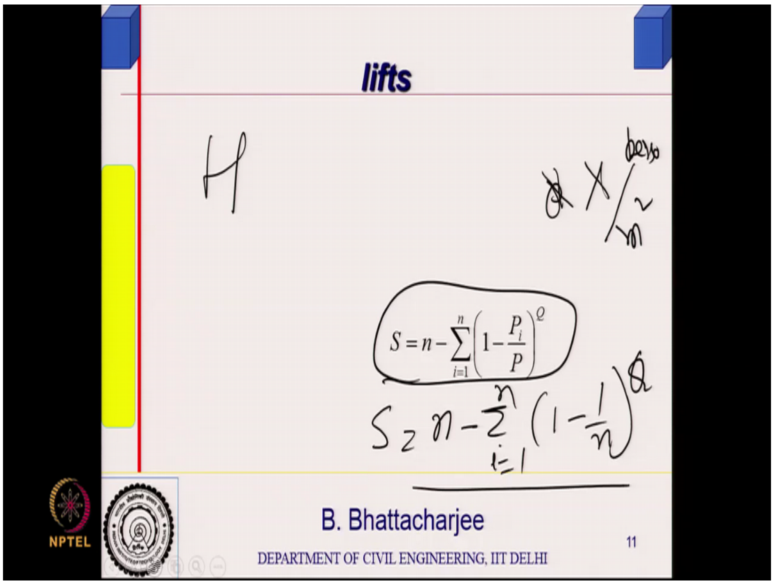

S then becomes n minus simply I going from 1 to n 1 minus 1 by n to the power Q, where population in all the floors are same. Which is generally may be the case because floor areas are same, and if it is a single purpose office occupancy is the same type therefore, we know the population is actually we can estimate the population, sum X you know, X meters X persons per meter square, X person per meter square.

For example, you know for residential it was 12 or 12.5 or such some. So, you know something similar to that 12 meter square per person. So obviously, it will be much less depending upon type of office. So, if the floor area is same of the each every floor then I am like you to find population being same, then I can use this simpler formula S, that is probable number of stops, probable number of stops, probable number of stops, right. So, probable number of stops. So, this is one part.

I need still one more thing that is my expected height of reversal or probable height of reversal. Similar logic will apply, similar heuristic logic we apply. You know similar heuristic logic will apply. Let us see what is the logic do we apply. You see first thing is I got to take look into probable you know say jth floor being the probable floor of reversal.

Now, when will it happen? When I have at least one person I mean they might go anywhere down below, but at least one person going to jth floor, but nobody above right, so finding at least one person going to jth floor and nobody above. Now, finding one person up to jth floor would depend upon you know probability of I mean I can find out like this supposing jth floor.

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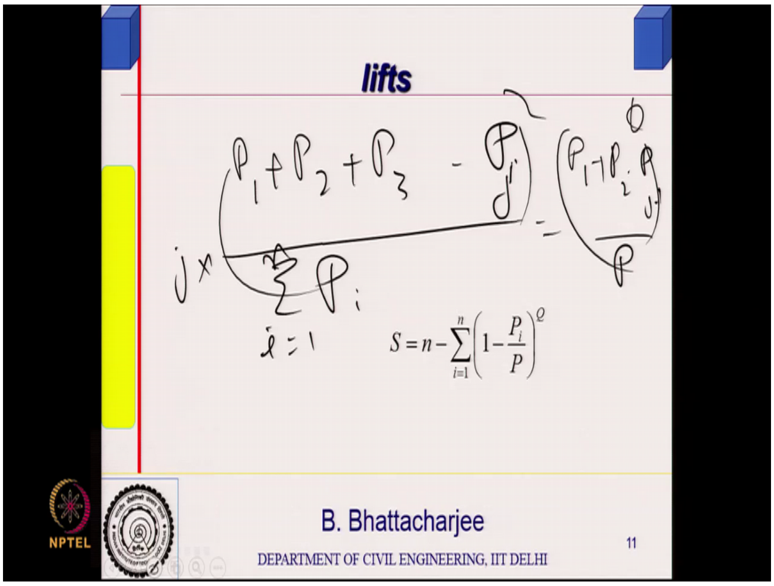

So, probability of finding a person up to jth floor. So, it would be P 1 plus P 2 plus P 3 P j divided by P j divided by P sum up, sum of P i's, i going from 1 to n.

