Energy Efficiency, Acoustics & Daylighting in building
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Lecture - 05
Introduction & Environmental Factors (contd.)

Good morning. So, we will continue from where we finished in the last class. A little bit of course, additional thing I will just tell you. This calculation we did in the last class.

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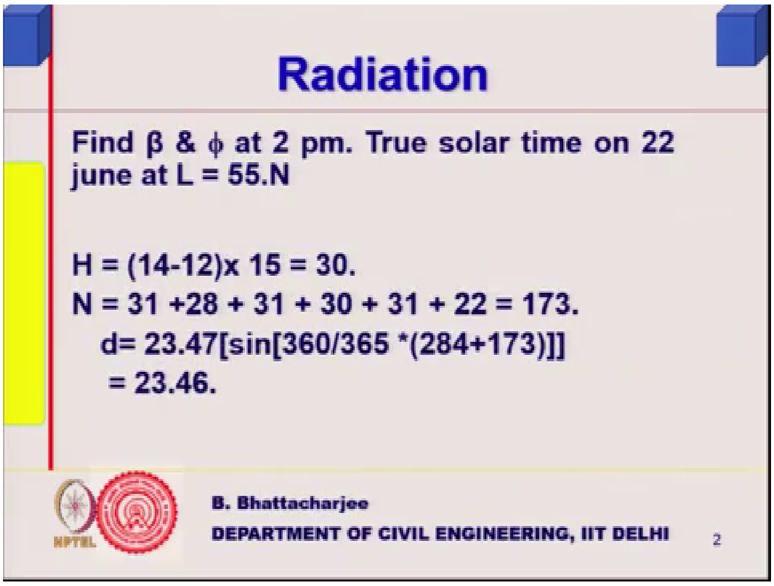

We did this calculation in the last class remember, and that is where we stopped actually. Then I was looking at celestial sphere and spherical trigonometry we will come to that.

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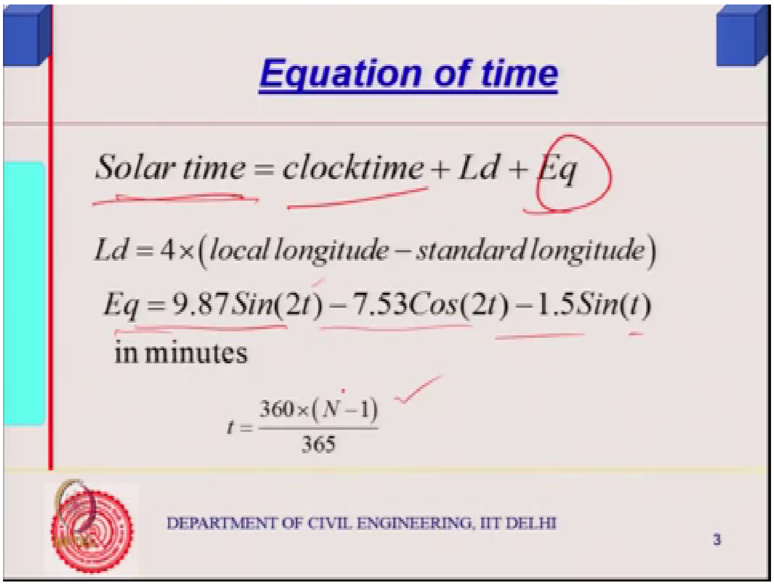

But just before that I want you to tell you about solar equation of time right, because anything you know hour angle is always calculated with respect to the solar time right.

So, clock time plus there a correction for longitude, and there is a correction for, you know there is a another, there is a another small correction for time exactly, time. So, this first one longitude correction is 4 into local longitude minus standard longitude. Why, because longitude is also whole of the surface of the earth, can be divided into 360.

Student: (Refer Time: 01:26).

Angles, degree angles. And in 24 hours earth covers 360 degrees. So, this 360 degree you know, since time is related to rotation of the earth. Therefore, 24 hours corresponds to.

Student: 360.

360 degree, 1 hour corresponds to 15 degree. So, 1 degree corresponds to how much, 1 hour 60 minutes corresponds to.

Student: (Refer Time: 01:51).

So, one; that is right. So, one degree will corresponds to 4 minutes. So, therefore, local longitude minus standard longitude 4 minutes basically, because 1 hour is 60 minutes corresponds to 15 degree. So, for 1 degree 4 minutes that is what it is. And there is a some corrections, you know there is some correction; that is given by, this correction is given by this one. So, that is what it is. So, this is some time we will be using this, or we may not use this, so you might be more directly into using this, but supposing you want to convert it from standard time, clock time for any country or any region, then you convert this to solar time.

