

**Energy Efficiency, Acoustics & Daylighting in building**  
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**Indian Institute of Technology, Delhi**

**Lecture – 41**  
**Sound within Enclosure (contd.)**


(Refer Slide Time: 00:18)

### Sound Absorption

*When noise source is inside the room itself, increasing absorption reduces noise level*

$$L_p = 10 \log \frac{W}{I_{ref}} + 10 \log \left( \frac{Q}{4\pi r^2} + \frac{4}{R} \right)$$

$$\text{Noise Reduction } \Delta L_p = 10 \log \frac{\left( \frac{Q}{4\pi r^2} + \frac{4}{R_1} \right)}{\left( \frac{Q}{4\pi r^2} + \frac{4}{R_2} \right)}$$



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10

Plus 10 log Q by 4 pi r square plus 4 by R. So, if I now change the R, you know, if I change the R, let us say let me change the R, R is what? What was R?


(Refer Slide Time: 00:31)

### Sound Absorption

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10

 $R = \frac{S}{1-\alpha}$

Room constant and we said, it was  $s \alpha \bar{a}$  divided by  $1 - \alpha \bar{a}$ . So, it is a property of the room. This is the total absorption in the room, and this is  $1 - \alpha \bar{a}$  average absorption in the room, right so, this is the property of the room itself. And as you can see if I increase the R,  $L_p$  will reduce. Particularly if you are away from the source, R is sufficiently large, right. So, you want to in you know you want a noise is generated within the room, and you do not want that is noise to, you know, that that is source to contribute too much to the noise then increase the R.

So, you will have R 1 and R 2, let us say I have changed the room constant from R 1 to R 2. So, this is what  $\Delta L_p$  will be basically I have changed.

(Refer Slide Time: 01:28)

**Sound Absorption**

*When noise source is inside the room itself, increasing absorption reduces noise level*

$$L_p = 10 \log \frac{W}{I_{ref}} + 10 \log \left( \frac{Q}{4\pi r^2} + \frac{4}{R} \right)$$

Handwritten notes on the slide:

- $R = \frac{S \bar{\alpha}}{1 - \bar{\alpha}}$
- $L_{p1} - L_{p2} = 10 \log \frac{W}{I_{ref}} - 10 \log \frac{W}{I_{ref}} + 10 \log \left( \frac{Q}{4\pi r^2} + \frac{4}{R_1} \right) - 10 \log \left( \frac{Q}{4\pi r^2} + \frac{4}{R_2} \right)$

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So,  $L_{p1}$  let me call it minus  $L_{p2}$  will be equals to  $10 \log w$  by  $I_{ref}$  minus again  $10 \log w$  by  $I_{ref}$ . This part will cancel out. So, this will cancel out, but this side will be there plus  $10 \log$ ,  $Q$  by  $4 \pi r^2$ , if the distance is same  $4$  by  $R_1$  minus  $10 \log Q$  by  $4 \pi r^2$  square  $4$  by  $R_2$ . So, this will cancel out, and this I can now combine write it  $10 \log$  this divided by this because the logarithm has a minus sign.

So, that is what we have done, oh that is what we have done, that is what we do.

(Refer Slide Time: 02:18)

**Sound Absorption**



***When noise source is inside the room itself, increasing absorption reduces noise level***

$$L_p = 10 \log \frac{W}{I_{ref}} + 10 \log \left( \frac{Q}{4\pi r^2} + \frac{4}{R} \right)$$

$$\text{Noise Reduction } \Delta L_p = 10 \log \frac{\left( \frac{Q}{4\pi r^2} + \frac{4}{R_1} \right)}{\left( \frac{Q}{4\pi r^2} + \frac{4}{R_2} \right)}$$

$$\text{For diffused field } \Delta L_p = 10 \log \left( \frac{R_2}{R_1} \right) = 10 \log \frac{\bar{\alpha}_2(1-\bar{\alpha}_1)}{\bar{\alpha}_1(1-\bar{\alpha}_2)}$$

$\frac{R_2}{R_1}$   
 $\frac{1-\alpha_1}{1-\alpha_2}$



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10

And that is  $10 \log \frac{Q}{4\pi r^2} + 10 \log \frac{4}{R}$  you know so, I can write it like this. So,  $\Delta L_p = L_{p1} - L_{p2}$ , initially it was  $L_{p1}$  and then it changed to  $L_{p2}$  when I have to interchange that  $R_1$  to  $R_2$ . So, this will be the ratio, and if the distance is sufficiently large, only diffused field I am looking at, then simply you know for diffused field only. So, when  $R$  is sufficiently large beyond room radius, I am not really interested in this part, only this part, this will be  $10 \log \frac{4}{R_2} - 10 \log \frac{4}{R_1}$  because 4 will cancel out. I will be left with  $10 \log \frac{R_2}{R_1}$ , right.

