Energy Efficiency, Acoustics & Daylighting in building
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Lecture – 37
Sound within Enclosure (contd.)

So, continue from where we were. So, we are talking about reverberant field in an enclosure.

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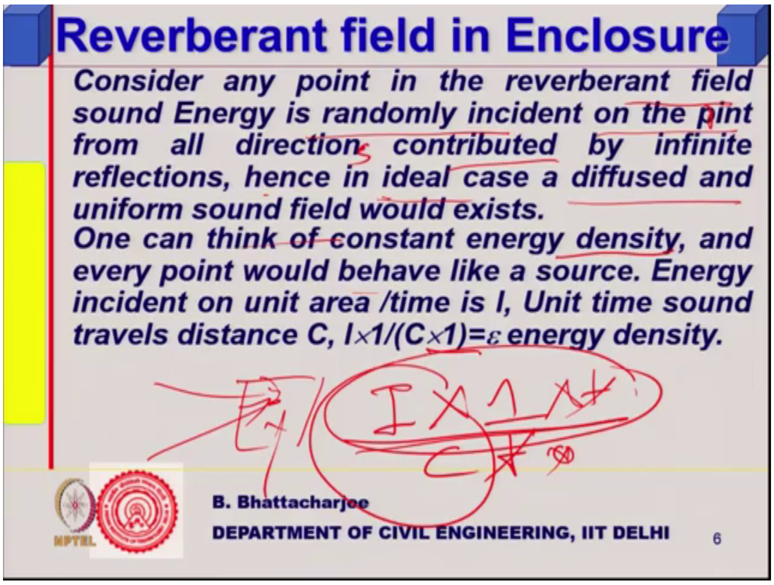

Now any point in the reverberant field basically if I consider in the whole volume of the room enclosure. So, it would receive I am keeping the direct sound out only the reflected sound and infinite reflections will reach to any point like that is what we have seen through this diagram if you recollect.

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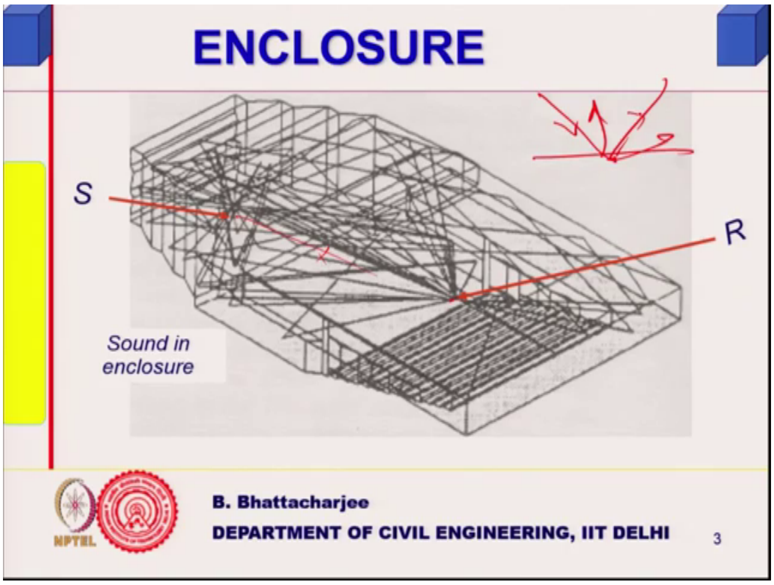

So, any point in the receiver if you look at it receiver you will get infinite number of reflections coming. So, I can consider any point and there will be infinite number of reflections coming. So, direct path I just eliminate out I do not consider this I am only talking of the field that is because of the reverberant you know reverberant field that is because of reflection coming from all direction, since it comes from all direction we call it diffused because specular means something like this specular reflection right diffuse means it will go in all direction.

For example if you look at light on a mat surface or a rough you know like rough surface it will spread all in all direction while if it is a mirror polished surface it will go in.

Student: One.

Only one direction, so that specular following loss of reflection and all that so, since it comes from all directions it actually diffused sound it is diffused right coming from all direction and coming randomly it is not you know it is not in one direction. So, random from all directions of that is diffused field. So, reverberant field is any point in the reverberant field if I keep the direct field out then it will receive sound from all directions and if you look at the whole room every point will be similar because every point will receive reflected sound form in finite reflections from all directions.

So, you have a diffused field in ideal condition it is uniform. So, that is what is written here consider any point in the reverberant field sound energy is randomly incident on the point it is not pinned it is on the point you know it is on the point from all directions contributed by infinite reflections and hence in ideal case a diffuse then uniform sound field will exist defused because it is coming from all direction uniform because it will be same throughout right and that is what is reverberant field and you can see that that is what if you remember this was reverberant field and this constant.