This generally we neglect, this part we neglect right, this general area. This will depend upon the day of the year. As you can see from here, this is something to do the day of the year. So, again 365 is corresponding to 360 degree angle. So, this corresponds to day of the year, because there is some variation with respect to day of the year. Essentially because of you know distance of the earth is not same from the sun every day. So, related to that, but we can neglect this part, we can just calculate with respect to this.

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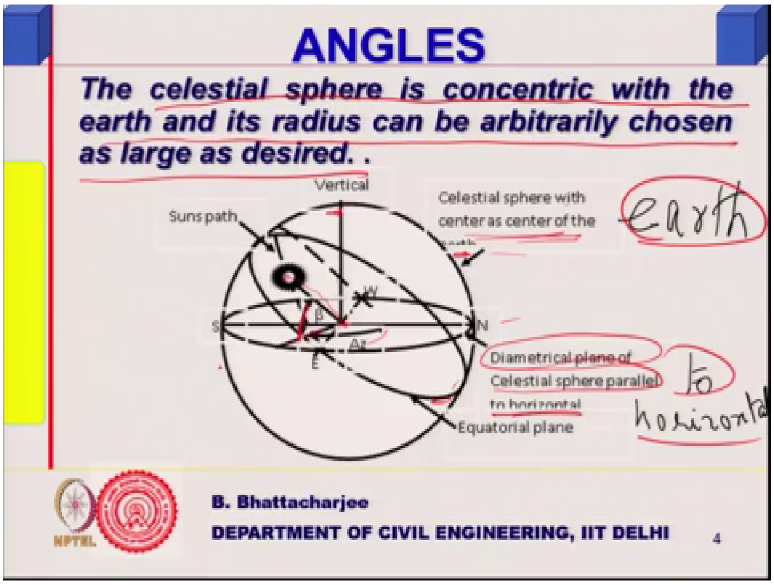

So, let us go back to our celestials angle, and in a celestial sphere and spherical trigonometry, which is required to relate altitude and azimuth angle, to declination. As we said in the last class, suns position is governed by altitude angle and azimuth angle right. And these two angles that is suns position, is dependent on location on the surface of the earth, which is given by, latitude of that location. Day of the year that is given by declination, and time in the day is given by hour angle.

So, altitudes angle is a function of all these three, and relationship between altitude angle in this. I can straight away give you the formula, but I thought I will also tell you about the basic principle behind it. So, that it, you know it remains in your mind more easily.

So, that can be derived. This relationship between altitude angle and those three factors hour angle latitude and declination can be derived through principle of spherical trigonometry. Similarly azimuth angle can also be there. And all other angles are derived based on those principles. So, I will just quickly discuss this principle. So, a celestial sphere which as, you know last class also just mentioned repeating this, celestial sphere is a concentric with the earth and its radius can be arbitrarily chosen, as last large as desired.

For example, this you consider this, you know this is a celestial sphere. Now this is the centre of the earth, this center of the earth, this is you know center of the earth right. So, this point is the center of the earth. And this circle or this circle or circular plane, is parallel to the horizontal surface. So, I conceived a diameter, diametrical circle at the center of the earth, and this diametrical circle is parallel to horizontal surface at some point right. So, this will be the vertical then, this will be the vertical. Earth might finish somewhere here because I said the radius can be arbitrary right. So, this is this will be the vertical, because horizontal is this, this is the horizontal right. In the horizontal I have north south east west. So, from the north if I go clockwise I go to the east, and follow further I go to the south, and there is west right.

Now, equator, you know equator will be somewhere here, this was equatorial plane, equatorial plane. So, this is horizontal celestial sphere with center as the center of the earth. You know celestial sphere is the center of the earth; this earth is not redrawn, so I just corrected it here. And diametrical plane of the celestial sphere parallel to horizontal surface, parallel to horizontal surface is not looking nice. So, I just you know not readable. So, I just made you this. So, this is parallel to the horizontal, at that particular location, normal to this is vertical. So, I have the center of the earth and their circle parallel to the horizontal at the center, perpendicular to that will be vertical. So, this is vertical.

