

Energy Efficiency, Acoustics & Daylighting in building
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Lecture - 42
Sound within Enclosure (contd.)

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Transmission

$$\tau = \frac{\frac{|p_t|^2}{\rho c^2}}{\frac{|p_i|^2}{\rho c^2}} = \frac{|p_t|^2}{|p_i|^2} = \frac{(\rho c)^2 u_0^2}{\left(\frac{M\omega}{2} + \rho c\right)^2 u_0^2} = \frac{1}{\left(\frac{M\omega}{2\rho c} + 1\right)^2}$$

At high frequencies $\left(\frac{M\omega}{2\rho c} \gg 1\right)$

$\tau = \left(\frac{2\rho c}{\omega M}\right)^2$

$$\frac{p_t}{p_i} = 1 + \frac{j\omega m}{2\rho c}$$

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Now, how did I define transmission loss? Transmission loss was $10 \log 1$ by tau. We will come to that.

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Transmission

$$(p_1 + p_2) - p_1 = P = j\omega m v$$

$$TL = 10 \log \frac{1}{\tau} =$$

$$20 \log M + 20 \log f + 20 \log 2\pi - 20 \log (2\rho c)$$

$$\tau = \left(\frac{2\rho c}{\omega M}\right)^2$$

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You can model this in this manner also assuming an equivalent analog circuit. I am not really interested in this because as we shall see, this is the theoretically derived formula. This is the theoretically derived formula. I know is this. So, $10 \log \frac{1}{T}$ by tau. So, it would be basically tau was tau. We have seen was you know how much was it rho c 2 rho c divided by square. I also was there. There is something like this, ok. Let us go yeah 2 rho c by omega m square m m m omega square.

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The slide titled "Transmission" contains the following handwritten derivations in red ink:

$$10 \log \frac{1}{T} = 20 \log \frac{1}{T} = 20 \log \left(\frac{2 \rho c}{M \omega} \right)^2$$

$$= 20 \log M + 20 \log f + 20 \log 2 \rho c$$

At the bottom of the slide, the following text is present:

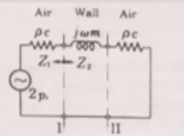
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Logos for NPTEL and IIT Delhi are also visible.

So, $10 \log \frac{1}{T}$ by tau would be $10 \log \frac{1}{T}$ by tau would be $M \omega^2$ $10 \log$ of $M \omega^2$ divided by $2 \rho c$ which will be simply $20 \log M$ plus $20 \log f$ plus $20 \log 2 \rho c$ minus you know same square. So, $20 \log 2 \rho c$, right. So, this is the expression it would be.

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Transmission



Air Wall Air
 βc $j\omega m$ βc
 Z_1 Z_2
I II

$(p_i + p_r) - p_i = P = j\omega m v$

$$Tl = 10 \log \frac{1}{\tau} =$$
$$20 \log M + 20 \log f + 20 \log 2\pi - 20 \log(2\rho c)$$



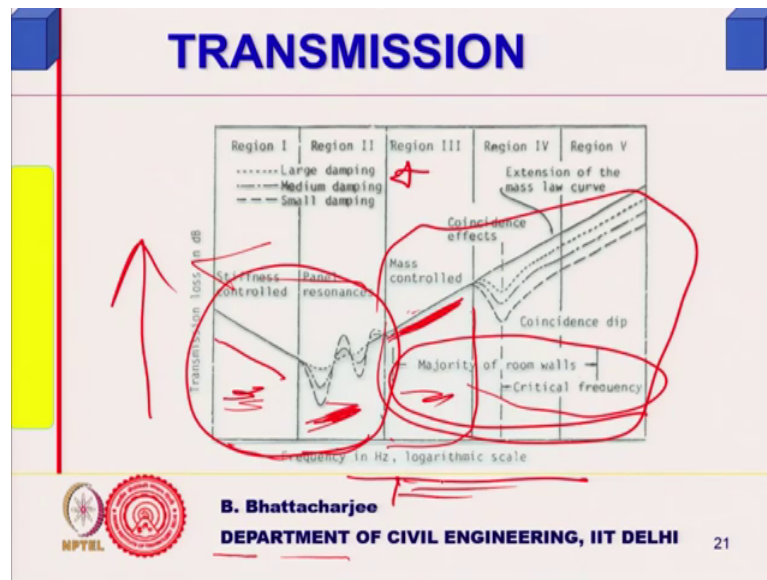
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So, if I write this it will come $20 \log M + 20 \log f + 20 \log 2\pi - 20 \log 2\rho c$ minus $20 \log$ because 1 over τ it was. So, this is what it is. That means, if I increase the mass per unit area, my transmission loss will also increase and it will be higher at higher frequency, right and this is the constant term because the ρc , we assume to be constant. However, what is observed is that it follows this law alright, but these coefficients are not practical. Coefficients are slightly different.