So, if I have higher  $R_2$  more  $R_2$  my  $\Delta L_p$  reduction will be more reduction will be more. So, I should not have more higher room constant, which means I should increase the alpha value, alpha bar value. So,  $\frac{\bar{\alpha}_2(1-\bar{\alpha}_1)}{\bar{\alpha}_1(1-\bar{\alpha}_2)}$ , if I increase this  $\bar{\alpha}_2$  is greater than  $\bar{\alpha}_1$ ,  $\bar{\alpha}_2$  is greater than. So,  $\bar{\alpha}_2$  is greater than  $\bar{\alpha}_1$ , this value will increase, and then noise will be controlled. So, that is how it is.

So, improving the if the sources within, one way of improving it is you know as I said a typewriter, somewhere in a corner. Or something similar, which may not be which is kind of a noise for other people who are sitting there in the room. Might be an office or similar sort of spaces, something is something generates noise anyone to control it then increase the absorption, that is what it says, right.

Now, if I know the frequency of the sources, then frequency selective absorption so, I can have. For example, you consider this is a you know, simply a porous screen in front.

(Refer Slide Time: 04:06)

The slide is titled "SPECIAL ABSORBERS". It features a diagram on the left labeled "Single thin porous screen" showing sound waves approaching a screen. To the right is a graph of the absorption coefficient  $\alpha$  versus distance  $d$ . The graph shows a periodic wave-like curve with peaks. Handwritten red notes are present: "Absorbent material do not improve insulation" is written across the graph. Below the graph, there are handwritten equations:  $4d/\lambda = 1, 3, 5$ . The slide footer includes the IIT Delhi logo, the name "B. Bhattacharjee", the "DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI", and the number "11".

This is porous screen in front and unlike, like something like this holds a screen. You know, a panel and I have made holes in that. So, that is basically a porous screen. So, this is you know this is how it is coming. Now it is response, alpha value the absorption coefficient is a you know is maximum, when  $4d$  by  $\lambda$   $4d$  by  $\lambda$  this is distance  $d$ ,  $\lambda$  is a wavelength of the sound.

So, when  $4d$  by  $\lambda$  is equals to 1, it is absorption is maximum or 3 5 etcetera one can show that. So, you see if I my frequency is such that, the  $\lambda$  is such a so,  $4d$  by  $\lambda$  should be equals to 1 let us say. In that case, I can find  $\lambda$  is nothing but  $c$  by  $f$ . So, if I know  $\lambda$ , I know  $f$ , I can find out  $f$ . So, frequency if the source is known. For example, if it is a typewriter, I know it is kind of frequency, remember we talked of equal loudness level contour. There you showed those frequencies. So, if I know the frequency,  $\lambda$  will be known to me velocity is 340 meter per second or close to 340, you know, 344 or whatever it is meter per second, I can find out what should be the  $d$  where I will have maximum absorption.

So, I can choose for a porous screen what is the distance, and provided you know, I know  $\lambda$ . So, that is what it is. So, this at this frequency it will be maximum, or this frequency it will maximum other frequencies is less. So, it is frequency selective, there is

a bandwidth, but still it is frequency selective, right. So, absorbent material this remembers that this is different than insulation. Insulation is transmission loss this separate his absorption do not come back. Absorption high means it will not come back, not necessary, it will not allow it to go other side. Because open window has got an absorption is equals to 1, but that will transmit everything to the other side as well.

So, that is it. So,  $d$  is equals to  $n \lambda$  by 4, that is the thing  $n$  equals to 1 3 5 etcetera etcetera. So, I can determine the  $d$  maximum absorption yeah, right. So,  $\alpha_2$  will be this value, here if you are you know you are very sure about this frequency, I mean this frequency, you are shared because  $\lambda$  is equals to nothing but  $c$  by  $f$ .