Equator will be somewhere there at certain angle right, and the sun path, sun will rise from east and go towards the west. So, sun path is shown by the dotted line, sun path is shown by the dotted line. So, that said that is the kind of a celestial sphere. Now I can define certain angle here. I can define all those angle here, because angle between the suns ray, and the equator is declination, and angle between suns rays. So, this is the suns ray. Sun position is as I said dotted line dotted line you know following. So, it is here right now. I joined at the center of the earth, and this is horizontal plane, so therefore, this angle is altitude angle sun join the sun ray at the center of the earth, and since its parallel to the horizontal plane this will be altitude angle right. And angle between suns ray and the angle between suns ray and this equatorial plane will be declination, we will come to that in a little bit later. And projection of the sun onto the horizontal surface is this. projection of the sun onto the horizontal surface is this, from geographical north.

If I take clockwise azimuth angle then this is a azimuth angle. So, you can see that, but its not necessary that all the time I take horizontal surface as a diameter. Only thing is that, say earth center and center of the celestial sphere should match. So, according to my convenience, I can choose the diameter, and then create, you know define the spherical triangle and then derive. So, let us see, let us go further.

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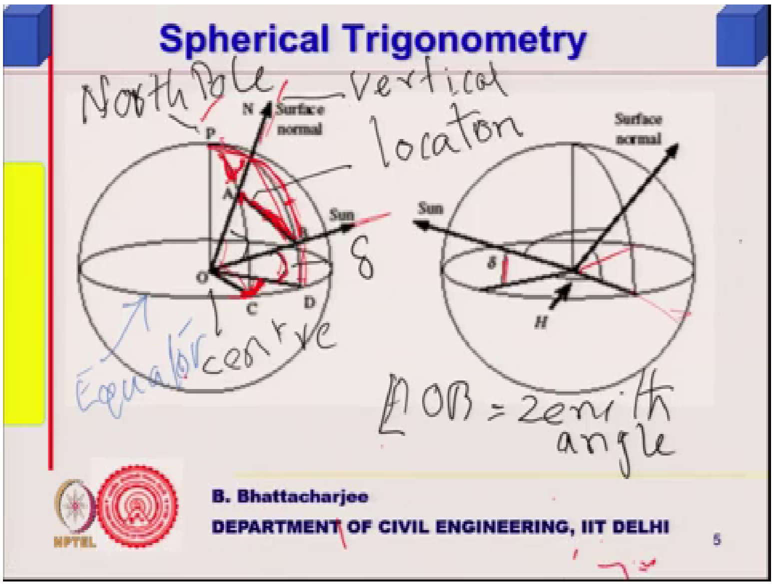

So, coming back here, you see this is my equator. This time I have done equatorial plane is that, you know equatorial plane as a ah center of the earth, an equatorial plane is a.

Student: (Refer Time: 08:21).

Diameter, yeah diameter, not horizontal, just it is a equivalent, you know earlier one I just did, I said horizontal plane, this is a equatorial plane. So, this I have my choice. So, here for my convenience what I do is, this is my equatorial plane right; that is the diameter. So, this P you know P would represent what North Pole. If it was horizontal plane, then you have been vertical here, this will represent the North Pole P, because normal to the equatorial plane will be North Pole, some where the polar direction. Surface this point is on the surface of the earth. So, if I draw a line from the center of the earth to the point, on the surface of the earth and beyond. So, this is the surface normal to it, that is basically vertical, that is basically vertical right that is basically vertical right. So, this is North Pole, this is vertical, this is vertical.

So, the point and center of the earth if I join, and extend, it will be a always normal to the sphere. So, this is vertical right, and you know this is. So, this is my location, let us say somewhere location projected into the celestial sphere. So, this is north pole, this is vertical, this is equatorial plane, and what will be then latitude, this angle will be the latitude, A O C would be the latitude, angle A O C will be is latitude, A O C would be latitude. If the suns direction is like this, suns direction is like this, then its projection is onto the equatorial plane is this much, and then that will be delta. So, I can define angles, did take a the way I take the plane. I can take different plane at the center of the earth as diameter, and then define various angles.

So, here I have defined delta. As you can see this is nothing, but the latitude because O C is the projection of O A here, and suns ray is B O. So, its projection is C O D. Therefore, B O D is nothing, but delta right, and its, you know its basically this is the vertical. So, this is angle O A B represents actually is zenith angle.

Student: Zenith angle.

Right sorry O B, you know our a same thing O A B, because this vertical, vertical to the sun, that is basically is zenith angle. So, hour angle would be. Now this you see when, when 12 noon sun is aligned to north south, sun is aligned to north south, at any other time the hour angle I can define. So, it would actually, it would be aligned to the, it will be the line joining the north pole, sun will be at 12 noon, sun will be at the north pole, and it will be joining the north pole, and this you know its in, I mean basically this is the highest altitude at that point of time. So, it is connects the north south, north south line. So, it will be joining this sun position we are joining this point, this point, you know suns position will be somewhere there.