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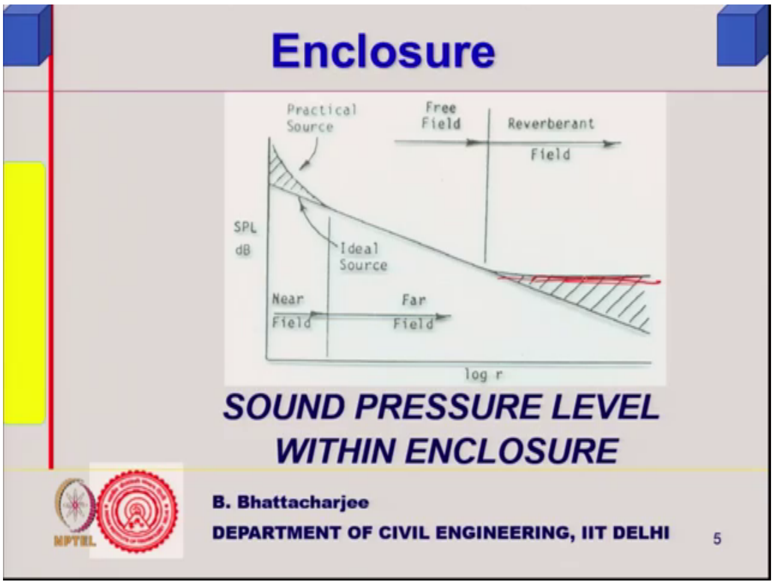

So, where reverberant field is dominant it is level is higher because we said that if I sum sum up 2 levels higher level always dominates because you know like you will take the either the intensity some of the intensity. So, 60 dB plus 40 dB 60 point something or 61 or whatever it is. So, since in this zone reverberant field will be dominant it is constant. So, we are assuming uniformly diffused field forever barrier field right.

Now, one can therefore, think of constant energy density at every point because it is uniform. So, constant energy density that is what we are saying last class also I as just mentioning and every point I can treat it as a point source for any other point or any other surface for any surface everywhere sound is coming from all direction and this sound will be traveling in again you know it will passing through that point and from that point it will again go to any location onto the wall from the reverberant field sound will also travel to the wall or ceiling or whatever it is because sound is reflected sound is reaching there and then it is somebody.

So, it can be treated as a point source every small volume every point in the reverberant field can be treated like a point source and then we said that it is constant the the you know energy is uniform I mean the is uniform therefore, I can talk in terms of a constant energy density of the reverberant field right and what is this energy density because if I take unity area on the reverberant field unit area let us say if I take some unit area on the reverberant field like the sound is received from all direction it is unit area.

So, intensity here of the reverberant field let us say is I. So, I into 1 is the power that is coming in per unit time and unit 10 sound travels to the distance C. So, if I talk in terms of energy you know energy per unit volume energy per unit volume right energy per unit volume. So, this is this is coming in T times. So, let me put it something like this is a total energy coming in T time and T time sound travels through a distance of C by T it travels to a distance of C by T velocity of sound is C.

Student: C.

Velocity of sound is C right. So, basically you know through this unit area sound this is basically the energy, energy density will be C travels in unit time C T will be the it is what will travel in T type. So, if I divide the volume in which the sound will be contained will be something like this C into T into 1 again because area is one. So, I can write it as I by C or I into 1 by C into 1 whatever it is because you need time if I take. So, instead of taking T is no I can take unit time. So, C into 1

So, some time it is denoted by epsilon is the idea clear that energy density is I by C because I is the intensity received on unit surface and this sound will you know this sound will be contained in a distance of C into unit volume, volume will be you know 1 is the area if 1 is a unit area C into 1. So, therefore, the energy densities I by C last class also we looked into this I am just repeating the same thing.

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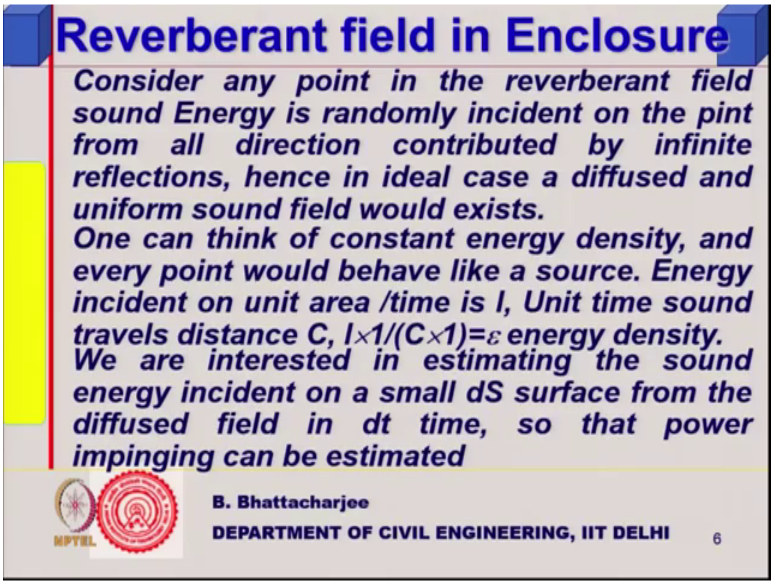

Our interest actually estimating the our interest is to estimate the sound energy incident on any surface I am assumed from the reverberant field, the reverberant field I can treat it as a point source right treat it as a point source every point and from that how much is the energy incident onto the different walls, why I am interested because some of this energy will come back, some energy will be absorbed by the surface itself.