(Refer Slide Time: 06:30)



**SPECIAL ABSORBERS**

Single thin porous screen

Sound AC

**Absorbent material do not improve insulation**

Maximum absorption at  $d = n \frac{\lambda}{4}; n = 1, 3, 5, \dots$



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11

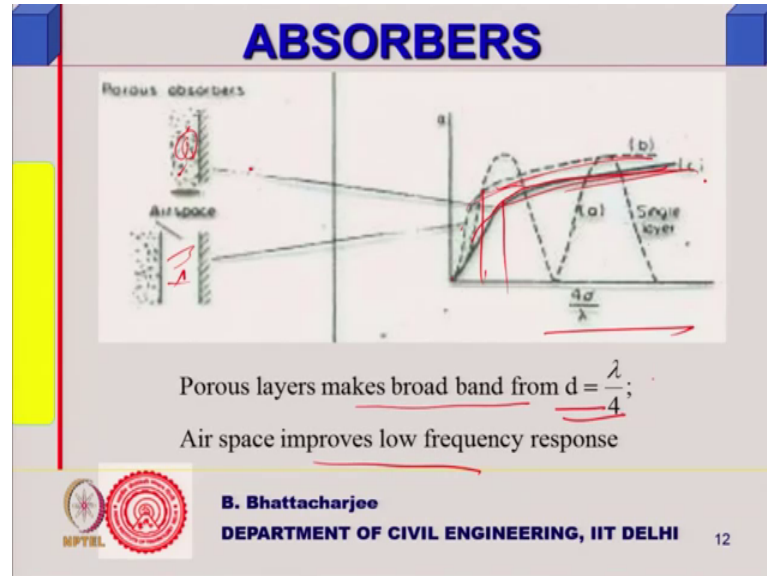
So, if you know this frequency you are quite sure of you can use this kind of an observer, right. So, that is  $\alpha_0$ ,  $\alpha_1$  is this value,  $\alpha_2$  is what is existing you want to improve over it by  $\alpha_2$ , right.

Student: (Refer Time: 06:58).

No, no, but if you have designed it well it is no problem. If you have designed it well, then you choose yeah, that then also if you are you are if you have actually envisaged or foreseen, that there will be some noise sources within the room, you can put right kind of observer to make see that maximum absorption occurs that given right. So, that this

improves supposing I want to improve this put you know this is porous material here, there is another way.

(Refer Slide Time: 07:25)



Now, this porous material is relatively, you know, so, porous layers make broadband from  $d$  equals to  $\lambda$  by 4 so, it is something like this. So, if it is a porous material it will be something like this curve for this case; that means, I have a backing wall I put a porous material on top of it, right. Support glass rule I mean mineral rule glass rule of course, can be a health problem it can you know handling, but it is fine steel glass or people use there is a thermal insulation also. So, you put some porous material right on top.

Now, this porous material this is the point where you know before beyond  $4d$  equals to  $\lambda$  by 4. So,  $4d$  by  $\lambda$  you find that you know beyond this it is higher frequency it absorbs more. And if you have a air gap, then this will be something like this will come even you know. So, it is much absorption at higher frequencies very low frequency absorption is less, not specific frequency. So, this is you know this is a single layer something like that air space was there. Air space should make something of this kind.

So, air space improves low frequency response. So, if I put an air space here, it improves a low frequency response. If I do not have any air space, just put porous material you

will get something like this, right? And that is how that is a one can you know. So, depending upon the type of source you are likely to have, you can use special absorbers.

(Refer Slide Time: 08:56)

**ABSORBERS**

*Membrane or Panel Absorber*

Membrane mass  $m$  ( $\text{kg}/\text{m}^2$ ); Resonant frequency

$$f = \frac{c}{2\pi} \sqrt{\frac{v}{md}}$$

$v$  = Poisson's ratio

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Membrane or panel absorbers just put a membrane, just put a panel here. So, this will depend upon this  $d$ , this will depend upon  $d$ , and mass of the material that is here it could be a porous material as well. So, you have actually  $d$  increasing along this direction, you will find that low frequency response increases. This is the mass of this panel here per unit area.

