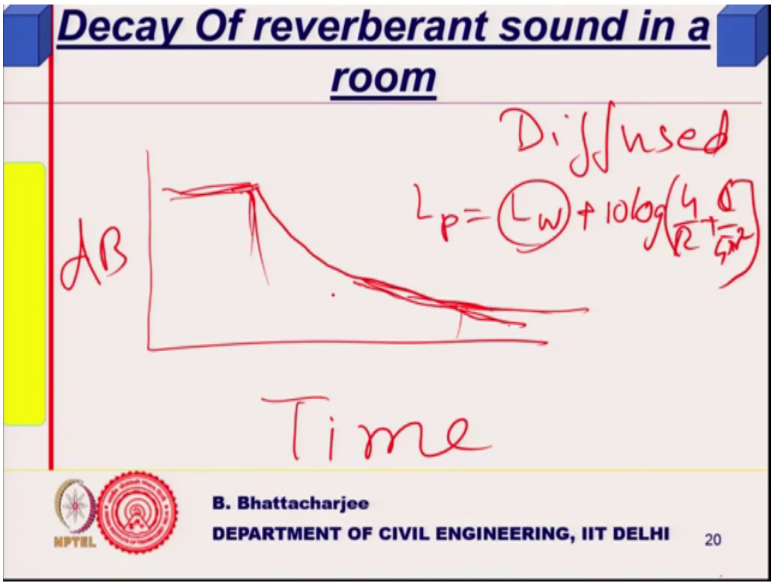
Energy Efficiency, Acoustics & Daylighting in building
Prof. B. Bhattacharjee
Department of Civil Engineering
Indian Institute of Technology, Delhi

Lecture – 39
Sound within Enclosure (contd.)

So, as we were doing it in the last class decay of reverberant sound in a room.

(Refer Slide Time: 00:27)



And if you recall then you know what we are saying is when I put off the with time, when I put off the steady sound some dB level it is only I am talking of diffused sound diffused field right, and it will decay slowly something like this, asymptotically to the ambient level asymptotically to the ambient level right.

So, it persists for some period of time, and if you remember we said that at this point of time L p is equals to L w plus just for the set of 10 log 4 by R plus q by 4 pi R square yeah, this is what we remember right. So, from there it will decay as soon as I put it off this is not there it will decay, and this decay persist for a certain period of time, and it ambient you know it reaches ambient asymptotically.

So, it takes quite a bit of long time to come back to the ambient level right, now this is important issue this you know, this decay phenomena is the important issue which you call as reverberation which I have mentioned earlier. Now this is an important issue because some cases some cases you might here, if you are say you know if your it if it is too long if it persists for too long, then it might blur the direct sound for example, you are sitting very close to me, then you get the direct sound straightaway, but you will also get that reflected sound you know diffused sound multiple infinite reflected sound and so on and so forth.

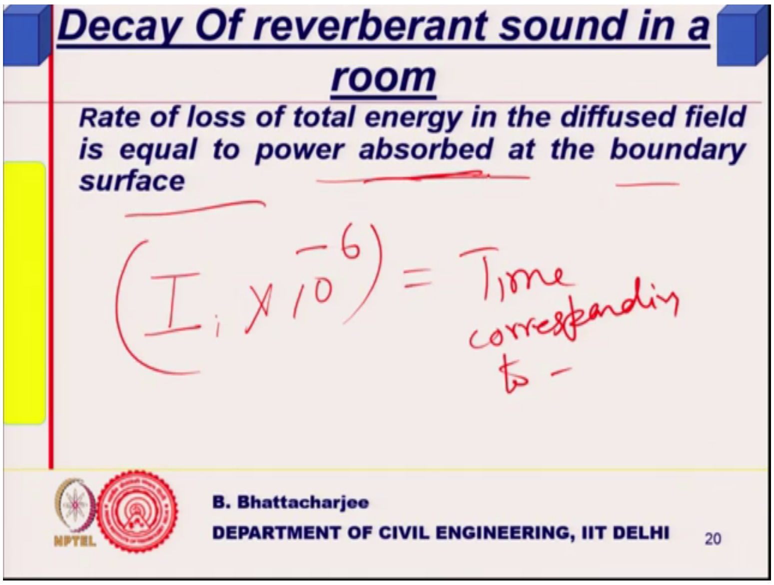
Now, if this reflected sound persists for too long time, then it can create a little bit of interference with the direct sound itself, this happens especially with speech, because speech is we speak in syllables right, we speak in syllables all right for example, a long word like university it has got university. So, supposing there is overlap with the last syllable, and the first last level from the direct sound and reverberant field is coming and disturbing it, then there is a problem. So, there is it is discrete speech is discrete.

On the other hand if you look at music particularly orchestra, or similar kind of lot of instruments and all that, since there is a rhythmic thing it purse you know persistence give you a better feeling. So, instrumental sound orchestras particularly; of the kind of you know the western classical type, zubeen meth I do not know whether your heart of him is supposed to be a famous conductor you know.