So, one can derive and show that, it is related like this, but finally practical equation is something of this kind you know.

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I will come to this, but I suppose I have a practical equation given here yeah before I discuss this. So, you see what if we take the full equation, complete equation over a large range of frequencies without neglecting damping, then stiffness part of it what is seen is that with frequency. The transmission loss value shows we assume that frequencies you know that frequency should be sufficiently high, right.

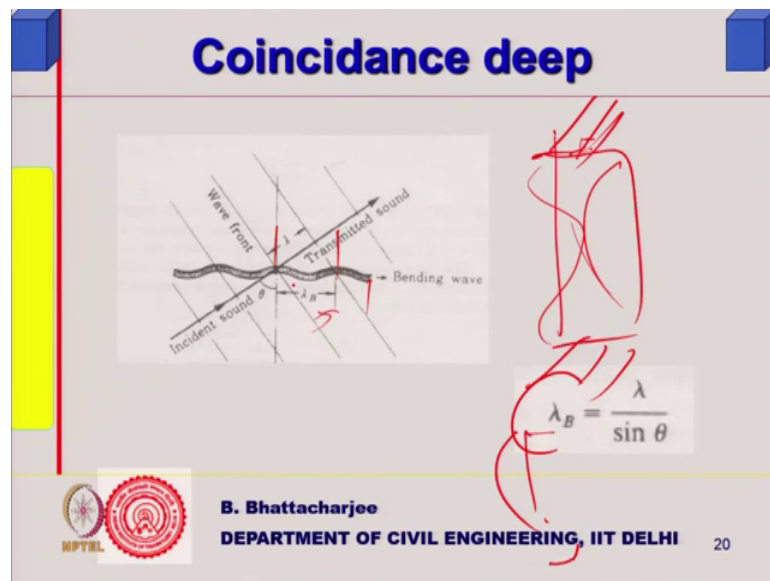
So, at low frequency it is been observed that actually stiffness $k \times$, we have actually did not take the stiffness of the system. Also, stiffness controls it at low frequency which is not acoustic frequency. Generally there can be some sort of a panel resonance because of the stiffness of the thing gets spring behavior of the panel itself or the wall and in such cases, you might have transmission loss gets reduced. Transmission loss reduces. So, this is not zone of our interest. Actually this is not zone of our interest. Acoustic related things are somewhere here, acoustic frequencies are somewhere here and majority of the room walls will be somewhere here, right.

So, that is why you did not derive it. Also region 1 and region 2 we even did not derive there. This is called stiffness control zone which is related to vibration of the wall. If it is there, but not of our interest related to acoustic frequency, then in this zone actually we can assume this frequency zone. We can assume that $2 \rho c$ over $m \omega$ is sufficiently large compared to one that assumption that we made and we get something like is proportional to as the mass increases transmission loss increases $20 \log m$, but the slope

is not $20 \log 20$. It is somewhat different. We will see, but something else happens and that is called coincidence dip. You know it should have gone straight like this with frequency $20 \log f$. You know this should have gone $20 \log f$.