So, I am trying to find out you know a kind of a reverberant field level that you know to in order to do that what I need is, I should be estimating the sound energy impinging on any small surface or any surface from a point then integrate for the whole room or the relevant volume because I am taking one point in the reverberant field and let us say it is falling on a small surface right dS surface small dS surface.

So, this diagram from the diffused field at let us say in dt time this sound comes. So, I am interested in finding out how much sound is actually incident upon dS surface any surface, let us say on floor or wherever it is from a point and then I can integrate that point appropriately to get from the whole room, how much the sound is impinging on small surface dS surface. So, let us say a small volume dV I take in the reverberant field because I said every small volume I can take it as a point.

So, that point corresponds to dV volume and energy density is constant. So, epsilon dV or edV would be you know whatever you notation.

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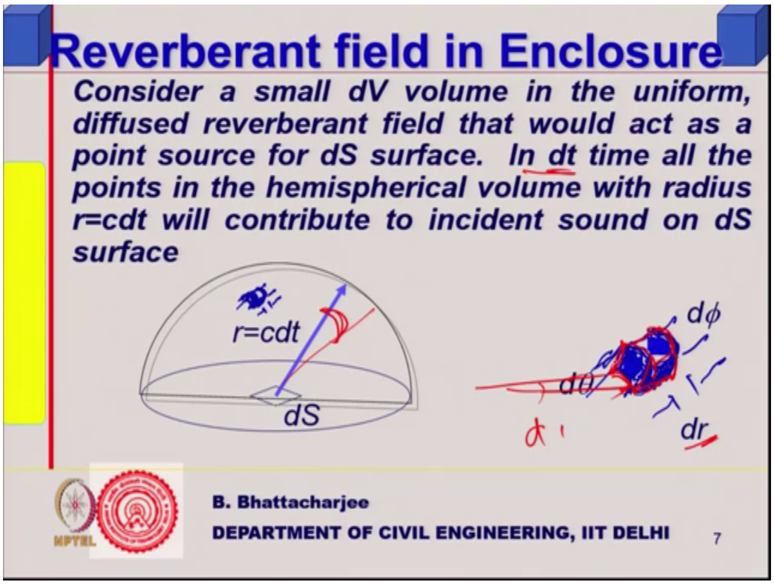

I am using is the energy in that small dV volume. So, dV volume let me consider and that would act as a point source for dS surface right in dt time all the points in the hemispherical how much no supposing I consider a surface like this surface like this said this is my dS surface. So, this is my dS surface this is my dS surface.

Now, in time you know if you are interested in dt time how much sound is falling onto this I take C into dt distance from here C is the velocity of sound into dt. So, it will come from all direction because it is diffused field. So, I can consider a hemisphere of radius cdt and all the energy coming from this volume will be incident upon dS in dt time are you getting my point see the you know this basically I am interested in finding out how much energy is impinging or incident upon dS small dS surface from the reverberant field which is uniform, throughout I take a small volume or then I got to integrate it over the volume from where it is coming.

The volume that will contribute in dt time, the energy that will come in dt time is simply from a hemisphere of radius cdt because velocity of. So, velocity of sound is c, c into dt is the distance. So, that will come during dt time that is what I am interested in now I am interested in finding out a small volume. So, take a small volume right that is what I have tried to draw, I am not very sure whether I have drawn it very good, but let me draw it slightly bigger then it is something like this you know this is something like this what I am trying to do is this is there is another are, there is another arc, there is something like this, something like this, something like this. So, this kind of a small volume I have taken right this kind of a small volume I have taken.

So, this is you know this is a blue colored I am trying to fill it in to show it that it is actually a volume and it will have 3 dimensions I can treat them as linear one will be d phi, d theta and dr . So, if I take any volume here radially outward will be dr, r going from 0 to cdt right finally, for the complete d phi would be if I take in azimuth plane. So, d phi is in this plane, this is d phi you know some azimuthal plane I can take d phi and these it have been elevation or altitude in the plane vertical.

So, these are all straight line actually. So, this is how I can make you know take a small volume, small volume I can take like this right.