This is the probability of finding person in any one of this floor 1 to j, right probability of finding any you know person up to jth floor you know. And if I do it for all Q of them the probability of finding all Q persons belonging to up up to jth floor, but if jth floor has to be the floor of reversal then I must find probability of finding a person in jth floor itself, which will be simply P 1, P 2 up to j minus P j minus 1 divided by P to the power Q. This will give me the probability of finding a person only up to one floor below.

And this difference gives me the probability of finding the people Q people who have boarded one person belonging to jth floor. So, probability of finding people who are boarded all you know probability of finding all people belonging to jth floor minus out of those that at least one person is there in the jth floor and rest all might be j minus 1th floor. So, I subtract this this is the probability of height of jth floor being height of reversal right. So, this multiplied by j gives me the jth expected value for jth floor.

You know remember expected value; expected value is nothing but, expected value is nothing, but the value of the random variable itself multiplied by the probability. So, we are trying to find out the probability of jth floor being the height of reversal exact probability of jth floor being the heights of reversal and multiplied by j and sum it up for all of them, all the floors, then I will get the H, then I will get height of floor.

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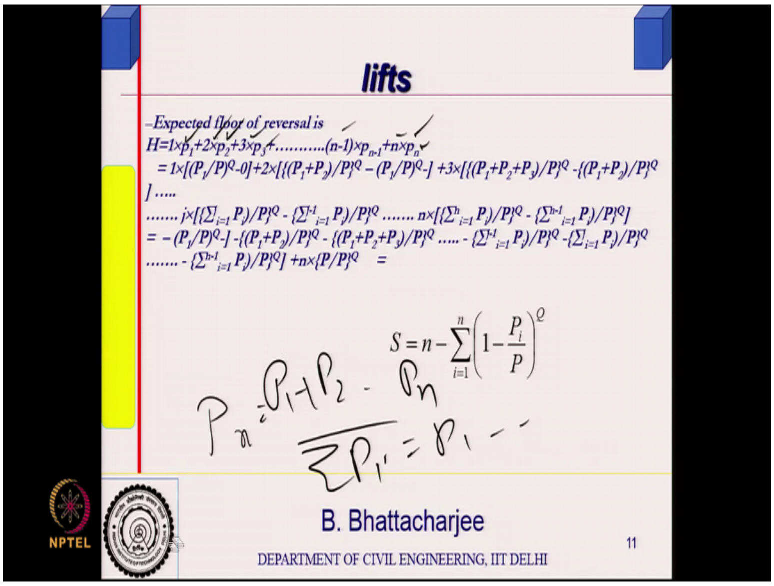

So, let us, let us see this, let us simply see this. You see the expected floor of reversal, so it says that I find out the probability of P 1 first floor being population up to first floor minus probability of finding you know this population one floor below, and thus probability of first floor being the floor of reversal.

Probability of second floor being the floor of reversal, I am not considering anything above because I am assuming all the population up to this floor and 1, minus 1 floor below. So, that multiplied by 2 because second floor, third P 3 and n minus 1 and n P n last one, this is the last one, right. So, n P n and this value will be 1 P n would be 1 because it will be sum total of P 1, P 2, P 3 up to P n divided by again P 1, P 2, P 3 etcetera.