So, if I plot with respect to frequency, I should have got $20 \log f$, but something else happens. It is called coincidence dip.

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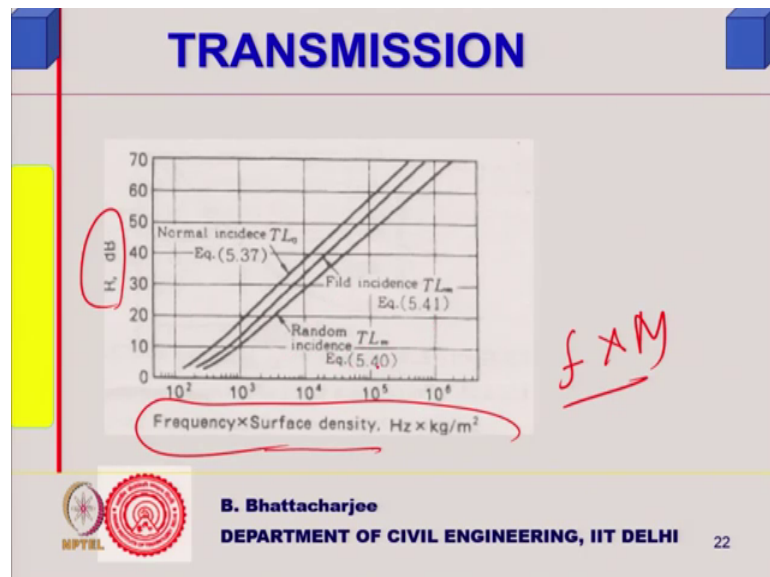
It is actually you know at certain frequencies, higher frequencies itself the bending wave of the panel much might match with the you know wave length itself or a projection of the wave length itself. That is called coincidence deep. In some case, you know it can bend the wall fixed on or some depending upon the support condition hinge or whatever it is, it can bend.

I mean I should draw it like this. It can bend or there could be bending of you know the other modes also and when this kind of a relationship is there, lambda of the bending wave, bending wave by mean is there to in a wavelength of the bending of the panel itself, right. It may not be visible, but it can bend structural bending of the panel itself. Somewhere $\lambda_B = \lambda \sin \theta$ lambda is that of sound when it matches, you find there is a reduction in the transmission loss and that is we call as there is a reduction in the transmission loss and that we call as coincidence deep.

So, coincidence dip occurs at some higher frequencies, at frequencies audible, frequency range largely most of the wall of the room, wall room. Walls shows mass law you know shows might you know frequency $20 \log f$ or some you know with frequency it increases and its proportional to transmission loss is proportional to mass per unit area somewhere at higher frequency. There can be a confidence dip and beyond confidence dip again it follows the same parallel law.

So, there is a reduction of the transmission loss somewhere here and beyond which it follows the same path again, right.

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So, that is what it is. So, frequency multiplied by surface density what is it? It is basically f applied by M is plotted on this side and this is a reduction transmission loss values. They are given for certain types of material various random incidents, then normal incident, another incident also one can look into.

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Transmission

Thickness h , cm

f_c , Hz

$$f_c = \frac{c^2}{2\pi h} \sqrt{\frac{12\rho}{E}} = \frac{c^2}{1.8hc}$$

$$Tl = 18 \log M + 12 \log f - 25$$

+18 log 2

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The critical frequency is given by this formula for various materials it is available. So, thickness H centimeter critical frequency for various materials; this kind of tables or just given by this formula or such charts are available actually.

