Energy Efficiency, Acoustics & Daylighting in building
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Lecture - 15
Heat Flow in Building (Frequency Domain)

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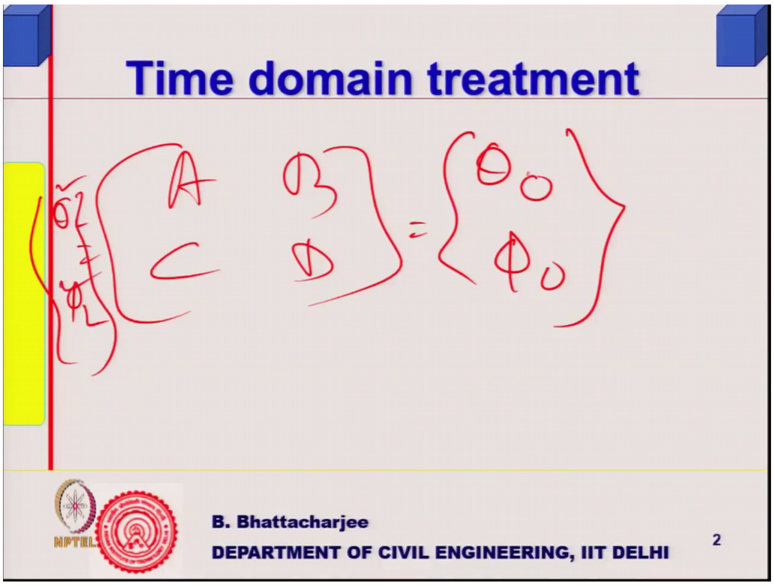

So, basically you know A B C D matrix that we know so, what we have seen is that actually theta l phi, l can be written in terms of theta 0 phi 0 something like this, we can write it now this boundary condition 2 of them must be known to find out the other 2 I can treat them in different manner, I said that 1 of the ways is doing it in frequency domain, but if I do it in time domain treatment which we are not doing in this class in details, but I will explain you what it is, because American society of heating refrigeration engineers good old days.

Now the computational capability is too large, but earlier days when even know you see this kind of ideas this came from I think milk bank you know frequency domain came from milk bank and Harrington lien, and I think time domain came from mostly from the united states I am not remembering the gentleman’s name. And now they used a different kind of treatment the treatment was something of this kind you know any function.

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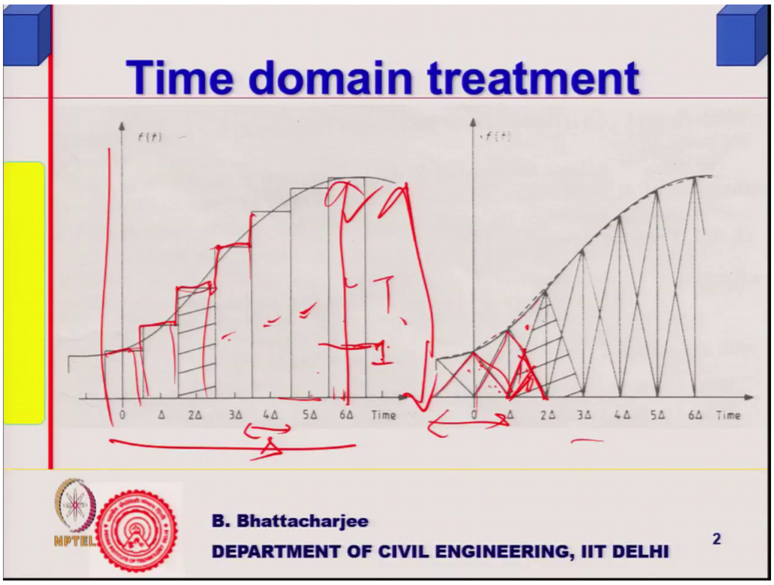

Any function if you look at it say temperature versus time, and if it is something of this kind you can actually approximate it by pulses take delta T delta time this is the delta space time, and small delta time and you can break it into rectangular pulses approximated by rectangular pulses right, and better would be if you take a triangular plus for example, this is you know this is 2 delta. And this is a triangle, then there is another 2 delta triangle. So, actually there is sup you know superposition of 2 of these and it actually fits better, you can see we very close where this is something like. This one triangle if you sum it up this 2 triangle this triangle you know 1 triangle like this twice of this triangle it fits fairly close to the actual curve.

There are other kinds of transformations available for this sort of situation, so what I do is I find out the response to single pulse, response to unit temperature pulse single unit pulse so, this is unity let us say and response to this unit pulse I find out and that we train of such pulses this is one followed by this this this train you know a complete number of such pulses would be there. In actually a 24 hourly temperature profile there will be number of such pulse pulses, and unit pulse if I find out multiplied by this value that will be the response due to this pulse at time which has come delta time earlier will be 2 things will happen, this will have a pulse response of this 1 will not be instantaneous I mean response will be something like this.

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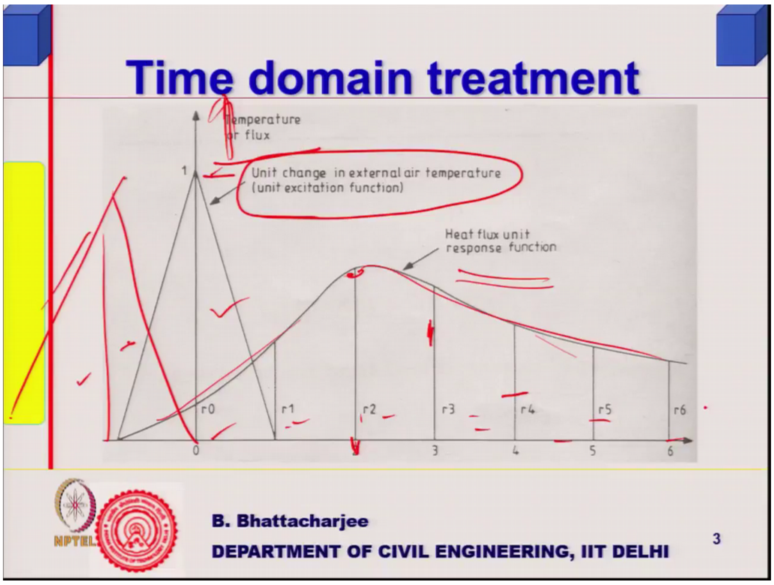

Let me see if I have response will be something like this for a triangular pulse for example, unit triangular pulse I will have response like this. And response will continue for quite some time, then it will dry die down gradually.

