

**Energy Efficiency, Acoustics & Daylighting in building**  
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**Lecture - 32**  
**Noise & Acoustic Fundamentals (contd.)**

So, that is what is a wave equation and then, we can define velocity get expression. For velocity see what we have done. So far as we said this is the wave equation  $c^2$  is related to. So, we can actually we can relate. How do I find out velocity? I mean I have to what I have. I have properties of the air, the medium because it will be a function of the medium itself. So, it is a function of medium.

Therefore, properties of the medium should dictate what should be the velocity as you know velocity of sound in air and that too through a metal is different or any other material like civil ending material such as concrete or soil. It will be different. So, it is a function of the medium itself. So, we can actually define this you know relate this  $c$  to medium properties and let us do that.

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**Wave velocity**

Direction of Wave →

**Strain =  $\delta y / \delta x$**

$$p = -E \frac{dy}{dx}$$

For a small element having pressure difference  
**Strain =  $\delta y / \delta x$**

The diagram illustrates a longitudinal wave moving to the right. It shows two vertical elements, A and B. Element A is at position  $x$  and has a height  $y$ . Element B is at position  $x + \delta x$  and has a height  $y + (\delta y / \delta x) \delta x$ . The distance between A and B is  $\delta x$ . A red circle highlights the  $\delta x$  label. A red arrow indicates the wave's direction.

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So, if I have they take a small  $\delta x$  element of the medium between A and B del you know partial since it is partial derivative. So, small delta  $x$  element we have taken at  $X$  plus del  $X$  distance.

Now, what is happening is, it will be compressed or expanding, right. It will be compressed or expanding. So, as if it is stressed, you might take unit area along the other direction, unit area along the other direction, right. So, you know some area along this direction. So, it will be compressed than expanded. So, this compression means there is you know it is kind of stress compression means there is a kind of stress, right. So, the stress is nothing, but the pressure stress is nothing, but the pressure above the atmospheric, right.

So, that is the stress and stress must be proportional to the strain. What is the strain  $\Delta y / X$  is our original dimension. If  $\Delta y$  is a particle displacement of this, this you know of this element is a small element. So, particle displacement, that means it has come out closer or gone away. So,  $\Delta y / \Delta x$  that is the displacement so, strain will be  $\Delta y / \Delta x$ , strain will be  $\Delta y / \Delta x$  you know. So, direction of the wave is like this. So, it has moved in this manner and you know the particle has displaced. So, particle was originally at  $y$  displacement.

Now, it is  $\Delta y / \Delta x$  you know that is what I can write. So, strain would be given by  $\Delta y / \Delta x$  and therefore,  $p$  must be proportional to  $-E \Delta y / \Delta x$  because  $p$  is the stress and proportional to  $\Delta y / \Delta x$  because compression. I am taking is positive you know compression shortening is shortening. Yes actually  $\Delta y$  is negative shortening means  $\Delta y$  is negative and that is my pressure positive, positive pressure. So, there is a negative sign  $-E \Delta y / \Delta x$  for a small element having pressure difference strain is equals to  $\Delta y / \Delta x$  and they can write pressure as this, right. Pressure I can write like this.

Now, my pressure is also available from elsewhere. So, rate of change of pressure I can find out  $\Delta p / \Delta x$  from my wave equation.

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**Wave Velocity**

For a small element having net pressure difference  $\delta p$  across  $\delta x$ ; the stress must be equal to mass and acceleration

$$-\delta p = \rho \delta x \frac{d^2 y}{dt^2} \therefore \frac{\delta p}{\delta x} = -\rho \frac{d^2 y}{dt^2}$$

*Handwritten notes:*  
-  $\rho \delta x$  is circled in red.  
-  $\frac{d^2 y}{dt^2}$  is circled in red.  
-  $\frac{\delta p}{\delta x}$  is circled in red.  
-  $-\rho \frac{d^2 y}{dt^2}$  is circled in red.  
- Below the equation,  $-\delta p/A$  is written, with  $\rho \delta x A$  circled in red.

