

**Geoenvironmental Engineering (Environmental Geotechnology):  
Landfills, Slurry Ponds & Contaminated Sites  
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**Lecture - 22  
Landfills: Some Solved Examples**

So, today we will discuss some solved examples relating to landfills, mostly relating to liners and covers. And the idea is that by doing about ten examples, you will get to know what kind of leakages occur? And you know we discussed action leakage rate that if the leakage is more than a particular value then, we have to take some action. So, from where does that value come? What is the logic behind it? That will also become a parent as we discuss these examples.


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**Example 1: Advective Flow Through OCL**

(a) If 0.3 m of leachate is ponded on a 0.9 m thick compacted clay liner which has a hydraulic conductivity of  $1.0 \times 10^{-7}$  cm/sec, what is the flow rate under steady state conditions in litres per hectare per day?

(b) How will the flow rate change if hydraulic conductivity is higher at  $1.0 \times 10^{-6}$  cm/sec?

(c) Compare with Action Leakage Rate and Rapid Large Leakage Rate.



So, the first example is advective flow through a compacted clay liner. As I talked to you in the very beginning that, flow through liners can be advective or diffusive. Advective is due to hydraulic gradient and diffusive is due to a gradient in concentration. And we know that for advective flow the Darcy's Law is valid. So now, what we are going to see is, that if 0.3 meter of leachate is ponded on a compacted clay. I am not talking of a composite liner, just a compacted clay liner which needs specification of 1 into 10 to the power minus 7 centimeters per second, then what is the flow rate, which occurs under

steady state condition in liters per hectare per day; liters because you understand what a liter is? You have seen a 2 liter bottle, the one in which you get your big coke or pepsi. So, when I say 20 liters, you are able to understand it is 10 such 2 liter bottles.

Per hectare; hectare is 10 to the power of 4 square meters; that is 100 meters into 100 meters all most like a football field or a cricket field per day. So, how many buckets or how many bottles of water come out per day?

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So, this is what has to be calculated as per the Darcys Law. What this question is saying, is that you have your clay and you have the drainage layer and leachate collection layer at the top and there is some leachate in that. So, this is 0.3 meters and the compacted clay is 0.9 meters and we want to know, if this situation occurs how much water will come out in a football field per day? And then take for example, it further the questions says, if by chance we wanted to have permeability of 10 to the power of minus 7, but the compaction was not good or the quality of soil was not good enough and by mistake you get 10 to the power of minus 6, instead what happens to this leakage rate? And finally, we would like to compare it with the action leakage rate and the rapid large leakage rate which the USCPA has stated.

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**Darcy's Law**


$$Q = k_s \cdot i \cdot A$$

where  $Q$  = flow rate through the liner, cm<sup>3</sup>/sec;  
 $k_s$  = hydraulic conductivity of the soil, cm/sec;  
 $i$  = hydraulic gradient; and  
 $A$  = area over which flow occurs, cm<sup>2</sup>.

If the soil is saturated and there is no soil suction, the hydraulic gradient is given by

$$i = \frac{(h+D)}{D}$$

where  $i$  = hydraulic gradient;  
 $h$  = leachate head over the liner; and  
 $D$  = thickness of the soil liner.



So, please use this formulae; you remember  $Q$  is equal to  $k_s i A$ , hydraulic conductivity, hydraulic gradient and the area right. And what's the hydraulic gradient in this case?  $h$  plus  $D$  over  $D$  where,  $h$  is the leachate head over the liner; in this case it is 0.3 meters and the thickness of the liner is 0.9 meters. So, can you please calculate  $Q$  for me, as per this formula it will come out in centimeter cube per second, but I want you to compute it in liters per hectare per day. Please note these two formulae and then I will go back to the problem and here is the problem back again. If you have noted the formula, 0.3 meter is the head, 0.9 meter is the thickness of the thing; the compacted clay layer and the conductivity is  $10^{-7}$  centimeters per second. So, if you put in these values, what is the value of  $i$  that you get?  $i$  is equal to?

Student: 1.33.

And with that you can do  $Q$  is equal to and all you have to do is converted to liter. So, 1000 CC equal to 1 liter, if you recall and  $10^4$  square meters is 1 hectare.

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Example 1: Solution

$$(a) \quad i = \frac{(0.3+0.9)}{0.9} = 1.33$$
$$Q = (1 \times 10^{-7} \times 10^{-2}) \times (1.33) \times (1.0) \text{ m}^3/\text{m}^2/\text{sec}$$
$$= 1.33 \times 10^{-9} \times 10^3 (\text{litres}) \times 10^4 (\text{hectare}) \times 60 \times 60 \times 24 (\text{day})$$
$$= 1.33 \times 10^{-2} \times 86.4 \times 10^3$$
$$= 1152 \text{ l/ha/day}$$

(b)  $Q = 11520 \text{ l/ha/day}$

(c)  $\text{ALR} > 935 \text{ l/ha/day}$

$\text{RLLR} > 9350 \text{ l/ha/day}$



So, can you give me a value which you get when liters per hectare per day? So,  $k$  is  $10$  to the power of minus  $7$  centimeters per second which becomes  $10$  to the power of minus  $9$  meters per second into  $1.33$ . So, this becomes, so many meter cubed per meter square, per second and this whole thing if you want to convert it into a hectare you have to multiply it  $10$  to the power of  $4$ . And if you want to convert it into a day,  $60$  seconds,  $60$  minutes,  $24$  hours. Is anybody able to give me a value here? What is the value that you get in liters per hectare per day?