If I have a unit triangular pulse, if I you know I can get a solution of this 1 assuming my boundary condition that there is a unit triangular pulse of temperature, let us say or heat flux whichever I want may be temperature. So, I give a unit triangular pulse and response to this I can find out over the period of time. Now I would have another triangle before that and there is another triangle before that.

So, this one response of this 1 at let us say I am interested in this time, response of this 1 will be this much response of this 1 actually will be this much, because it is delta time earlier right, and response of the previous 1 will be this much do we get this point response you know so, the since there are trains of pulses. So, several pulses this is the last current pulse I know and there are many pulses before. And let us say I take unit, but multiplied by the actual amplitude or actual height because this is a linear system. So, like I show you here this is unit pulse if I multiply this response by this value, then I get the effect of this pulse at this point of time this pulse it is you know effect I can find out at same point of time, because it would have come 1 delta earlier.

So, I can sum them up series of them I can sum them up and triangular pulses they are better ones. As I said this is the basic philosophy behind it I am not solving them, because that is not amenable to hand calculation easily as I had tables available in their book earlier. Now of course, many I mean the programs are available I believe that early doe you know twos program in photon for might have been based on they I am not sure, but there are modern ones many of those softwares which are available are based on finite element or finite differences, because better numerical methods are available, but it really does not make difference I am just do I will do that one which is amenable to hand calculation or excel spreadsheet simple.

So, that you understand the thing software you can easily use, and if you know what is inside how to give the boundary conditions and how to give you know your inputs that is good enough, but basic principle is something else. So, time domain treatment is done this seminar as it is said temperature or flux, unit change in rectangular excitation heat flux, to unit response function that is known to us.

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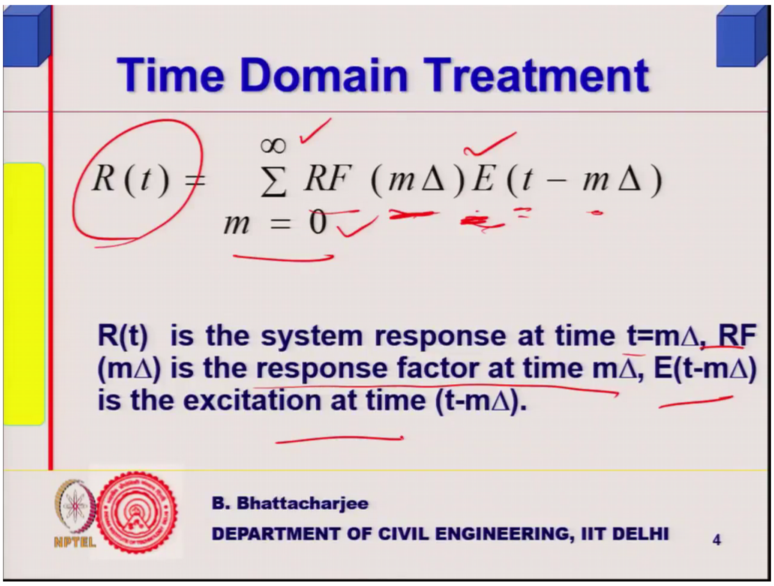

And then you know any response for a temperature it would be sum of m equals to 0 to infinity minute; that means, infinite number of pulses, starting from the current pulse to the previous pulse m delta right, m delta because each 1 is delta and its excitation is et minus m delta.

So, actual you know multiplied by the excitation I mean as I was just telling you so, this I am calling as E, this is E excitation. So, excitation current 1 will be you know current 1 will be m equals to 0 current is t. So, m equals to 0 and then m equals to 1 one delta time earlier excitation value E, and the response also earlier time. So, you sum it up the response of the system you can obtain for any construction you can obtain, and store them keep them recorded for your purpose for future uses, that is what was done R t is the system response at time excitation m delta these are response factors is response factors at time a delta this is the excitation at time t minus m delta. So, this is the time domain solutions; time domain solution, the principal is this and you can actually use this. Now the you know the solution to temperature this must be known to you.

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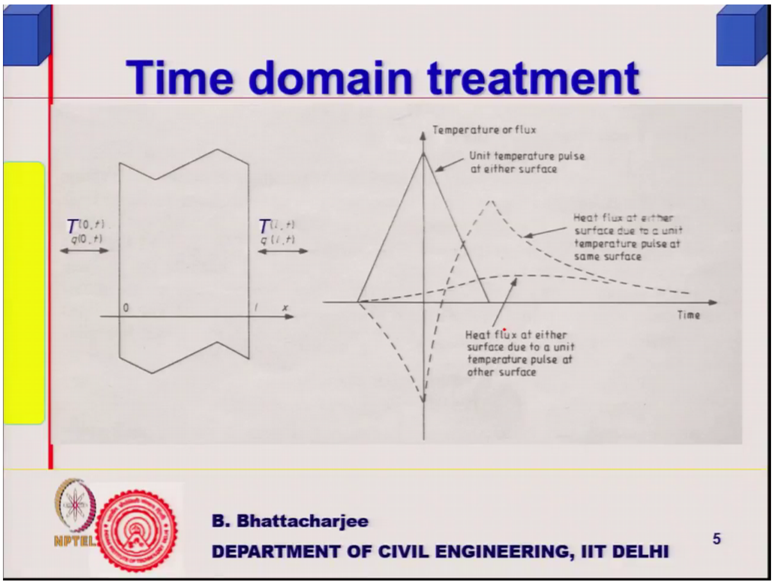

And this you solve that A B C D you know matrix for A B C D is related to the section, and it is a function of s A B C D is this matrix coefficients they are you know they are essentially function of you know their section and s right. So, they will be function of s if you when you for the pulse for a unit pulse the boundary condition now theta 0 is units pulse, you take Laplace transformation of triangular pulse and k into s buyer or k p Laplace transformation of first derivative of temperature d t d x that will be phi. So, that you take and their responses can be solute so, in a problem can be solved the A B C D is known you can solve this differential equation and solve values from those the response values were obtained, and then stimulated you can if you want you can find out in tables.

