Energy Efficiency, Acoustics & Daylighting in building
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Lecture – 43
Isolation

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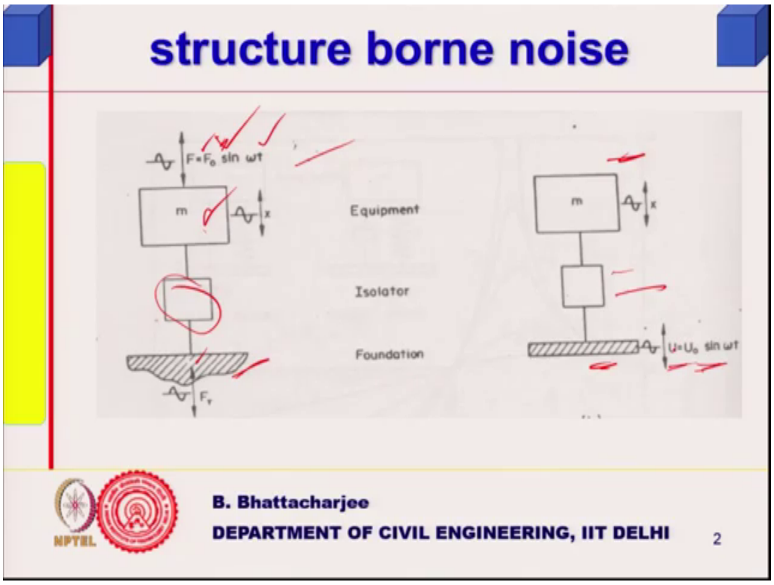

So, you got structure borne noise in the last class. And we said that structure borne noise we usually take care of by providing isolation, you know, for those sources whose frequencies are known. For example, for a machine we know the rpm, and you know it would be imparting the force and that rpm. So, anyway we know the frequency of the forcing function, the one which is actually causing vibration of the structure.

So, we actually provide isolation in between, and let us say this is our machine mass is somewhere here. Machine mass is somewhere here, this is our machine mass, and we have a force, because it would be periodic rotating machine mostly or reciprocating machine. So, they will exert periodic forces. So, force would be something one can express possibly in terms of some sin omega t, and this is my mass of the machine, and isolated I put in between this is my floor.

So, this isolator actually what it does? It does not allow all the vibration or all the forces to be transmitted here, all the forces to be transmitted here. It reduces around the force, and the decibel we can define transmissibility in this you know in some manner the amount of force that will transmit, and this is the forcing function which has got a amplitude of f 0, and then where it is sinusoidally, because it will be 0 at some time and then go to the other direction and so on so forth.

So, this is the isolator this is the mass, let us say I do not have the forcing function now, and I have put the isolators the mass would be there this, mass results in some sort of static deformation or static deflection of the isolate yourself which might be something like a spring which might be something like a spring, right. Spring or similar sort of system, the idea is that if it is a spring it stores the energy, a spring will store the energy and release it.

Now, it stores the energy the moment the energy comes from here it stores this energy, and then sometime you know releases with kind of a phase lag or time lag you know releases here, some amount gets attenuated and also releases at certain at certain ah, you know, at certain some kind of a time lag or at certain time later. So, therefore, finally, it results in a kind of a vibration of the floor or foundation u 0 sin omega t.

So, there is a you know force here would be u 0 sin omega t. So, that is the idea that is how we actually stop structure borne noise, particularly coming out from machine or similar sort of thing, whose frequencies are known. For those ones which are random type and whose frequencies we do not know, we handle them in a slightly different manner we will see that later on.

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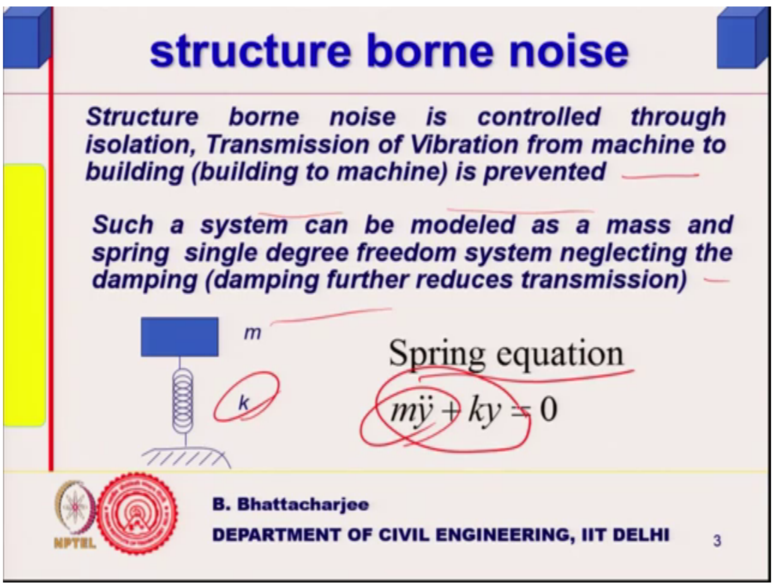

So, through isolation that is transmission of vibration from machine to building is prevented.