Anybody has taken out a value? So, my calculation is  $1152$ , but you will have to confirm. Is there anybody got  $1152$ ?

Student: Yes Sir.

Ok, great. So, what we have to see is, that if I have a good well compacted clay liner, I will get a  $1000$  liters per hectare, per day and if a bucket is  $20$  liters how many buckets is that? A  $1000$  liters?

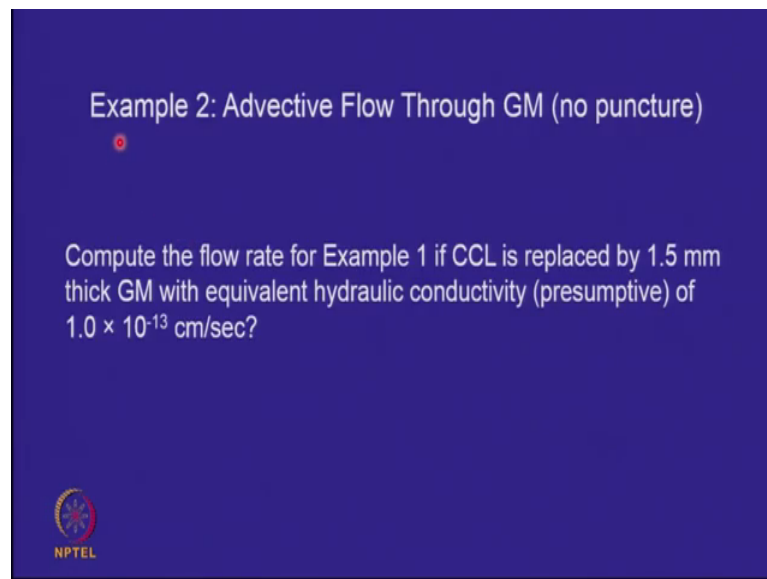
Student:  $50$

$50$  buckets per. So, if you compare it now with the action leachate rate which we had talked about, that it is  $935$ .  $935$  is the same as a  $1000$ . So, the  $1000$  comes from this; that means, if it exceeds a  $1000$  you have something to worry about.

The next question is, if by mistake or by construction practices which are not very good you have not done good compaction, you started with the soil which has significant plasticity, but from the border area the soil which is coming now does not have that much elasticity. Then, maybe the  $k$  can become  $10^{-6}$  centimeters per second that is above. If this becomes  $10^{-6}$ , what happens to  $Q$ ?  $Q$  becomes  $1.152 \times 10^6$ ; that means, it has come ten times what is acceptable. And large leakage rate if you recall that was  $9.35 \times 10^{-6}$ . So, this ten times mean that you are having a liner which is not even performing like a 0.9 meter thick; compacted clay liner. So, that is the basis of fixing up the action leachate rate and if you have large leakage then what happens? You have to shut down, you have to take action to reduce the leakage otherwise you have to plug the liner system.

Let us say that we were using a geo membrane alone. So, the second example that we are going to do is advective flow through geo membrane.

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Example 2: Advective Flow Through GM (no puncture)

Compute the flow rate for Example 1 if CCL is replaced by 1.5 mm thick GM with equivalent hydraulic conductivity (presumptive) of  $1.0 \times 10^{-13}$  cm/sec?

NPTEL

Now do remember that, I do not have a precise hydraulic conductivity value for geo membrane. I have some presumptive values because I use some water vapor transmission test and on the basis of the water vapor transmission test, I can get an equal and hydraulic conductivity for ease in calculation and just like we said that, the clay has a hydraulic conductivity of  $10^{-7}$ . Let us say the presumptive hydraulic conductivity of geo membrane is  $10^{-13}$ . Can you now

tell me what is the  $q$  coming from the geo remembrance? Same formula, but now the thickness instead of being 90 centimeters has become 1.5 millimeter. So, if I just take a 1.5 millimeter geo membrane and put some water on it which has a leachate head of 0.3 meters and a equalent presumptive hydraulic conductivity of  $10$  to the power of minus  $13$ , what kind of  $Q$  will come out? We know that for clay it is about a 1000 liters per hectares per day. Can you please compute the  $Q$  which will come out through a geo membrane with no puncture? Same formula, except you substitutes  $k$  with  $10$  to the power of minus  $13$  centimeters per second and the hydraulic gradient will change. In fact, can you tell me, what is the hydraulic gradient here?

Student: 2 0 1.

2 0 1 and what is the hydraulic gradient in the clay?

Student: 1.33,

1.33. So, the hydraulic radiant is very high. So consequently, what do you get as your  $Q$  in liters per hectares per day? So, with the hydraulic anybody would like to give me a value.

Student: 0.17.