So, so such kind of a some hundred violins you know piano, and then a so many so many things going on and it actually the sound itself is of such a kind that you rather like that it should persist longer in area. So, while for music you would require to have good reverberation for or it should remain there, for speech you would like that it should be within control. So, this is important that is what I say. So, this is important this issue is important actually therefore, we must find out somewhere to have a measure of the space, you know it should be it is the property of the space.

So, some measure through which I can say they can compare at least distinguish 1 space from another in terms of its holding or persistence of sound. So, this is done in terms of reverberation time, reverberation time this is done in terms of what is called reverberation time right. Now reverberation time is defined as that time do you know required for original sound to reduce to 1 millionth of its value that is what it is.

(Refer Slide Time: 04:37)

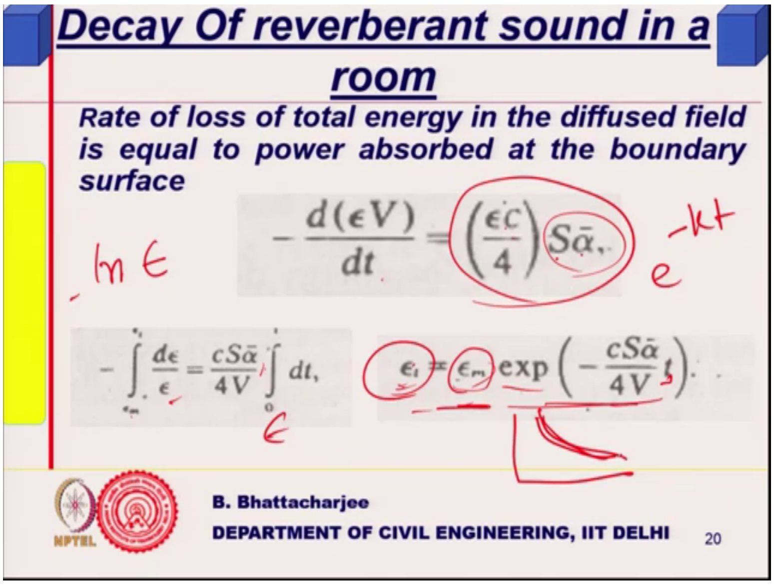


So, if it was originally so you know it is that is mathematically of course, is the power absorbed at the boundary, but once what we would like to know supposing initially intensity was I say i initial and then after a certain period of time, it reduces down to 10 to the minus 6 value of this, the time corresponding to this time corresponding to this time corresponding to this we call as reverberation time, there well you know this we call as reverberation.

So this will we will try to look at this. Now to understand this you must look into rate of loss of energy in the diffused field, and how it will be lost because supposing output of the source. Now sound is absorbed in the boundary sound is absorbed in the boundary, and nothing to replenish it replenish it I am talking only of the diffused field, remember when I talked of earlier, I said that contribution to the diffused field comes after first reflection.

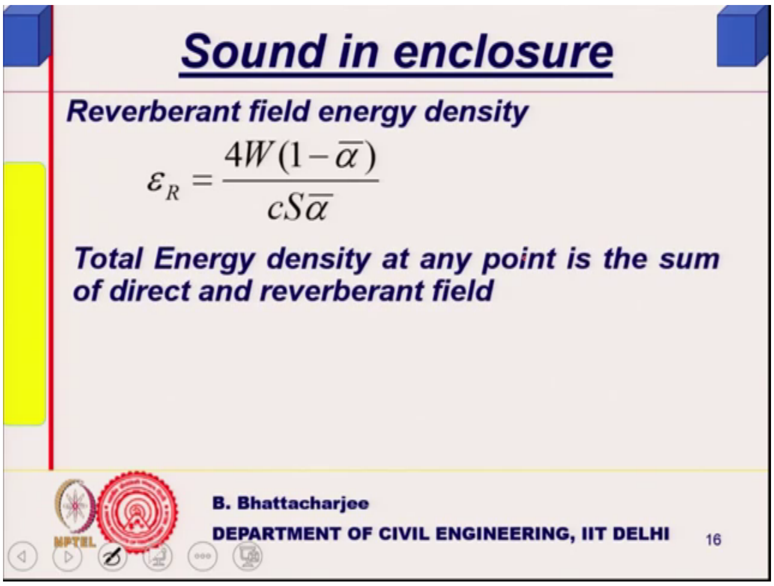
Now, this will not be existing, now this will not be existing, because after you know it will not be existing after first reflection nothing will come. So, there will be no sound coming to the no sound coming to the no sound coming to the reverberant field, because I have put it off, but sound will be lost from the boundaries sound is absorbed in the diffused field is absorbed at the boundary surface that is what we have seen. So, this rate we you want to find out we want to find out this rate we want to find out this rate.