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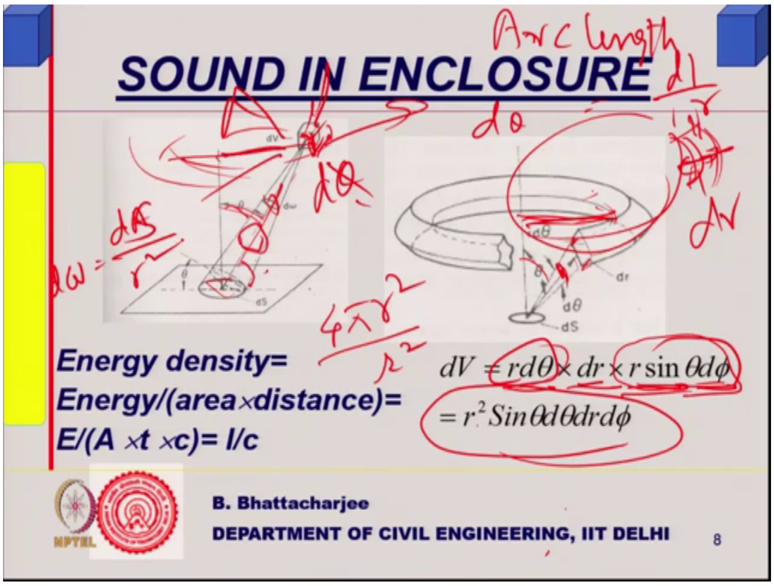

So, this is what I was trying to do in the last class energy density is I by c I take a small volume there dV and I said that dV can be written as rd theta because this angle is theta right and it makes an angle of d theta here this angle is theta. So, this, this normal to the surface and this point is theta and I am calling this, this covering it d theta this part is d theta.

So, this height will be rd theta this height will be rd theta this is part of the part of the circle this part of the torus element you know if I am talking of. So, this is part of this chord circle that you know any whatever is the radius. So, if this distance is small r. So, I take this volume at a distance r from that dS small r from the dS small r from the dS r root of course, vary from 0 to cdt that is what we have seen.

So, r I take at a distance of r and dr is that it you know radially outward this is this one I am calling it d phi and this vertical one is d theta. So, is this idea alright? So, this is theta d theta, this 3 my theta and this d theta and you know d phi is one of them is r d theta, this is d theta, this d phi, this is dr. So, 3 of them would be there. So, rd theta dr and this one would be r sin theta because r is this distance this angle is theta. So, this is r sin theta. So, this is our sin theta.

So, r sin theta, d phi you know because I am taking from the circle volumetric a ring ring having it sizes defined by dr along this direction a ring will have 3 dimensions is not it. If I take a small portion from a ring or if I take a ring it will have one the the ring will have one direction like this because I am taking a ring and it will have I mean if I if I take the full ring then this will this is one dimension and full ring if I take. So, it will have a vertical height and as well as you know this, this, this dimension as well this dimension as well. So, vertical height and this this I am calling as dr this I am calling as dr you know and this vertical one is d theta rd theta.

So, rd theta and this is small d phi, phi will vary from 0 to 2 pi, phi will vary from 0 to this azimuthal plane and r sin theta d phi that is what I am writing. So, for this small volume r sin theta d phi, dr is the outward towards the radius and r d theta is the along the elevational plane. So, that is the volume. So, volume this one is r square sin theta d theta dr d phi is it right is the volume understood, this is the volume now from this volume it contributes to the dS surface.

So, these are point source this is a point source and it is actually the sound energy is incident onto the dS surface. Now in order to find out how much sound will be actually coming it will radiate equally in all direction because it is diffused field a point source radiates equally in all direction in order to find out how much sound will be reaching on dS surface I got to find out the solid angle subtended by dS angle dS surface on the point solid angle subtended you know from d s..

So, this is the solid angle this is the solid angle and solid angle is what analogy to our regular 2 dimensional angle, angle is defined as arc length divided by radius. So, for a circle it is twice pi because total Arc length is the perimeter of a circle which is twice pi r divided by r. So, 2 pi that is a constant actually that is how you know now if you extend this idea to 3 dimension not for surface for a sphere 2 dimension of the fail you know the perimeter you are looking at now you take to the sphere then it will be 4 pi r square is a surface area divided by r square.

So, solid angle subtended at the center of a sphere is 4 pi stay radians that is right now analogy from planar angle here we define solid angle as the surface area normal to the direction of the radius there it was Arc length normal to the direction of the radius. So, normal area normal to the direction of the radius divided by radius square earlier it was Arc length, d l Arc length I can say d l divided by r that is not the d theta that is what we write. So, here we will talk in terms of d omega we write it d omega is equals to dA divided by dS surface here divided by r square dS divided by r square.

Now, let me just write it separately.