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So, also pressure is nothing, but mass into acceleration into you know per unit area mass into acceleration per unit area. So, if my area is A, the force is  $\delta p$  into A and  $\rho \delta x$  into  $\delta A$ . So, therefore you know like  $-\delta p$  will be equal to  $\rho \delta x A$  and here is A. This is the force, this is the mass, A is an area I am taking unit area. So, this is equals to 1. So, that is why multiplied by 1 here also multiplied by acceleration.

So, this is nothing, but the mass for unit area. This is the pressure that I have applied. So, that must be equal to the particle acceleration. Therefore,  $\delta p / \delta x$ , I can write as minus  $\rho \delta x A$  you know  $d^2 y / dt^2$  if or  $d^2 y / dt^2$ . Actually I should be writing because partial derivatives they are in fact both are partial derivatives. So, this should be written in partial derivative terms and partial derivative terms, right and this should be this is corrected, right and followed by this.

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**Wave Velocity**

For a small element having net pressure difference  $\delta p$  across  $\delta x$ ; the stress must be equal to mass and acceleration

$$-\delta p = \rho \delta x \frac{d^2 y}{dt^2} \therefore \frac{\delta p}{\delta x} = -\rho \frac{d^2 y}{dt^2}$$

$$\frac{\delta p}{\delta x} = \frac{\delta}{\delta x} \left( -E \frac{dy}{dx} \right) = -E \frac{d^2 y}{dx^2}$$

$$\therefore E \frac{d^2 y}{dx^2} = \rho \frac{d^2 y}{dt^2}$$



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Del p del x is nothing, but earlier p we have found out was E dy del y. You know this also should be like that. So, earlier we have found out p was nothing, but minus E dy dx with a strain. This was a strain. So, del p del x I can get an expression from here like this and I can get an expression like this and remember these two ratios of these two one was what wave equation c square. These two are related through c square. Remember else do you know del square y del t square is related to del square y del x square through c square.

So, if I take a ratio you know dp dx, I have found out and these two dp dx I have found out and rápida x is also you know it is also this and these two are equal. So, these two, from these both consideration. So, E del square y del x square must be equal to rho del square y del t square, right and these two ratio of these two now I can use c square. So, in other words, I can get a relationship between ce and rho. Rho is the medium property is also bulk modulus of the medium, right because I am talking of volume compression and all that.

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**Wave Velocity**

$$E \frac{d^2 y}{dx^2} = \rho \frac{d^2 y}{dt^2} \quad \left( \frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} \right)$$
$$\therefore E = \rho c^2 \quad \therefore c = \sqrt{\frac{E}{\rho}}$$

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So, this I can write this is again why I have written I think it should be like that. So, this can be written like this and this is correct now this time. So, replace this from the earlier equation, the wave equation which was this. So, c square must be equals to rho by you know c square must be equals to E by rho c square must be equal to E by rho. It follows from this c square must be equals to E by rho. So, E goes to rho c square whatever it is and c square. So, c is equals to E by rho. So, that is what it is c square is equals to E by rho, right.

So, velocity is dependent upon the elastic modulus or I mean this is a bulk modulus actually and rho is a density of the medium you know this comes from earlier that we have derived. This is earlier we have derived. Now, we have done this and therefore, we can relate this and this comes out the way, right. So, c is under root E by rho and that is what we use elsewhere also. You know you want to find out the velocity of sound in say any medium steel E by rho air, it is 3 by 4 around 340 meter per second. In steel, it will be 5.5 or 5.6 kilometer per second order or order of you know kilometer per second.

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**Acoustic impedance**

Acoustic impedance is the ratio of pressure to particle velocity

$$Z = \frac{p(x,t)}{u(x,t)}$$

$$p = -E \frac{dy}{dx} = -E \frac{d}{dx} (y_0 \cos\{\omega t - kx\})$$

$$= -Eky_0 \sin\{\omega t - kx\}$$

$$Z = \frac{p(x,t)}{u(x,t)} = \frac{-Eky_0 \sin(\omega t - kx)}{-\omega y_0 \sin(\omega t - kx)} = \frac{Ek}{\omega}$$

$\frac{E}{\rho} = c^2$

$\frac{Ek}{\omega} = \frac{E}{\rho} \frac{k}{\omega} = c^2 \frac{k}{\omega} = \rho c$

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Now, all our purpose was to define something called acoustic impedance analogy to electrical impedance and also thermal also get something. Now, in case of electrical current is driven by the potential difference. Here the particle you know alternating current, right. It is different driven by potential difference.