As I said will not solve this will not do this because you have to sum up so, many of them. So, it is a relatively cumbersome process you know you have to if you have even if it is stored summing up is to be done well, when computer was not there it was really cumbersome. Now it is not that cumbersome, but still we will do simpler 1 which gives you better understanding.

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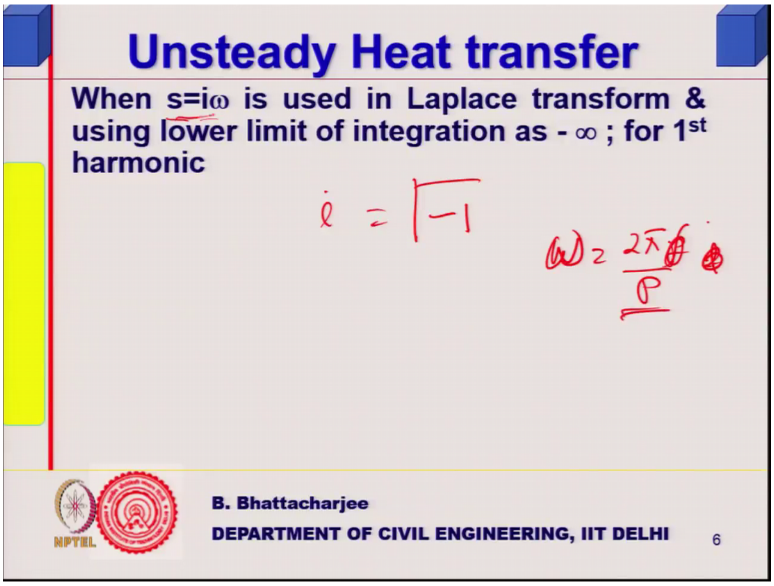

The simpler better 1 when frequency domain comes when i replace s by i omega, i is the hundred minus 1, i is nothing, but you know under root minus 1. So, imaginary and omega is twice pi F twice pi F right, twice pi F by p we are using you know twice pi by p F is 1 by into T, obviously I mean omega I am saying omega will be time period. So, capital T in many other places, you have used we used earlier p. So, maybe notations are getting capital p I used. So, twice pi by p. So, if I replace this by Laplace transformation lower limit of integration as minus infinity for first harmonic you can see that.

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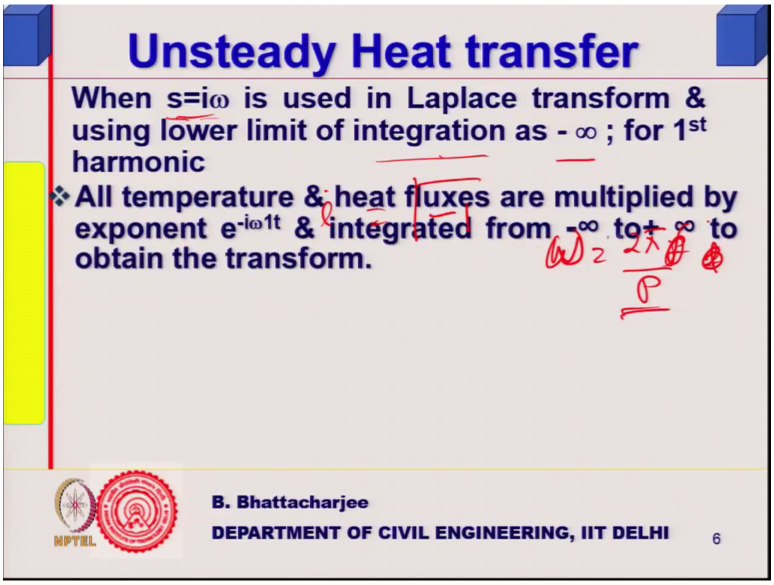

All temperature and heat fluxes are multiplied by exponent E to the power I is let us see, how we do it.

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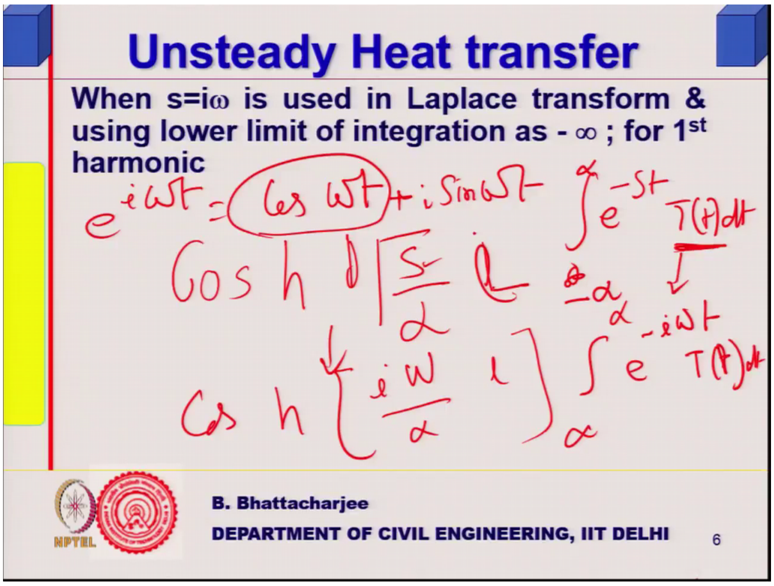

See if I am doing it let us do it for cos hyperbolic what was it under root S by alpha, and you know it was l or whatever it is that is right that was it. Now S I am writing cos hyperbolic I omega by alpha l right, that is what it will be and Laplace transformation would be what of temperature would be E to the power minus s t t as a function of t d t and this was from 0 to.

Student: Infinity.

Infinity well I then you know this is I can use minus infinity, here minus infinity to 0 the function is 0 right. So, I can use like this and what is E to the power s t minus s t, s I am replacing by so, minus infinity to plus infinity I omega T d t what is the E to the power i omega t, this will change like this and temperature Laplace of temperature will become something like this. So, what is E to the power i omega t what is E to the power i omega t, E to the power i omega t is cos trigonometric function cos omega t.