Now, such a system can be modeled as a mass and spring system that I have just shown you the single degree freedom, system neglecting damping. There will be some damping, some loss of the energy right, some loss of the for example, it can be you know it could be some kind of viscous viscous damping. The energy is now actually transmitted in a way you know energy, energy is some energy will not be transmitted, because there is some viscous damping can occur in the sense, which is generally proportional to velocity right which is proportional to velocity, or as you know viscosity is related to deformation rate therefore, is proportional to velocity, and the force is actually proportional to our stress shear stress is proportional to velocity gradient, you know, as we have seen earlier in different definition of viscosity.

So, there can be some viscosity and we have all loss of energy etcetera, etcetera, but we are neglecting that in this particular case because we are considering the simplest of all cases right. So, this is our system can be modeled, and this is the mass, this is my spring and this is my foundation. So, spring equation we know mass into acceleration if y is a displacement along this direction. So, y is x acceleration y double dot is the acceleration, must be equals to spring stiffness, because you know the force in a spring is proportional to the displacement. And this is the equation of spring that you would be knowing.

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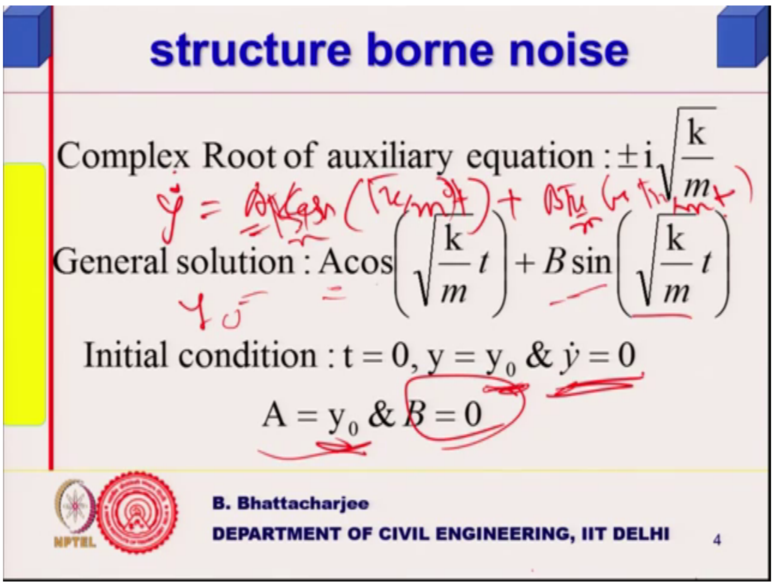

And therefore, if I look at this equation, and if you remember the auxiliary equation will be d square plus k is equals to 0 because it is you know m sorry k by m, m y double dot plus k y equals to 0.

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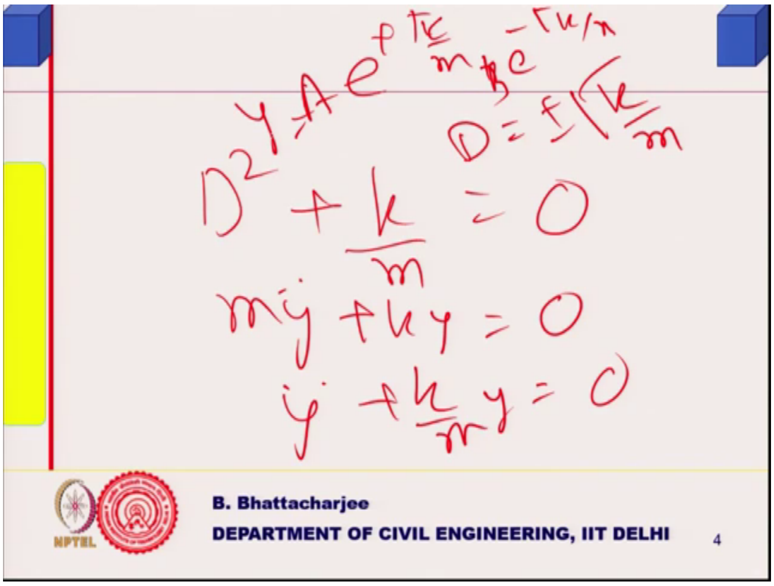

So, y double dot is simply is equals to k by m, y equals to 0. So, this is my auxilary equation, right? And root of this one will be simply plus minus under root k by m. So, solution would be simply e to the power plus k by under root k by m plus e to the power minus under root k by m A plus something like A plus B as you know that will be the solution of y right. So, this we discussed something similar earlier, because we know just for those of who still do not you know recollect your solving basic maths.

