Energy Efficiency, Acoustics and Daylighting in building
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Lecture – 11
Heat Flow in Buildings (contd.)

So, we continue from; we said that heavy mass, heavy section would have higher time difference and also amplitude would be lower.

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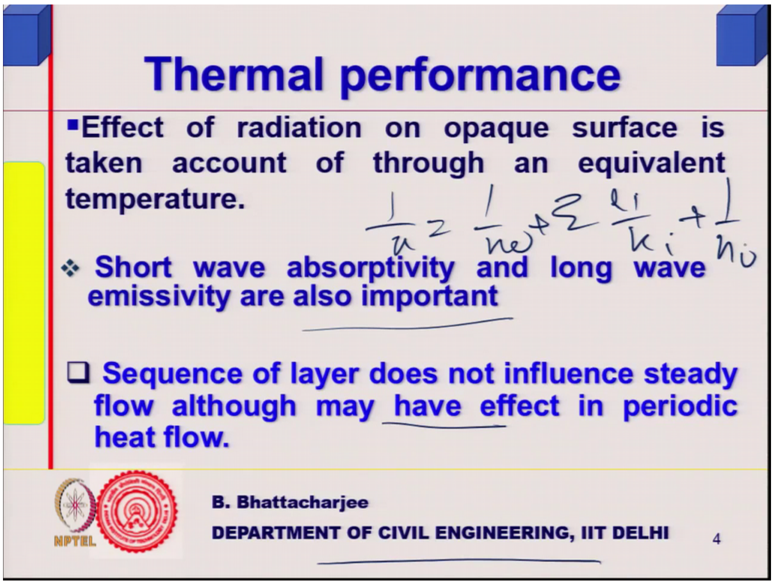

And similarly effect of radiation on the surface can be taken care of through an equivalent temperature we have seen. And these are important; now suppose just sequence of layer does not influence the steady flow, that is what we have seen because we said 1 over u is equals to 1 over h of; plus simply sum of l i by k i plus h i. So, this sequence whether you know l 1 is l 2 or which sequence it does not matter because I am simply summing it up.

But in periodic heat flow or steady, unsteady state heat flow sequence matters. Because suppose I have a heavy thermal mass outside, it will absorb more and then only it will transmit. But if you have somewhat relatively less; lower thermal capacity mass outside it will allow more heat to come in and it will heat up the other one. So, sequence matters in case of unsteady state heat flow, but does not matter in case of steady state heat flow. The other factor is that these are important; so if you have a wall and you do not want heat to be received in inside the room; a high emissivity and low absorptivity wall is good. It should have low; short wave absorptivity, but high long wave emissivity.

Alpha should be small; alpha shortwave alpha with respect to solar radiation should be small. For example, a white wall surface it has got a value of alpha you know somewhere close to 0.4, but its emissivity is around 0.9; while absorptivity of many other let us say if I have a black surface that will absorb more. So, alpha will be high that is not good; so all those colored surfaces if decoratively put they are likely to black particularly is likely to absorb more heat.

Brick for example, has got higher rick colored one; has got higher absorptivity although emissivity is around point nine similar. So, these are important issue and sequence as I said it does not matter and this is what we have seen earlier.

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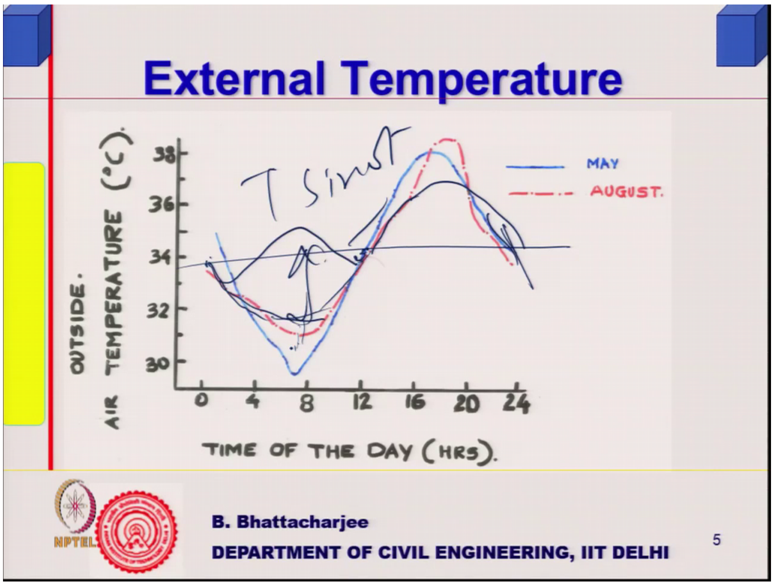

And if I am looking at this unsteady part of it periodic part of it.