So, this you know membrane or panel absorbers, they can also work by and large although it is broadband, but the peak is somewhere only at a single point. So, if you know the frequency, you can choose the right kind of absorbers. So, membrane mass is  $m$  kg per meter square, this is the membrane mass. And this at this frequency, it resonates and therefore, it absorbs maximum energy. Because it will vibrate itself will not allow sound to come back yeah, right.

So, that is what it is what this is poisson ratio  $md$  is this value. As you increase the  $md$  frequency reduces, frequency goes down. So,  $m$  or  $d$  you increase it will actually this  $f$  you reduce  $f$  gets reduced. So, actually if you put a membrane simple put a plywood shit in front, if you are you know I am saying if you are diagnosing, writing the planning design stage one can do that you can choose, you know, what are the kind of sources and therefore, you can choose.



So, choose the kind of absorber you want to put. And if it is all random you do not know the frequencies, you put you know there is nothing this kind of absorb especially absorbers do not work. If you know the frequencies, then this sort of absorbers will work. Because it has got a peak at a given point and very high value while, other kind of absorbers just you know putting casually something like some fiber boats. And all as would have done in this particular case, this will definitely reduce you see noise.

In fact, I you know like they are possibly possibly one should not have put this perforated edge if some boats here, you know, in front of the source; this is all same they put in. A good acoustic design, see we are using mike for such a small room which is really not necessary, if it is done in a proper manner. This should have some amount of reflector, I mean on top of this up to certain point. As you shall see in case of auditorium design because whatever is spoken here, that should go back to the end.

So, if I put a correctly angled reflector that will send to the end. So, acoustic design if it is done properly, right in the beginning can put even special absorbers, if you know the frequency of the source frequency of source, if you do not know then; obviously, this kind of special absorbers you can use they are useful in machines place, where there is a machine, because their frequencies are very well known, right. Speeches it will be very difficult to handle in this manner. This is called Helmholtz resonator.

Student: Sir.

Yeah.



(Refer Slide Time: 12:00)

The slide is titled "ABSORBERS" in large blue letters at the top. Below the title, there are two main sections. On the left, under the heading "Resonator absorbers", is a diagram of a "Simple Helmholtz resonator". It shows a rectangular box with a smaller circular cavity inside. A red arrow points to the circular cavity, and a red circle is drawn around it. The diagram is labeled with 'a' for the cross-sectional area of the neck and 'l' for the length of the neck. On the right, there is a graph showing a resonance curve. The y-axis is labeled '10', '5', and '0'. The x-axis is labeled '100', '1000', and '5000'. A sharp peak is shown at a frequency of approximately 1000. Below the graph, the resonant frequency is given by the equation: 
$$f = \frac{c}{2\pi} \sqrt{\frac{a}{lv}}$$
 where 'c' is the speed of sound, 'a' is the cross-sectional area, 'l' is the length of the neck, and 'v' is the volume of the cavity. Below the equation, it says: "v = volume of the cavity; a = cross-sectional area & l = length of entry". At the bottom of the slide, there are logos for NPTEL and IIT Delhi, and the name "B. Bhattacharjee" and "DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI". The slide number "14" is in the bottom right corner.

Student: (Refer Time: 12:00) depending on the mass per unit area.

Of this membrane, mass per unit area of the membrane, this membrane.

Student: Whether this a particular frequency correspond to the resonant frequency.

Yeah, they are at the resonant frequency, because of it is resonance. It will not allow energy to come back. It will absorb that energy. The membrane itself will resonate it will vibrate quite a lot and will not pass back that energy to that space. So, it is in a way the structure itself holds the energy together, so.

Student: (Refer Time: 12:35) occurring is the.

Occurring there in the membrane itself. That is related to, you know, the poisons ratio of this one because something is impinging there then. So, this directional movement, mass and the distance between this one, because air also will resonate together with this, this cavity you will have actually high level of sound if you can put a microphone there. But it will not be there into the room outside, here it will be less.

So, basically if it is resonating within this column this area, sound is resonating forming a standing wave let us say within. Inside is forming a standing wave. In that case you will have high sound level here, but the space will have much less, space is outside. It is

within that you know this space right. So, I do not mind some resonance there to put someone inside, that this distance is relatively small.