It is exponential function is one which if you differentiate it again it will come back. So, if it is y dot some a y dot plus you know B y is equal to 0, simply if you it is it is the type of function which will satisfy that kind of equation is exponential. And if it is double dot, y double dot e to the power minus x can also if you differentiate it twice, if you come back to the same one again.

So, y double dot some my double dot plus k y. So, you know and that the to the power pollution would you can show that very easily that complex route it would be because there is a minus under root k by m. So, under you know I under root k plus m, and solution of this one comes out as A cos k by m t B sin k omega k by m t, right? And if you differentiate it A and B will come from the boundary condition, because y double dot is involved. So, you to solve that 2 constants of integration will come if you are integrating them therefore, 2 boundary from the 2-boundary condition A and B can be solved.

So, you can see that cos function if you differentiate it once you get sin again you differentiate you will get cos back again. So, you know and similarly sin is so on. So, that is that is how it is it comes, and if you have you must have looked into this solution. So, initial condition t equals to 0 y is equals to y 0, and y dot is equals to 0, that is initial condition spring has deflected, static deflection somewhere it has deflected 2 y equals to y 0, and velocity is equals to 0.

Now, supposing I consider the spring without the mass and just pull it a little bit. So, y 0 is the initial, I am only looking at the property of the size so, later right now, right. There is no force so, I just pull it y 0 to y 0 and release it. So, at time t equals to 0 y 0 and release it velocity is 0 at that part. And if I solve this you know put this here, when t equals to 0, this term becomes 1, right cos 0 is equals to 1 and this term is 0. So, y is equals to y 0, therefore, A is equals to y 0. Because this would be 1, A this is y is equals to this y is equals to all this and this time goes to 0.

So, when t equals to 0, y 0 is equals to a. So, a equals to y 0, and if I differentiate it, you know, then I get y dot. So, this will become sin this will become cos and y you know y dot is equal to 0 at t equals to 0. So, therefore, B must be equals to 0, that will satisfy, you know, because B with this will become cause after differentiation, right, after differentiation this will become cos y dot if I y is equals to A cos under root k by m t k by m t plus you know. So, y dot will be A sin something, you know, this one and multiplied by this factor coming out side.

Similarly, this will become B cos, cos function of t after differentiation y dot. And therefore, when I put t equals to 0, B under root k by m, you know, this term under root k by m will always be there, here this will be cos under root k by m t, sin if you differentiate you get cos, cos you differentiate and get minus sin ah. So, cos you differentiate, you know, this will become sin actually if I differentiate it. So, when t equals to 0, this term goes to 0, this term will be one, but y dot is equals to 0. So, therefore, this is a situation. I mean is there any problem in this, it is pretty simple there is no let me just do it again if you are not comfortable. Let me just do it again, let me just do it again, I am saying I am saying, y is equals to A cos under root k by m t n plus B sin under root k by m t, that is right.

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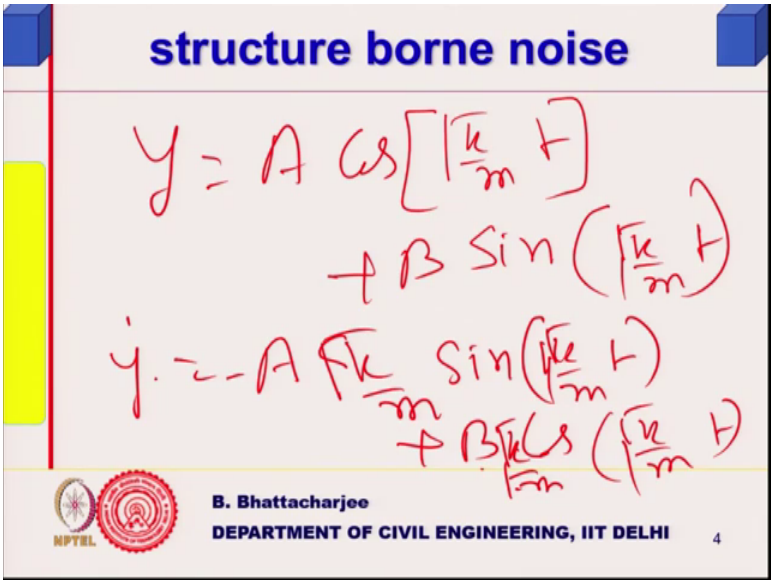

So, if I differentiate it I get y dot is equals to a minus s under root k by m sin under root k by m t plus, am I right? Is there is it ok, cos under root k by m t and here again you will have under root k by m right under root k by m, is it correct? There is any problem in this. So, when I put t equals to 0, this term goes to 1, this term goes to 0. So, my B and all this k under root k by m is not equal to 0. So, B must be equal to 0 B must be equal to 0 that is all I am getting it this I am getting it, right that is how I am getting it.