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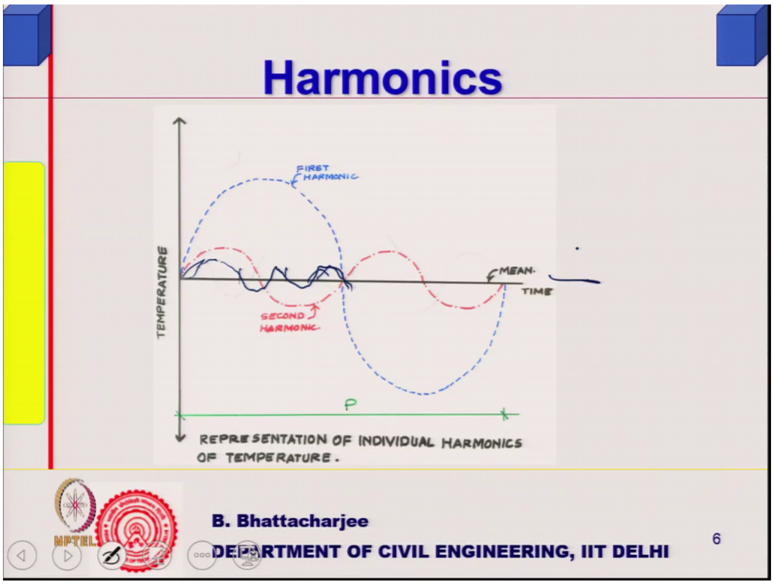

I can actually you remember from basic, some mathematics earlier Fourier series expansion of any periodic function.

So any periodic function can be expressed as sum of set of sin or cosine functions that is easily understandable. How it is easily understandable? Suppose I have I have this; so I just fit first any one of them, I just fit it sin to this; sin for some, with this amplitude, some sin function; sin T sin omega T or whatever it is.

Now there will be some error; so what I do? I put a second harmonic which will have a periodicity half of the previous one. Now, if you had periodicity half of previous one; that means, it will be 0 here and 0 here. And if I have one third periodicity; still it will have 0 here and 0 somewhere in between also. So, if I go on summing them up the errors that would be coming, that will be minimized.

So, you can approximate any periodic function by sum of; I am talking of the fluctuating part only; obviously, the mean is there, then the fluctuating part; I am talking about fluctuating part. So, the fluctuating part you can always express as sum of; series of sin and cosine functions sin and cosine functions well you do many approximation of a similar kind.

For example Taylor series expansion of a function around a point is the value of that function at that point plus first derivative; the gradient multiplied by delta x, the distance that you are talking about. Now then rate of change of gradient multiplied by delta x etcetera etcetera and you know like many functions actually in mathematics, we can do it you can you can always large number of terms you take, it tends to fit into closer and closer, so Fourier series expansion is one of those kind.

So, you can actually approximate a function by mean plus first harmonic, second harmonic, third harmonic could be coming something like; it will be less sorry I think it is not correct it is a series resist out third harmonic could be third harmonic could be something like. Third harmonic could be three part; I should divide by three part and it would be something of something like this, going like these and going like this and my drawing is not so good. But the third harmonic, fourth harmonic, fifth harmonic you can go on adding them up and more you add more close you would be to the real variation of the temperature.

So, this is the physical explanation band Fourier series expansion. So, this is one of the ways of handling this temperature; this is called frequency domain treatment. This is what we will do more in our class, there are other ways of doing it that I have not discuss because that is not amenable to hand calculation. But many computer software uses more complex you know or complex or other form of treating this external temperature.

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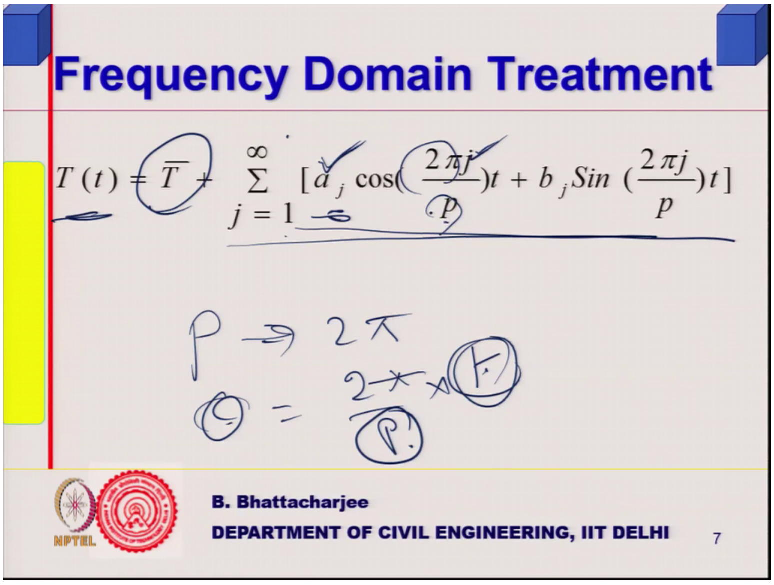

So, this is called frequency domain treatment where we express time; temperature as a function of time; as a sum of mean and in finite series; in finite terms a series of infinite terms having different amplitudes because as I said I had.

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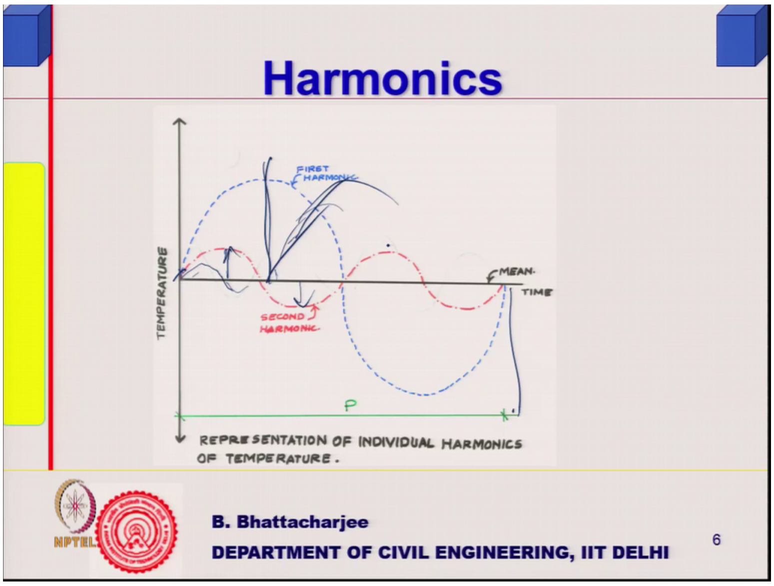