Similarly, this Helmholtz resonator, it will actually resonates within this right, but nothing will happen? Outside it will not win. So, a very specific freedoms even an order of narrow band but actually, 100 percent absorption at a narrowband frequency. So, this is something like this if this distance is  $L$ , cavity you know this length is  $L$ , this is the area of the cavity, area of the neck of the cavity. Then this is given by  $c$  is a velocity of sound  $a$  is this area as I am showing, and  $L$  is the length of entry, length of entry and  $v$  is a volume of the cavity.

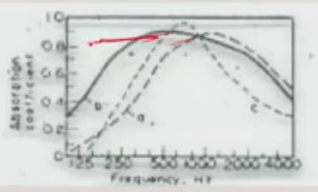
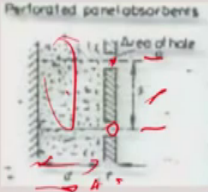
So, this is the volume of the cavity is known, at this frequency you have almost 100 percent absorption. You can derive this one can derive equation for this, but we are not interested in that. So, depending upon this volume, at certain frequency it will you know it will resonate. So, sound will resonate within this cavity, and this simple Helmholtz resonator. And it will absorb everything, but nowhere else. So, if you know you can make this actually you can make this not a problem this is the wall actually this is your wall, you know, there will be another something like this, I mean, you know another sorry, sorry, my drawing is not I am not doing a good job as far as my drawing is concerned.

So, it will be extend like this, then something of this, kind then there is another one. You know, the wall is something like this, backing wall etcetera etcetera. So, I have actually holes in the wall, cavities in the wall with entry, right. So, this is one there is a next one, and these are all walls. And again, again there will be another cavity. So, series of cavities in this manner, and with a entry point available right.

Now, this kind of things should specifically absorb at a given frequency given by this formula depending upon area of the entry and length of the entry, and volume of the cavity cell. So, one can derive this formula, but slightly different is if you want to make it Broadway's what you do is you have porous material, make this cavity this itself is the cavity might put porous material or may not put porous material.

(Refer Slide Time: 15:44)



## ABSORBERS



**a, b, c with varying d, a etc**

Resonant frequency;  $f = \frac{c}{2\pi} \sqrt{\frac{a}{dts^2}}$

using Helmholtz resonator concept



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15

But you have holes larger holes, earlier we had a perforated one was very small, there is the larger holes, and this you know, formula is again given a b c with varying d a etcetera.

So, this is d, this is area is that same entry radius spacing is there this is the spacing. So, formula is given as s as the spacing between 2 entry center to center. T is the thickness of this one, t is the thickness of this panel in front. And a is the area this area as I talked about, d is this distance. So, this is relatively broadband, well, Helmholtz cavity resonator as it is called is very fixed to a specific frequency.

So, one can use that if you know the frequency, otherwise this is broadband, and this you know this changes depending upon value of a d etcetera, you can get different absorptions. So, but they absorb quite a bit significantly they can absorb 0.9 or whatever it is high absorption signal though. So, they are used for specific frequencies here, here of course, is relatively broader. So, these special absorbers are used in those places where I know the frequency of the source, right, know the frequency of the source there I can use them, alright.

So, that is it so, that is absorbers. So, it can use those absorbers, and control the noise generated within the room itself. If I do not know the frequency, well, generally I it will be known because it will be machines or something of that kind, right. It will be machine or something, right if I do not know the frequency, then of course, it is random noise, like

in lecture theaters auditoriums there we do something different we reality we use this kind of this kind of absorbers, why? Because frequencies are not known.

Supposing it is a multipurpose auditorium, I can have a speech, I can have music, dramatics so many thing, and they are you the frequency is varied. So, you have to have broadband, not very specific frequencies use absorbers which will absorb large in many frequencies and so on. So, quite often people do it by simple as I said you know just use some more. But it is better to look into absorption coefficient. You know, get the information about the absorption coefficient at different frequencies before you apply them into the space that you are interested in right.

So, that is what that is what it is. So, this was related to observers in fact, they give you free absorption at 5 different frequencies at mid frequencies of certain octave band, and average of that is specified in many such materials, right. Many such materials so, that is one way many many you know if you if you are asking for some absorbers to be purchased, they will give you a free what are the absorption at different frequency sometimes they give you the average at 5 Octavian's 125, 250 etcetera, etcetera, you know certain a certain octavians at the center frequencies what are the assumptions they will give you, and average of that will be given. So, that is called I think noise reduction coefficient or something ok.