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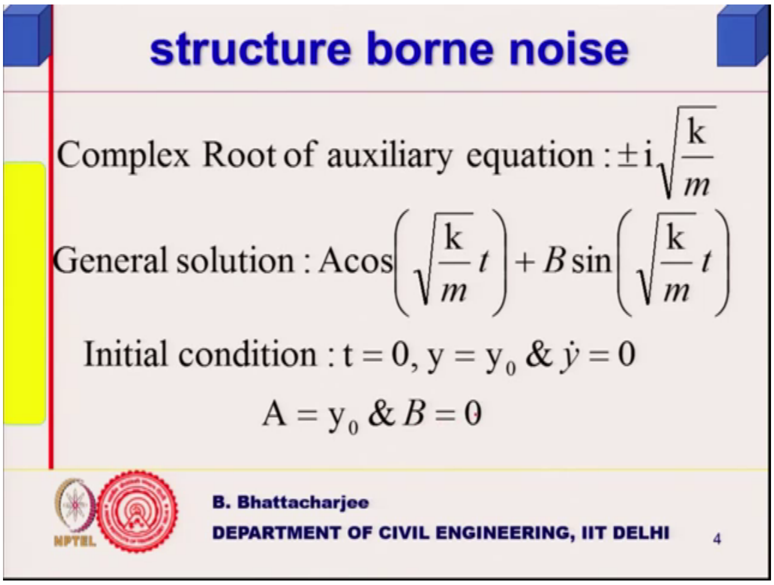

So, therefore, you know a equals to y dot 0 B equals to 0.

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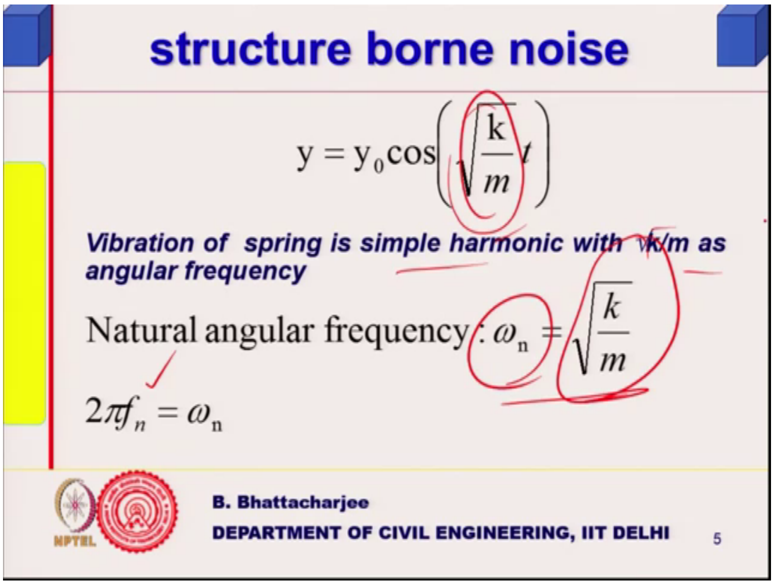

So, finally y is equals to y 0 cos because A is equals to y 0, cos under root k by m t. So, if you know, this is I am not put any load or any force on to this isolation, and this is nothing but omega. So, if I pull this isolation, put it to some value y 0, and there is no damping then it will vibrate from it will vibrate from y 0 to y 0 it is displacement will go from y 0 to minus y 0 and omega t. So, it will repeat itself after this is omega 2 pi, you know. So, this is basically omega. So, this is a natural frequency of the system, this is the natural frequency of the system, I have taken only the spring and spring will right.

So, k by m if I put in a mass and then I have started causing the mass just pulled it up to y 0 leave it you know. So, just pull the mass a little bit and leave it. So, for that system the machine I have pulled it and then left it no, forcing function I am not applied any function, then it will vibrate at with this natural frequency right so, right. So, it will vibrate with this natural frequency or angular frequency and natural frequency is f n.

So, vibration of spring is simple harmonic with k by m as angular frequency under root k by m as angular frequency. And at when I have just put in the machine it deflects by delta, spring you know static deflection deflects by delta right in the beginning.

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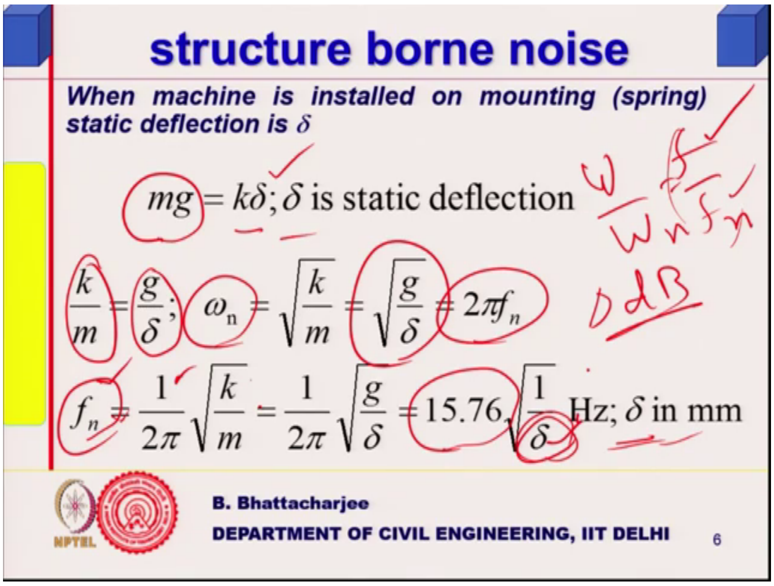