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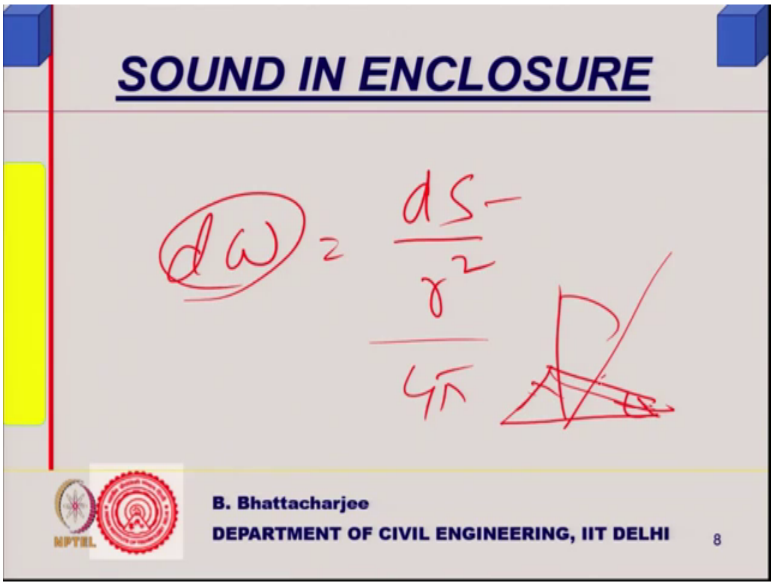

So, that is what we are saying dS divided by so d omega will be equals to dS divided by r square. So, solid angles are defined in this manner it is the area now that the energy contained in d omega is a total fraction of the energy that will be incident upon dS total energy goes out through 4 pi. So, this divided by 4 pi write is a fraction of energy that will be incident on dS, but I must take the normal surface. So, dS surface must be taken normal, but you see your surfaces like this ray is something like this. So, I must take the normal surface; that means, it will dS cos theta, dS because this you know this, what was this, this angle is theta. So, this angle so, it will be actually dS cos theta because I have to take the normal surface.

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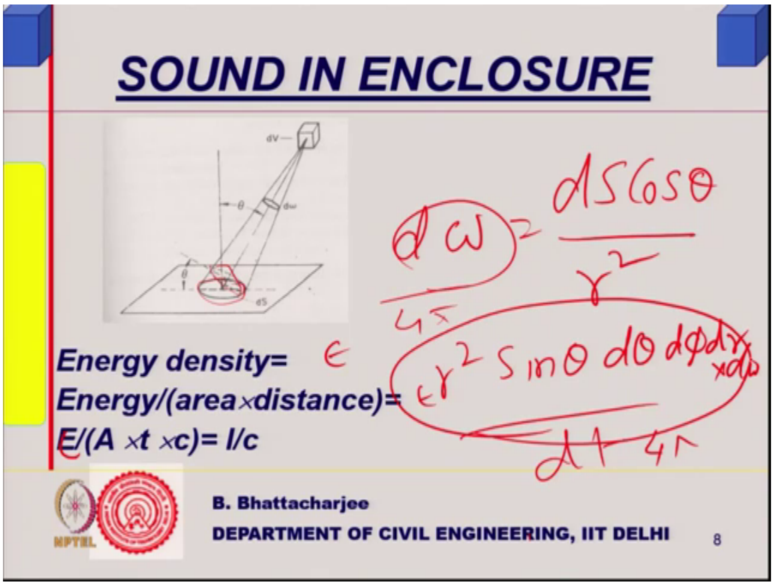

So, dS is this, dS cos theta will be this much. So, d omega will be given by dS cos theta divided by r square right and how much will be the total energy going out from this, volume we know the volume, volume was how much we said r square sin theta d theta d phi d r right something like this those are volume multiplied by the energy density if I am calling it epsilon I mean e by this energy density of epsilon if I am calling it. So, multiplied by energy density multiplied by energy density that will be energy that will be the energy and fraction of this energy reaches there.

So, that I can actually find out d omega divided by 4 pi r, 4 pi that is what it should come. So, fraction of energy incident on this dS surface I can find out right, because this is the volume multiplied by epsilon is the total energy contained in this one which is power of the I mean which is the source this divided by dt will give me the power, this is the energy this divided by dt will give me the power, this we divided by dt will give the power that will come out of this and w by 4 pi this divided by 4 pi into d omega will give me the fraction that will be incident upon this dS and dt time am I right am I did I make myself clear what I am saying is this energy density I found out multiplied by the small volume dV that is the energy this divided by dt if I am interested in finding out how much energy will come through come in dt power.

So, power of the source will be w in what should be this divided by dt and this divided by 4 pi goes along 4 pi r square goes along all direction, multiplied by d omega is the fraction which will be received at this one. So, this is what is.