So, this value will be actually 1, if you remember because we said that P you know P n up to P n is P 1 plus P 2 plus P 3 up to P n and this is also sigma P i which is nothing, but P 1 plus P 2. So, this value will be equals to 1. So, it is simply n, right. So, this value is 1 this is simply n ok, or, right. So, let us, let us look at that. So, we can actually come to write ok, we will come to that later on.

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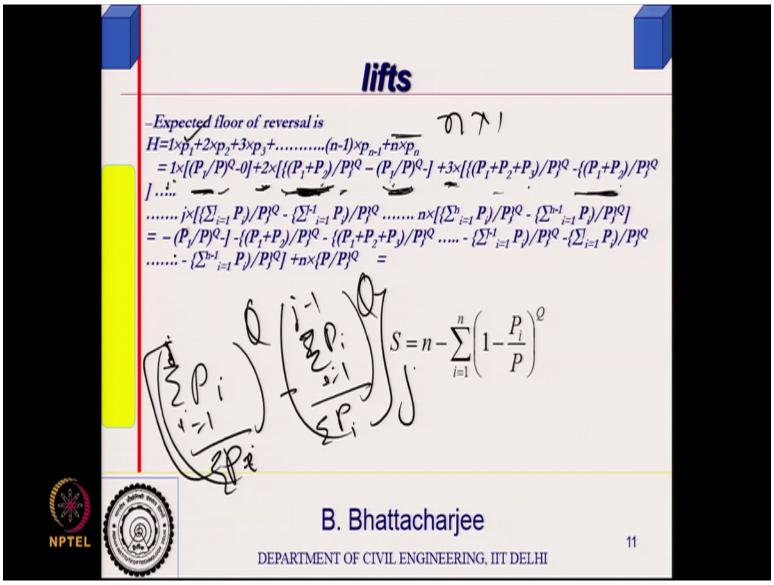

So, now notice P 1 I have explained to you, I explained to you P 1 P 2. What is P 1? Population up to P to the power Q minus 0 because that can be ground floor multiplied by 1. This is population up to P 1 plus P 2 divided by P minus P 1 by P to the power Q into 2 and similarly for all this sum.

Now, interestingly if you see in this sum you will see something P 1 by P comes here and it is there in the next time also right, next term also but here it is negative and multiplied by a factor 2, here it was multiplied by 1. If you look at this; this is multiplied by 2 and here it is multiplied by 3, but with a negative sign, right. And in the last term if you see this is a positive term, this is you know this is a this is a positive term, positive term n into 1, because it will not get cancelled in the next one there is nothing above n.

So, this is the term which remains which is n, which is simply n into 1. So, this remains, rest all you will have one term each cases there is a negative term because this is twice of this here thrice of this with a negative term. So, what I find is what I will find it, I mean if I take the jth term then I will get again the sum going from i going from j P i divided by P 2 the power Q and this going from i going from 1 to j minus 1 you know. So, this is going j minus 1, for example, this was 2, this was j minus 1, j equals to 2, this is 1, j equals to 3, this is 2 and so on. So, this is generalized term.

This is a general term, general time will look like which I just explained a couple of minutes earlier, sigma P i, i going from 1 to j right divided by P to the power Q minus sigma P i, i going from 1 to j minus 1 divided by P which is sum of P i you know which is sum of P i right, to the power Q into j is the intermediate term any term and last term would be something like this, right. So, that is what we see.

So, there is one negative term remaining from everywhere, and if I collate this negative terms if I collate all those negative terms I get something like this.