(Refer Slide Time: 06:20)



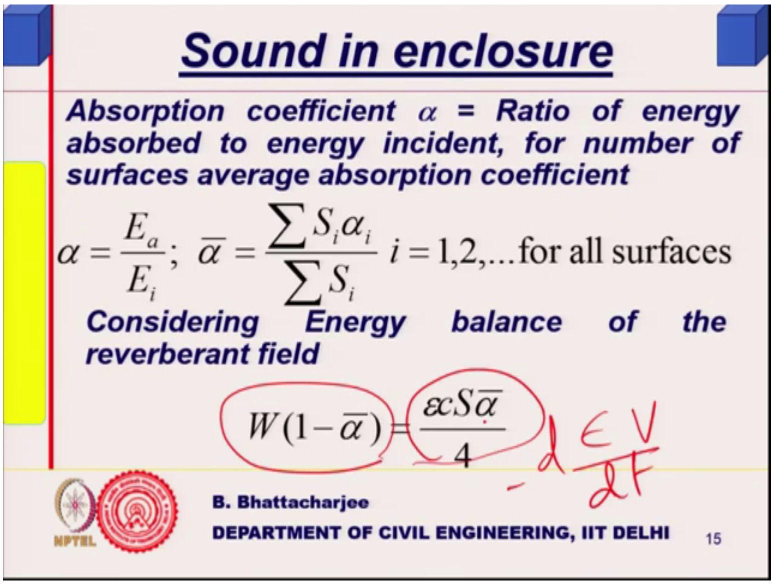
So, epsilon V epsilon is energy density into volume of the room, and rate of change of this one, because it is reducing there is a negative sign, you remember V was given by c you know at the surface e is you know energy density, induce you know the volume you know this energy the rate of energy density absorbed will be this remember this formula this is actually what was this, this we said you know this is this the energy e c by 4 was this you know the rate of change of energy.

(Refer Slide Time: 07:04)



If I go back to earlier expressions perhaps, this will become clear.

(Refer Slide Time: 07:10)

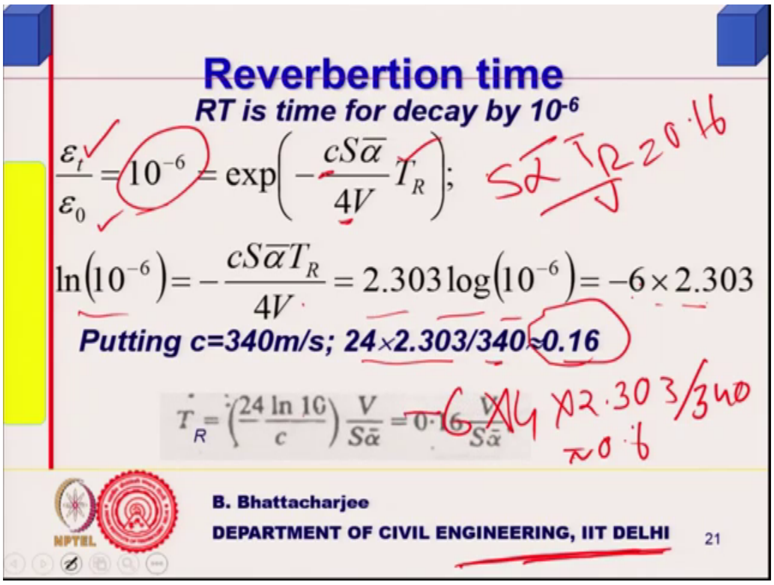


We said the energy density energy density, in the if you recollect we said that the amount of energy absorbed is equals to e c S alpha bar by 4 remember this, we derived this formula.

So, energy absorbed in all the surfaces will be given by epsilon c S alpha bar by 4, and that was equals to the energy coming into the field W was the power source 1 minus alpha bar is the energy coming after first reflection. So, this is the energy absorbed. So, this is this energy is absorbed in the field. So, energy at any state is epsilon into V. So, differential or rate of change of this 1 must be equals to whatever is absorbed on all the surfaces. So, following from here then, we go straight away there again back. So, this is the amount of energy absorbed in all the surfaces, as we have derived earlier, and if I integrate it I get you know d epsilon by d t epsilon comes here is given by this.

So, epsilon at any time t if I take from initial to some final state right e t is a final state or at any time t, and epsilon m is a original you know state basically 0 to t time I will get if I integrate this I will get it l n epsilon is equals to all this multiplied by t, and there is a minus sign here. So, e can be written in terms of exponential form minus c S alpha by 4 b c S alpha by 4 V comes here. So, this is what it is and t was you know after integration t would have come there is a negative sign. So, epsilon initial and epsilon final they can be related in this form of exponential form.