So, hour angle I can define from here actually. So, hour angle; see therefore, this is the center of the earth A O B is a zenith angle as I says, A O B is a zenith angle, and hour angle is defined with respect to 12 noon position where this is the north pole in this projection onto the earth. So, this, this is at 12 noon, any other time sun will be somewhere there and that is hour angle.

So, at 12 noon sun direction will be something like. It will be almost you know it will be matching it somewhere along in this plane suns ray will be there. So, at any other time it would be moving. So, hour angle can be you know. So, this is delta, this is delta, this is hour angle. So, hour angle will be 12 noon, north south direction coincides with this; you know suns position is on the north south plane north polar to.

So, basically from, you know polar to the equatorial plane, it will be it will be actually on that itself. At any other time it will shift and hour angle is defined like this. So, this is how we can define all the angles

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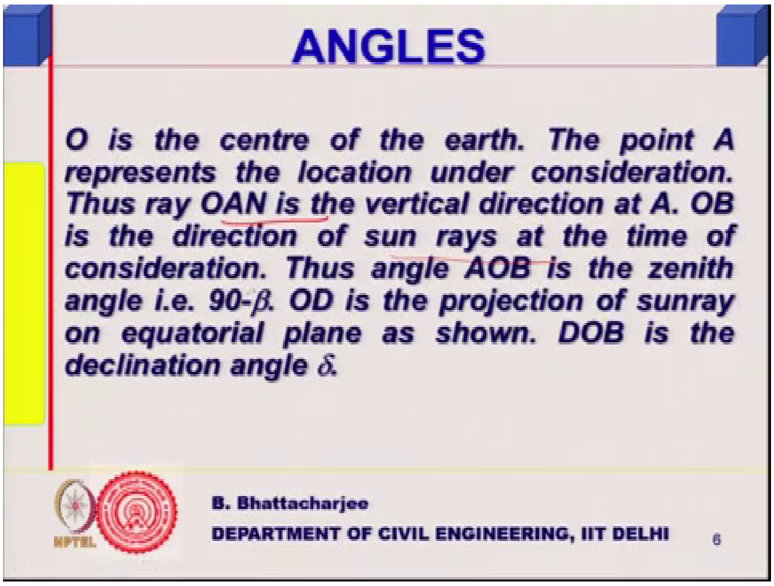

So, this is written here actually, I already explained to you, O is the center of the earth. The point I represent the location under consideration, point A represents the location under consideration. So, that you can read it and understand it, and that is O A in is a vertical direction of A. So, O you know this is a O A N is its a P Ps, you know this is the vertical direction O A N is a vertical direction, O A N is a vertical direction that is what is stated here, is the vertical direction at a.

O B is the direction of the sun rays. So, O B was a direction of the sun rays at that point of time, O B is the direction of the sun rays. So, you come back again, O B is the direction of the sun rays at time of consideration that is angle A O is the zenith angle that is what I just explained. So, angle A O B A O B will be zenith angle, because this vertical this is the sun. So, angle between vertical and suns ray, there is zenith angle right. So, that is how we define A O is zenith angle that is 90 degree minus altitude angle.

What is the projection of sun ray on the equatorial plane as shown? So, D O B is the declination angle. So, that is what I explained D O B is the declination angle all right. So, this is, this diagram is explained through this statements. Now point P represents North Pole continue with that. So, angle B O P is thus, what is this angle. Angle B B O P, this is North Pole right, this is the suns ray. So, angle between suns ray and equatorial to pole what is this angle, the 90 minus.

Student: (Refer Time: 14:00).

Declination, angle between equator and suns ray is declination, and north pole is normal to the equator. So, this is 90 degree minus declination 90 degree minus delta. So, basically, you know the angle, angle B O P or P O B is nothing, but delta that is what is being said in a 90 degree minus delta, B O P is thus 90 degree minus delta. A O C is the latitude angle, because A O C would be latitude angle again going to this A O C will be latitude angle this my location, this is equatorial plane this latitude angle. So, this angle, you know A O P or P P O A, whatever you call it this 90 degree minus latitude. So, this angle is 90 degree minus latitude, what is this angle, zenith angle 90 degree minus 90 degree minus.

Student: (Refer Time: 14:51).