So, mass into mg must be equals to k delta, because I said that I have put in the machine and then pulled it up the system is mass, then the isolation then my on my foundation.

So, there is an initial static deflection whenever I have put it. So, that is called static deflection, and m by k or k by m is g by delta right. So, omega by n g by delta that is equals to twice pi f n. So, twice 1 by twice pi you know f n k you can get it by 1 by twice pi under root k by m etcetera, etcetera and delta is in millimeter. So, one can find out the natural frequency of the isolation, and the machine put together right isolation machine put together.

So, that is the system, when the machine is operating it in imparts vibration to isolation periodically that which applies a force and then equation becomes something like this.

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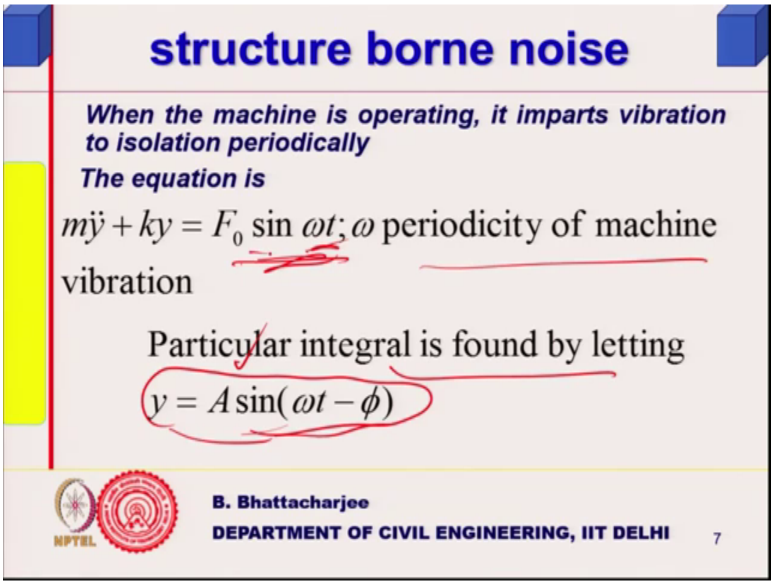

Because earlier there are no force I have just pulled it up and it is it started moving about this mean position of equilibrium, machine plus isolation together, now I have just started forcing it the machine is on and it forces continuously imparts force onto the isolation.

So, periodicity of the machine this is the periodicity of the machine at which it for applies force, and this you can solve by through particular integral, you know, solved through particular integral. So, y is equal to this will be the particular integral for this particular this specific case, A sin omega t minus phi, because if you put this here, and differentiate it twice you know, if you put this one, because it has to be A sin of cos function. Or you know like exponential which will repeat itself.

So, here it will be sin or cos function, for this kind of sinusoidal function particular integral is of this kind. If you differentiate it twice plus k y if you put it in right side if put f 0 equals 2 sin omega t, you will get the value of a. Differentiate it twice, multiplied by m, then k multiplied by this. If it is this particles to this then you will get the value of a.

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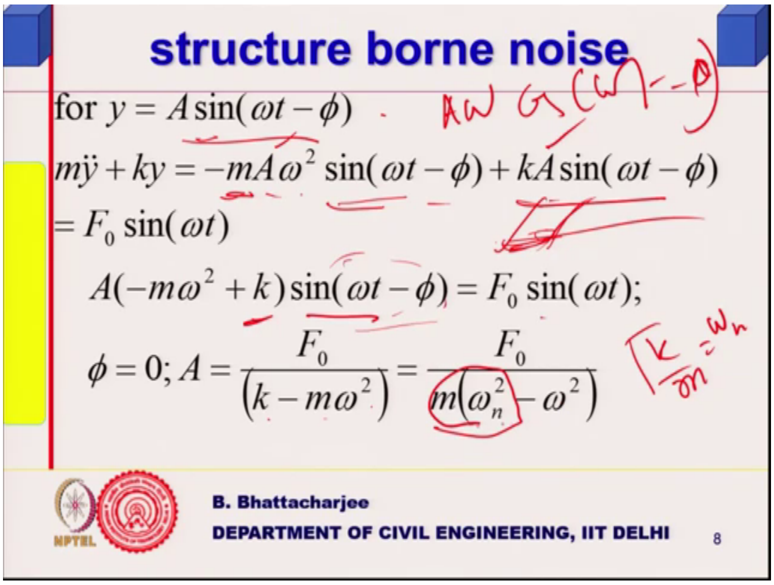