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Example 2: Solution

$$i = \frac{(0.3 + 0.0015)}{0.0015} = 201$$


$$Q = (1 \times 10^{-13}) \times (201) \times (1.0) \text{ m}^3/\text{m}^2/\text{sec}$$

$$= 2.01 \times 10^{-11} \times 10^3 (\text{litres}) \times 10^4 (\text{hectare}) \times 60 \times 60 \times 24 (\text{day})$$

$$= 2.01 \times 8.64$$

$$= 17.4 \text{ l/ha/day}$$

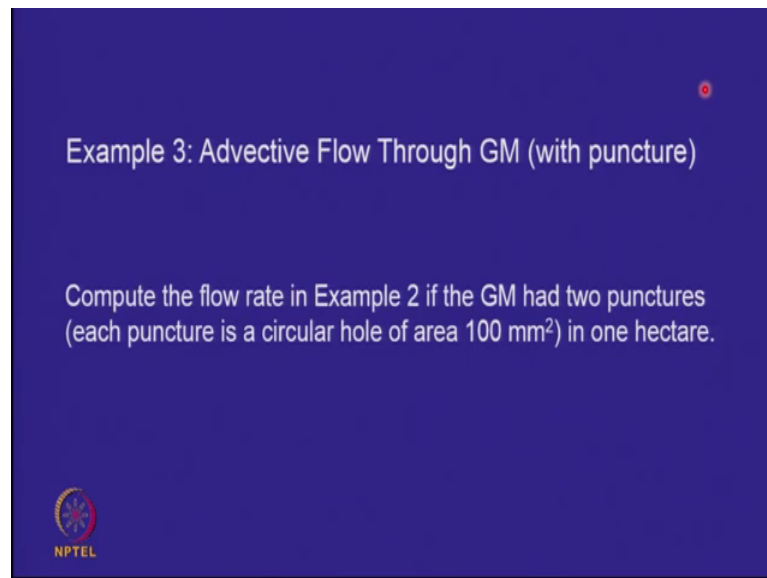
Actual observed flux is controlled by water vapour transmission rate and is reported as 0.1 l/ha/day.



0.17. So, you should get 0.17, which is also what is found from water vapor transmission. Here there is an error because a 100 has not been applied to the 10 to the power of minus 13, but anyways the solution that we get is 0.174 hectares per day, a liters per hectare per day.

So, right from 1000 to 0.1, that is what is the effect of a geo membrane, but let us make a hole in this geo membrane.

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Let us make a puncture. If I am make a puncture, what happens? Water will flow through an RFS and let us assume I have a circular hole and we have to give the area of the hole, but then I have the Bernoullis Equation for flow through an RFS. So, the same example I would now like you to take at 2 punctures per hectare and this circular hole of area 100 millimeter square; that means, 1 centimeter square in one hectare. So, if I have two holes which is very good quality construction, if you will go recall in one football field, if I were to get two holes which I have 1 centimeter square area is very very good excellent quality of construction.


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Bernoulli Equation (flow through circular orifice)

$$Q = C_b \cdot a \cdot (2 \cdot g \cdot h)^{0.5}$$

where

- $Q$  = flow rate through geomembrane, cm<sup>3</sup>/sec;
- $C_b$  = flow coefficient with value approximately 0.6 for a circular hole;
- $a$  = area of a circular hole in geomembrane, cm<sup>2</sup>;
- $g$  = acceleration due to gravity, 981 cm/sec<sup>2</sup>; and
- $h$  = liquid head above the liner, cm.



The formula that has to be used is the formula for flow through a circular RFS. And please have a look at this formula and note it down  $Q$  is equal to  $C_b$  into  $a$  where,  $C_b$  is the flow coefficient and we take it 0.6 for a circular hole,  $a$  is the area of the circular hole in centimeter square,  $2 g h$ ;  $g$  is the acceleration due to gravity and  $h$  is the liquid head above the liner. So, same problem we are now looking at, if I make two holes what is  $Q$ ? Please do this computation for me and in this formula you can use  $C_b$  equal to 0.6 and you will have to multiply this with 2 because this is per hole,  $g$  is 981 and  $h$  will be 30. For compacted clay liner we had got 1 1 5 2, what do you get for geo membrane with two holes?

Student: 2.29.

Point.

Student: 29.


That is very low. I can immediately feel that if it is a hole 1 centimeter square, what is going to come out very fast? I am going to fill several buckets. Let me see what the calculations reveal here? Anybody has got this value?



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Example 3: Solution

Compute the flow rate in Example 2 if the GM had two punctures (each puncture is a circular hole of area 100 mm<sup>2</sup>) in one hectare.

$$Q = 0.6 \times \frac{100}{100} \times (2 \times 981 \times 30)^{0.5} \text{ cm}^3/\text{sec} \times \text{no. of holes}$$
$$= 145.6 \times 2 \text{ cm}^3/\text{sec}/\text{ha}$$
$$= 291.1 \times 10^{-3} \times 60 \times 60 \times 24$$
$$= 25000 \text{ l}/\text{ha}/\text{day}$$


So, it is about 25000. So, you jump. A geo membrane is virtually impermeable, very little water comes out of it, but if you make a hole or two in it, everything flows out. Anybody who is not reconcile to this value? And I have to remember action leakage rate was 935.

So let us see, do we have any evaluation methodology for a composite liner? So, this is from the Darcys Law. Darcys Law, flow through an RFS. For a composite liner, we have some empirical results, some empirical formulae which have been given to us.


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Example 4: Advective Flow Through Composite Liner  
(with puncture in GM)

Compute the flow rate in Example 3 for composite liner (1.5 mm GM; CCL 0.9 m thick) with two punctures (circular 100 mm<sup>2</sup> each) in one hectare for

- Good contact (few wrinkles / waves)
- Poor contact (large no. of wrinkles waves)

Assume hydraulic gradient is 1.0.



So, if I use the same geo membrane and the same compacted clay 1.5 mm thick geo membrane, 0.9 meter thick clay. Again two punctures in one hectare, then I have some empirical formulae for good contact and for poor contact if the hydraulic gradient is one.