(Refer Slide Time: 09:01)



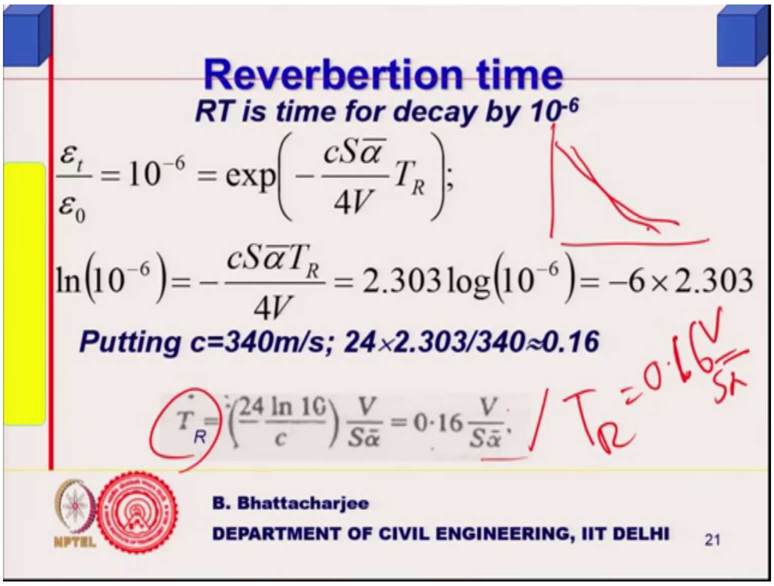
And that is what I am saying so, I said that et by e 0 initial was e 0 at any time it is et that should become 10 to the power minus 6 that is what I was saying that it should reduce to 1 millionth of its original value. So, e 0 is the original, and this is given you know this was given e t was given as e 0 into e to the power all that.

So, this last expression if you recollect this was there et is equals to e 0 starting or 0 whatever it is you know as it exists at the steady level. So, this is this is this is what it is. So, this time will then corresponds to the reverberation time T R, I am defining reverberation time it will correspond to you know T R. So, l n 10 to the power 6 then I can write is equals to this which will be 2.303 log 10 to power minus 6 which will be minus 6 into 2.303, because log of this is minus 6 into 2.303, because it was a napierian to natural log, I mean napierian you know log to the base 10. So, this is what I get, and putting velocity as 340 meter per second.

So, velocity I am putting as 340 per meter per second divided by 4, and divided by 4 you know 4 and multiplied by this so 4 into 6 is 24 into 3 divided by 340.016. So, minus 6 this 4 goes here into 4 into 2.303, and the c I just divided. So, I get this divided by 340 this divided by 340 I get approximately equal to 0.16.

In other words S alpha bar T R by V is equals to 0.16 0.16 is equal to 0.16 right. So, I am writing this down. So, this you know this is simply. So, T R then is equals to 0.16 by V by alpha you know from this it follows, because what I said was T R S alpha bar by V is equals to 0.16. So, T R is equal to 0.16 V divided by S alpha bar. So, this is this is expression for reverberation time, this is called Sabine's equation, this is called Sabine's equation, this is called Sabine's equation right just this is you know.

(Refer Slide Time: 11:35)

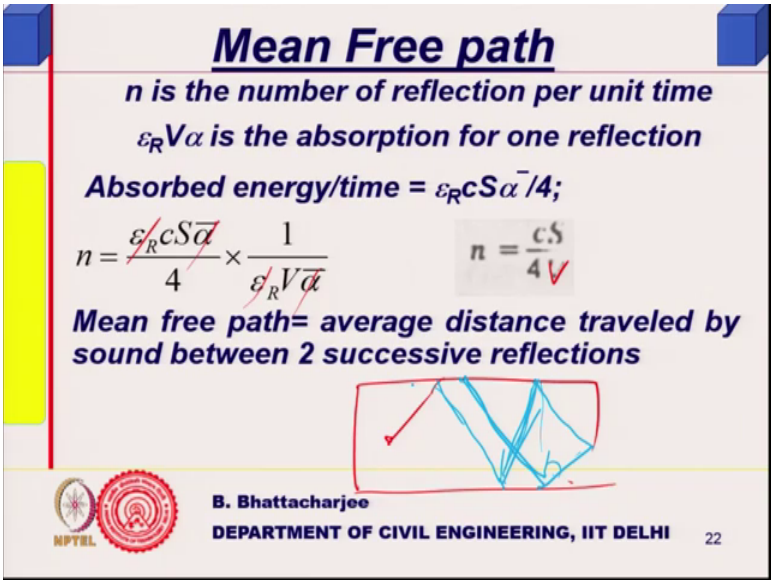


So, T R is equals to approximately 0.16 V by S alpha bar, V is the volume of the room V is the volume of the room right. So, this is reverberation time equation for reverberation time this the simplest of all there is more complicated once a little bit I will talk about them right.

Now, another aspect is defined you know you look at inner in some other form, this can be derived in another manner as well, but you can understand that to this you know it decreases, there is an exponential decrease the curve that I showed. So, it decreases exponentially. So, this is dB rate of change is equals to you know as a function of rate of change itself was a function of the time itself, rate of change is a function of time itself you know. So, d V d I mean not time.

So, the rate of time is a absorption self. So, from this it follows that it actually since it is. So, it decreases exponentially e to the power something negative as a function of time, e to the power minus some k into t. So, this form is a exponential decrease form that is what it comes right.