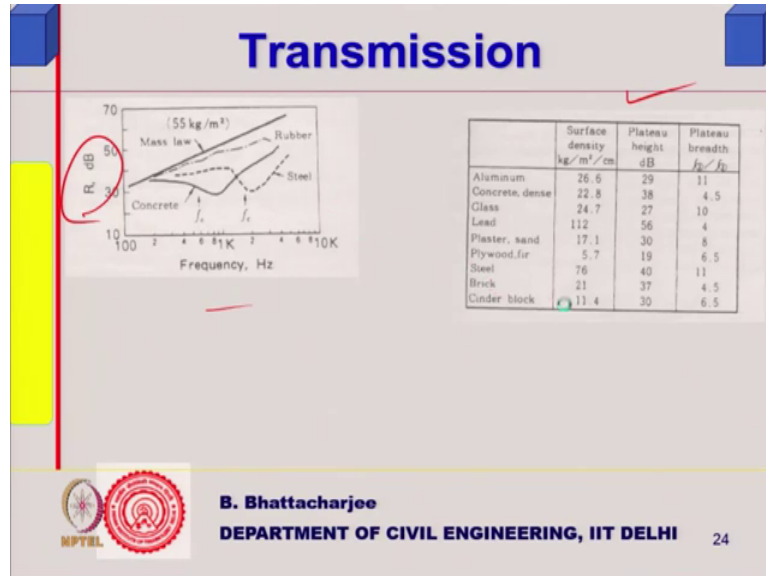
So, you can find out where coincidence deep will occur and this given by velocity of sound H is a thickness, thickness of the wall, rho is the density, e is a elastic modulus. So, one can actually find it out in this manner some formula is available. C square approximately velocity of the sound divided by 1.8 h c s velocity in the solid itself.

So, velocity in the solid itself, therefore, you can find out where the coincidence deep will occur and beyond that again it follows the mass law. Anyway you need not consider too much into it. Actually you can do detail you know work onto it actually an equation that is followed most of the time is 18 log M plus 12 log f minus 25. You know it was 20 log M. If you remember plus 20 log e f minus, there are some minus term all the. So, this is you know this is a practical equation. That is why I said that you can derive it, get an idea that is function of mass per unit area. As you increase the mass per unit area transmission, loss increases, but not 20 log m. It is 18 log m and also as you increase the frequency, it reduces, but not 20 log f 12 log f.

So, that is what is practically used. That is why the derivation of it is, ok. You understand the physical phenomena, but I cannot derive. This found is more of a very empirical somewhat semi empirical solo and you can use this formula for calculating out the

transmission loss. So, for example, how it will be using supposing I know this room noise I do not want it to go to the other room.

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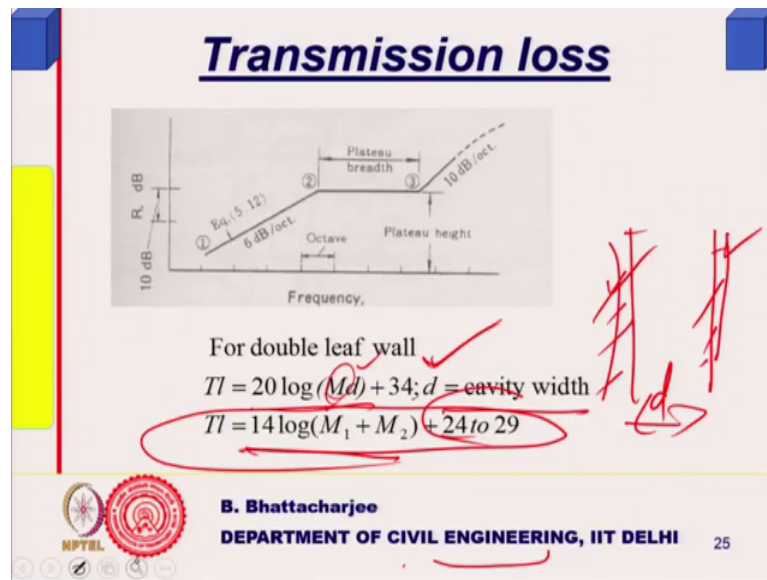


So, one way is you take a approximate formula which I will give you. Again I think there is an approximate formula, yes $14 \log m$ plus something. Not here, this. There is another approximate formula which takes care of all average frequency which is $14 \log m$ plus some value or minus some value plus or minus some value you know. So, this is good enough you know what supposing I know the critical frequency at which I should design.