Here the particle displacement or velocity is driven by pressure difference. So, we define impedances potential by the flow, right potential by flow. So, p ux you know this should be this is in complex form because not a periodic say in sine function cosine function. So, it can be expressed in periodic function E i omega etcetera. You know that form. So, it will be in complex form and Z is defined x i t. So, acoustic impedance is a ratio pressure to particle velocity acoustic impedance is you know ratio particle velocity and p you remember, it was something like this. I think del x.

So, this and p is something like this. Remember p was something like this. If I write it from the stress strain relationship, I get something like this and velocity I have got omega y 0 sine omega t cakes. These two are common. So, therefore my acoustic impedance is given as you know omega 0 is not there. These are not there. So, E k over omega and how do k is what omega by omega by c c omega by c. So, this is omega by c. So, what was E? What was E related to c, ok.

Lets us write it. So, E rho E by rho is equals to c square. So, E is nothing, but rho c square. So, I write it as rho c square and k is omega by c and this side is also omega. So,

this goes away and this remains what is this. So, acoustic emit in acoustic impedance of the medium is rho into wave velocity, right. Acoustic impedance is simply rho c.

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**Intensity of sound**

Intensity is defined as the average energy passing over unit area (normal to direction of propagation) in unit time (power).

Power=force×velocity.  
Power/area=pressure×velocity

$$I = \frac{1}{T} \int_0^T p(x,t)u(x,t) dt$$

$$= \frac{1}{T} \int_0^T p(x,t) \frac{p(x,t)}{\rho c} dt = \frac{p_{rms}^2}{\rho c}$$

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So, it is rho c. Simply it is rho c, all right. So, this gives me a way to relate actually pressure to velocity pressure. So, this gives me a way to relate pressure to velocity. Now, this has got some relevance all that I have done. So far I am try to get some kind of a relationship of this kind, right. Now, pressure is measurable, right. I can measure the pressure. I have stands you know various kind of transducers which can convert pressure into electrical signals and therefore, I can measure, right electro dynamic type piezo type are these days.

There are so many of them, pressure transducers are there, but my ear actually it does not depend upon pressure, but it depends upon I mean what I hear, how much do I hear in crude sense or rather in subjective sense, what do we call loudness. Loudness, it depends upon how much energy per unit area is falling on to my ear. That is right. I mean the energy, density energy you know so, similarly the intensity that we talked about in case of heat, right.

In case of heat we talked about intensity. So, intensity is nothing, but energy per unit area and how much you know the temperature is of course, sensible heat, but heat flow per unit area that dictates how much energy is coming into my body or whatever it is, we are looking into or into the system. Here also energy density is important and loudness is

perceived, right. You know more energy coming on to per unit area you will feel the thermal stress more. Similarly more energy, they coming on to per unit area on a year you will feel the loudness more. So, it is related to the loudness is related to energy density.

So, an acoustic impedance of course will come and we can relate this. We will see that now this is a characteristic of the medium for air. It is about 400 or 5 you know reals. So, kg per meter square per second. So, that is how it is defined. Now, intensity of sound is nothing, but same thing energy density and you remember what per meter square power coming in per unit area for heat. Same thing here, it is defined as an average energy passing over unit area normal to the direction of propagation in unit time that is power per unit area. So, you know that the loudness I feel similar like as I was just explaining loudness I feel would depend upon the energy per unit area or energy density. It will depend upon the energy density. That is right.

Now, power per unit time, right and per unit area. So, power is basically force into velocity. You remember force into velocity force into distance divided by time, that is your power. Power per unit area is nothing, but pressure into velocity. So, power per unit area is pressure into velocity.

Now, my pressure is varying with time. Therefore, you know when I am doing, I am dealing with it, I cannot I can take instantaneous pressure, but that makes my life more complicated. I am really interested in energy coming per unit area per unit time. So, what I got to do, I got to integrate it over average energy coming per unit area per unit time. Since, it is periodic, it will vary from 0 to  $t$  and  $u \times \Delta u \times t$  velocity pressure into velocity that will give me the power  $1$  by  $t$  will give me what you call intensity.