Student: (Refer Time: 11:56).

Plus i.

Student: (Refer Time: 12:00).

Right now if I multiplied by cos omega t this function what do I get can you recollect. If I multiply this t by cos omega t or sin omega t I get Fourier's coefficient, how did I obtain Fourier coefficient by multiplying this one, we collect this Fourier coefficient a j b j etcetera, I multiplied this by cos omega t I get the coefficient of cos. If I multiply by sine then I get the coefficient of sin whichever b a j and b j. So, you can see that this becomes this becomes actually nothing, but you know this becomes nothing but Fourier's coefficient simply Fourier's coefficient you know a j b j this is nothing, but Fourier's coefficient.

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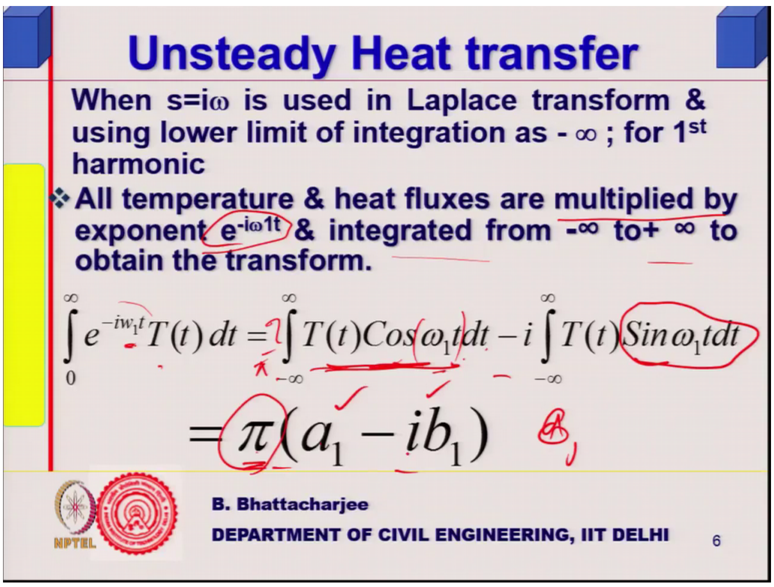

In other words we will just see this sorry we will see this all temperature heat fluxes are then, I will I also have a term multiplied by E to the power i omega T integrated from this to obtain the transform. So, you will have something like this E to the power i omega 1 first harmonic T t is nothing, but T t cos omega t dt because this is cos omega t this is you know cos I am multiplied by this.

So, cos omega t dt sin omega t dt this is what it is. So, this is nothing, but a 1 a j. So, this 1 I have taken only the first harmonic a 1 minus i b 1 right, and pi term will come pi term will time come after integration actually, because this was remember it was pi by 2, and now I am integrating from minus infinity to infinity so, it a pi term comes you know and remember that it was a j was pi by 2 I have to divide therefore, a 1 by you know like it was pi there was a pi involved in that equation if you remember, I multiplied by cos omega t T t and a j is nothing, but this integration divided by 2 by pi pi remember that.

So, a j multiplied by pi will come a and then integrating it full so pi a 1 minus i b on that is what it comes. So, this is nothing, but b 1 is i this 1 is nothing, but same Fourier's coefficient same Fourier's coefficient right. So, I have no you know inverse transform of Laplace transform or anything simply instead I take E to the power i omega t. So, I get something like this.

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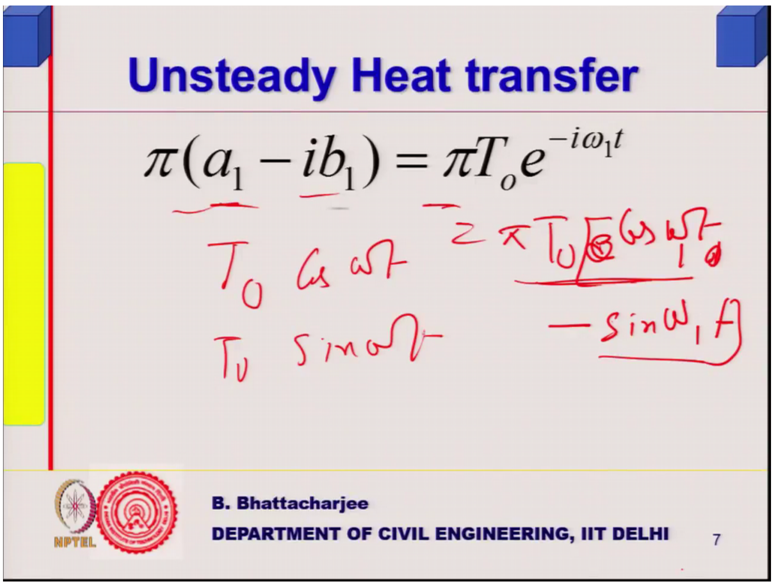

And then this can be written as pi t 0 E to the power i omega t, because you know if I is simply a 1, I can write in terms of T 0 T 0 cos omega t and b 1 is equals to T 0 sin omega t right and therefore, if I take square of this or let me let me write it like this earlier 1 this ones. So, this would be phi a 1 i b 1 and phi remains this I expand what does it come to pi T 0 E to the power I mean this becomes cos omega 1 t, cos omega 1 t minus sin omega 1 t and T 0 is common there. So, T 0 cos T 0 cos omega T 0 sin omega t there will be a phi multiply there should have to be a 5 multiplied some other has to be a T is equals to I just replacing this a 1 square by plus b 1 square right.