So, some you know they give it. So, all that I am trying to point out, if you want to use them the sources inside, you want to control that noise you want to put in absorbers, find out the frequency at which your source are likely to it is a known source, and then use that for use that specific frequency related absorption now how much absorption is that frequency. But if it is all different kind then use as broadband as possible, you know which should observe at different frequencies of total overall the average absorption at different frequencies that you can look into.

So, that is related to control of the source, if it is within the room, same room. If it is another room then I should have transmission loss values I should look into. So, that is transmission, and let us look into the transmission tau, how does it depend on top partition wall properties?

(Refer Slide Time: 20:14)

**Transmission**

Velocity  $v$   
Mass  $m$

Incident sound  $p_i$       Transmitted sound  $p_t$   
Reflection sound  $p_r$

The vibrating air molecule induces acceleration of wall particles;  
The equilibrium equation

$$p_i + p_r = 2p_i = M\ddot{y} + 2\rho c y$$

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16

So, basically you can model it will like this, you know. I have an incident sound pressure level  $p_i$  excess pressure in the air, and there will be some reflection, and we assume for this particular case our walls are mostly very rigid.

So, that the pressure reflected pressure and incident pressure they will be by and large same but they are acting in the opposite direction. So, net pressure acting on to this will be actually  $2 p_i$ . One along this direction and the others right, now this actually sets the particle in motion, particle within the wall in motion about their mean position of equilibrium, and vibration is transmitted through the solid through this medium. And this in turn causes particle to vibrate the air particle to vibrate or the other end, that is how the sound is transmitted.

So, pressure here will be  $p_t$  pressure here is  $p_i$  and then reflected is  $p_r$  that is force per unit area. And that must be if I know mass per unit area, mass per unit area that must be the you know particle acceleration inside, particle acceleration inside, right. And particle velocity inside, and particle displacement inside all 3 will be there. And that we are talking about. So, the vibrating air molecule induces acceleration of wall particles, and these results in mass into acceleration  $y$  dot I am calling it because displacement we use  $y$  if I remember. So,  $y$  double dot means acceleration of the particle  $m$  is the mass per unit area.  $M$  is the mass per unit area,  $m$  is mass per unit area. Because pressure I am

talking of, and  $y$  is velocity. And remember if I have velocity, acoustic pressure must be equals to acoustic impedance multiplied by velocity.

So, I have both acceleration and velocity, and this is you know this is  $\rho^2$  is  $\rho c y$  that is. So, the  $2\pi$  is a force that is acting here, the particle will move mass and acceleration mass and acceleration plus the dam you know acoustic impedance into the particle velocity. So, n this is this will be the actually equation of you know motion there equation of motion there.

So this.

Student: Sir.

Yeah.

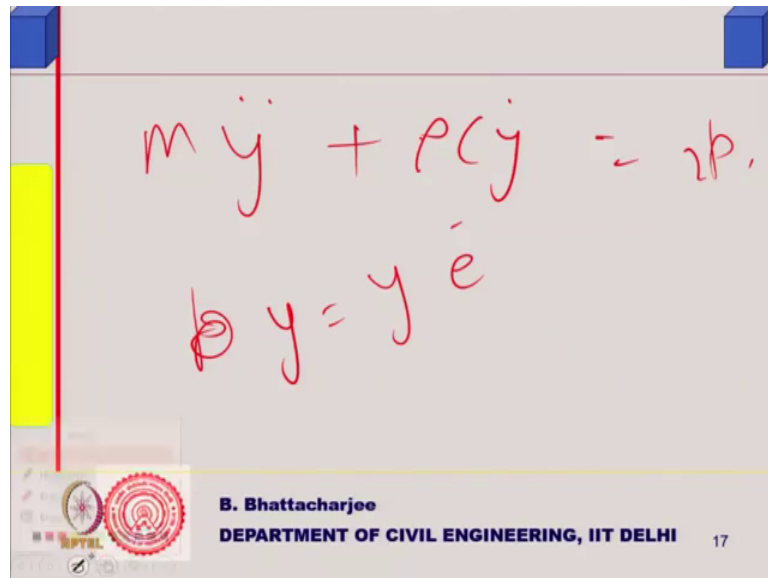
Student: Why it will be? (Refer Time: 22:47).