So, intensity is the average energy per unit time per unit area, right in a medium and that will impinge on my ear as well. So, in a medium, in any medium if I am looking at it will be this and remember you can be written like this because my acoustic impedance is  $\rho c$  velocity  $p \times t$  divided by  $u \times t$  was  $\rho c$ . So, therefore I can write  $u \times t$  in terms of this and it comes out to be  $p^2$  square by  $\rho c$ . So, this comes out to be  $\int_0^t 1$  by  $t p \times t$  square, right  $p \times t$  square because  $p^2$  square is coming divided by  $\rho c$ .

Now, this if I integrate this  $p^2$  square over the period, time period divide by  $t$  you know I am squaring it up, then taking its mean  $1$  by  $t$  means average and if I take root of that, I



call it root mean square  $p_{rms}$   $p_{rms}$  square. So, I am writing it as  $p_{rms}$  square. If I take  $p$  average, average of  $p$  it will be 0 because your sinusoidal or course you know it is a sinusoidal. It will be 0, but  $p$  square is not 0. So, intensity is related average when I am dealing with average energy that will tell me how much is the loudness that will tell me how much energy is received on my year and related to loudness. It exactly is not loudness, but related to that we will see that.

So, it is we can call it  $p_{rms}$  square divided by  $\rho c$   $p_{rms}$  square by  $\rho c$  and you can actually find out sine square  $\omega t$ . If you integrate you know sine square will convert into  $\cos 2 \omega t$  etcetera and you can find out electrical technology. Also you might have done similar sort of thing because alternating current will be the same thing. So,  $p_{rms}$  square by  $\rho c$   $p_{rms}$  square by  $\rho c$ .



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**Intensity of sound**

$$P_{rms}^2 = \frac{1}{T} \int_0^T p^2(x,t) dt = \frac{1}{T} \int_0^T p_0^2 \sin^2(\omega t - kx) dt$$

**For ideal point source, strength of the field**

$$I = \frac{W}{4\pi r^2}$$



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So, intensity of sound is given by  $p$  square  $\Delta t$   $1$  by  $t$   $p_{rms}$  square will be written like this  $p_{rms}$  square. I am saying  $p$  square, right  $P$  square  $x$   $t$   $\Delta t$   $1$  by  $t$   $0$  to  $2$ . So, this integral of this right and you can for ideal point source. So, you can integrate this and find out what is the factor, but we are not interested. We do not need to look into that. We are not interested really in this. What we understand is that intensity is  $p_{rms}$  square by  $t$  naught instantaneous  $p$  at any point, but it is  $p_{rms}$  square by  $\rho c$  is my intensity at any point. You know it has, its intensity is essentially energy per unit area per unit time. So,

its scalar quantity, its scalar its quantity, right. So, this is a scalar quantity for ideal source of course.

So, therefore, this is how we define p rms square and intensity. This is how we define, right. Well let me see if I have slight other, but I think we have already said it is an average energy passing through unit area in unit time for a medium average energy passing through unit area. So, unit time and in unit time will have number of cycles. Therefore, you have to find out you know because it will depend upon the frequency. So, that is why you are averaging it over the time and in unit time that is how it is, right. So, prm square that is how its coming. Now, supposing I have a point source just a point source ideal point source you know. So, let us say power.

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**Intensity of sound**

$$p_{rms}^2 = \frac{1}{T} \int_0^T p^2(x, t) dt = \frac{1}{T} \int_0^T p_0^2 \sin^2(\omega t - kx) dt$$