So, I can find out what should be the transmitter loss and remember for two rooms, the partition wall if I have to design for noise control, it was related to that $T I$ of that room that wall plus absorption in the receiver room. I gave you a formula earlier. So, there the $T I$ is required. So, you can obtain that $T I$ here and for various frequencies whichever is desirable whatever with the frequency values. So, you can design the partition wall based on this formula. If you double the mass, so you actually increase you know twice M . So, your transmission loss will increase by $18 \log 2$ $\log 2$ is 0.3. So, about 6 db I mean you know less than slightly 5.7 db.

So, if you double the mass, it will be simply about 5-6 db reduction. 5-6 db reduction is quite significant because you know intense. So, you can use this in design, right. So, these are some of the formula frequency and for various kind of materials surface density and this is actually the reduction that is there for various kind of materials is there ah.

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This is again some transmission loss values for three against frequency beyond this is this is related to confidence deep where it would not change in factor with transmission loss would not 6 dB for every frequency reduction nearly 6 dB after that it could be ten db between some place there may not be any change because of confidence deep somethings like that, but it follows something of this kind right some some practical examples.

Now, for double leaf wall if I have two leaves and there is a air cavity inside modeling, this is fairly complex right, but I am straightaway given empirical equation for this class. So, this is given by this formula $20 \log M d$ plus 34 d is a cavity width. So, I have one leaf here, another leaf here. This is the d cavity width. So, mass total, mass per unit area of the wall including this $20 \log M d$ plus 34, right $20 \log M d$ plus 34. So, you see it is a function of d also.

Now, it cannot go on increasing the mass. I cannot make the walls thicker and thicker. What I can do, I can make same wall separate into two parts, put a cavity inside, it will increase. This is another formula $14 \log M_1$ plus M_2 plus 24. You know transmission was $14 \log M_1$ plus M_2 plus 24. This is average over all the frequencies. This also averaged over all the frequencies. So, one can use this you know you can take $14 \log M$ simply for a single leaf scenario plus this you know somewhere 24 to say empirical equation.

So, you can actually use this formula to calculate out the partition thickness $E1$ because mass per unit area is known to you if you know the material.

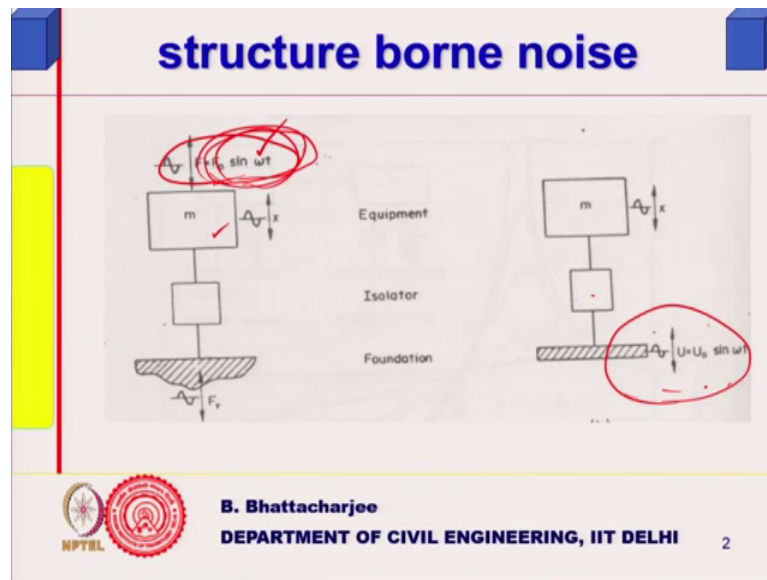
Yes, but considering the mass here and mass there again an empirical equation, right empirical in an empirical.

Well this 24 and 29 would have taken into account. 24 to 29 would have taken into account you know. So, that is all, but this anyway is a better formula you will give you might get some more formally somewhere because it is empirical. So, this one can use for cavity. You can use knowing the cavity depth. You can use this. So, this was related to transmission loss.