You know this one had a different amplitude, this one had a different amplitude and third one would have somewhat different amplitude and so on so forth. You know third one would have different amplitude and fourth one would have different amplitude and I can sum them up for example, this was a sin function; a Cos function would be because this is 360 degree or 24 hours periodicity.

After 360, any trigonometric function repeats itself, that is how you can express them in trigonometric series. So, another function might start; I can actually have another one a Cos term as well, sin in the Cos term. So, let us make it life easier to make this as close as possible to the actual variation pattern. So, I can approximate this; so these are the amplitude difference; j goes from 1 to infinity.

There are infinite number of terms; so, a j is the amplitude term, sin j equals to 1; that means, twice pi by p. Because sin function that is trigonometric function, so sin 360; is sin 0 sin 720 is sin 0 again; so it repeats itself. So, basically sin functions are all periodic; Cos functions are also periodic

So, how do I convert because they would be should be expressed either in radians or degrees. So, if I want to express in radians or degrees P must be equals to twice pi. So, if you know P corresponds to twice pi, then theta will corresponds to 2 pi by like any other angle; 2 pi by P into t will be basically time is 24 hours, this 24 hours say 12 hours will correspond to how much? 0 corresponds to 0, 24 hours corresponds to 2 pi 12 hours will corresponds to pi, 6 hours will correspond to my pi by 12. So, that is why; t divided by P; so that is what it is; 2 pi by P and j stands for the harmony; for the first harmonic, periodicity is 24 hours.

So, 2 pi by P; if it is second harmonic 2 pi by 12 because second harmonic; if you recollect, it had a periodicity of 12 hour. Third harmonic will have a periodicity of 8 hours; so if this is 3 by 24; you know 24 by 3 will be 8. So, 2 pi corresponds to 8 hours and so on so forth. So, this is the Cos function; this is a sin function, I can just convert this T into trigonometric functions, convert this time using this. So, that is what it is, so this T can be expressed as a function of a mean plus the fluctuating component like this right sin and this.

Now dealing with sin and Cos together; it is a little bit problematic; sometimes we simply deal with Cos. We will see that how do I do it? But then if I want to express this periodic function in this manner, I must find out values of a j; p is known, p is known 24 hours in our case and I should find out b j; a j and b j, I should find out.

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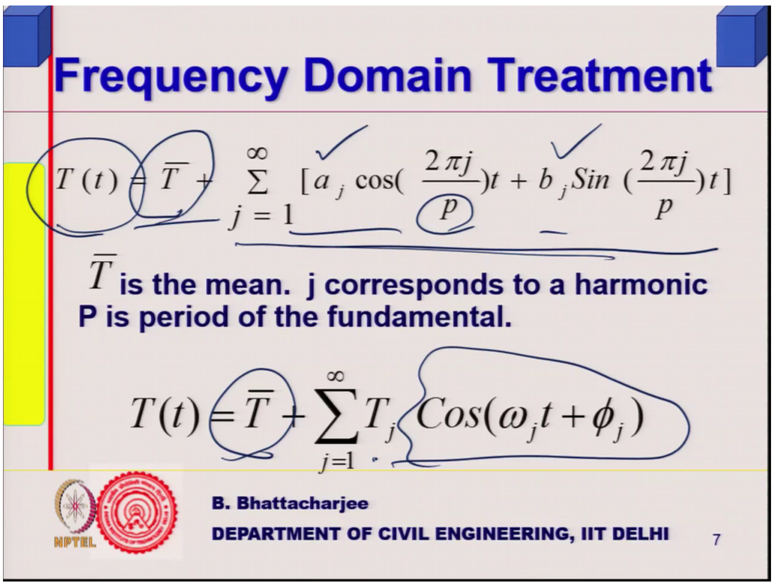

So, I must evaluate a j and b j; now we have a simple way to do it T bar is mean. So, if I have 24 hourly value given which is the case usually, for any other thing you can do it in different manner. But if it is 24 hourly discrete value is given; you simply sum them up and divide by 24, you get the mean.

If 12 values are given two hourly; then what it will do? Each value you know sum them up and divide by 12. So, arithmetic mean is simply T bar; so that is very simple. Now, so this you can find out; as I said sometimes we express it in terms of Cos function; we will do that. But first let us find out a j and b j and then we will express it in terms of this sort of a Cos function.