(Refer Slide Time: 12:47)



So, let us look at it there is a concept called mean free path, if n is the number of reflection per unit time, and distance between average reflection I know. So, for n number of reflection what is the travel time I can find out. So, you know for 1 reflection what is a what is the absorption, epsilon diffused field multiplied by v that is the energy density into volume into alpha. So, I am talking of reverberation so that is why after I have used here, that is the amount of energy absorbed in 1 reflection, and if I n is the number of reflection, then total absorption in n reflection will be so, total you know number of if I know if let me remove this now this is yeah.

So, absorption per unit time would be epsilon R c S into alpha bar by 4 that is what we have seen earlier absorbed per unit time, and number of reflection therefore, I can talk in terms of this is the energy this is the energy absorbed in unit time that is what we have got earlier we have seen epsilon c divided S alpha bar by 4 right, and this is absorbed in 1 energy 1 reflection. So, total number of reflection is given by this, number of reflection you know because number of reflection where per unit time or in 1 second, would be given by this because this is the energy absorbed in unit time, and you know absorbed in unit time, and in 1 reflection energy absorbed is this much because epsilon R into V is the energy contained in the whole volume multiplied by alpha.

So, n is the number of reflection. So, n is the number of reflection therefore, 1 can get an expression for n equals to see the this you know n will be equals to from this it follows 4

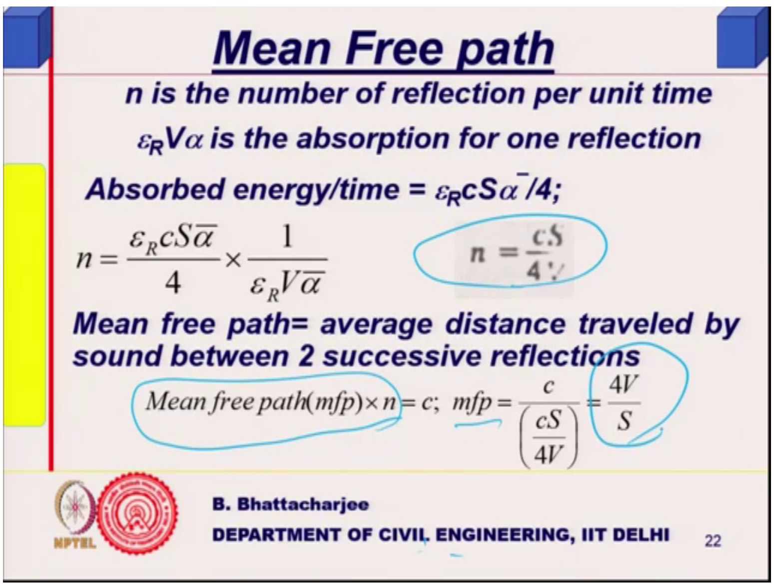
V alpha goes away right epsilon R goes away, I am left with c S by 4 V c S by 4 V. So, number of reflection is equals to c S by 4 V number of reflection is equal to c S by 4 V right. So, mean free path is defined as I average distance travelled by sound between 2 successive reflection, in the reverberant field.

So, there will be a infinite number of reflections, and their part length will be different because in 1 case I am might I am just drawing see this is the source. So, the after fast reflection it comes, and then this is the reverberant let us say this is the river in the reverberant field. So, this is you know distance travel between 2 successive reflection, next 1 it might travel much less distance depending upon angle of incident, and sometime it could be as you know like near vertical and so on.

So, there are infinite and sometime it goes back there hits here. So, this is the distance between two reflection, and then it might take much longer path depending upon what is this angle. So, actually distance between two successive reflection varies, it will vary it can be small as small as this or can be larger depending upon what is the angle of incident right, but average distance table by sound between two successive reflection I can talk about right, average distance between two successive reflection, and that is we call as mean free path that we call as mean free path right that we call as mean free path.

So, mean free path is equals to average distance travelled by sound between two successive reflections right.

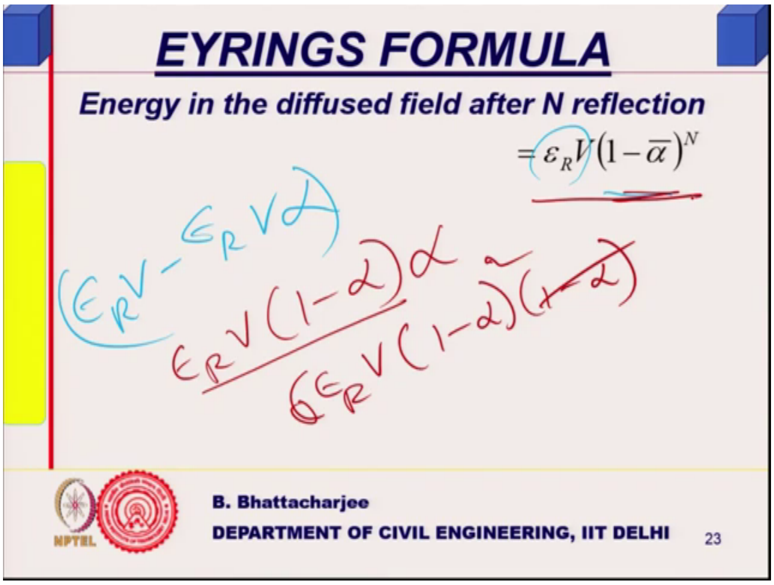
(Refer Slide Time: 16:52)



So, can we now get some expression for this, mean free path into n that is number of reflection per unit time must be equals to the velocity between 2 successive reflection it travels mean free path, and into number of reflection per unit time is a distance that will be covered in unit time.