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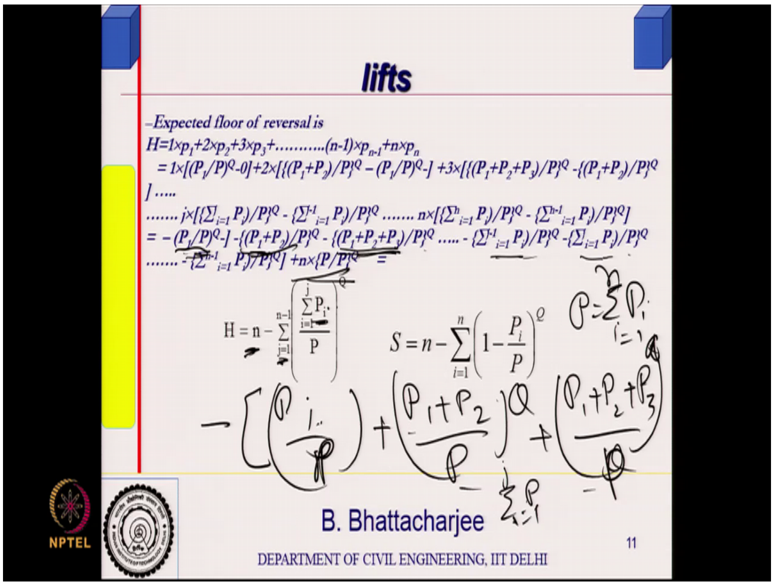

This I get 1, every you know I will get 1 only because 3 minus 2 is 1, 4 minus 3 is 1. So, I get a negative term, here I get another negative term of these and so on you know, so i minus 1 going to j minus 1 and P. Last term will be simply P i divided by P sum you know every. So, last term will be simply n into this P by P to the power Q.

So, then if I do a little bit of algebra out of this, if I do a little bit of algebra out of this, right. If I do a little bit of algebra out of this what do I get? If I do a little bit of algebra out of this I will get, you know I will get I do a little bit of algebra from this I will get H equals to sum this up, this last term comes as n. All other terms because there is only one term remaining, so it will come as you know j going from 1 to n minus 1 right because j going from 1 to n minus 1 the internal sum is this, there is an internal sum which is P 1, P 2, P 3 up to j divided by P and this is summed up now. So, this all these are summed up you know there were sums were like this I had something if I may write it like this minus sign is very much there. P 1 by P to the power Q P capital P is nothing but sigma P i. So, I am just, you know I going from 1 to n total population. So, this is the first term.

Next time will be P 1 plus P 2 divided by P again to the power Q and P 1 plus P 2 P 2 plus P 3 divided by Q, which in other words my j will be going from 1 2 3, you know j would go. So, there is a j going from 1 2 3 etcetera etcetera. So, there is a summation from i going, i 1 to j and j goes from 1 to n minus 1. So, this term I just each term you know I can just overall sum it up.

So, this if I sum it up internal 1 this will write it simply it is going from, i going from you know P, i going from 1 to j, right. And 1 to j divided by Q 2 divided by P to the power Q divided by P to the power Q right, that is what is coming so to the power Q. So, i to j it is going, and then j goes to 1 to n minus 1. So, for j equals to 1 first term j equals to 1 it is this term, j equals to 2 j equals to 2 it will be only this term, j equals to 3 it will be this term, j equals to 4 it will be this term. So, whole thing can be now written in a compact manner, compact manner we can write in this manner.

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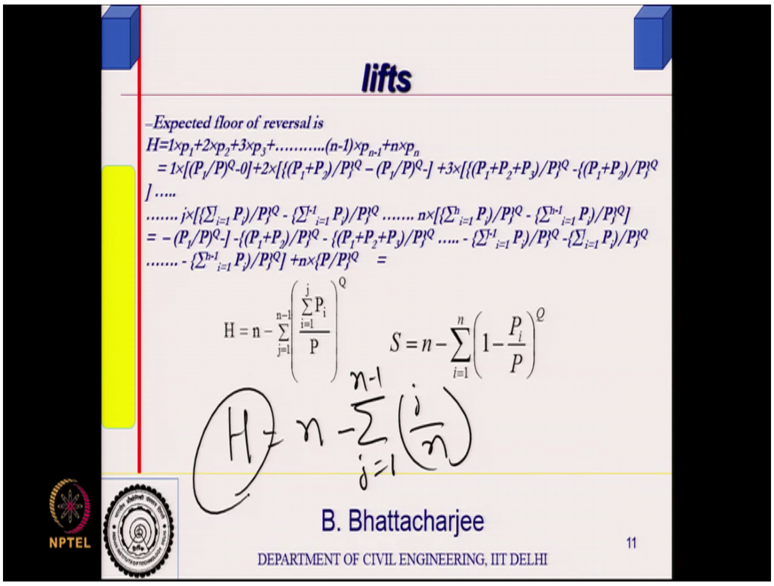