**For ideal point source, strength of the field**

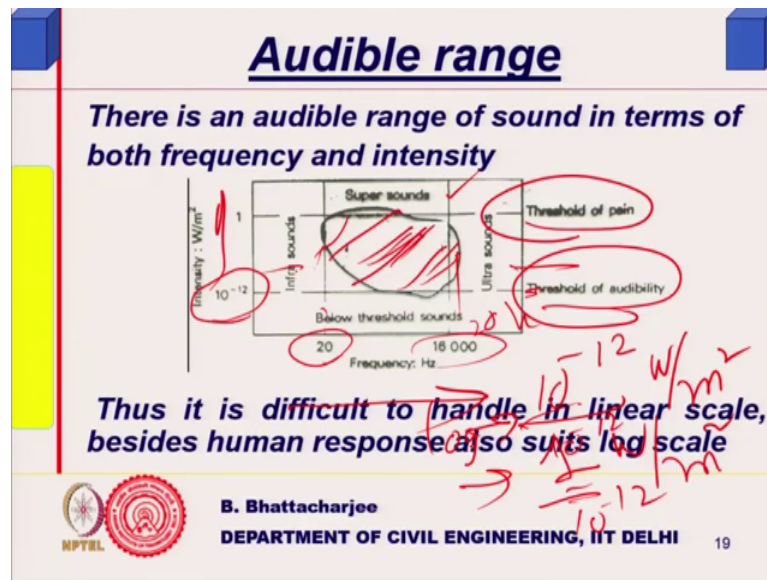
$$I = \frac{W}{4\pi r^2}$$

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It is power is w watts. It is emitting w watts in all direction. So, the wave is propagating along all direction equally, right. So, w is the power generated constant power source. You know constantly w power sound power; it is everything in all direction sound power. I am talking of sound energy emitting. So, at any distance r from this how much will be the intensity, intensity its power. So, energy per unit time so, per unit area I should do. So, this should be simply I divided by w divided by 4 pi r square. So, simply 4 pi w divided by 4 pi r square. So, simply 4 pi r square you know. So, intensity for a point source at any distance r from it will be given by w divided by 4 pi r square, right.

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So, it will come back to this, but all you know intensity some at very small intensity. You do not be able to hear at all. As I said your ear is sensitive to the energy per unit area per unit time and that is why we are so much bothered about intensity. So, higher the energy coming in you perceive and very high energy person comes into your ear like a lot of people became deaf during the atomic explosion 6th August 19 whatever it is 45 or you know this Hiroshima Nagasaki permanently. Therefore, many died, but even it because of the sound impact. You know sound can make you deaf. It is beyond certain level or if you stand behind a jet engine directly 120, you know it is very huge quantity of sound.

So, jet blasts I mean you may not be blown away, but your ear will surely go away if you are not along the line close to it. So, there is a point beyond which your eardrum get permanently damaged. So, all intensity is you cannot hear. Similarly very small intensity you do not hear like you like some insect moving on to this. You do not hear well you know sometime at dog taking off. Still you are able to hear that you know this or scratches or something. So, what I am saying there is a minimum level beyond you know which you cannot hear and there is a minimum level beyond which you cannot hear. Again I mean because it will be permanently damaged. So, there is a lower limit and there is an upper limit as well this is one issue.

The other issue is not all frequencies you can hear like I said when I was talking of thermal infrared heat here; we also looked at a glass. There is a visual range. So, certain

portion your ear, eye is sensitive to the spectrum of electromagnetic waves. So, you see certain wavelengths, certain frequency not lambda values you can see visual light rain. Similarly, sound also you see certain frequencies only. So, the frequencies beyond which you do not hear, we call it you know beyond hearing audible range, we call it ultra sound and you will have similarly infrasound.

You know you remember infrared ultraviolet. So, similarly infrasound and ultrasound and ultrasound is used for very other mechanical you know mechanical diagnostics and similar sort of thing, mechanical diagnostics and health diagnostics both. So, we are not interested in that at the moment. So, there is an audible range of sound in terms of both frequency and intensity, both frequency and intensity and this is what is you know.

So, supposing I plot frequency along this direction frequency along this direction. I hear slightly below 20 and up to 20 k. I hear 20 cycles per second to 20000 cycles per second. So, that is audible range of frequencies. This side could be ultrasound, this side will be infrasound and then, there is a bottom line beyond which I will not hear at all intensity wise and there is a high intensity beyond which you have pain and still go farther, you will have permanent damage.

So, this is called threshold affordability. This is called threshold of pain and it is in watt meter square if you see intensity in watt per meter square because power per unit area per unit time average power. So, then 10 to a minus 12 what you can barely hear and watt per meter square, it starts paining, right. So, this we call as threshold of audibility, this we call as threshold of pain and you know this is the audible range starting from this.