So, that is velocity. So, mean free path this is equals to c and number of reflections we have already calculate out to be c S by V. So, mean free path is equals to 4 V by S mean free path is equals to 4 V S. So, mean free path is equals to 4 V right so 4 V by S ok.

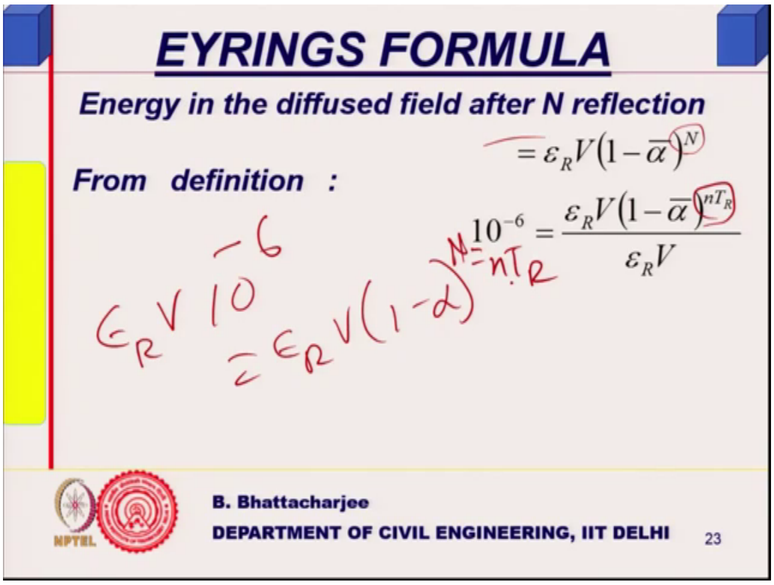
(Refer Slide Time: 17:33)



Energy in the diffused field that if you still after n reflection therefore, will be how much initially it was like this after n reflection it will be after first reflection it will be epsilon R V right, what will be lost is this much, what will be left after first reflection is this. So, after second so this is what after first reflection which I can write like that you know which I can write like epsilon R V 1 minus alpha.

Now, this enters and then goes to a second reflection. So, what will we lost this multiplied by alpha, what will be remaining in the field after second reflection epsilon R V 1 minus alpha into 1 minus alpha. So, that square basically to the power right, and this I continue. So, if there are n reflection after n reflection epsilon R V into 1 minus alpha bar to the power n alpha bar to the power n and right so after n reflection will be given by in the diffused field after n reflection will be.

(Refer Slide Time: 18:37)

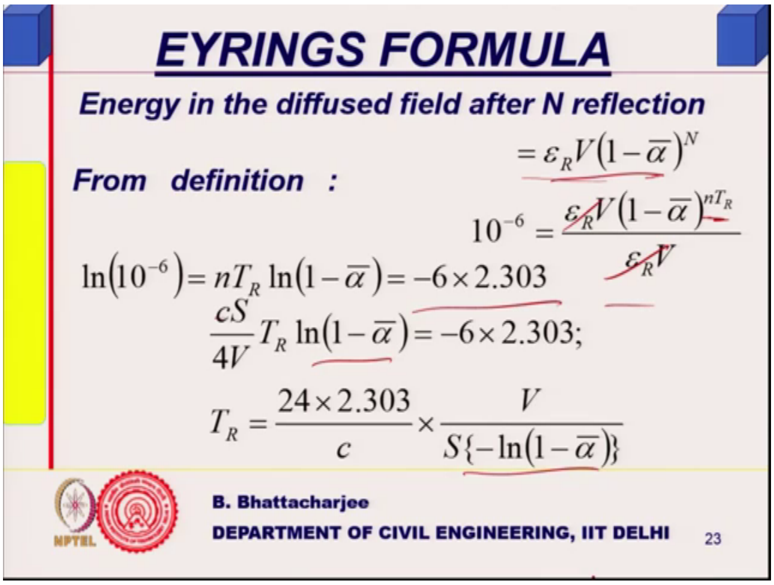


From definition you know this must be equals to n into T R if I define reverberation time. So, I am interested in reverberation time deriving it in another manner. So, n T R T R is the reverberation time into n is the number of reflection required for it to decay by 1 millionth of a time. So, originally it was epsilon R epsilon R V into 10 to power minus 6. So, epsilon R V into 10 to power minus 6 is the final right that must be equals to what has been lost in 1 minus alpha to the power N reflection, and n is equals to small in to clear where n is a number of reflection per unit type right so, into T R so that is how the definition.