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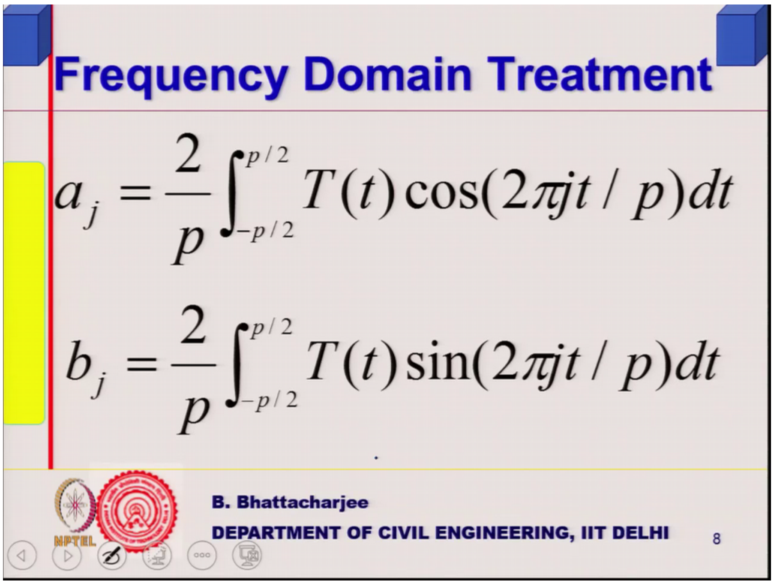

So, a j can be easily found out; how? Do I have this or I have to you know by this expression, but how this expression comes?

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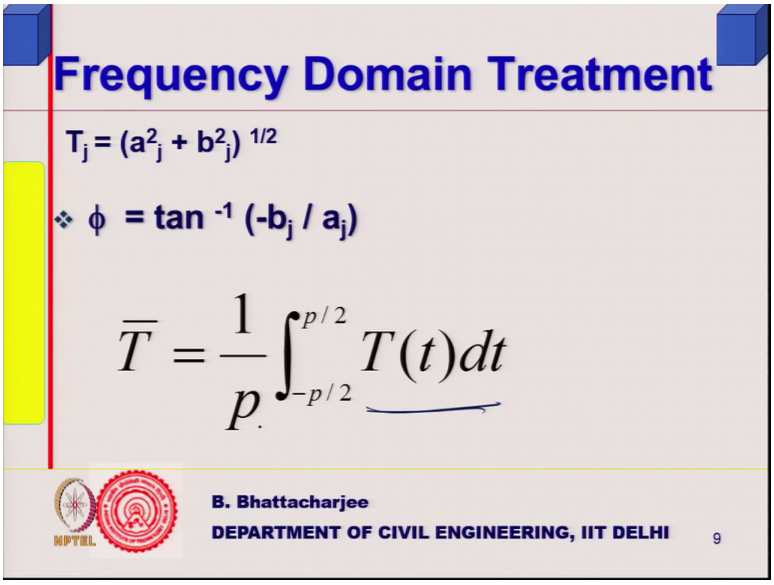

Let me; perhaps I finally, explain and come back to this not come to the solution, now how do I find it out? If I have to do that.

Now, suppose; I multiply this by Cos; some Cos term multiplied by some Cos term. Let us say a j; you know I multiply by some Cos term. So, Cos let us say multiplied by a j I want to multiply by Cos this term itself and this also by same term. And then integrate this from 0 to 2 pi, so if you look at this.

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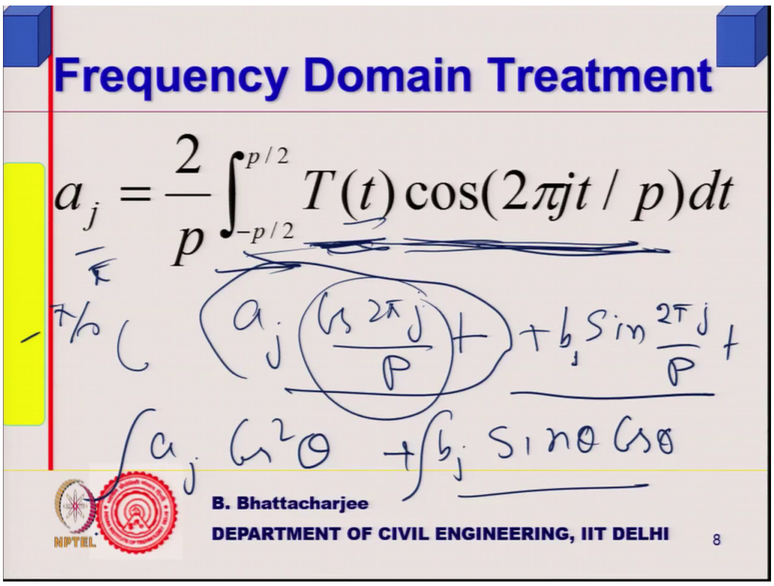

If I multiply this by Cos term; then temperature values are known to me and multiply this temperature values because T; t is nothing, but this is a j for a j value I am trying to find out. So, a j Cos 2 pi j by p into t; plus sin b j, sin 2 pi j by p into T. Only first harmonic I am taking; first term j equals to 1, I am not taking anything else just to explain.

So, if I multiply this with same term again; so I will get a j Cos square. Let me write this as theta or t or whatever it is plus b j sin theta Cos theta. Now if you integrate this; this term will go to 0, because it will be you know sin 2 theta by 2 and integrate from pi by pi minus pi to plus pi, which corresponds to p by 2, minus p by 2 by; 2 plus p by two. Then you will find that; this term, sin theta; Cos theta sin 2 theta. If you integrate, what do you get? Cos 2 theta by whatever it is; some product. Cos 0 is 1, Cos pi is minus 1. So, 1 minus minus 1; it will give you 0.