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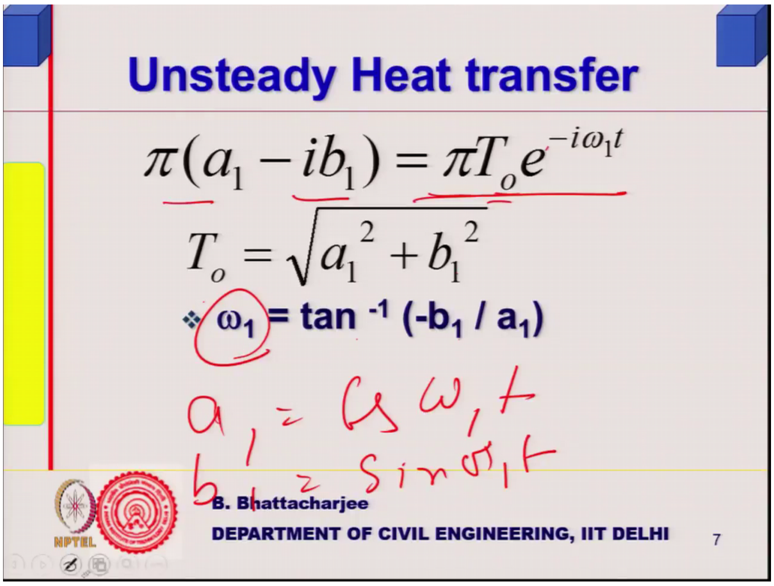

So, T 0 is equals to if I write it in this manner,, then there is a complex number which I am write it in this manner, then T 0 will be a 1 square plus b 1 square you know a 1 square plus b 1 square, and omega is 10 inverse of b 1 by minus b 1 by a 1, because this a 1 I can write as sum cos omega 1 t b 1 you write as sin omega 1 t, then a square plus b square is equals to 1. So, T 0 and E to the power ten inverse of b by a gives me this angle omega 1, 10 inverse you know see if I write it in this manner 10 inverse of b by 1 gives you omega 1.

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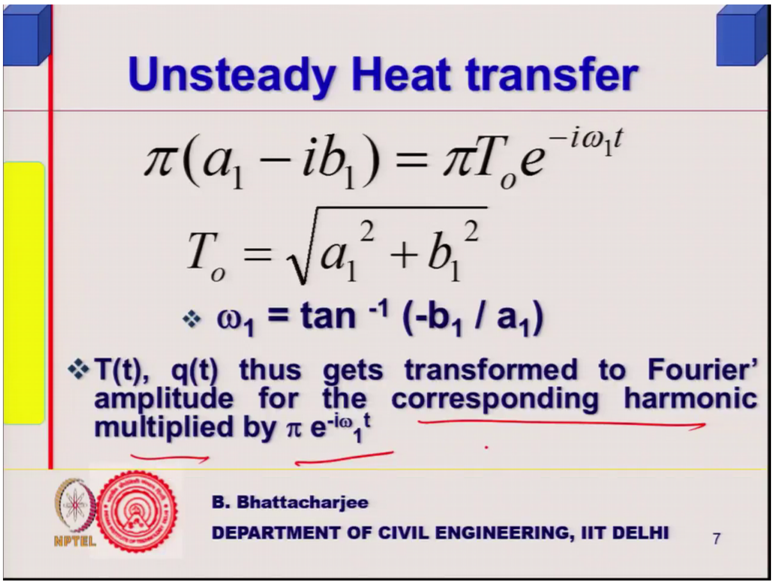

So, it actually follows from here it follows from thus T q t, q t thus gets transformed to Fourier's amplitude corresponding to harmonic multiplied by if i 1 in omega t. So, you see just let us go back to this.

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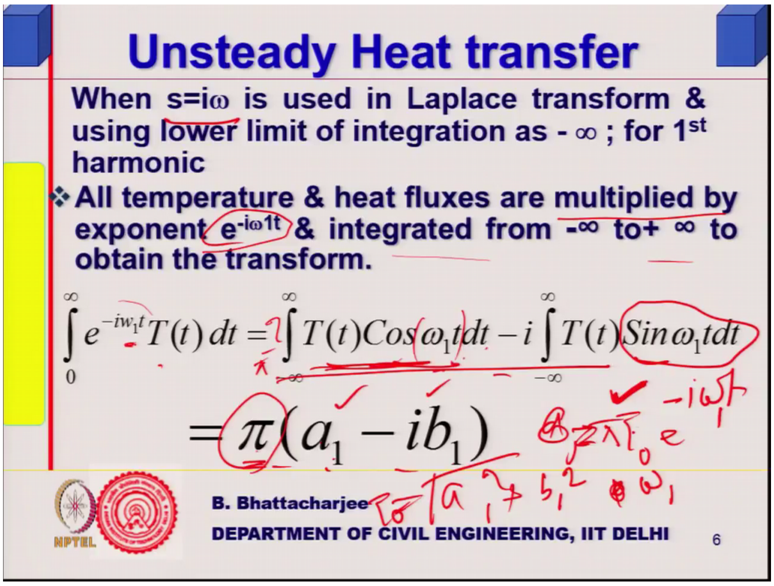

This is equals to phi 1 minus i b 1 right and this I can write as phi T 0 E to the power minus i omega t omega 1 t right, if phi this can I write as phi E 0 i omega 1 t. So, basically if I multiply phi t 0 omega 1 t there is the there is basically amplitude in fact, a 1 plus b 1 square under root will be T 0, and phi would be I mean theta would be whatever it is the omega 1 would be given by minus 10 inverse of b 1 by a 1. So, this when I multiply use this term s is equals to i omega the whole thing gets converted into my temperature gets converted into phi t 0 E to the power i omega t.

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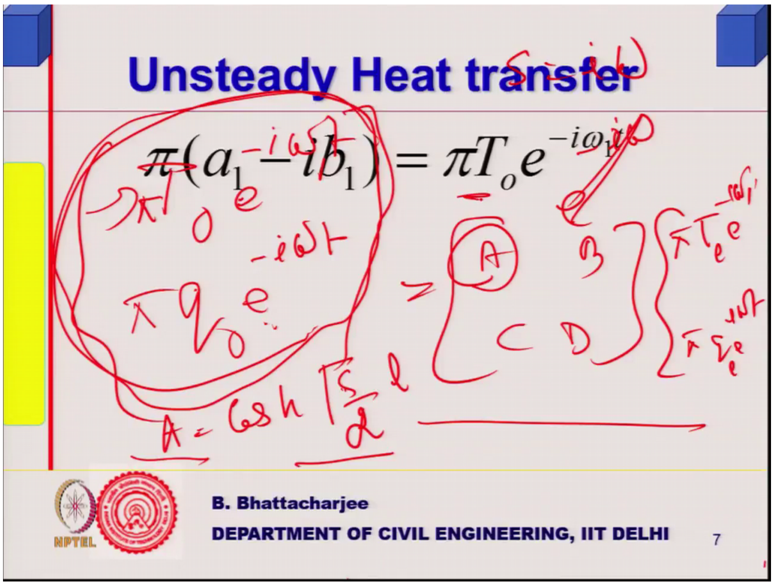

Similarly so, my temperature transform temperature, I mean equivalent of theta it is not theta actually. So, if I multiply everything you know take i omega it I will get something like this T 0 E to the power minus i omega 1 t phi. And similarly q 0 phi E to the power I omega t, because I have taken instead of Laplace transformation I have you know I did this minus i omega.