So, I can get you can see that I get an expression for T R again, in another manner right from this, because 10 to power minus 6 into e R we must be equals to whatever it will be lost in during T R time, and in T R time I have got n into t R a number of reflections and each reflections you know after n reflections that is what I get. So, that many number of reflection would have occurred during this period of time T R, and after that the final sound that is there in the field is given by this.

So, that must be equals to e R V into 10 to power 6 by definition, because I am defining reverberation time as that you know reverberation time is that which will reduce it to 10 to power 6.

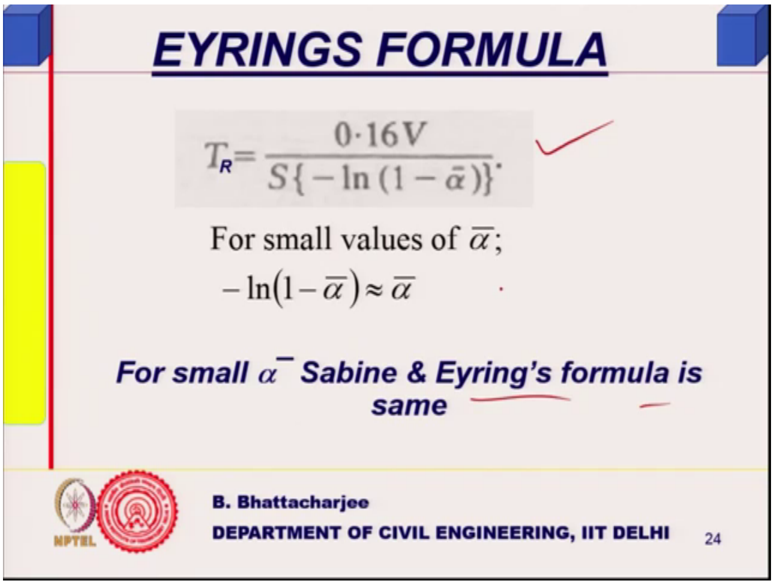
(Refer Slide Time: 20:18)



So, this is the expression I take log of both sides I take log of both sides. So, I get this will cancel out anyway, I am left with n T R 1 minus alpha bar equals to l n 10 to power 6, which is a 6 into 2.303, and then you know n what we have already obtained as c S s by 4 B number of reflections in unit time. So, I just put in here and I get, then you know 24, 2. 303 c S l n 1 minus alpha 1.

So, you can see this is how much is this how much is this same 0.16 V 24 by 2.303 in SI units of course, it is because I am taking 340 meter per second. So, this is this is 0.1 0.6 V. So, T R is equals to V divided by l n minus l n 1 minus alpha bar that is the derivations in slightly different way.

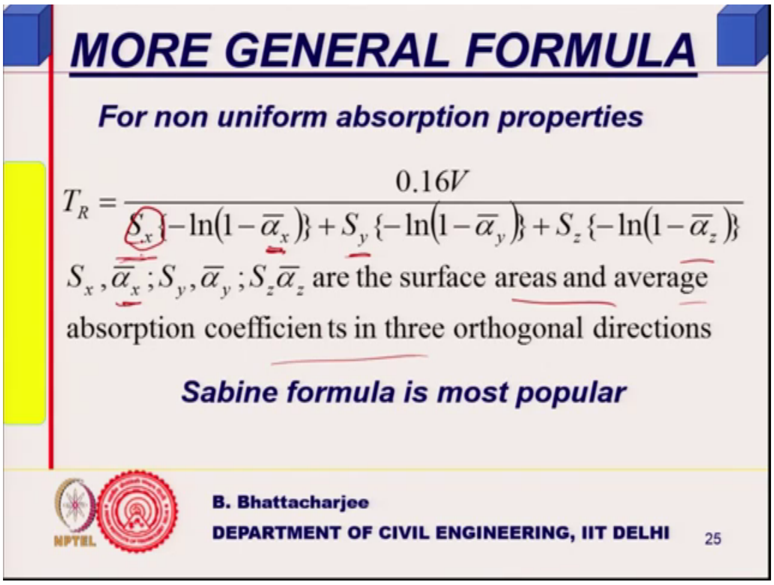
(Refer Slide Time: 21:20)



And this is called this is called Eyring's formula, this is called Eyring's formula, actually the Eyring's formula, but small value of alpha bar l n you know minus l n 1 minus alpha bar is equals to alpha bar.

For small values of alpha bar, 1 minus alpha bar is equals to approximately equals to alpha bar this you can prove of course, from algebra. So, these 2 formulas same when alpha is small Sabines formula was the earlier 1 Eyring's formula is slightly different for small absorption Sabines formula is same as Eyring's formula you know for example, if you look at it 1 minus I mean l n 0.9 l n 0.9 a negative value of l n 0.9 would be very close to 0.1, or you can express this as series and you can see that approximated to alpha bar that you can do for many you know, but there is want something more than that actually, both if I use this formula both assumes that both assumes that my absorption average absorption coefficient alpha, which means that by and large by ever absorption in all the surfaces should be of similar order not too much of difference is there, otherwise there will be some more absorption in 1 corner some less absorption in another corner.