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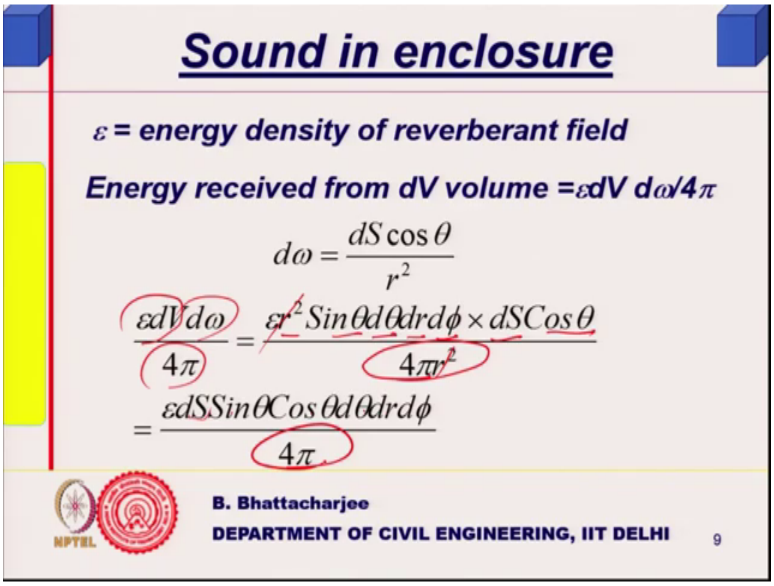

Now, this is what is? This is what is written you know further written. So, this is what is further written energy density of the reverberant field. So, that is what we are saying. So, epsilon dV d omega by 4 pi that is the energy that will be received this divided by d omega is dS cos theta by r square I have already explained you and epsilon dV d omega 4 pi.

So, that is the fraction of energy that will be received that divided by dt will be the power that is received. So, you see that if I write epsilon I have written dV was r square sin theta d theta dr d phi d omega is dS cos theta divided by 4 pi r square. So, r square gets cancelled I am left with epsilon sin dS is there any way sin theta cos theta d theta dr d phi divided by 4 pi. That is the fraction of energy you know fraction of energy that will be received if the epsilon dV fractional energy that can come in there right.

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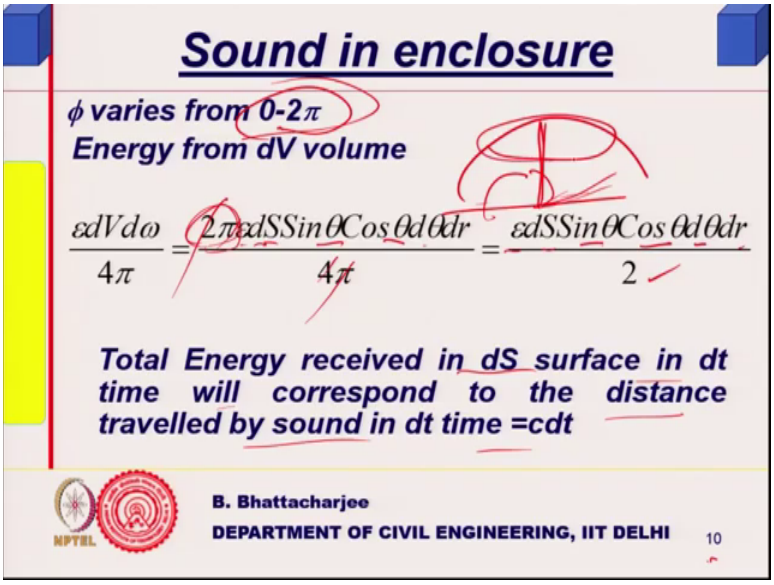

Now, this divided by delta dt will give you that power that is received. So, if I am interested in intensity of; obviously, phi varies from 0 to 2 pi I have already told you and this this is this, we have written like this 4 pi. So, this 2 and 4 will cancel, come something like this 2 pi 2 was there not. So, twice pi by putting this 2 pi d phi where phi varies flow easily I can write instead of d phi I am writing 2 phi dS epsilon dS sin theta cos theta d theta dr right divided by 4 pi. So, 4 cancels out and also pi cancels out.

So, I am left with epsilon dS sin theta cos theta d theta dr that is the fraction of energy coming from a complete ring because 0 to 2 pi now I have taken. So, it is coming from the full ring phi is out rest of the things still remaining. So, total energy received in dr surface in dt time, this will be correspond to the distance traveled by sound in cdt coming from the whole hemisphere. So, when I integrate this with time right integrate with time then I will integrate r will go from 0 to cdt and theta will go from 0 to 2 pi by 2 because hemisphere I am taking. So, 0 0 2 pi by 2 theta will go from 0 to pi by 2 right and phi I have already taken going from 0 to 2 pi.

So, this is hemisphere I am taking no. So, my hemisphere I am taking this angle. So, 0 to pi by 2 it will be automatically cover the other side also because I have already taken the ring I have taken this full ring 2 pi. So, if I take 0 2 pi by 2 whole of the hemisphere will be covered. So, that is what I am doing.