So, this is the zone which you can actually, this is the zone which you can hear. Hearing range is this, right. So, typically this is what it is. So, this is below threshold sound, this super sound and so on. So, that is how it is. So, there is a range that is difficult to handle. So, therefore, it means that you see if you look at 10 to the power minus 12 to 1, now it will be difficult to handle in linear scale 1 to 10 to power 12 times. So, if you take log of this, then this becomes you know minus 12 to 0 log 1 will be equals to 0 minus 12 to 0.

That is also not a very good range and if I divide by this value, then just divide both of these by this value. This will become and take log, then this will become 0 dividing 10 to power minus 12. This becomes 1, this becomes 10 to power 12. Then, I vary from 1 to 12 multiplied by 10, then you get 1 to 220. So, that is a distance scale and therefore, you

know this was after the name of Alexander Graham Bell. So, we actually express sound levels as we call it intensity levels in a scale called decibel scale which is a relative scale reference is threshold of audibility.

So, the intensity divided by threshold of audibility take log of it multiplied by 10 take log of it is a bell multiplied by 10. It is the same.

Decimal. So, that is why the decimal is. So, there is another reason why we do not handle it in linear scale. Remember we talked of factors law and we said that it is also logarithmically related to your sensation. So, sensation is logarithmically related some or other and therefore, we use it. So, decibel is a relative scale and you might use it for many cases, not here many kind of such signals people use relative scale decibel.

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**Intensity Power & Pressure levels**

$dB = 10 \log (I / I_{ref}) = SIL(L_I)$

$SPL (L_p) = 10 \log (p^2 / p_{ref}^2) \text{ dB}$

$PWL(L_w) = 10 \log (W / W_{ref}) \text{ dB}$

*Handwritten note:  $I_{ref} = 10^{-12}$*

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So, for example, even in a microwave attenuation you will be talking in terms of decibel. So, decibel is  $10 \log I$  by ref and we call it sound intensity level. So, intensity levels are expressed in decibels and quite often we denote by  $L_i$ . Some people might denote by something else, but here we will keep it  $L_i$ , right. So, sound now if it is easy, intensity is not directly measurable, but we can measure pressure and if I divide it by the reference pressures square actually as we shall see divided by some reference thing take log and  $10 \log$  that will be a relative scale. So, even pressure level I can express in terms of decibel pressure level. I can express in terms of decibel.

So, you can see that  $L_p = 10 \log \frac{p^2}{p_{ref}^2}$  because intensity is nothing, but proportional to  $p_{rms}^2$ . Now, whenever I am talking of  $P_i$  only mean  $p_{rms}$ . I am no longer talking of  $p_{xt}$  or  $p_a$  you know amplitude, I am not talking about amplitude, I am talking in terms of  $p_{rms}$  per  $p_{rms}$  because that is what the output you will get from any instrument or anything of that kind and we are dealing with that situation only and you remember  $p_{rms}^2$  or  $p_{ref}^2$ .

So,  $p$  this would be related to  $I_{ref}$  actually which was  $10^{-12}$  to the power minus 12, right. So, this can be pressure level can be written like this  $10 \log \frac{p^2}{p_{ref}^2}$ , right and this reference taken 2 into  $10^{-12}$  I think and similarly for a source, source its power is  $w$ . This is for medium somewhere in the air. I will measure the intensity somewhere in the air, I will measure the intensity and that will fall into my ear and this is also because in sound pressure level can be measured. Intensity measuring is difficult because energy measuring per unit area pressure is easy. Sensors can measure them. So, sound pressure level we measure and the other one is power level. Power level is for source.

So, sound power level is for the source and that we call as  $w$  divided by  $w_{ref}$   $10 \log$ . So, power level is for the source pressure level is for the field. So, is the intensity level. Pressure level is what normally would be talking about because that is what is measurable, right.

These are related. Actually we will see that how they are related, right. So, power level is for source. So, if you have, if you talk in terms of a source, you talk in terms of power level dB for the field, you will talk in terms of pressure level dB unit is same. Unit is same because they are relative units. So, that is what it is right, ok. So, any question if you have while I will take these questions and then, next class will look into it.