90 degree minus beta right and what is this angle 90 degree minus delta 90 degree minus delta because this is delta you know suns ray to the equator and if you connect for ray an a pole to the suns position. So, this is 90 degree minus delta this is 90 degree minus beta and this is 90 degree minus latitude angle, this 3 arcs now a spherical triangle as I said, I mean a spherical triangle you know. So, by definition this is not it. So, arcs A B B P and P A lie on the great circle on the celestial circle therefore, they form what is called spherical triangle, you know a spherical triangle is.

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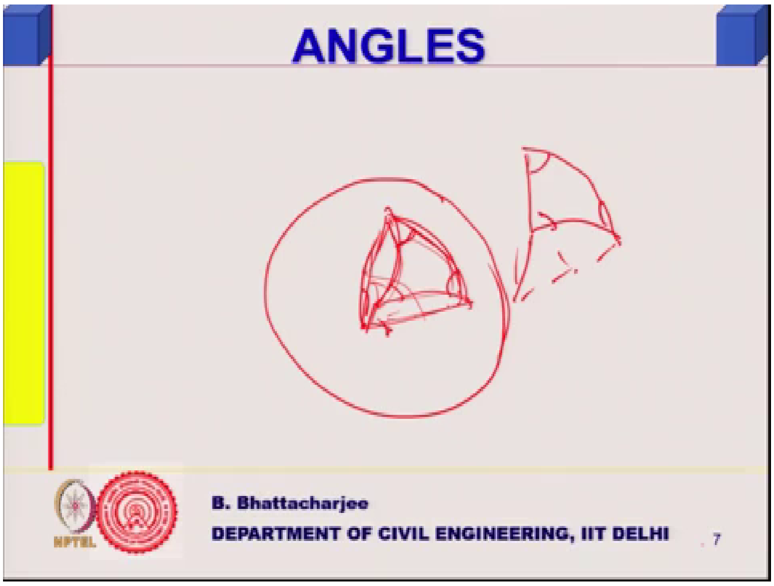

If this my drawing is, this is a sphere right. So, any arcs lying on to the, this might be my center my drawing is not all that good; that is why you know he is not drawing it.

So, from the center I draw 1 arc, another arc and there is a third arc between these three. So, a spherical triangle, there are three planes, actually one is covering, one of the arcs this arm, another arc three arcs, they must lie on the same circle. So, this is one, this is another, and the third one is a bigger one, and then there are in between angles right. There are in between angles. So, that is what is a spherical triangle, that is what is a spherical triangle you know. So, there are three arcs, essentially three arcs. If this is my radius, you know this is my radius is one, this is another and the angle between these two is other, and contained angles are this angles right.

Now, relationship between these angles can be found out from principles of you know spherical trigonometry. Let us see what are those. So, this I just said now at 12 noon plane P O C A coincides is a plane P O D B, you know at 12 noon 12 noon P O C A; that is basically line joining the pole to the equator right. This is projection, you know this is O C is this O C A certainly P, is a point this is the location point. So, this vertical plane, sun will be coinciding with this at 12 noon, because its right at the right, at the top right, is that you know what is the maximum altitude. So, the maximum altitude would be this. So, this is where it will be coinciding with this.

But other time it will shift. So, hour angle come into picture, other time it shifts and hour angle will come into picture. This angle at P thus represents the hour angle, angle at P represents the hour angle, and is same as C O D. So, hour angle is same as C O D. Let us see what is C O D, go back C O C O D is a hour angle. This is a hour angle you know this is hour angle, this is hour angle, this is delta, this is hour angle. So, the delta would have been, it would be the sun would have been here, but now somewhere it is there, earlier its in the eastern side actually showing. So, you know its. So, this is what represents the hour angle.

Hour angle we always be right in term. So, absolute time term you know 12 minus. I mean the hour minus 12, whichever is the difference absolute value we take. So, it is here. So, this is hour angle, if it was on this side still absolute value we take. So, this is hour angle. So, basically at 12 noon it will coincides with the line joining north pole, and its projection onto the equatorial plane, because 12 noon is the time you know corresponds to declination is defined with respect to that. So, any time, any other time in will be shifting right. So, 1 1 consider ok.