So, this is the expected height of reversal because we have multiplied by the floor number, multiplied by their probability of that floor being the height of reversal and so on. So, this is the expected height of reversal. So, this is how we find out the expected height of reversal.

Now, if the population are same this will be simply written as j into P i, you know this will be simply written as n minus sigma j going from 1 to n minus 1, and within bracket this will be simply j into P i and divided by n P i. So, P i's will cancelled out I will be left with j by n. So, this will become you know this sum will become j going from simply it will become you know it will get simplified I will write it like this. So, for all population being same I will simply write this term as j by n, j by n and j going from 1 to n minus 1; j going from 1 minus you know 1 to n minus 1.

In other words let me just rewrite it. So, for population being same this would be n minus sum, j going from 1 to n minus 1 and inside is j by n, j by n. So, this is how it you know this simply simplifies to a simple, simple thing height of reversal.

Now, having done that now I know H and S and therefore, I can calculate out the round trip time, I can calculate out round trip time right, round trip time, I can calculate out. If I go back my earlier slide if I go back to my earlier slide you know so H known, S is known I can calculate out round trip time. Best of course, is an example calculation. So, let us see that example calculation. Let us see the example calculation. There is an example calculation I have.

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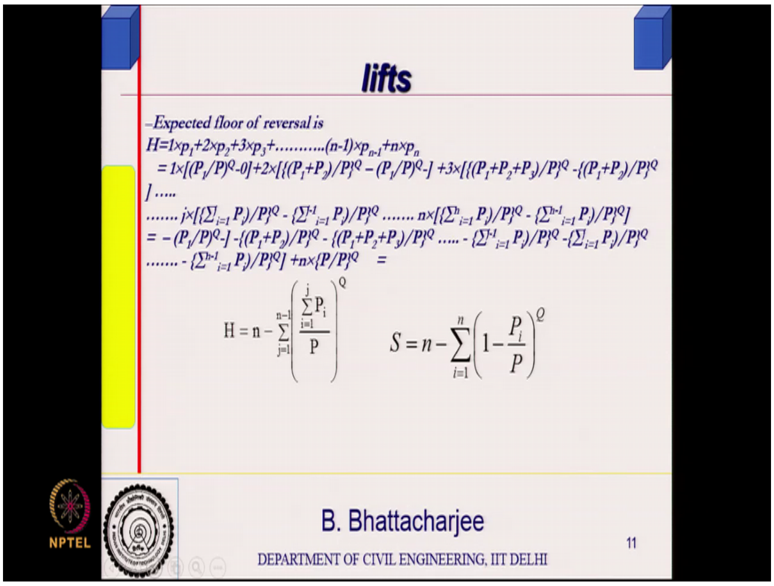

So, this is how we you know we find out S and H, this is how we find out S and H, height of reversal and S heuristically. Of course, one can find them out probabilistically more you know with person's process as you process, we will discuss sometime later on. And now let us look at the example calculation.