So, therefore this term will all go to 0; leaving this term around. So, you can get an expression for a j by multiplying that is a 1, a 2 etcetera by multiplying the function or temperature value by corresponding; by multiplying this by multiplying this value. If I multiply first term with this value and all other times will also go to 0 because you can show that they will go to 0 because this will be Cos; when this is equals to 2, this is nothing but Cos theta by two multiplied by Cos theta.

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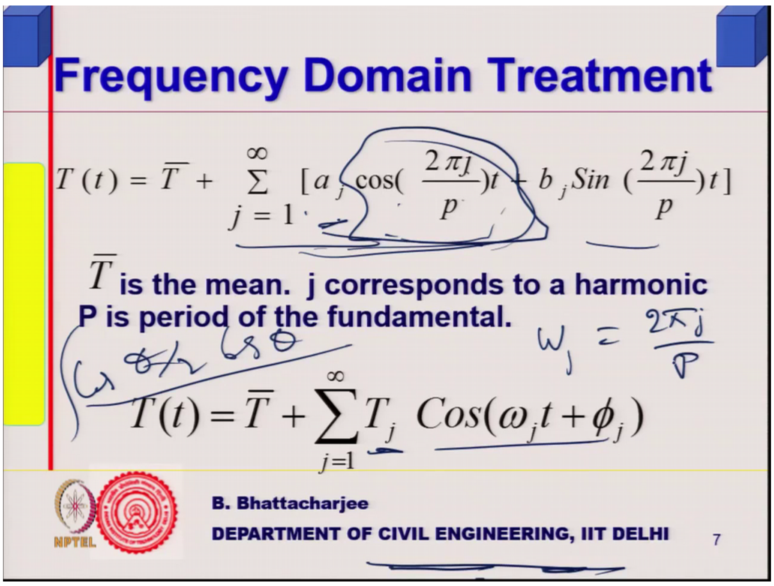

Because I am multiplying by this; so this integration of this one also will go to the sub angles; they will also go to 0. You can actually see this; see in any one of books

Engineering Mathematics by Erwin Kreyszig or anyone of this book Fourier series expansion is there. I am just trying to explain because some of you have forgotten and some may not have done it also.

Those who are architecture background may not have done it also, but does not matter it is not very complex thing. All I am saying is; if you multiply this function with this series, you expand this; multiply with this term; then only a j a one will remain rest all will go to 0. Because when you integrate within these limits; they will go to 0.

So, therefore, a j can be evaluated by multiplying T t by this term, I have discrete value then how do I do it? I have a table; I will show you. Similarly b j can be found out; by multiplying by sin function because it will be this sin square will remain. Now Cos square will become Cos 2 theta minus 1; divided by 2. Similarly sin square is again Cos 2; theta plus 1 divided by 2.

So, therefore some time will be always remaining there and therefore, you can evaluate b j or a j in this manner. This is an exercise, you please do it because you are supposed to do a little bit of self study yourself you might do this yourself. And you can see that you can do it and mean I can find out simply if I have a continuous function; I can find out the mean by simply this manner; integrate it from p to you know.

So, that is how we find out the mean, but then if I have discrete value very simple I will take; so in through an example. And if I have to express this in form of a simple Cos function like this; if I know a j and b j; a 1 and b 1 and I want to find out a T 1; express the whole thing as Cos function, then what will I do? You know suppose I have a function a 1; Cos theta plus b 1 sin theta.

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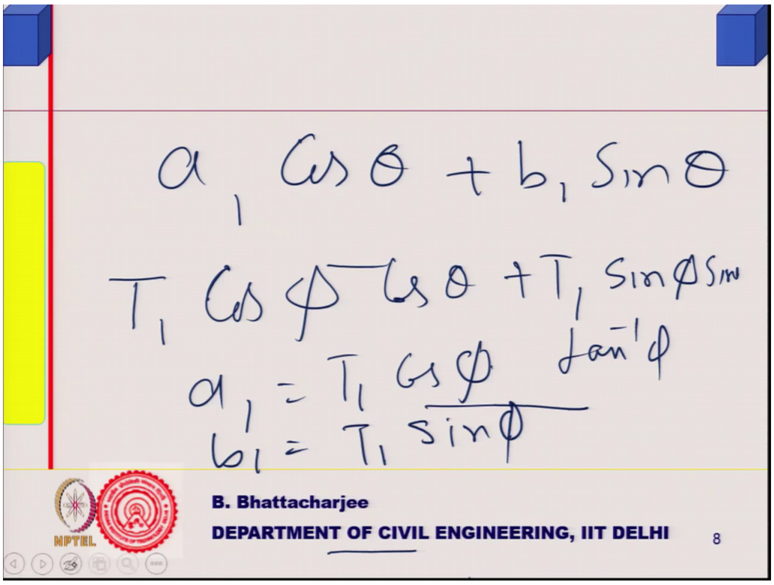

I want to express it again as a function of Cos function only. So, I write this as say let us say T 1 Cos; I want to express only in Cos. T 1 Cos pi let us say and Cos theta plus T 1; this I write a sin pi, sin theta. So, this would mean that; what it would mean? a 1 is equals to T 1; Cos pi, b 1 is equal to T 1 sin phi.

So, a 1 square plus b 1 square will be equals to T 1 square and pi will be what? tan inverse of; phi I can find out b 1; possibly a minus sign comes in somewhere; I will see that. So, that is how we can express, so that is what we are doing.

So, this you know; so right tan inverse of minus b j by a j will come tan inverse of; so, this is how we can find out T j because Cos function is always; that is right because you know Cos function will be Cos omega j t.