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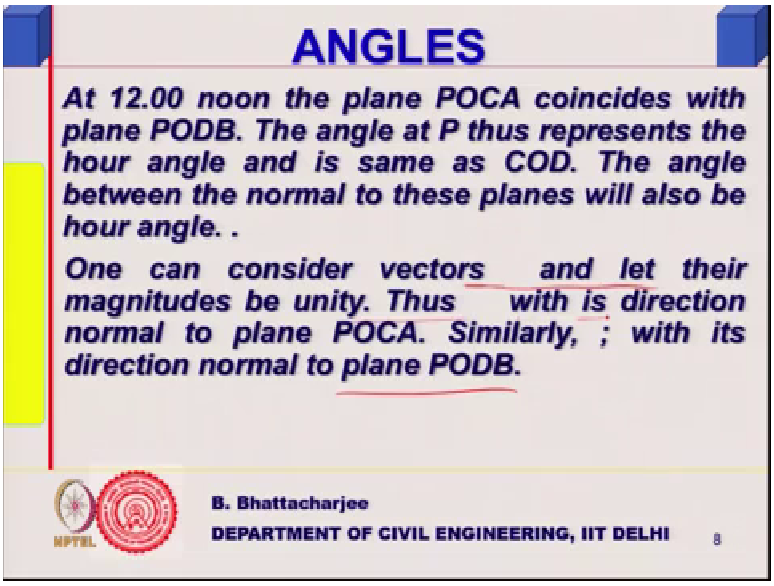

Now, how do you find out? You should find out relationship between journey.

So, you know the, one can consider vectors and let their magnitudes be unity, so.

Student: (Refer Time: 19:02).

In spherical triangle what you do is let me just.

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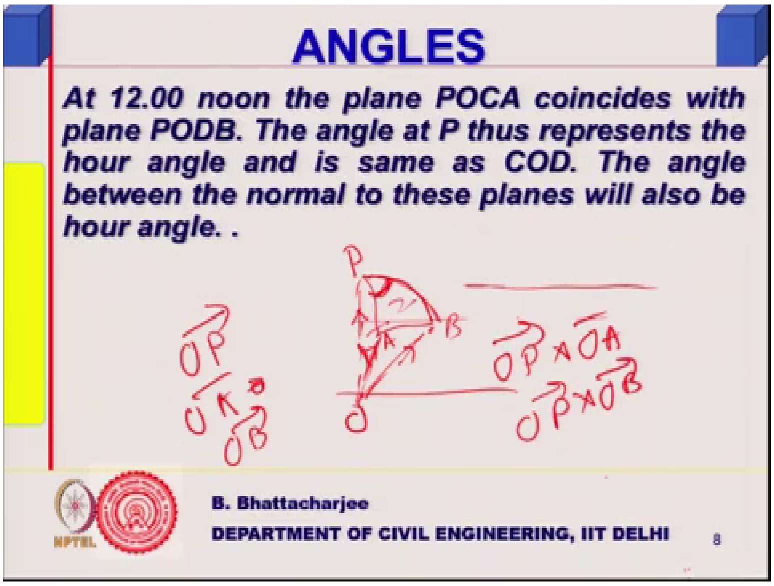

You know. So, these are the triangle, this was a spherical triangle as I said, and these are the arms right this is. So, what we do is. We consider, you need vector along this direction, you need vector along this direction for example, this is O P. So, O P vector this was I think O A. So, O A vector, O A vector, and this is O B or whatever it is. So, you need vectors, we consider that is their magnitude is unity what we do is, we take. So, if we take the cross product of this two vector, what will I get cross product of this two vector, cross product of two vectors are the magnitude of each one of them into sin of the.

Student: Angle.

Angle included angle. So, sin of the included angle is this. So, you can see that this is nothing, but this arc right, this arc. Similarly if I take, if I am interested in this included angle, then I will take cross product of this vector with this vector again back right. So, I will get this angle, and if I take dot product of these two, it will have cos of this included angle right. So, first I take unit vector, therefore, value is not important, because radius I am taking unity. So, it will all whatever visa. So, that will go away. So, I will have O P vector, O A vector, and I think I called it O B is it, O B may be. So, O B vector each are unity. So, if I take cross product of O P and O A, then I get a vector which is normal to this plane.

Similarly, I take cross product of O B and O P and O B, I get a vector normal to it, and dot product of this two will give me the cos of included this angle, because they are normal to both of this, they will make same angle as those two planes are making. So, this is exactly what we do. Let us see the, you know understanding between you know thus