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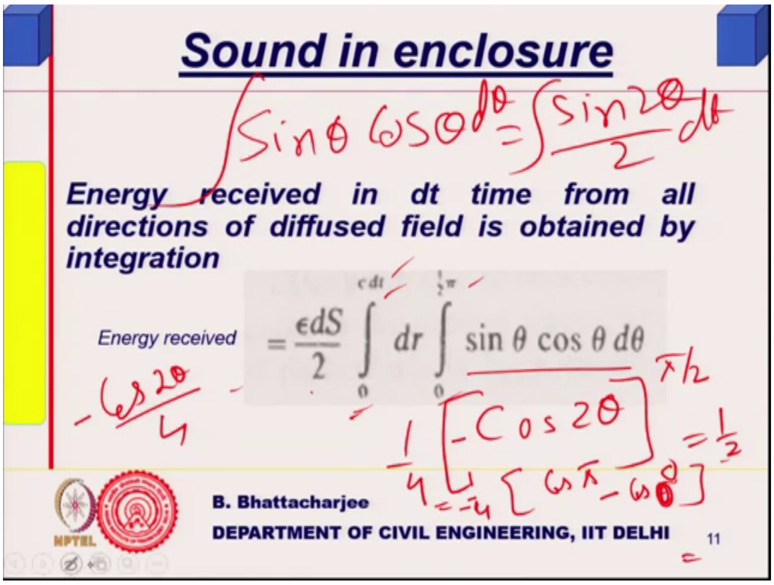

So, energy received in dt time from all direction will be give from the direction is diffused field is obtained by 0 to c t, 0 to pi by 2 epsilon dS by 2, epsilon dS by 2 was already there epsilon dS by 2 was already there you know epsilon dS by 2, I separate it out.

So, I get sin theta cos theta d theta and r goes from c to 0 to you know 0 to c dt etcetera. So, energy received I only integration that I got to do is here and this will become simply epsilon dS c dt . Now sin theta cos theta integration is, what sin theta? Cos theta can be written as sin 2 theta by 2 sin 2 theta by 2. So, if you integrate it integrate it you will get.

Student: Minus cos.

Minus cos 2 theta by minus cos 2 theta by 4 and theta going from 0 to pi by 2 so, I will have actually you know it will be cos 1 by 4 let me keep it out there is a minus sign cos 2 theta, this goes from 0 to pi by 2. So, this will be simply 1 by 4 remains or minus 1 by 4 I can take out cos 2 pi cos 2 pi I mean sorry cos pi minus cos 0 cos 0 sorry cos 0.

So, cos pi, pi is how much.

Student: Minus 1.

Minus 1 and this is again minus 1 cos you know minus 1. So, minus 2 this minus will go out this will turn out to be simply half because this will be 2. So, simply this will turn out to be simply half this will turn out to be simply half am I right sin 2 theta by 2 is this going to cos 2 theta by 4 this is pi by 2. So, pi by 2 by 2 that becomes pi and minus cos 0. So, minus 2 you know minus cancels out I am finally, left at half and finally, with half.

So, if I look at this one, if I look at this one, I will get this half this one cdt. So, energy received will be given as.

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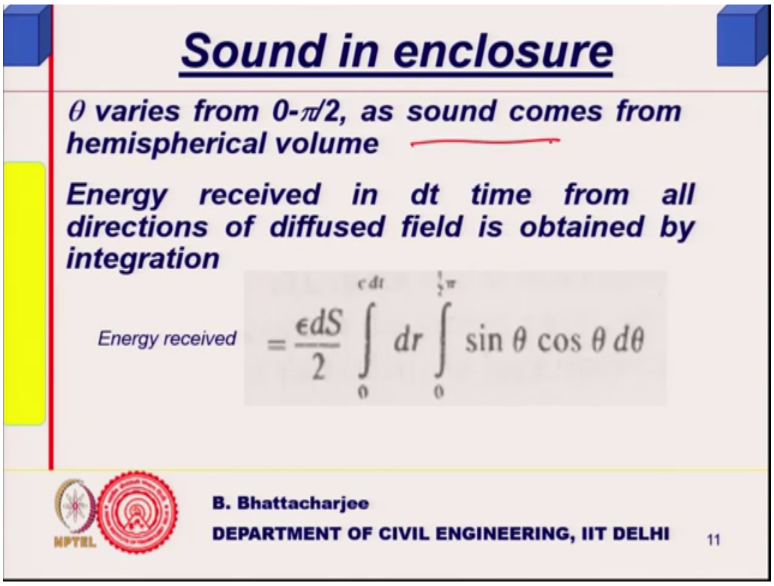

So, 0 theta varies from 0 to pi by 2 as sound comes from hemispherical volume and after integration oh it is shown here again full and this is equals to half.