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Empirical equations as per Giroud and Bonaparte (1989) and Giroud et al. (1989)

For "good" contact conditions,  $Q = 0.21 \cdot a^{0.1} \cdot h^{0.9} \cdot k_s^{0.74}$

For "poor" contact conditions,  $Q = 1.15 \cdot a^{0.1} \cdot h^{0.9} \cdot k_s^{0.74}$

where  $Q$  = leakage rate through a hole in GM component, m<sup>3</sup>/sec;  
 $a$  = area of a circular hole in geomembrane, m<sup>2</sup>;  
 $h$  = liquid head on top of the GM, m; and  
 $k_s$  = hydraulic conductivity of the low permeability soil component of the composite liner, m/sec.

These equations assume that the hydraulic gradient through the soil is 1.0. The equations are not dimensionally consistent:  $Q$  (m<sup>3</sup>/sec),  $a$  (m<sup>2</sup>),  $h$  (m), and  $k_s$  (m/sec).

These were studies which have been done by Giroud and Bonaparte and Giroud. So, these are empirical, these equations are not dimensionally consistent. So, you have to use the values which are given here, same problem as before.

If I have good contact, it becomes 0.21 a to the power of 0.1, h to the power of 0.9 k, h to the power of 0.74. So, if you use the area in meter square, if you use the head in meters and if you use k s in meter per second then you will get the Q in meters cube per second right. So, per hectare we have to compute, what is the quantity of leachate? So, please note the first formula, substitute the values and this is for one hole and if there are two holes in a hectare. So, we will have to multiply it by 2 and what is good contact and what is bad contact?


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**Example 4: Advective Flow Through Composite Liner  
(with puncture in GM)**

Compute the flow rate in Example 3 for composite liner (1.5 mm GM; CCL 0.9 m thick) with two punctures (circular 100 mm<sup>2</sup> each) in one hectare for

- i) Good contact (few wrinkles / waves)
- ii) Poor contact (large no. of wrinkles / waves)

Assume hydraulic gradient is 1.0.



Good contact means few wrinkles and poor contact means large number of wrinkles. So, in a way when they did the testing in the laboratory, this was the compacted clay. A good contact meant this and a poor contact right. So, what was done was, the wetting front in a poor contact was much wider and a wetting front in a good contact. So, consequently you would get larger k. So, please apply the formulae, take it is an empirical formula and give me the values for.


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**Example 4: Solution**

For good contact,  $Q = 0.21 \times \left(\frac{100}{1000000}\right)^{0.1} \times (0.3)^{0.9} \times \left(\frac{1.0 \times 10^{-7}}{100}\right)^{0.74} \times 2 \text{ holes m}^3/\text{sec}$

$$= 0.21 \times 0.40 \times 0.34 \times 2.19 \times 10^{-7} \times 2$$
$$= 0.125 \times 10^{-7} \text{ m}^3/\text{sec/ha}$$
$$= 0.125 \times 10^{-7} \times 10^3 \times 60 \times 60 \times 24$$
$$= 1.08 \text{ l/ha/day}$$

For poor contact,  $Q = 1.15 \times \left(\frac{100}{1000000}\right)^{0.1} \times (0.3)^{0.9} \times \left(\frac{1.0 \times 10^{-7}}{100}\right)^{0.74} \times 2 \text{ holes m}^3/\text{sec}$

$$= 0.685 \times 10^{-7} \text{ m}^3/\text{sec/ha}$$
$$= 5.91 \text{ l/ha/day}$$



So, the competitions shown here give you 1.08 liters per hectare per day for good contact and 5.91 liters per hectare for poor contact. Please confirm this, anybody has got these values  $Q$  into  $0.21$  into  $a$  to the power of  $0.1$ ,  $0.3$  to the power of  $0.9$ ,  $10$  to the power minus  $9$  to the power of  $0.7$  into  $2$ . You will have to use the function  $a$  to the power of  $b$  in your calculator or you have to do log. Is anybody confirming this value? You also good. So, for good contact I am getting  $1.08$  and this is about  $5$  times. So, what do these examples show us? The example show us that, our action leakage rate is about a  $1000$  liters per hectare per day which is what we would get with the compacted clay liner, which is relatively thick and has a head almost equal to the thickness of the leachate collection layer on top.

If I use a geo membrane alone it looks very good, but no geo membrane can be laid without a puncture. So, a geo membrane with puncture is bad. However, if you put both of them together even, if there is puncture in the geo membrane the leakage is very very small. So, that is why we use composite geo membrane plus compacted clay liners at the base of a landfill, any questions?

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Example 5: CCL and GCL – Advective Mass Flux