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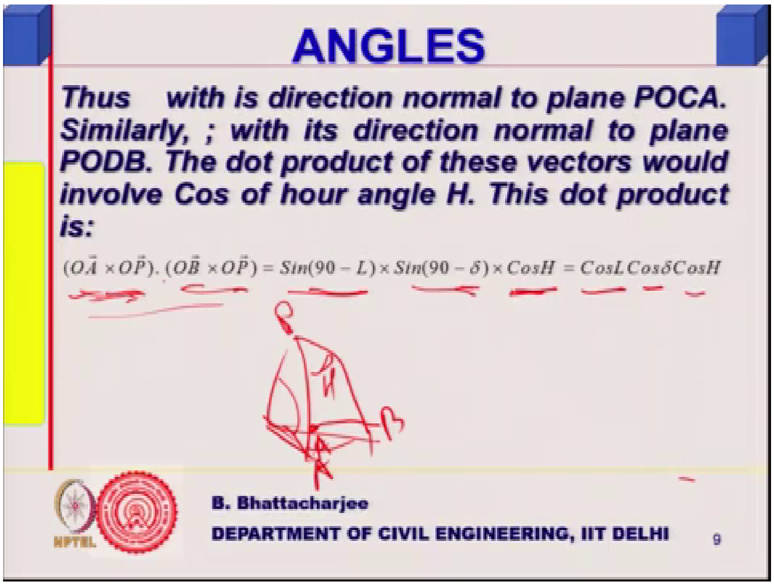

with its direction, normal to the plane of P O C A. Similarly with direction normal to the P O D B is dot product of these vectors would involve cos of hour angle H. Now what was hour angle, we defined that this was hour angle. If you recollect this here, this was a hour angle my spherical triangle is, this is my spherical triangle P B and O, and the angle this is nothing, but the hour angle, because this is the angle between this plane, and this plane. So, that is the hour angle.

So, cos of hour angle, you know cos of hour angle will be involved, when I take vectorial products as they are, you know vector products, products of vectors as they are. So, you can see that O A O P I said dot O B O P, and included angle is hour angle. You know it was like this. If you recollect another one is like this, and another one was like this. Now the angle between these two, their projections nothing, but this is hour angle, because we said the sun ray will coincide, or it will lie in the plane joining north pole and equator, you know north projection of the north pole, pull onto the equatorial circle. So, it will lie on that plane at 12 noon. Any other time it will be deviated by hour angle.

So, included angle is hour angle, and that is what it is. Now remember this, this was how much O A O P will be the angle between O A and O P O A, you know angle between this this this one, the angle between this, and you know this, this, what was this angle. This is the North Pole P, and this was a location. So, this is 90 degree minus you know this is A is the location here, and this is B. So, this is 90 degree minus.

Student: (Refer Time: 23:13) latitude.

Latitude.

Student: Latitude.

With a north pole, it will be latitude when it is the sun it is 90 degree, you know minus in this case a 90 degree minus delta, because equatorial plane the angle is delta. So, this is O B O A P sin of this, and sin of between O O B and O P, which is nothing, but 90 minus delta cos hour angle, this cos l cos delta cos hour angle right. So, this product is, we can find out in this manner cos latitude, cos delta, cos hour angle.

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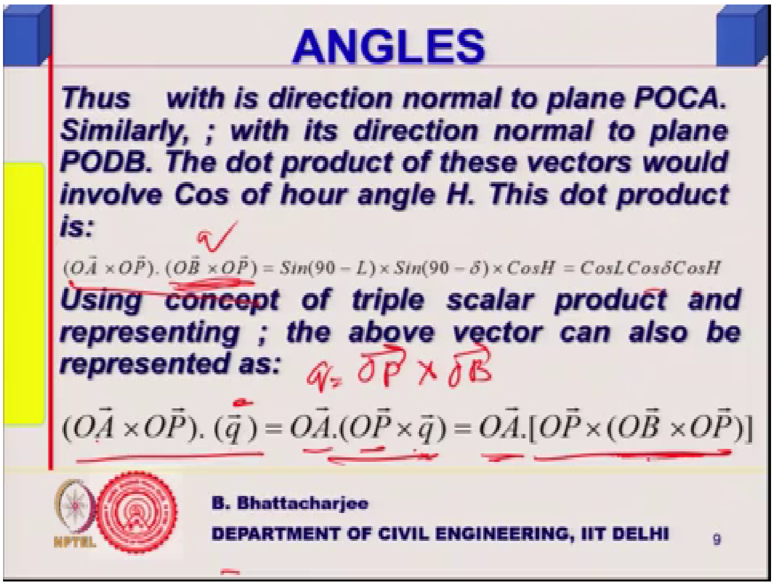

Now this one is nothing, but I can also find out from another process, another way taking what is called triple scalar product principle of, you know in vectorial vector algebra. If you look at it triple scalar product.

So, using concept of triple scalar product, if I write this as q, let me write a single vector q, I will write it like this. So, I am writing this dot q, you know this is a cross, and this is a dot. So, when you combine this, this is called scalar products. This will be from you know you can derive this look into anywhere, you will find it out. Also it will be given as O A O P into cross q O A dot O A dot O P cross q.