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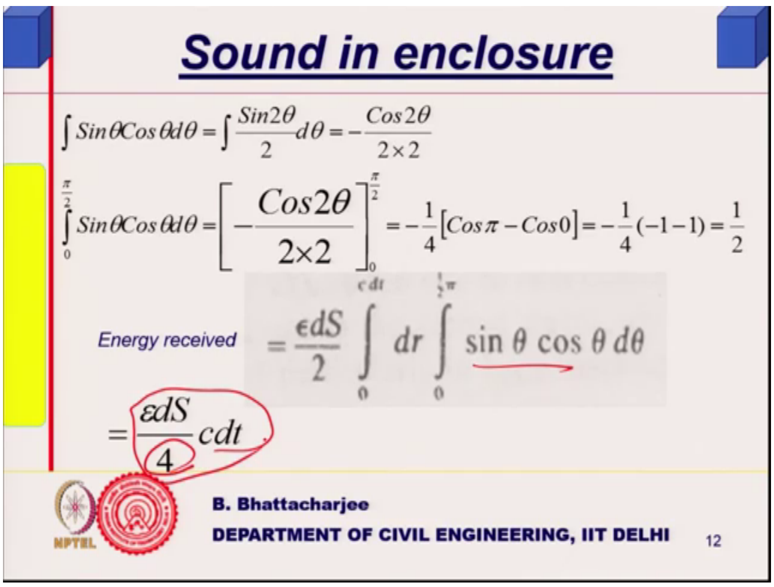

So, energy density will be given by this and this will be written as epsilon d dS remains there is a half coming from there. So, it becomes 4 and this is cdt energy received in dt time.

So, what is the intensity there, divided by dt. So, intensity, intensity at the surface or energy density you know intensity is energy density or energy come in per unit area per unit time on dr surfaces this divide this divided by right.

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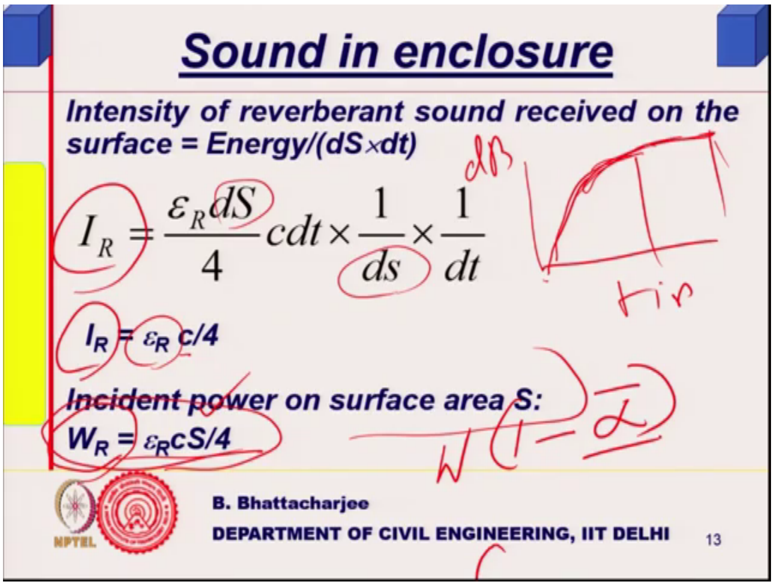

So, that is what it is. So, energy from the reverberant field will be given by you know and you divide by dS to get the intensity. So, you get simply epsilon R by epsilon R c by 4.

So, reverberant field intensity on any surface is given by epsilon R which is from the reverberant field c by 4 c by 4, if I am interested in energy coming in you know power received in dr surface it multiplied this by dS because I is power unit area multiplied by dS. So, anyway, so incident power on surface area of S, now I do not take dS I take full let us say larger one is epsilon R c S by 4 just by 4.

So, by doing this what I have been able to do, I am been able to find out from reverberant field, how much it will be will be the energy incident upon surface areas of the wall or ceiling surface of the roof, you know walls ceiling roof or whatever you call it ceiling basically . So, this is the power received on all the surfaces if S is a total surface area of the room, if S is a total internal surface area of the room, then this will be the total power received from the reverberant field into that into all the internal surface area, where S stands for all the internal surface area right.

Now, whatever power is received a part of it is absorbed in the wall and rest on is reflected back at any time whatever is you know a static condition for example, is existing like that I showed earlier that when you have you know which time you have decibel level here and we said the growth in growth and decay of steady field it will attain some value remain steady.

So, under such condition of steady condition before I put it off then it decays. So, this under this steady condition sound coming in must be close to sound gross burner or an average manner. Now how much is the sound coming in, how much is the sound entering into the reverberant field no how much is a sound it is only talking of reverberant field.

So, how much is a sound entering into the reverberant field, that sound after first reflection enters the reverberant field see direct field, then after one reflection it just enters the reverberant field and continues to get reflected from anywhere some of you might be playing squares right, but then you take after one reflection if you just allow it to go for a large number of reflection. So, after first reflection whatever is the energy comes in that is that is what is that you know sound enter you do the reverberant field how much is going out whatever is absorbed in the boundary.

So, right so, in see in you know this is the amount incident upon the boundary per unit time epsilon R c S over 4 is incident upon the boundary, incident upon the boundary in unit time right and during this time the sound that will enter will be given by if the power of the source which is causing this reverberant field because you have to have a dark field which is first have direct field.

So, this multiplied by 1 minus alpha bar where alpha is absorption coefficient because first reflection after first impression how much will come out w alpha will be absorbed direct sound will be absorbed 1 minus alpha will enter into the reverberant field that must be equals to the amount of energy lost out of the reverberant field.