So, if you do that for y is equals to differentiates. So, differentiate it twice, you will get sin first omega t. So, omega comes and again differentiate it you will get sin back, minus sin is cos and cos is minus sin. So, you will get a first time you differentiate this you will get a omega cos omega t minus phi. Again, you differentiate it you will get a omega square, and it must be multiplied by m, and then k into this only k into this is goes to f 0. So, I can evaluate a out of this. I can evaluate a out of this. So, if I take a common from this 2, I get m omega square plus k sin omega t and right-hand side also f 0 sin omega t.

So, a at phi is goes to 0 if at when there is a you know there is a phase angle is 0. it is because it is it starting point could be different. It is starting point could be different; phi 0 means, it is starting point is different if you remember when we talked about that it is in times you know one wave is following the other let us say. So, this would be in terms of some phi. So, if at phi goes to 0, a equals to f 0 k m omega m m omega square. And k you remember was related to omega n square, because under root k by m is goes to omega n right.

So, k is nothing but m omega n square. So, k this is k and there was m here. So, I can take m out. So, it is something like this. So, A is this, and general, solution therefore, would be y 0 cos omega n t that is for the natural frequency part of it.

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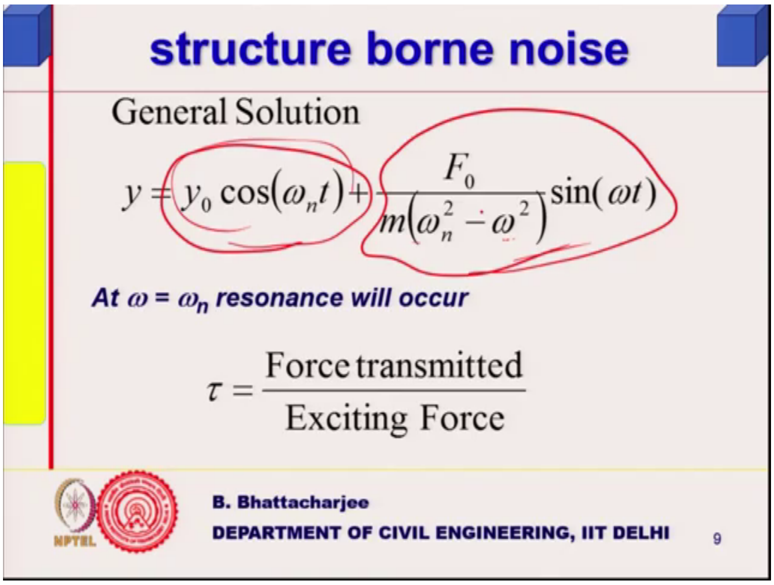

And this is because of the forcing frequency, forcing frequency right. So, what we are saying is that, we assume that they are not there in phase, when they are in phase that is the worst situation, and f 0 omega n minus sin omega t.

So, when this, this is natural frequency part of it, but when this towards same omega n and omega is same, then this term becomes infinity; that means, displacement tend to become very large, because this is a small portion compared to this, and when both of them are in phase. So, forcing frequency and natural frequency there is phase. Then you get you know you get resonance will occur; that means, it will have very high amount of vibrations transmitted through.

So, we define a function called or rather in parameter or property of the isolation, which we call force transmissibility, like transmission coefficient we talked about, we talked about transmission you know transmission loss was 10 log 1 by tau. So, where tau was transmission coefficient you know transmission coefficient. Here we define something called because it is not force per unit area we are not dealing with pressure. So, we talk in terms of force transmitted divided by exciting force, f 0 sin omega t and how much is transmitted u 0 sin omega t or whatever it is, that we define as tau.

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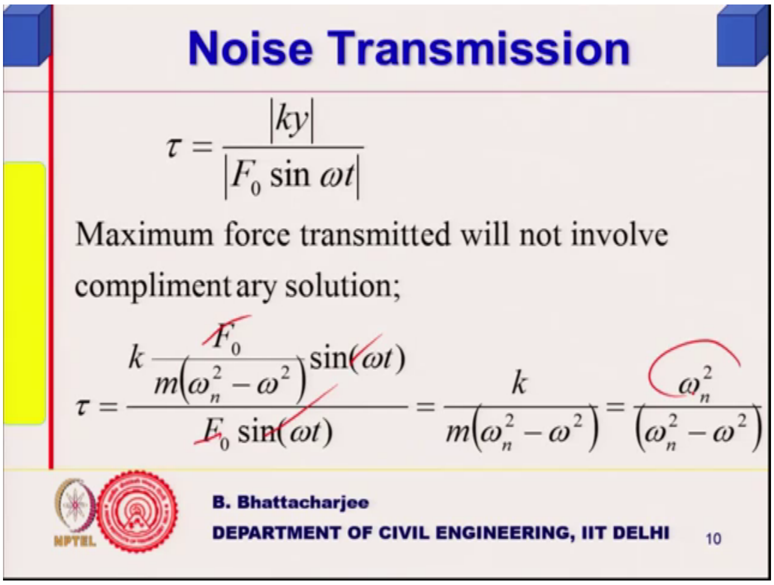