So, when I transform use this transformation s equals to i omega, then I get this equals to A B C D this side also phi T l E to the power minus i omega t, and phi q s l E to the power i omega t, because instead of using s if I use s i omega my function gets after integration function gets transformations this, but this I will have to change a was cos hyperbolic under root s by alpha into l. So, s has to be also replaced by i omega.

Student: I omega.

I omega which I will do right now; so does it make sense that if I use i omega use not only from 0 to infinity, but go from minus infinity to infinity because minus infinity to 0 function as I am assuming it is 0 which I can do, then instead of Laplace transformation; this transformation becomes something like this, that is what I have shown that this will become this is nothing, but Fourier's a 1 and Fourier's a 1 can be written in terms of you know some single function T 0 some single temperature T 0. So, I will write it in this manner.

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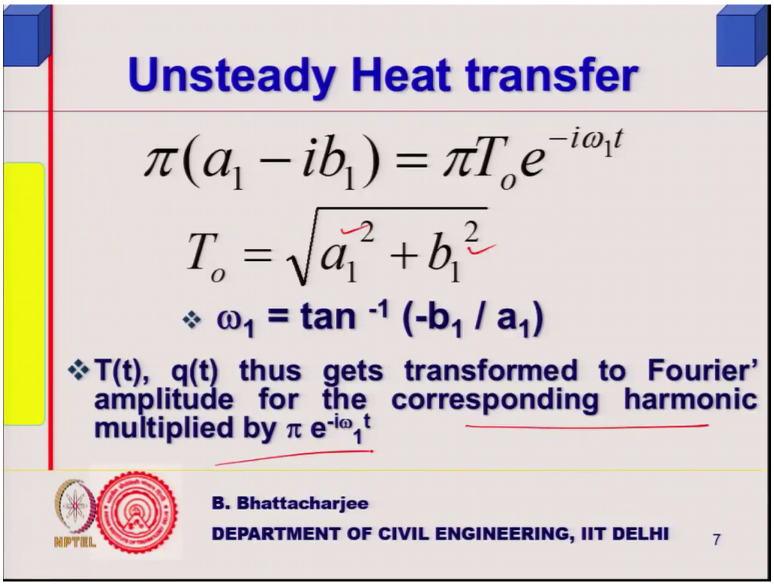

This transformation can be written in this manner, and you will have T equals to a 1 plus b square where a 1 b 1 is a Fourier's coefficient omega 1 represents minus b 1 by a 1 T q t this can be transformed to Fourier's amplitude for the corresponding harmonic multiplied by pi E to the power i omega t.

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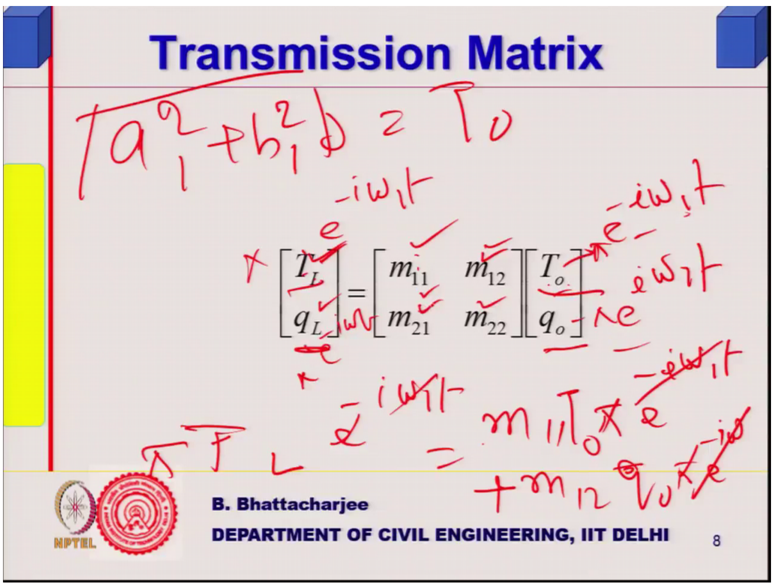

So, my equation would look like transmission matrix will look like T L q L, and here E to the power minus i omega 1 t, here also E to the power minus i omega 1 t pi, here also E to the power minus i omega 1 t, here also pi multiplied by E to the power i omega 1 t.

See everywhere this term is coming therefore, it will cancel out because T L will because 2 m 1 1 into t 0 you know T L, E to the power i mean equation look like this pi T L E to the power i omega 1 t is equals to m 1 1, E to the power minus i omega 1 t, T 0 plus m 1 2 right. So, q 0 pi E to the power minus i omega. So, every place this is coming; this will come, with cancel out from all those places.

So, I will be left with only T L q L is equals to m 1 1, m 1 2, m 2 1, m 2 2, T 0 q 0 which means that I do not have to actually do the transformation at all this is simply the amplitude, you know which I can find out from Fourier's sin and cosine ones coefficients a 1 square plus b 1 square that should be under root of that should be equals to T 0. So, if you have that Fourier's coefficient which you have found out earlier you know we did a tabular form same 1 you can use take square of each 1 of them a 1 and b 1 square take under root of that is this one.

Similarly, outside if you are its known then you can take it this way, you can find out how much q is coming at the inside, or you can also find out how much heat is entering into the system. So, you can actually solve this now only thing remains that expressing this in terms of omega i omega instead of s let us do that. So, is this idea clear what is happening is this idea clear right?