(Refer Slide Time: 22:49)



We are assuming all diffused field. Now this formula can be extended actually for non uniform absorption properties. So, Sabine formula also assumption that is all absorption, uniform diffused field there are that kind of assumptions they are both in there both of them, and if you extend this idea of different absorption in different surfaces, then you can extend the Sabines formula in this manner, I am sorry Eyring's formula in this manner.

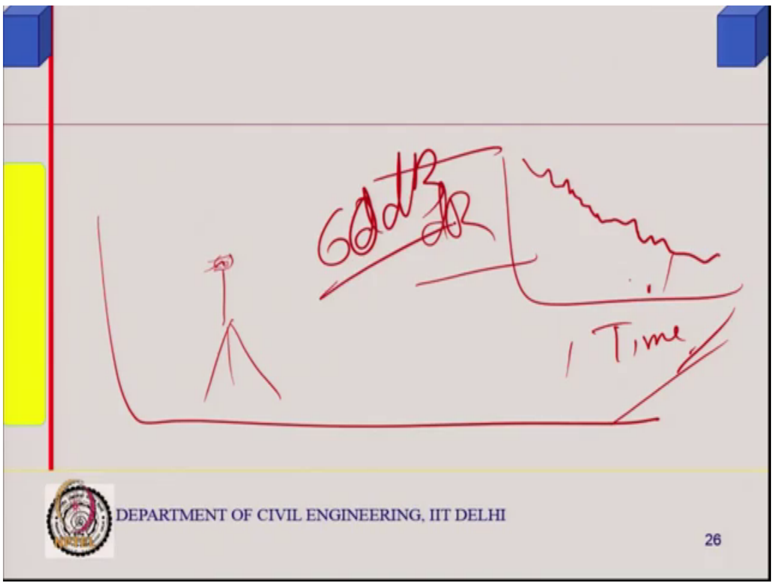
So, that gives us a way assuming discrete reflections, and taking the loss in every reflection rather than assuming a continuous decay, this gives us a way to actually extend that formula also. So, if you take this is that S x let us say x y and z direction, I can consider 3. So, surface areas in this wall and that are the wall opposite wall. Let us say that is S x and they are likely to have similar kind of absorption, similarly S y is let us say the you know the floor and the ceiling, and as you know S z is let us say floor and the ceiling as y is the other direction, and there absorptions are likely to be similar or ever is absorption I can take, then I can get an expression for somewhat none of the non you know non uniform absorption as well.

Assuming that every absorption in two opposite surfaces. So, 2 3 sets of them I get an extended formula right. So, that is what it is. So, where S x alpha x etcetera are the surface areas an average evolution coefficient in three orthogonal directions right so, it gives us a way to actually take account of that scenarios. However, we use this only for our purposes of auditorium design, and similar sort of thing, but I think this idea is important, if your absorption coefficient or large and something you want to extend this and there may be better ways, actually there are better much better ways today what is called image tracing actually there are software's which can do it.

So, it would do not like the diagram fast diagram I showed, there is a point source and going in ever every direction. So, you can simulate this in a software and it is available I mean it is even you can write your own program it is not a big deal it can be just simply you can write it. So, this alpha x bar is the average absorption coefficient corresponding to 2 surfaces, three orthogonal direction I have said assuming it is a rectangular body by enlarge, and rectangular parallelepiped.

So, in 1 orthogonal direction I have got 2 surface, other orthogonal direction I have got another 2 surface, and average absorption in these 2 surfaces and average absorption in the other 2 surfaces, and average absorption in the other 2 surfaces, and corresponding surface area. So, in three orthogonal direction I am taking the surfaces, and there are the averages absorption right, but then finally, we use most of you know we do not want to get into that kind of complicated, you can verify it and it is possible to measure the reverberation time also, I am not sure whether I have slide here or not, but just before I can just mention to you see what you can do is you can put a source at a given point. So, what is the source it would be a microphone put a microphone some are there right.

(Refer Slide Time: 26:16)



Within the space room put a microphone, and then measure you know it has to of course, the level meters some level meters which can trace track the sound and record it also.

So, meter with recorded the modern day instruments all are possible. So, what you do you put it off we say it is connected to a constant power source, and a constant frequency any frequency you want to test, then measure the decay at 1 point other points you can repeat it. So, time versus dB level at certain distance you measure with the sound level meter and recorder. So, you will find that it reduces in some manner like this right, there will be and then you can find out measure the time required to reduce it to by 1 million, how many dB is there 60 dB.

Student: 60 dB.

60 dB. So, how much time it requires it to reduce to 60 dB by 60 dB that is right. So, will break and then again I start.

Thank you.