Because you remember,  $p$  by  $y$  dot was equals to  $\rho c$ , right. So, some component of the pressure because I have acceleration, and velocity together it is moving and also it is accelerating. So, some component pressure must be balanced by velocity because medium has got  $\rho c$ . The impedance of the medium is  $\rho c$ , multiplied by the velocity. So, that is the medium, you know, medium kind of medium related to the media. And mass of the thing, per unit area into acceleration. So, this is a component both the components will be acting simultaneously balancing the pressure.

A part of it would cause acceleration, part of it would cause velocity, you know is there these 2 things are occurring together. It is accelerating also at any particle if I take at any point, it is accelerating as well as it has got a velocity. So, corresponding to velocity depending upon the acoustic impedance of the medium, there will be a pressure component. So, total pressure that is impinging on to the wall must be balanced by some velocity which is occurring of the particle at the place and also the acceleration. So, that is why you are doing it like that, right.

So, I can assume, because this is the solution of this one, you know, if I have pressure, pressure you know, the this is the pressure impinging pressure, and what we are saying is, I have got  $y$  double dot  $y$  dot.

(Refer Slide Time: 24:46)


$$m \ddot{y} + \rho c \dot{y} = 2p,$$
$$y = y_0 e^{i \omega t}$$

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So, if I want to find out the solution of this, this is second order differential equation, solution of this one, you know,  $y$  in terms of rest of the thing.  $P_i$  is external pressure that is coming in. So, if I want to find out something like something like this  $y$  double dot like  $y$  double dot plus you know some mass,  $y$  double dot  $\rho c y$  dot, equals to some constant  $p_i$  whatever it is  $2 p_i$  and so on.

So, this is nothing but it is a second order differential equation. So, I can assume  $y$  is  $y$  actually will be equals to  $y_0 e^{i \omega t}$ , I mean  $\omega t$  you know the way we do exponential function. Similarly pressure I can express, because pressure if you remember that we have we can relate displacement to the pressure and pressure, and velocity there in the same phase in the beginning we talked about.

So, instead of simply instead of using  $y$ , as you know.



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**Transmission**

Velocity and pressure are in phase

$$p = p_0 e^{i\omega t}; \dot{y} = u = u_0 e^{i\omega t}$$



$$\ddot{y} = \dot{u} = u_0 i \omega e^{i\omega t}$$

$$2|p_i| e^{i\omega t} = M u_0 i \omega e^{i\omega t} + 2 \rho c u_0 e^{i\omega t}$$

$$|p_i| = \left( \frac{M i \omega}{2} + \rho c \right) u_0$$

$$p_t = \rho c \dot{y} |p_i| e^{i\omega t} = \rho c u_0 e^{i\omega t}$$

$$|p_i| = \rho c u_0$$



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17

Why is it go yeah? So, that is what is so, y dot is equals to u and u 0 i omega t. So, we assume, because second derivative of the pressure also will be wave equation shows a second derivative of the pressure or displacement all will be related in the similar manner. So, what we do is we assume p is equals to p 0 e i omega t it is periodic, it is periodic the impinging pressure sound pressures, you know, it is like this velocity is u 0 i omega t. And y you know y so, this is this, what it is and y dot is equals to e 0, because y double dot will be given like this.

So, I can write it in this manner, because y dot and y double dot. In fact, I could have solved it in a slightly different manner also, but does not matter this itself is good enough. everything since this is periodic the, you know the wave that is impinging onto the surface pressure, this is periodic and I can assume it to be p 0 i omega t, t velocity is also in phase therefore, e 0 i omega t. The acceleration will be acceleration will be given in this manner, and I can write this equation ok. I can I could have done something also , instead of y doubt I could have written it some another variable. Then this will get converted into a first order equation actually, but does not matter for our purpose this is good enough.