Now, how do I control noise within a space? If it is generated within inside, then I put a lot of observers. If it is coming from outside from the next room or from outside, then I provide insulation and qualitatively I can say I should put heavy mass, right. See even if you are not accurately calculating because there are so much of uncertainty of the noise that is coming in you know if it is all random noise coming from traffic or something like that. I will not get same frequency all the time. I might know the frequency range and therefore, there is empirical equations are good enough right, ok.

So, if it is coming from outside, I put in insulation if it is generated within. I put absorbers, but I do something related to planning also that I will come in begin later, but supposing it is structure borne noise, you know it is in this floor, coming from a floor, above or coming from another room because some machine is placed in the structure, then I have to provide isolation between the machine and the structure, right and let us look at how do we do it. Here we have advantage as I said because we know the frequency. So far we talked of a airborne noise control , let us look at structure borne noise control.

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I have a mass equipment which has got a mass. This is an isolator which could be springs or some sort of resilient material like rubber pads or whatever it is.

So, this is isolator and this is my structure, the foundation or the structure whatever we will call it. So, basically this will be you know this would be actually oh oh this would be since they are machines and I know the pattern of their vibration isolation is usually for machines, structure borne noise comes from machines, but structure borne noise also can be random. We will look at that later on. Let us see the one which is known. So, the machine vibrates, it is the force it imparts.

Now, the equipment it imparts it is the own mass. So, this is actually vibrating. So, forcing frequency is ω let us say and $\sin \omega t$. It is because most of the machines are periodic reciprocating or you know cyclic sort of. So, there will be periodic again and they will be imparting the noise with certain frequency. So, that is the forcing frequency and I can possibly assume it to be $f_0 \sin \omega t$.

So, that is the force which will come onto the mounting, right equipment and then, when I have an isolator, it has got a machine on top. So, there is a static deflection first it if the isolator will deflect. So, that is a static deflection and it will transmit this vibration somewhere there. So, it will transmit in some another you know whatever the frequencies we can find it out.

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structure borne noise

Structure borne noise is controlled through isolation, Transmission of Vibration from machine to building (building to machine) is prevented

Such a system can be modeled as a mass and spring single degree freedom system neglecting the damping (damping further reduces transmission)

Diagram of a mass-spring system: A blue rectangular mass labeled m is suspended from a fixed support by a coiled spring labeled k .

Spring equation
 $m\ddot{y} + ky = 0$

Handwritten notes in red ink: $D^2 = -\frac{k}{m}$ and $y = \frac{k}{-m} y$

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So, we can write the equation of motion for this. So, structure borne noise is controlled through isolation. That is what I am saying. Transmission of vibration from machine to the building that is prevented and such a system can be modeled as mass and spring system neglecting the damping part of it, right and single degree freedom system there is one would have done in vibration machine foundation design takes a lot more complicated thing, but I am not interested in this. We are trying to only control the noise.

So, we can neglect because it will further reduce it. Damping means some energy loss would be there. Supposing it is sports in the pores, there will be air motion some frequency you know. There will be air motion, some heat will be generated. So, the viscous damping could be there, there could be frictional losses and that is the damping basically. So, that would reduce further, but for noise control we model our isolation like a spring. So, this is my mass; this is the spring. So, isolation is a spring and this is my structure and we simply write this equation $m \ddot{y} = -ky$. That is equals to 0, right. We have not taken the forcing function. First we are trying to find out the natural frequency of that spring itself, ok.

So, complex root this, this one if we want to solve it, it will be $d^2 = -\frac{k}{m}$ auxiliary equation $d^2 + \frac{k}{m} = 0$. You know I can write $\ddot{y} + \frac{k}{m}y = 0$. So, this will be auxiliary

equation will be written as d^2 is equals to this. So, root of this one is it will be imaginary plus minus k/m . So, root of this equation is $\pm i \sqrt{k/m}$.