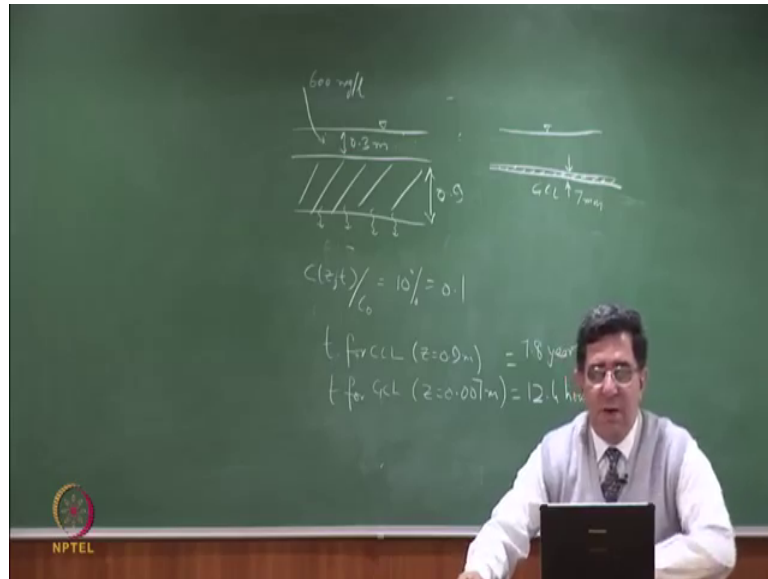
Compare the advective mass flux ( $J_A$ ) through a CCL of thickness  $90$  cm and  $k = 1.0 \times 10^{-7}$  cm/sec with that of GCL of thickness  $7$  mm and  $k = 1.0 \times 10^{-9}$  cm/sec . Assume steady state condition and leachate head of  $30$  cm and chloride concentration of  $600$  mg/l. Compare the ratio of  $J_A$  in GCL to that in CCL.



If there are no questions then, we would like to go to the issue about comparing compacted clay liners with geo synthetic clay liners. You recall that we had said that if you can get low permeability thin material then, it can also perform as well as a compacted clay liner. So, I am going to compare four aspects of this comparison,

Advective mass flux is the same as advective flow. So, if I have a compacted clay liner which is 90 centimeter thick has  $k$   $10$  to the power of minus  $7$  centimeter per seconds. The head is 30 centimeters and I compare it with a geo synthetic clay liner of thickness 7 millimeters. So, a 90 centimeter thick clay has been replaced by 7 millimeters and because it is got Sodium Bentonite in it, its permeability is 100th of the compacted clay.

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So, what are we comparing? We are comparing same leachate head, but now that is your 7 millimeters is thicker than a geo membrane. It is thicker than a geo membrane 1.5 mm, but the GCL is posed to be self healing. So, if you have a puncture, the Bentonite will expand heal itself. So, we are now going to compare the  $Q$  through both of them right, but this problem is presented. So, please understand once again your hydraulic gradient is going to be very high here just like your 201. You are going to have a high hydraulic gradient here, but the permeability of this is much lower. Now we will do this comparison in terms of the advective mass flux. We are saying that this leachate has 600 milligram per liter of chlorides; of chloride ions. So, we want to know, how much chlorides will come out of the compacted clay liner? And how much chlorides will come out of the GCL and compare the two?


So, when you are doing a advective mass flux then the equation that you use similar except that you multiply the concentration with  $v$ .

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When the concentration of solute of concern in the landfill leachate remains constant, the one-dimensional advective mass flux (Darcian hypothesis) of solute per unit area can be expressed as

$$J_A = C_o \cdot v = C_o \cdot k \cdot i = C_o \cdot k \cdot \left( \frac{H+L}{L} \right)$$

where  $J_A$  = advective mass flux, mg/cm<sup>2</sup>/sec;  
 $C_o$  = concentration of solute in water at top of liner, mg/cm<sup>3</sup>;  
 $v$  = average seepage velocity of flow through the liner, cm/sec = Q/A;  
 $k$  = hydraulic conductivity cm/sec;  
 $i$  = hydraulic gradient;  
 $H$  = leachate head on the liner, cm; and  
 $L$  = thickness of the liner, cm.



So, the average seepage velocity of flow through the liner is  $Q$  by  $A$  which is  $k \cdot i$ . So, the advective mass flux in milligram per centimeter square area per second is  $J_A \cdot C$  naught into  $v$ . Please note this down, which becomes  $C$  naught into  $k \cdot i$  and which become  $C$  naught into the  $h$  plus  $L$  over  $L$ . So, just note down this formula and please be consistent with your units milligram, per centimeter square per second, milligram per centimeter cube. In our case, I have given you the value which is normally we used in the Indian standards in terms of milligram per liter because that works out like parts per million. So, using this concentrations, you can compute the  $J_A$  both in the compacted clay liner and in the GCL. So, compute the advective mass flux using this formula, you have  $C$  naught 600 milligram per liter. You have  $k$ , if it is compacted clay liner it is  $10$  to the power of minus  $7$  centimeter per second, if it is GCL it is  $10$  to the power minus  $9$ ,  $H$  plus  $L$  vary  $H$  is  $0.3$  meters in both the cases, but  $L$  is varying.


So, what is the advective mass flux for the first case in the compacted clay liner and then the second case in the GCL?

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Example 5: Solution

$$J_A(GCL) = \left(\frac{600}{1000}\right) \times (1 \times 10^{-9}) \times \left(\frac{30+0.7}{0.7}\right)$$
$$= 26.3 \times 10^{-9} \text{ mg/cm}^2/\text{sec}$$
$$J_A(CCL) = \left(\frac{600}{1000}\right) \times (1 \times 10^{-7}) \times \left(\frac{30+90}{90}\right)$$
$$= 80 \times 10^{-9} \text{ mg/cm}^2/\text{sec}$$
$$\text{Ratio (GCL : CCL)} = \frac{26.3}{80} = 0.33$$

GCL passes less advective flux than CCL



In the GCL, please note 10 to the power of minus 9 centimeters per second is the k. This has the 600 milligram per liter. So, we are trying to convert milligram per centimeter cube. So, 600 by a 1000, head is 30 centimeters, thickness is 0.7 centimeters and you should get this value of advective flux through GCL. Please confirm that you are getting this value. Anybody is confirming it?