Now, omega j is nothing, but twice pi j by p; I am calling it as omega angular frequency term. So, omega j t plus pi it will be Cos omega j t; Cos pi j minus am I right? Minus sign, there is a minus sign involved. So, if I have a minus sign you know from there it comes at minus. So, from there it follows; pi is equal to tan inverse b j by aj. So, you can express therefore, mean you can find out mean.

So, some 24 data; hourly data is available to you, you can find out the mean and you can find out the Fourier coefficients and convert them into an equivalent temperature term as a co function you know coefficient of Cos function. How do I do it? This is a table which shows you.

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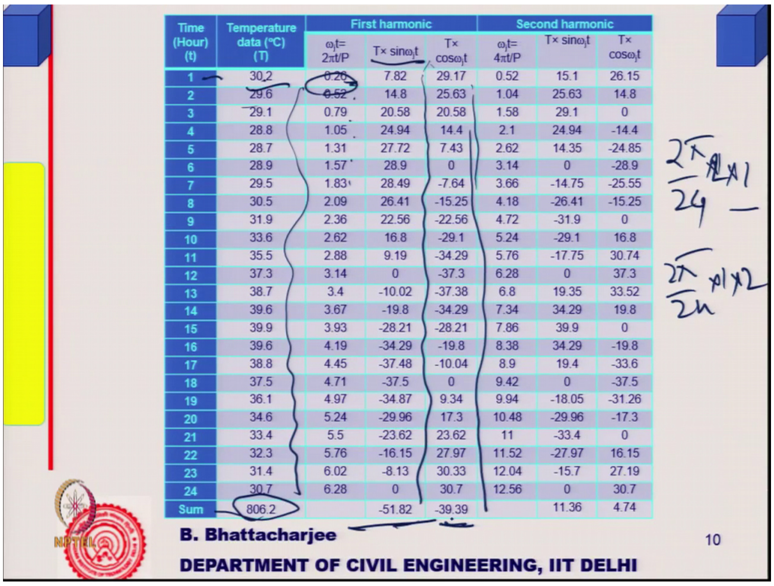

Time (Hour) (t)	Temperature data (°C) (T)	First harmonic			Second harmonic		
		$\omega_j t = 2\pi t/P$	$T\times\sin\omega_j t$	$T\times\cos\omega_j t$	$\omega_j t = 4\pi t/P$	$T\times\sin\omega_j t$	$T\times\cos\omega_j t$
1	30.2	0.26	7.82	29.17	0.52	15.1	26.15
2	29.6	0.52	14.8	25.63	1.04	25.63	14.8
3	29.1	0.79	20.58	20.58	1.58	29.1	0
4	28.8	1.05	24.94	14.4	2.1	24.94	-14.4
5	28.7	1.31	27.72	7.43	2.62	14.35	-24.85
6	28.9	1.57	28.9	0	3.14	0	-28.9
7	29.5	1.83	28.49	-7.64	3.66	-14.75	-25.55
8	30.5	2.09	26.41	-15.25	4.18	-26.41	-15.25
9	31.9	2.36	22.56	-22.56	4.72	-31.9	0
10	33.6	2.62	16.8	-29.1	5.24	-29.1	16.8
11	35.5	2.88	9.19	-34.29	5.76	-17.75	30.74
12	37.3	3.14	0	-37.3	6.28	0	37.3
13	38.7	3.4	-10.02	-37.38	6.8	19.35	33.52
14	39.6	3.67	-19.8	-34.29	7.34	34.29	19.8
15	39.9	3.93	-28.21	-28.21	7.86	39.9	0
16	39.6	4.19	-34.29	-19.8	8.38	34.29	-19.8
17	38.8	4.45	-37.48	-10.04	8.9	19.4	-33.6
18	37.5	4.71	-37.5	0	9.42	0	-37.5
19	36.1	4.97	-34.87	9.34	9.94	-18.05	-31.26
20	34.6	5.24	-29.96	17.3	10.48	-29.96	-17.3
21	33.4	5.5	-23.62	23.62	11	-33.4	0
22	32.3	5.76	-16.15	27.97	11.52	-27.97	16.15
23	31.4	6.02	-8.13	30.33	12.04	-15.7	27.19
24	30.7	6.28	0	30.7	12.56	0	30.7
Sum	806.2		-51.82	-39.39		11.36	4.74

For example, I am not very sure whether it is very much visible to you. If it is visible to you it is fine, but I will just explain it; say time 1 hour; this is the temperatures are given in this line. Say, first hour is 30.2, second hour 29.6 etcetera, etcetera; then I want to find out the mean; what I do? I simply sum it up; divide by 24, I get the mean. Then omega j t or twice pi T by p i, calculate out here p is 24 hour early period for first harmonic if I am doing it; j is equals to 1.

So, twice pi T by p; so this will be twice pi by 24 because j is equals to 1 in not twenty of j equals to 1; into 1 into first hour. For second hour, it will be twice pi by 24 into 1 into 2; so this is tabulated here; twice p. Then I find out T sin omega sin omega j t; now this is in radians sin of this one I can find out, multiplied by the T value; that gives me the table force T sin omega T; sum it up that is integration, summing at is up is integration from minus p by 2 to p by 2. And Cos omega T I find out like this, so this is for the first harmonic; if I am doing for second harmonic then this will be simply 2.