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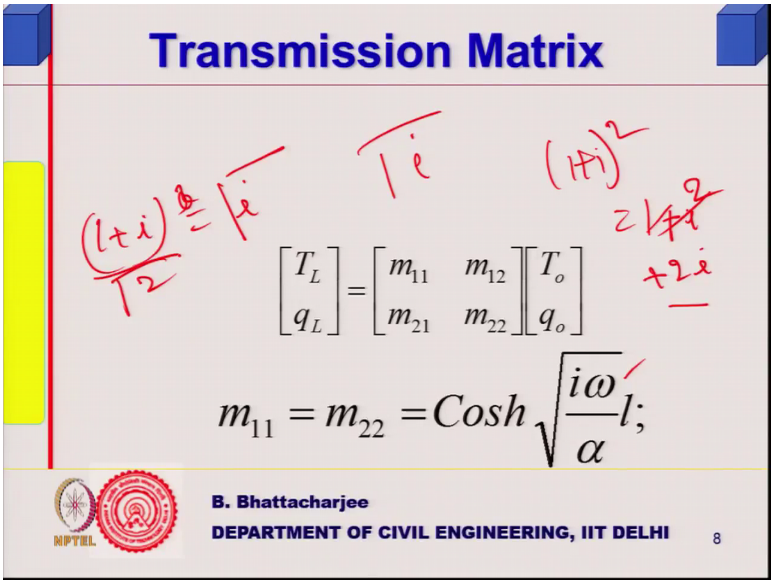

So, let me erase everything of m 1 1 2 is equals to replace s by i omega. Now under root i is interesting 1 plus i square is equals to what 1 minus.

Student: (Refer Time: 23:22).

1 plus i square plus twice i.

Student: (Refer Time: 23:30).

Right, so what is this i square?

Student: Minus 1.

So, right so 2 i is equals to 1 plus i square. So, 1 plus i square is equals to by 2 is equals to i. So, under root i is nothing, but 1 plus i by 1 plus i to the power 2. So, we make use of this and do this exercise.

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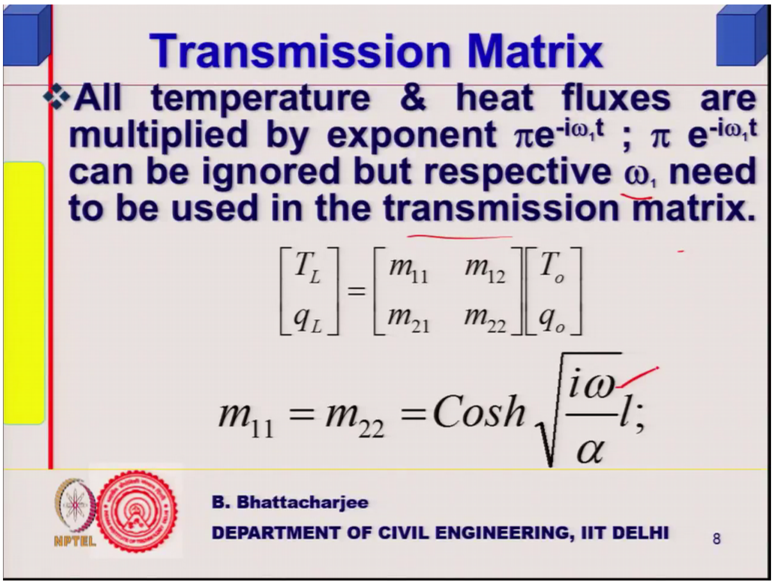

So, all temperatures and heat fluxes are multiplied by exponent, if I into E to the i omega t and can be ignored, but respective omega 1 need to be used in transmission matrix. So, if it is first harmonic you will use omega 1, if it is second harmonic then use omega 2 and so on so forth right.

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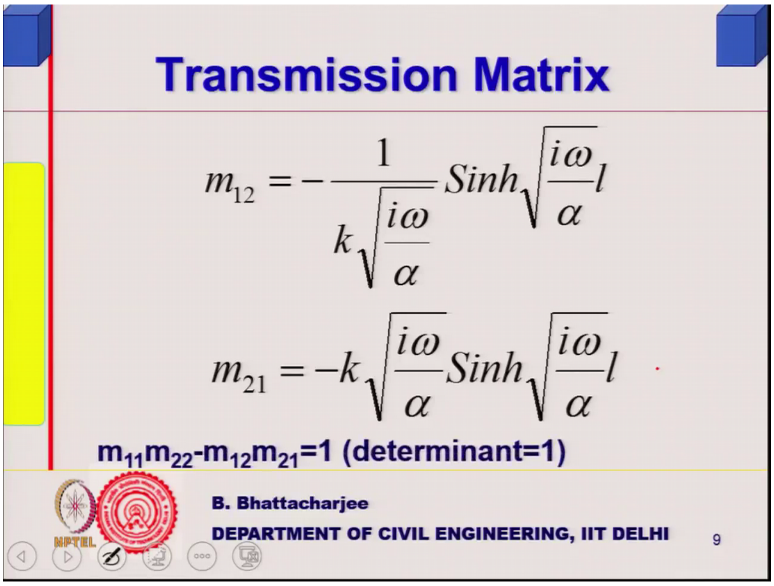

So, that is what it is so, m 1 1 sin you know all of them can be in therefore, written in the same manner i omega 1 right, and under route i is that same determinant remains same there is no problem about that.