And tau is simply then k y divided by f 0 sin omega t, right. X f 0 sin omega t you know the force that the force that is imparted, and k y will be the displacement of this isolator which will be transmitted to the is the spring.

So, that is the force it will transmit to the foundation or the base or the floor foundation, because whatever k y is the force in the spring that would be transmitted to the floor. So, k y divided by f 0 someone we take modulus. So, maximum force transmitted will not involve complementary solution. So, we forget about the sin omega t, because that would be small natural frequency part of it, and therefore, we write it simply this part, we write simply this part, we take this part only and you know maximum force transmitted would be given by this.

So, which is simply coming out as this will cancel out with this so, I left with k by m omega n minus omega and what is k by m omega n square. This I can write all neglecting damping and all that, this I can write as one divided by you know divide everything by. So, it will be one omega by omega n square omega by omega n square right.

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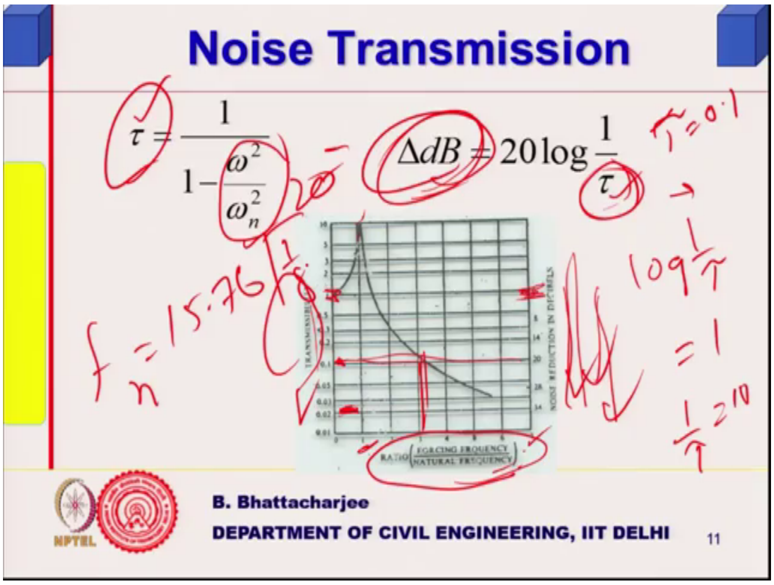

So, tau is given by 1 by 1 by 1 minus omega square by omega n square or you know the difference one can take in both sides. And we define transmissibility as tau, and one 10 log or 20 log 1 by tau will be because 10 log pressure square was there so, 10 log 4 square. So, here 20 log you know 20 log 1 over tau will be the delta d v 20. Because we are talking in terms of force like pressure square was there. So, force would be r square. So, 20 log 1 by 20 and this will be delta dB.

So, if you can see that if these 2 are same then tau becomes infinity. So, this is the point you know this is the ratio of forcing free currency, and natural frequency omega forcing frequency omega n by omega here it is forcing frequencies omega by omega n, and when this is equals to 1 the transmitted ability is maximum. If this is, you know, the ratio is quite large then transmissibility reduces on both the sides.

So, here also it reduces when the ratio is small. So, therefore, and corresponding dB, you can find out because supposing it is one 20 log of 1 is equals to 0. So, force transmissibility you know in this point if this you know essentially if transmissibility is one tau is equals to 1 delta d B is equals to 0. And on this direction, it increases, so, here is this rate transmissibility is when this transmissibility is 0 2 that is 1 by 0 2 delta dB is more it is inversely proportional to so, as this value is smaller and smaller, delta dB noise reduction increases. Noise reduction increases as this value becomes smaller and smaller because delta dB is this.

So, we try to keep this away from the natural frequency. I mean sorry, forcing frequency. So, natural frequency of the isolator should be 3 to 4 times than thereof.

Student: (Refer Time: 23:35).

Yes, so, choose the isolator accordingly this is a simplest of all these structure dynamics or vibration equation simplest case we are talking of. So, simple isolator because you want to reduce down as much noise as it is. So, for known omega decide omega n for given dB reduction. For known omega m determine isolation from graph or from equation. You see this graph gives us, for example, it gives us natural frequency versus static displacement, or simple 15.76 under root you know that that 1 by delta, the formula that I gave you earlier.