So, and I can find out; so, sum of the T sin omega j t sin of the T Cos omega j t; I am able to find out. And if you recollect; go back to this one; this was nothing but integration, this divided by 2 by 24; this sum divided by 12 will give me corresponding a j value.

This sum divided by; second sum divided by 1 of them, this divided by; Cos values divided by 12 will give me a j value.

The sin coefficient (Refer Time: 22:14) sum up divided by 12 will give me the b j value. Similarly, for the second harmonic a j and b j values are here. So, you can check this calculation; this is simply in an excel sheet, pretty easy and find out these coefficients. But actually we will see that we do not use them too much; what we do is we look at consider it to be first how many can leave out the second, third harmonic etcetera etcetera.

Six harmonics are absolutely good enough to have very small error six or more harmonics if you take you will fit the card more or less same, but even first harmonic does a good job for us, we will see that how you go about it.

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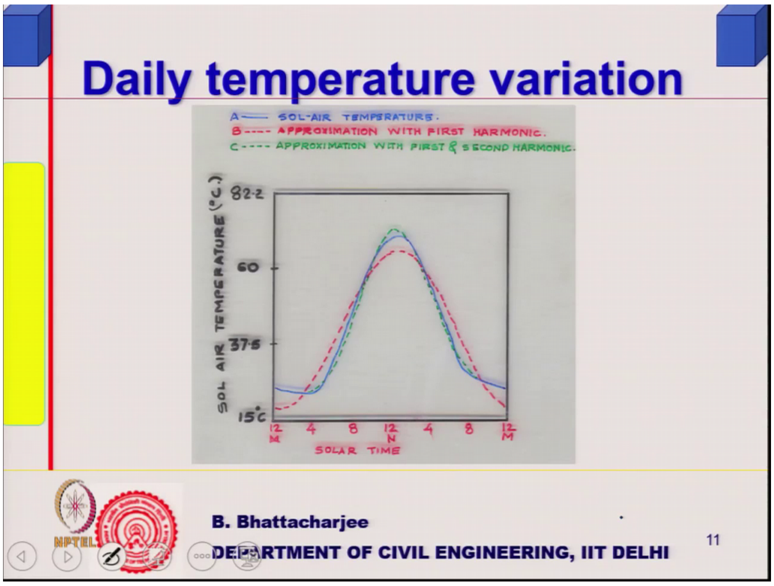

So, you can see that once you have done the harmonic analysis; a is the sol air temperature, this blue colored one. Approximate with first harmonic is a green colored one; this is with the approximation with first harmonic and approximate with the second harmonic. First harmonic is a red one sorry which is pretty back; with the second harmonic if you see it comes fairly close, this dotted line approximately first and second harmonic the green one.

Only with first harmonic is the red, where blue is a real one. So, when we have done added second harmonic, we have gone closer to it. If you had third harmonic; we will go still closer to it, fourth harmonic will go and six harmonics are go around; so, that is what is. So, daily temperature this sol air temperature of a given surface, whose surface that I was talking about from our own measurement in the campus sometime.

So, sol air times 12 hours; even sol air temperature you can fit into the harmonic series. So, that is it; that is basically related to how we handle the opaque bodies and you know from this one how do you go further? So, now what we have done? We said that we have; what we said is that, we can express the temperature.

Mean part we have already seen how we handle it because we said steady. So, you U a delta T; if I know outside temperature, if inside temperatures are unknown then you have to do a little bit of things. But we understand, if I know the inside temperature and both are constant then I can find out the heat flow inside. Or if inside is not known, if I know the heat flow then I can find out the inside temperature; we will find to see that how do you do it?

Now, when it comes to the fluctuating part; we can approximate it by a simple cosine function or sum of cosine functions. So, this was my basic heat flow heat conduction equation; I am still dealing with conduction. I will come to ventilation heat transfer later on; which are simpler. So, this you remember this we discussed you know the heat coming in minus heat going out right; this is the flux.

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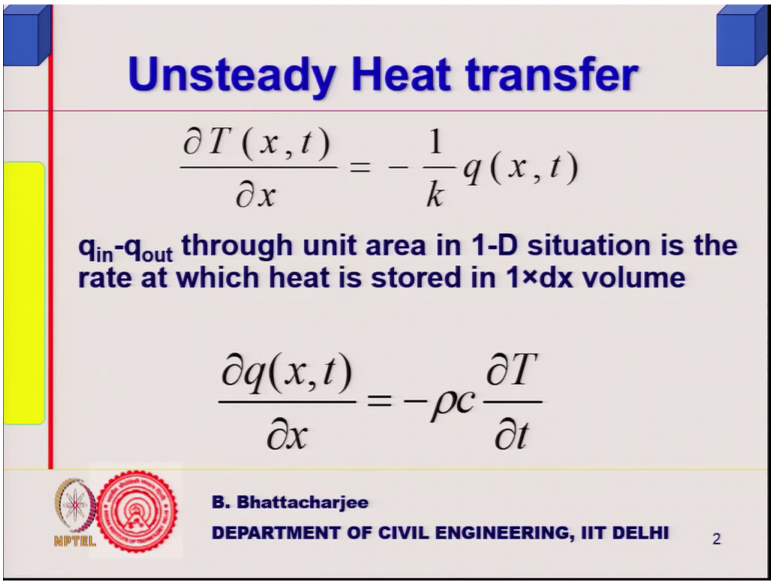

This was the flux and this you remember; we talked about this rate of change of you know this we derived earlier.