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structure borne noise

Complex Root of auxiliary equation : $\pm i \sqrt{\frac{k}{m}}$

General solution : $A \cos\left(\sqrt{\frac{k}{m}} t\right) + B \sin\left(\sqrt{\frac{k}{m}} t\right)$

Initial condition : $t = 0, y = y_0$ & $\dot{y} = 0$

$A = y_0$ & $B = 0$

$y = A \cos\left(\sqrt{\frac{k}{m}} t\right) + B \sin\left(\sqrt{\frac{k}{m}} t\right)$

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So, my solution would be when I have plus minus i a $\cos km$ by t plus $b \sin k$ by t you differentiate it twice it will give you the you know same equation back you can write it. So, general solution is like that initial condition at t equals to 0 y is equals to y_0 that's the static deflection and velocity is equals to 0 right it has is you know like initial condition when it starts vibrating. So, it has gone to the extreme you know extreme end and from there it will start.

So, a equals to y_0 , then if I put this condition here t equals to 0 . This would be $\cos 0$ is equals to 1 . This will be equals to 0 . So, t equals to 0 y is equals to this. So, y is equals to y_0 and a is equals to y_0 and b you know when t is equals to let us say you know what t equals to $\pi/2$ at t equals to $\pi/2$ y dash that is if I it you know if I take the derivative of this one and find out y dash y double y dot equals to 0 , put this two boundary condition and I will get a goes to y_0 b equals to 0 . You know if differ the first derivative of this one will be $a \sin$. Am I right? Under root k by m t plus b under root k by m will come here anyway and b under root k by m yeah I can write that, but I am just writing it. It is for me at the moment under root k by m equals to t .

So, I am just trying to find out b . So, if at t equals to 0 \dot{y} is also equals to 0 , b will be equals to 0 because y dot this is y dot. So, t equals to 0 . So, b will be equals to 0 ,

right. This is what we study. I differentiate this. This is y equals to this general solution y goes to this and then, put first differential of d y d t equals to 0, then I get b equals to 0. So, a equals to y 0 b equals to 0. So, my equation then becomes y is equals to y 0 cos under root km by t cos under root k by t, right.

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structure borne noise



$$y = y_0 \cos\left(\sqrt{\frac{k}{m}} t\right)$$

ω_n

Vibration of spring is simple harmonic with k/m as angular frequency

Natural angular frequency: $\omega_n = \sqrt{\frac{k}{m}}$

$2\pi f_n = \omega_n$



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So, y is equals to y 0 cos km by t and anyway i omega n is k under root m this is what this is the frequency at which it will vibrate this is you know this is equals to omega n. So, k omega this is the natural frequency of the system of the spring because if you know this is nothing, but this is equals to omega cos cos omega t. So, omega and t. So, this will vibrate if I put the spring, we push it and release it. I mean just release it. So, it will vibrate at this omega n is 2 k by n and this is twice pi f n.

So, natural frequency of the system I can find out from here. So, vibrational spring is simple harmonic with k by m as angular frequency that we know.

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structure borne noise

When machine is installed on mounting (spring) static deflection is δ

$mg = k\delta$, δ is static deflection

$\frac{k}{m} = \frac{g}{\delta}$, $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{\delta}} = 2\pi f_n$

$f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{g}{\delta}} = 15.76 \sqrt{\frac{1}{\delta}}$ Hz; δ in mm

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When machine is installed on the mounting, I will have a static deflection and this static deflection is δ . So, if the spring stiffness is k , you know static deflection is δ spring stiffness is k , then mg mass of the machine into g must be equals to $k\delta$. In other words, if I know the static deflection, you know I can find out k by m from this equation because that will be g by δ .

So, when I put in the machine what is the static deflection if I know I can find out k by m . In other words, I can find out what will be the natural frequency of the system. So, if I know the static deflection, I can find out the natural frequency of the system, right. So, f_n is 2π under root k by m etcetera and you can simply you can express it in 15.761 by δ . δ is in millimeter putting g you know in appropriate units I get. So, if I know the bound thing which I am modeling as a spring is deflection, when I put in the load that, then I can find out its static deflection and then, I can find out its natural frequency.