So, because this is periodic, this is periodic, this is a general equation because you have a cost function as well as i sin function, e to the i omega t. So, what we do is, we you know we express it in this manner. And we get pi is equals to from this we get pi is equals to

$Mi\omega$  by  $t$ , if I differentiate and put them second differential  $y$  double dot I put it here,  $y$  dot I put it here,  $p_i$  put it here, then I get  $p_i$  is equals to  $M e \omega t$  will cancel out from everywhere. I will have  $m i \omega$  right and there is  $u_0$ , is coming here  $u_0$  is coming here. So, this is taken here  $u_0 u_0$  is taken out here, and  $Mi\omega$  by  $2$  plus  $\rho c$   $u_0$  is outside.

So, this is how I can write from here. So, pressure incident pressure can be related in this manner, right? And transmitted pressure is what? Transmitted pressure, yeah, because so, what we do is basically the mass you know acceleration is caused by the force that is coming in. On the other side, other end it is actually the velocity is no transmitted pressure we write in this manner.  $P_c y$  there, and similarly this also you assume it to be  $e$  to the power  $i\omega t$  multiplied by some amplitude of this. And we can write it in this manner.

So,  $p_t$  is written as  $\rho c u_0$ ,  $p_i$  is written as same as  $\rho c u_0$  the amplitude of the velocity and if I take the ratio of this  $p_t$  by  $p_i$  by you know  $p_t$  by  $p_i$  ratio of this  $p_t$  by  $p_i$ . So, then I get it in this manner you know this divided by this. So, I will this  $u_0$  will not be there  $e i\omega t$  any wouldn't be there and this divided by this part. So, I will get  $\rho c u_0$  will cancel out to  $\rho c$  by you know  $\rho c$  this  $p_i$  divided by  $\rho c$   $1 + Mi\omega$  by  $2$ , you know, this is what will come to.

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**Transmission**

$$\frac{p_t}{p_i} = 1 + \frac{j\omega m}{2\rho c}$$

$$\tau = \frac{\frac{|p_t|^2}{\rho c^2}}{\frac{|p_i|^2}{\rho c^2}} = \frac{|p_t|^2}{|p_i|^2} = \frac{(\rho c)^2 u_0^2}{\left(\frac{Mi\omega}{2} + \rho c\right)^2 u_0^2} = \frac{1}{\left(\frac{Mi\omega}{2\rho c} + 1\right)^2}$$

**At high frequencies**  $\left(\frac{M\omega}{2\rho c} \gg 1\right)$   $\tau = \left(\frac{2\rho c}{\phi M}\right)^2$

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18

This is what it will come to actually or let me write it in this manner. If I write it separately  $p_t$  by  $\rho c u_0$  divided by  $u_0$  multiplied by  $M_i \omega$  by 2 plus  $\rho c$ .

So, this is  $p_t$  by  $\rho c$  if I divide by this would be 1, here I will get  $\rho c u_0$  will cancel. So, I will have  $m y n m_i \omega$ , you know, and  $p_t$  by  $\rho c$  I can write it in this manner, how does it come? So,  $p_c \rho c$  if I cancel out, or take  $\rho c$  out yeah same thing will come, then  $p_i$  by  $p_t$ ,  $p_i$  by  $p_t$  this is  $p_t$  by  $\rho c$  would be yeah this is our value writing it  $j I$  that is exactly the thing. So, it comes out something like this  $1$  by  $m$ .

So,  $p_t$  square by  $\rho c$  square, if I write it because that will give me the decibel level reduction, it comes out something like this. And if this is too large compared to 1, then I can write it simply at high frequencies, because  $\omega$  is twice  $\pi f$ . So, if this is too large compared to 1, then I can write it in this manner at high frequencies, and  $\tau$  will be equals to because  $p_t$  square by  $\rho c$  square is  $\tau$ .  $p_t$  square by  $\rho c$  square this is nothing but  $\tau$ , this is nothing but  $\tau$ .

So,  $\tau$  can be written in this twice  $\rho c$  by  $\omega m$  square. From this one for at high frequencies twice  $\rho c$  by  $\omega m$  square you see, what we have done is we have neglected, several things here the stiffness part of the thing we have neglected. And at higher frequencies we are dealing with. So,  $\tau$  can be related to the mass per unit area at high frequencies that is of our interest. So, when frequency is high it is proportional to you know inversely proportional to mass per unit area. And acoustic frequencies these values in this efficiently, you know, this value will be sufficiently large compared to this ok. So, we will just break for a minute, and come back again.