Student: Yes.


Ok, great then, if you go to the compacted clay liner the concentration is still the same. Two things happen; permeability is 10 to the power of minus 7 and the hydraulic gradient changes. And this causes the advective mass flux to be 80 into 10 to the power minus 9 milligram per centimeter per second. So, what does this show us? Which is transmitting more? Compacted clay liner is transmitting more. So, GCL is better because you want less mass flux. We want less chlorides to flow out. So, this shows that the ratio of GCL to CCL is 0.33 which is less than 1. Therefore, the geo synthetic clay liner passes less advective flux than CCL, but the other thing which I would like to study is, how quickly does it come out?

So, suppose we have two liners; if I have a geo membrane, the moment it is punctured how quickly does the leachate come out? Instantaneously, but if I have clay, it takes some time for it to travel to the soil. So, it takes may be a few months may be a year or so. So that is the next aspect we are going to compare.

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Example 6: CCL and GCL – Advective Breakthrough Time

Compare the advective break through time for CCL and GCL in Example 5 given that porosity ( $n$ ) for CCL is 0.5 and for GCL is 0.6.




We would like to compare the advective breakthrough time; that means, in how much time does the leachate come below the liner? That means, you see as far as your design is concerned this is an impervious layer. One is thin, one is thick, one is low permeability, one is a little slightly higher than the low permeability of the GCL. Now what we are saying is, compare the advective breakthrough time for CCL and GCL in the previous example. The additional data that is given to you is that the porosity for compacted clay liner is 0.5 and the porosity for the geo synthetic clay liner is 0.3. That is the additional data, you still continuing with the same example and the way to tackle this is as follows.

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$$v_s = \frac{v}{n} = \frac{k \cdot i}{n} = k \cdot \left( \frac{H+L}{L \cdot n} \right) \quad T_B = \frac{L}{v_s}$$

where  $v_s$  = seepage velocity (i.e., average velocity of flow through the voids of the liner), cm/sec;  
 $v$  = average seepage velocity of flow through the liner, cm/sec;  
 $n$  = porosity;  
 $k$  = hydraulic conductivity cm/sec;  
 $i$  = hydraulic gradient;  
 $H$  = leachate head on the liner, cm;  
 $L$  = thickness of the liner, cm; and  
 $T_B$  = advective breakthrough time, sec.





We know that  $v$  is equal to  $k i$  and  $v$  is called the average seepage velocity of the flow through the soil or through the liner. We first compute not the average seepage velocity, but the seepage velocity through the voids of the liner. So,  $v_s$  is defined as the seepage velocity through the voids which is nothing, but  $v$  divided by porosity. So, I can get the value of  $v_s$ , if I know the value of  $v$  the leachate has to travel the length  $L$ . which is the thickness of the liner.

So, the breakout time is  $L$  by  $v_s$ . So, in both your problems first find out  $v_s$ . You can compute it in centimeters per second or you can compute it in meters per second and then convert it into meters per year. You will get an idea is to how quickly it is traveling, how many meter per year is travelling.

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Example 6: Solution

$$v_s(CCL) = (1 \times 10^{-7}) \times \left( \frac{30 + 90}{90 \times 0.5} \right)$$

$$= 2.667 \times 10^{-7} \text{ cm/sec}$$

$$= 8.4 \text{ cm/year}$$

$$v_s(GCL) = (1 \times 10^{-9}) \times \left( \frac{30 + 0.7}{0.7 \times 0.6} \right)$$

$$= 0.73 \times 10^{-7} \text{ cm/sec}$$


$$= 2.3 \text{ cm/year}$$

$$T_B(CCL) = \frac{90}{8.4} = 10.7 \text{ years}$$

$$T_B(GCL) = \frac{0.7}{2.3} = 0.3 \text{ years} = 110 \text{ days}$$

$$\text{Ratio (GCL : CCL)} = \frac{0.3}{10.7} = 0.028$$

Advective breakthrough time is much shorter in GCL than CCL



So, with this we can find out the break through time. So, let us first look at CCL in 10 to the power of minus 7 centimeters per second, 30 plus 90 over 90 into 0.5, this is the porosity effect and the seepage velocity will work out to be 8.4 centimeters per year. So, the important thing which I am trying to tell you is,  $Q$  is large; how much is the  $Q$ ? 1 1 5 1 liters per hectare per year; the question is when does it start to come out in?

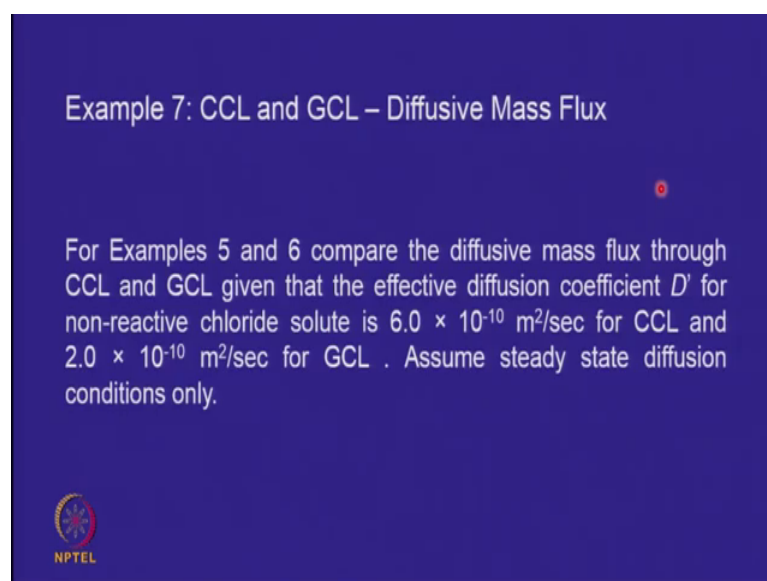
If you look at this our  $v_s$  in the compacted clay liner is 8.4 centimeters per year and how many centimeters does it have to travel to come out of the liner? 90. So, 90 divided by 8.4, 10.7 years is the amount of time when your leachate will come through. So, you will not see any leachate at the bottom of the compacted clay liner till 11 years of past. That is