You know this dot can change. So, this is equivalent, and this I leave it like this. This I replace q, because q was nothing, but q was q. I defined as you know q was q u was what O P into O B, O P cross q is nothing, but O P cross O B. So, principle is this in my case q is nothing, but because if you remember this was a case, this is. I am writing as q. So, q is O P cross O B. I just place it back again here. So, this is, now this is triple vector product.

So, using principle of triple scalar product, I can write it like this, using principle of triple vector product. I can express it, I can expand this, actually this product O P cross O B cross O P right, and yeah this is, this is what it is, this is what it is. So, it is this now.

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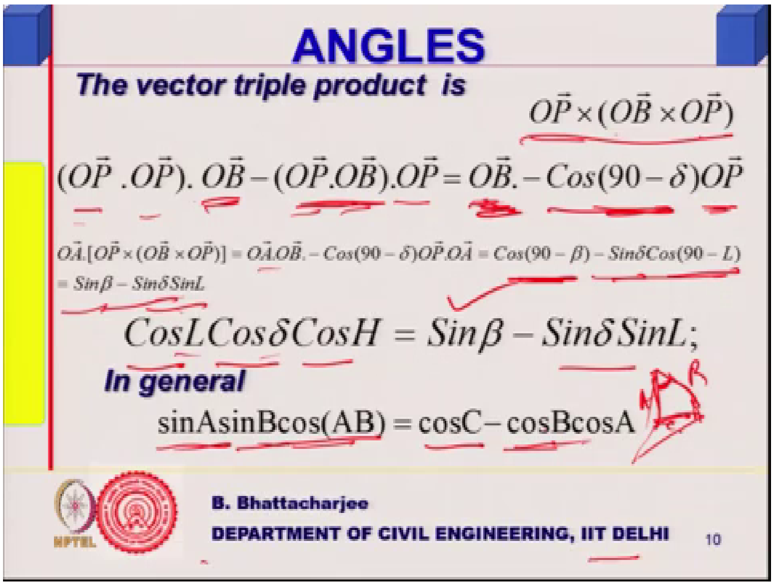

O Ps. So, the vector product triple product is O P dot O P minus. So, this is a, you know this, this vector product is given by O P dot O P from vector algebra. I could have derived it, but I did not do it, you can look into any one of them, any one of the, you know it just search net, you will get it triple vector product, you will get it or triple scalar scalar product. So, its given as O B bar, I mean O P dot O P vector and O B vector, you know this is the formula O P dot O P dot O B minus O P dot O B dot O P. So, this is how it is.

So, this is basically you know, O P dot O P is how much.

Student: (Refer Time: 26:17).

Is 1, because cos 0 1. So, this is nothing, but O B dot product of the vector on the same back vector is cos 0.

Student: 1

One and unity vector. So, its one, this is O B, and what was this O P and O B between this cos, you know O P was north pole, B was the sun. So, 90 minus delta and O P bar, what is the O P bar, what is O P bar, what is O P bar, what is O P bar? This is yeah O P bar is unity simply. So, this I can write as you know O B bar. So, this would be written as O A O P O B etcetera, will be written as cos delta etcetera.

So, finally, it will turn out to be cos 90 minus delta and O B bar is the angle unity multiplied by the angle will be there. So, this comes out to be sin beta minus, because 90 minus beta sin delta cos 90 minus. So, this is what it is. So, yeah. So, this would be, this would be this. I can now equate with my formula that I had cos latitude cos delta cos hour angle is equal to sin beta minus sin theta sin latitude.

So, I am interested in this altitude angle. So, I can express this sin beta is equals to sin declination sin latitude plus cos latitude cos declination into cos hour angle in general. This expression you will, you know its if A is the arc angle, B is the arc angle included angle is A B, then sin A sin B cos this came, because these are all 90 degree 90 minus, that is why they came as cos here cos A B is equals to cos of C, the third angle and cos beta cos you know.

So, this this again B and A, any threes in a spherical triangle you will find this relationship will be valid, if this is my A angle, this is my, you know this is A angle, this is the 3 angles. This is a contained angle A B between, this is B, and this is C. So, there are three angles representing these three arcs contained angle, I call it A A B angle. So, this sin of A, this angle sin of this angle and cos of included angle is equals to cos of this arc angle minus cos of this angle again, and cos of a angle again.

So, this is a general relationship, this general relationship. So, I think we just stop for a minute, and then I will, you know we will start from the next one right. So, we will start from the next one; look for in general principle related of this.