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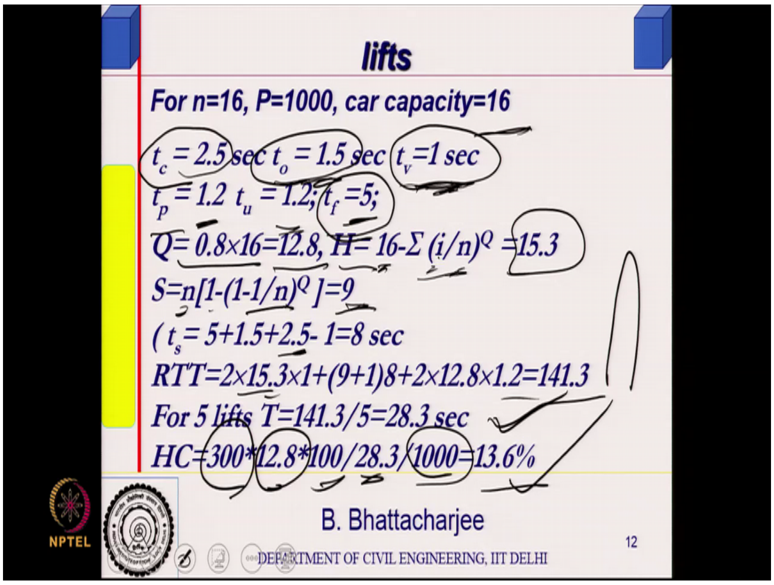

So, example calculation is something like this. Consider 16 floors total population 1000 for simplicity, car capacity is 16 right, car capacity is 16. Door closing time has 2.5 seconds, door opening time has 1.5 seconds. And height you know constant speed time TV, remember we talked of TV earlier you know at constant velocity floor every floor would be jumped, it is called one floor jump term is 1 second. And person entering is 1.2, person getting unloaded loading and unloading 1.2 and one floor jump term time is 5 seconds.

Now, what is one floor jump time? I think I have explained it earlier. One floor jump time is you start from a floor it accelerates goes to the constant speed and then stops. So, it has got an acceleration and deceleration and of course, a little bit of constant speed would be in between. So, that is the jump time and we have explained this earlier.

Now let us see next. So, Q is 80 present of 16, 0.8 of 16, 0.8 of 16 which is 12.8, all right. And H therefore, since we are assuming equal population all over the place you know all floors. So, simply i by n to the power Q, so 16 minus you know sigma 1 by 16 plus 2 by 16, 3 by 16 up to 15 by 16. So, if I sum this up I get 15.3.

Yes, same formula because I said that when it is all equal, then I get something like this that comes out to be 9, right. You can check this calculation. So, n is equals to 16, 1 minus 1 minus 1 by 16 to the power Q, that is your 12.8 comes out to be 9 and a t s is the some time that is what we said is a 5 seconds. This t f plus t f plus 1.5 second opening time and door closing time and minus 1 you know constant speed one floor jump term time that is your t s. So, RTT therefore, by putting in the formula that we talked about a plus b plus c plus d a b c d etcetera etcetera, if I put it and I get 141.3. That is the round trip time for a lift round trip time for a lift right.

And if I have 5 lifts then my waiting interval becomes 141 divided by 5 which is 28.3 seconds, 28.3 seconds. Therefore, handling capacity I can find out, remember handling capacity was 300 into the capacity Q into 100 for percentage into divided by waiting interval and total population is here. So, we express it in terms of percentage of population that comes out to be 13.6. So, that is how we actually find out the hanging capacity.

Now, what we will do is we will you know this how we find out the handling capacity, this is how we find out the handling capacity right. So, this is how we find out the handling capacity. So, what we will do is next time we will look into what this, one thing we have assumed the arrival of the people and their service the rate is same. The rate at which for example, every to put you know every 5 minutes people coming hours, every 1 minute, number of people coming all of them are being served. You know it may be 28.3 seconds or whatever it is there is a lift. So, whatever people come in 28.3 seconds all of them will board and they are equal to 12.8 or on an average, something of that kind, right. So, this is an assumption made.

Without this assumption also we will be able to calculate out we will see that later on. So, I think we will have a break right and after that we will follow up to the next lecture, right, ok, then. So, that is it.

Thanks for time being. I will take some question from the students here.