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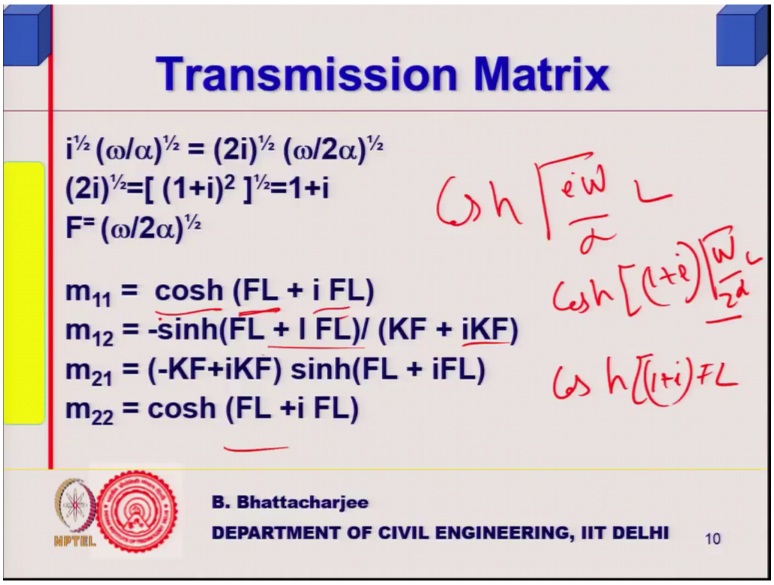

So, under root you can write it like this 2 i to the power half remember that is what I did 1 plus i square right is equals to 2 i. So, you know 2 i to the power half, I can I just it a little bit of i to the power under root i was there, and also under root omega by alpha. So, I just add here 2 alpha, this becomes something like this 2 i to the power half omega plus (Refer Time: 25:16) right, and then 2 i to the power half is 1 plus i.

So, this i call as F this we call I mean this is the notation given by you know people who use it they use F. So, (Refer Time: 25:35) I will also continue to use the same. So, F is omega divided by 2 alpha to the power half right. So, under root s by you know alpha which we wrote as under root i omega to the power alpha, then turns out to be 1 plus i to the power omega by 2 alpha, which becomes 1 plus i to the power into F.

Because this I am calling as F and get me written as F plus i F, and if you have L multiplied it will be F L plus i F L and so on. So, that is what we do so, cos hyperbolic F L plus i F L, because it was cos hyperbolic under root i omega by alpha L. So, this I am calling as this will become cos hyperbolic, you know 1 plus i into under root omega by 2 alpha into L and this I am calling as F. So, cos hyperbolic 1 plus i into F L. So, F L plus i F L and so on for others and plus 1. So, you can actually find out. So, if you use each individual harmonic you do not have to go to Laplace transformation or anything as long as you use you know single harmony right use a single tone, and single you know sinusoidal or co sinusoidal does not matter, because there will be only phase angle change 90 degree out of phase. So, he assumed a single sin function or cos function, I can simply use this no solution nothing is required, and we can from calculate out the heat coming in or you know heat going out or whatever it is.

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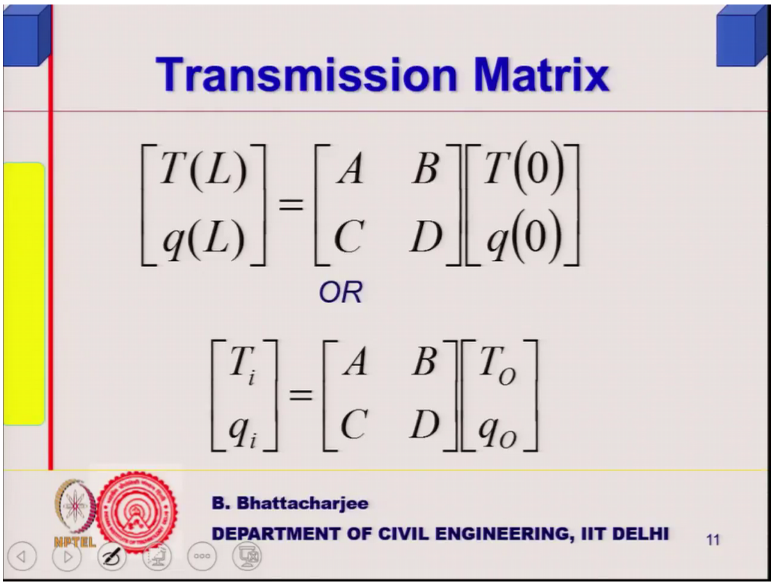

So, followed from this m 1 1 2 and T L q L I can write as A B C D T 0 q 0 where let us say 0 stands for outside q stands for outside, and accordingly A B C D matrix you find out or you can you know that is what I saying T L is T i q i T outside, A B C D you can find out right, accordingly A B C D you find out.

So, let us say take up with an example because, but you must the basis was important that is why I gave you the solution to the differential equation how it has come although final calculation really does not require all that right, but understanding for understanding purpose you must under this . So, example let us say simple example thermal conductivity 1 watt per meter degree centigrade l is 100 mm.

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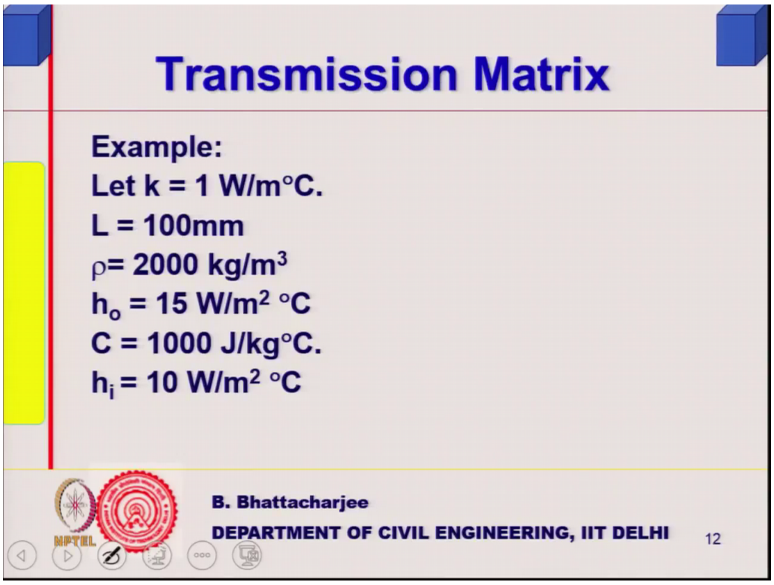

Rho is 2000, I have deliberately put it in figures such that you can sorry h is 15 watt per meter square degree centigrade C specific, it is density was 2000 specifically, a 1000 joules per kg, h i is 10 watt meters. So, it is a simple problem. So, we want to calculate out the matrix. Now first of all I must find out F L and F L, I should find out right. So, we will just break and then solve this problem right. So, F L plus i F L will find out solve this problem break and solve this problem.