So, natural frequency the mounting I can find out some ideas are given δ as a function of ω_n for various materials are known. For example, if I know δ for a cork board, right correspondingly what will be because it will depend upon its stiffness sense you know like E modulus of elasticity and all. So, this sort of graph is given in national building code for various material ω_n as a function of I mean you know δ relationship depending upon δ versus ω_n , you know these relationships are available.

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structure borne noise



When the machine is operating, it imparts vibration to isolation periodically

The equation is

$$m\ddot{y} + ky = F_0 \sin \omega t; \omega \text{ periodicity of machine vibration}$$

Particular integral is found by letting

$$y = A \sin(\omega t - \phi)$$

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So, when machine is operating it imparts force $F_0 \sin \omega t$ forcing function, right it imparts vibration to the isolation periodically. So, then the equation becomes $F_0 \sin \omega t$ where ω is the period. This is another machine vibration. Now, just look at if it is ω_n same as my natural frequency of the system, it will simply amplify it like I was talking about a hammer hitting a pendulum at the same frequency, natural frequency of the pendulum itself and then, noise will be transmitted maximum, but if like I do it some mind you know natural frequency enforcing frequencies are quite different, then obviously it will not transmit.



So, you can find out the solution of this equation putting y is equals to a $\sin \omega t$ minus ϕ because there is a phase difference. So, $A \sin \omega t$ you know put with this how we find out solutions of such differential equations $A \sin \omega t$ minus ϕ .

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structure borne noise

for $y = A \sin(\omega t - \phi)$ = $\omega A \cos(\omega t - \phi)$

$$m\ddot{y} + ky = -mA\omega^2 \sin(\omega t - \phi) + kA \sin(\omega t - \phi)$$
$$= F_0 \sin(\omega t)$$
$$A(-m\omega^2 + k) \sin(\omega t - \phi) = F_0 \sin(\omega t);$$
$$\phi = 0; A = \frac{F_0}{(k - m\omega^2)} = \frac{F_0}{m(\omega_n^2 - \omega^2)}$$

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If I do that for y is goes to A sin omega t. Just put it y double dot would be sin again back with A minus sign and omega square. So, you know sin omega t minus phi i. Just differentiate it twice. So, first time I do, I will get a cos you know omega A cos omega t minus phi, right. Am I right and then, this will give me A square. A remains as it is there is a minus sign coming because again sin will come back and plus k into sine omega t.

So, that should be equals to if this is a solution particular integral because of the forcing function, this must be valid and therefore, I can find out this value of A. So, I can find out A you know from this A, I take common i get minus m omega square plus k plus k sin omega t must be equal to sin omega t. So, in other words, when phi is equals to 0 is equals to have 0 km minus omega and I can get an expression like this.

So, I can get expression in this manner, I will come back to this in the next class again at this point of time.

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structure borne noise

General Solution

$$y = y_0 \cos(\omega_n t) + \frac{F_0}{m(\omega_n^2 - \omega^2)} \sin(\omega t)$$

At $\omega = \omega_n$ resonance will occur

$$\tau = \frac{\text{Force transmitted}}{\text{Exciting Force}}$$

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So, I can write the general equation as $y = y_0 \cos \omega_n t$ because of the natural frequency point of view and this is because of the forcing function. So, earlier without the forcing function, this was the case and then, we define something called force transmitted ability. It is transmitted, force transmitted divided by exciting force and this now is slightly different than what you looked into because there we are doing and ω_n^2 by you know ω^2 . This is from the wave isolation actually and we can derive from this. Actually we can derive an expression for tau.

So, what should be the properties of mountain that we can find out is a simple case of vibration. Isolation case can be quite complicated in foundation design and things like that.

So, I think we will stop here and next class we will relook into new thing.