a huge amount of time. Just if you had a much slower a much lower permeability of the soil, let us say 5 into 10 to the power of minus 8 then, it would have been even longer.

Let us look at these seepage velocity, in the geo synthetic clay liner. Well here we are, this is the lower permeability value, but the hydraulic gradient is higher and you get 2.3 centimeters per year. So, the seepage velocity through the geo synthetic clay liner is lower; however, the thickness of the geo synthetic clay liner is lower. So, the break out time in the geo synthetic clay liner is 0.7 centimeters divided by 2.3 a 110 days. That is about 3 to 4 months. So, the leachate will come out of the geo synthetic clay liner much faster than it will come out of the compacted clay liner. So, once it starts to come out the mass flux is going to be higher in the compacted clay liner, but till it comes out it is all inside the clay. So, in that sense because the break through time is much shorter in the GCL, the performance of CCL is better as far as break through time is concerned and this you have to realize, that the performance when you are trying to do equivalence you are not only trying to do equivalence in mass flux, but you are also trying to do the equivalence in when it comes out at the bottom.


So, the breakthrough time is much longer in compacted clay liner. Anybody is confirming these values for me? Just so that, I have not made an error in computation. Can we move forward? Anybody has a doubt?

(Refer Slide Time: 33:44)



Example 7: CCL and GCL – Diffusive Mass Flux

For Examples 5 and 6 compare the diffusive mass flux through CCL and GCL given that the effective diffusion coefficient  $D'$  for non-reactive chloride solute is  $6.0 \times 10^{-10}$  m<sup>2</sup>/sec for CCL and  $2.0 \times 10^{-10}$  m<sup>2</sup>/sec for GCL . Assume steady state diffusion conditions only.



We have so far focused on advective mass flux; that means, due to difference in head. We can also look at diffusive mass flux. The diffusive mass flux requires us to use the Ficks Law and we have not gone into detail about the Ficks Law in a classes, but I will give you two formulae which we are going to use here and we are going to use parameter called the effective diffusion coefficient.

So, for the same problem we are still comparing the CCL with the GCL. We want to find the diffusive mass flux through the CCL and the GCL, given that the effective diffusion coefficient  $D$  dash for the non reactive chloride solute. If it was reactive it would be attenuated and therefore, it would be a different value all together. So, for the CCL, it is  $6 \times 10^{-10}$  meters per square per second and for GCL, it is  $2 \times 10^{-10}$  meter square. Please note permeabilities, can vary by a 100 times or a 1000 times.  $10^{-7}$ ,  $10^{-9}$ ,  $10^{-13}$  right, but diffusion coefficients within the soil will vary not more than that an order of magnitude of 2, 4, 6, 8. So, here it is 6 and here it is 2.


Now, assume steady state diffusion conditions. So, this is an idealized problem, steady state diffusion condition means there is a  $C$  at the top and there is a  $C$  at the bottom and it remains fixed. Transient means that as it comes through  $C$  at the bottom will start to rise you know if I have a concentration of 600 milligram per liter at the top and 0 milligram per liter at the bottom. If I am saying that this is a steady state condition which means I am flushing the bottom all the time and this is remains as 600 and this remains at 0, but as it starts to come though this 0 will begin to rise and that is transient case.

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Fick's first law of diffusion, for one dimensional transport (steady state transport of non-reactive solutes):

$$J_D = D' \cdot n \cdot \left( \frac{\partial C}{\partial z} \right) = D' \cdot n \cdot \left( \frac{\Delta C}{L} \right)$$

where  $J_D$  = diffusive mass flux, mg/cm<sup>2</sup>/sec;  
 $D'$  = effective diffusion coefficient, cm<sup>2</sup>/sec;  $n$  = porosity;  
 $\delta C / \delta z$  = concentration gradient; and  
 $\Delta C$  = difference in concentration between top and base of the liner, mg/cm<sup>3</sup>, or  $\Delta C = C_o - C_e$ ;  $L$  = liner thickness, cm  
 $C_o$ ,  $C_e$  = solute concentration at the top and base of the liner, mg/cm<sup>3</sup>  
The effective diffusion coefficient depends on the chemical of interest (e.g., ionic size, charge of solute) and diffusion conditions (e.g. diffusion through free solution vs. soil pore water, tortuosity of pores). Solutes diffuse more slowly through soil pores than in free solution



So, we look at the transient case a little later. At the moment we are assuming steady state diffusion conditions only and the formula that we use is called the Fick's law of diffusion.