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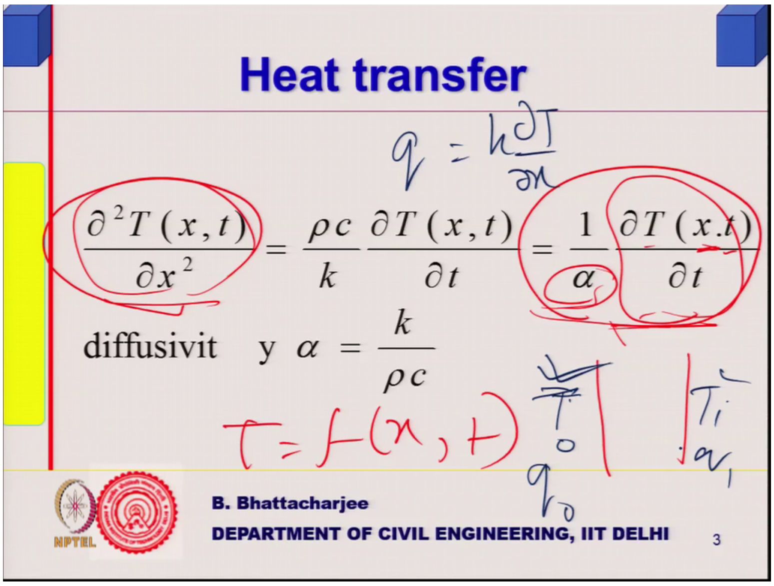

And from this it follows all this I have just discussed I am just trying to recollect this. And this then turns out to be a second order differential equation it turns out to be a second order differential equation of this form.

Temperature is a function of space and time within the construction, it will be changing from one point to another and say function of time as well. And this is a partial derivative with respect to time, this is the thermal diffusivity; maybe some notations are overlapping, but that is what people conventionally use. Alpha is used for both absorptivity as well as thermal diffusivity; some book might use a, but generally you know quite often it is like this.

So, this is a second derivative with respect to space; this first derivative with respect to time. So, I need some boundary conditions because I can integrate this with time and with space. There are many ways of solving this one for different boundary conditions closed form solutions are available and you can even solve numerically. But then you got to integrate this one; so time twice with space and then you will have constants of integration.

So, therefore you need one initial condition and two boundary condition to get temperature as a function of x and t; solution of this one temperature is a function of x and t. So, you need two boundary condition and one it is an initial condition; we assume an initial condition that everything is at constant temperature. And that is one way of doing it and then boundary condition are either I have four possibilities.

One if this is my wall; outside I might know the temperature and all might also know the heat flux. Because the second derivative and we know q is nothing, but k, d T, dx you know in this case partial derivative that is why I am writing like this.

So, either I know the heat flow per unit area how much heat is coming in or I know the temperature. This might be unknown to me T i and qi; then the knowing these two because then two boundary conditions I have known; I can solve it. Or if I know this and this then I can find out what are the fluxes or if I know this you know; out of this four if I know two of them.

For example, if there is a radiating source say; hypothetically only sol air radiation is falling there. Temperature is all constrained inside outside only sol air region then I have got a flux boundary at the outside; some radiative flux boundary and outside. Or you know that is flux worried, but that is not the real case anyway, but outside temperature as a function of time that would be known to me.

So, four possibilities are there flux for this one; two of them must be known to find out the other two. But first I must solve this out; let us see how do you do it.

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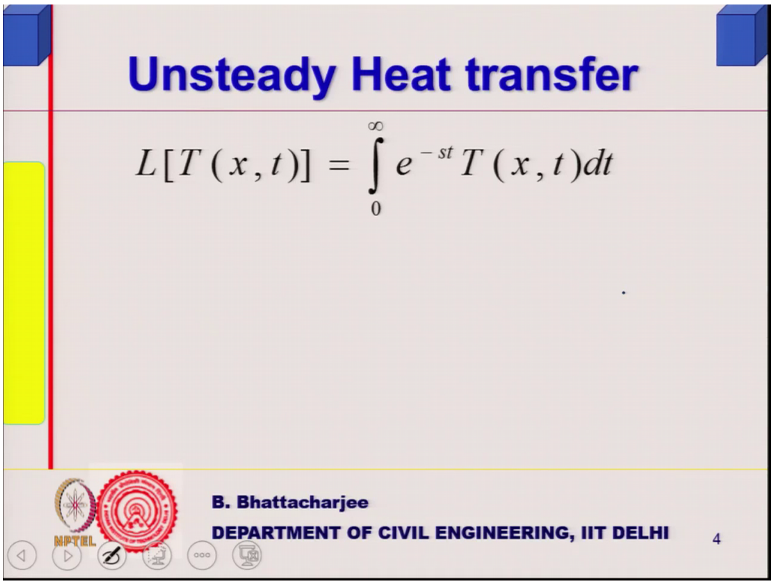

So, you know to do that we actually resort one of the ways; there many ways you can solve it. As I said you can use numerical like finite element, finite difference techniques or we can use time domain solutions. What we will be doing here is frequency domain solution; both time domain and frequency domain solutions. I will explain you what they are, but frequency domain handling is amenable to hand calculation. So, to do that what we do is; we go to Laplace transformation.

Now, Laplace transformation we will see that actually it makes easier whole thing is Laplace transformation is essentially converting time domain into a domain of Laplace s domain; s variable.