So, one can determine from this, remember this formula we talked about or this is given in national building code, I mean this was given in national building code or anywhere you can actually need not do look into the code. But look at the material and for the given load so, delta is the static deflection. Given machine weight, how much is it? Is static deflection of the isolator, which could be a cork board, could be a neoprene rubber, or some kind of system depending upon whatever it is, right.

So, find out if under the static load, how much delta it would be. So, you have got to know it is k value in a way that should be known to you. And this will be the natural frequency, now forcing frequency of the machine you know so, the ratio now we can choose. So, what should be the f n, you can find out because you know f omega by omega n that is same thing omega by omega n is same as f by f n, and delta dB is known to you, right.

Therefore, how much ratio you should maintain you can find out, because delta dB is equals to 20 log 1 by tau, and tau is equals to 1 by you know tau is equals to you I mean delta dB 1 by 1 minus this ratio. So, this ratio you can find out depending upon delta dB, and this is known to you, you find out f n, choose an isolator which will have deflection required for fn, because fn is equals to this deflection.

So, under the machine load the deflection should be this. You can choose materials like as I am showing you materials like materials like cork board, materials like you know steel springs are somewhere there rubber 15 centimeter thick cork board etcetera. So, this would be this would be you know steel spring typical spring should be something of this kind rubber 15 centimeter thick they show static deflection under per kg or whatever it is known.

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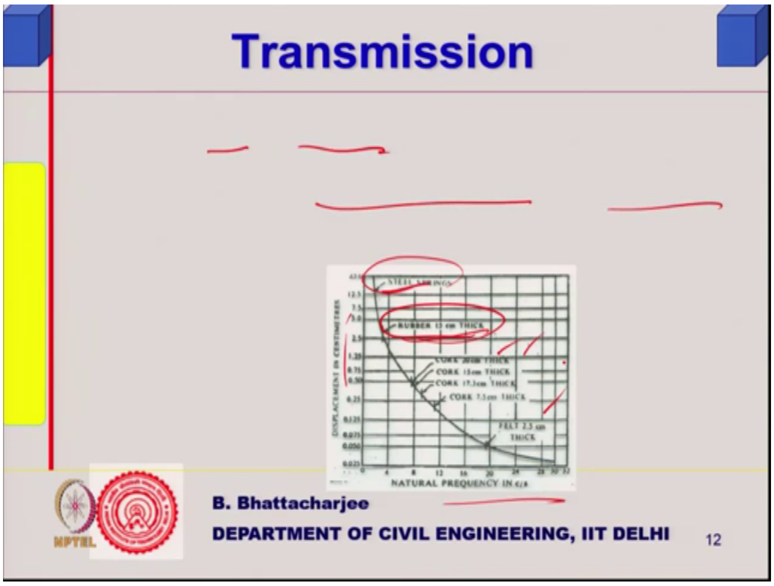

So, different bolts then cork, cork board then felt. So, specific materials that you are using. So, it is thickness, how much will be it is static reflection for the given machine load that you can find out right. So, let us go back to this equation. So, I can find out tau how much first I note? Tau I will find out from this equation. Let us say I want 20 dB this. So, corresponding to this, tau will be equals to corresponding to this, log 1 by tau will be equals to 1, right.

So, how much is tau would be? How much the tau would be? This you know this is equals to 1, which means that 1 by tau is equals to 10, am I right? 1 by tau is equals to 10, anti-log of one is how much? 10 so, 1 by tau is goes ten; that means, tau is equals to 0.1, tau is equals to 0.1. And for tau is equals to 0.1, how much is this ratio? You can find out either from this curve, or straight away can find out for 20 dB with this 0.1, right? And you want this much then the ratio should be 3. But then no these days nobody looks at the curve, because you do in excel calculation or somewhere else.

So, 3 times your natural frequency ratio of this 2 should be you know, forcing frequency by natural frequency should be 3. So, choose the natural frequency of the system fn accordingly, then you know for this machine load fn is equals to 15.76, 1 by delta. From the set of materials that is available cork with different thickness, may be spring rubber whatever it is if they are forced springs in parallel you can find it out you know all this, this I do not think I will go into that details, but this is from basic school physics you can find out, and return determine isolation, right.

So, that is how we take care of machine noise or noise with known frequency. Because f f must be known, forcing frequency small f or omega must be known to me. If it is unknown which can happen. So, let us look we into that, it can happen for example, just for sake of example footfall noise. Kitchen noise utensil falling, these are random noises, they are no machines right.

So, if there is a kitchen there would be noise just generated and they are random. I mean let us say a hotel kitchen in first law right, or something somebody says some workshop, some of them might be you know dropping something or doing some work on to the you know some work on to the floor or similar sort of thing. So, why do you expect random noise? You can you do not know that omega, then you do not know that natural frequency of the system I mean sorry forcing frequency you do not know. So, you have to then think it in different terms.

So, you look into that in the next.