We have done  $J_A$  earlier  $J_D$  is equal to the effective diffusion coefficient  $D'$  into the porosity and that is the concentration gradient, please have a look. So, here we are saying 600 milligram per liter at the top 0 milligram per liter at the bottom. Please tell me, what is the diffusive mass flux? So, you have been given  $D'$  you know  $\Delta C$  600 minus 0, you have the liner thickness. So, having these components you can find out the  $J_D$ . Do remember that the diffusion coefficient will depend on the contaminant that you are tracking. We are saying this is the coefficient for chlorides, which is the faster one which travels because it is non reactive. It will depend on also the type of soil the tortuosity of the pores that is why you find that the diffusion coefficient for a GCL is lower because the tortuosity of the pores is larger. So, one case it is  $6 \times 10^{-10}$ , in the other case it is  $6.2 \times 10^{-10}$ .

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Example 7: Solution

$$J_D(CCL) = \left(\frac{600}{1000}\right) \times (6 \times 10^{-10} \times 10^4) \times \left(\frac{0.5}{90}\right) \text{ mg/cm}^2/\text{sec}$$
$$= 2 \times 10^{-8} \text{ mg/cm}^2/\text{sec}$$
$$J_D(GCL) = \left(\frac{600}{1000}\right) \times (2 \times 10^{-10} \times 10^4) \times \left(\frac{0.6}{0.7}\right) \text{ mg/cm}^2/\text{sec}$$
$$= 102 \times 10^{-8} \text{ mg/cm}^2/\text{sec}$$
$$\text{Ratio (GCL : CCL)} = \frac{102}{2} = 51$$

GCL passes much larger diffusive flux than CCL



So, just apply this and give me the values of the diffusive flux and compare that two. So can somebody confirm for me this value? You got it independently? Great.

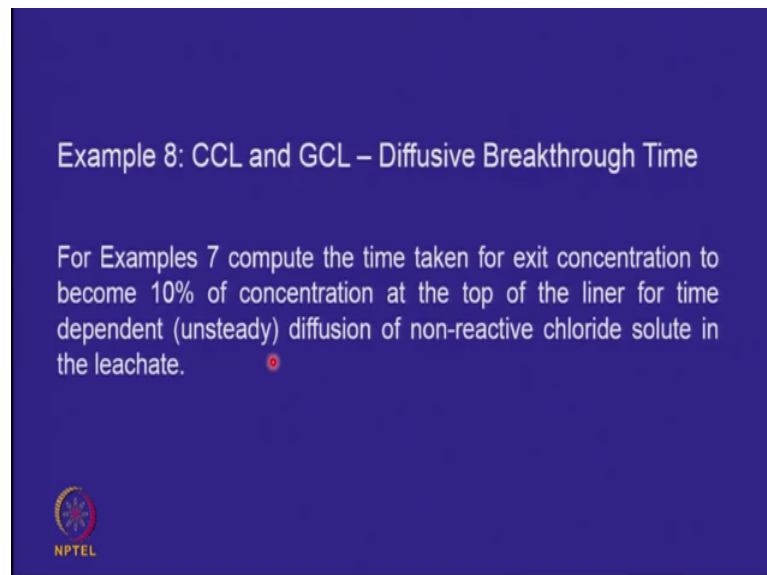
So, this is 2 into 10 to the power of minus 8 and GCL is 102 into 10 to the power minus 8. Why did that happen? Because the gradient is over, here this drops over 90 centimeters concentration gradient, here the concentration gradient drops over 0.7 centimeters. So, the concentration gradient is high just like the hydraulic gradient was high. In the case of advective flux, what happened? Your hydraulic gradient was high, but it was offset by a order of difference in permeability of 10 to the power of 2; that means, one was 10 to the power of minus 7 and the another was 10 to the power; so 100. Here what happens? The order of difference between the 2 is not 100, but 3, where as this is almost a 120 times. So, that is why the diffusive mass flux is larger, 51 times. Let us quickly go back to our advective mass flux and see what we did we calculate there.

So, advective mass flux was 2.6 into 10 to the power of minus 8 and 8 into 10 to the power of minus 8. 2 into 10 to the power of minus 8 and 102, two things I want to make statement here; one the advective mass flux and the diffusive mass flux in compacted clay liner or similar right. 2.6 into 10 to the power of minus 8 into 2 into 10 to the power of 8, but the diffusive mass flux in GCL is much more than the advective mass flux. So, GCLs, when the permeability becomes very low, the diffusion will predominate. When the permeability is high advection will predominate. So, the cutoff is about 10 to the

power minus 7. At  $10^{-7}$  centimeter per second permeability and diffusion coefficients in the order of  $10^{-10}$  meters per square per second advection and diffusion have about the same role to play, but if you go to  $10^{-9}$  centimeter per second permeability then, you find that the diffusive flux is much more.


So, in comparison once again the GCL is passing more through diffusion. Agreed? Finally, we look at we talked about advective breakthrough time and we are now going to talk about diffusive break through time.

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Example 8: CCL and GCL – Diffusive Breakthrough Time

For Examples 7 compute the time taken for exit concentration to become 10% of concentration at the top of the liner for time dependent (unsteady) diffusion of non-reactive chloride solute in the leachate.



So, advective break through time was 10 years, 10, 11 years compacted clay liners and a few months in GCLs. To find out diffusive break through time it is little tricky, in the diffusion because you now have to use transient formulae, you have to use transient formula.

So, here what you will find is, that we are going to use Ficks Second Law. The first law was for steady state condition  $\Delta C$  by  $L$  is a constant as simple to do.


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For time-dependent (unsteady) transport of non-reactive solutes in soil, Fick's second law:

$$\frac{\partial C}{\partial t} = D \cdot \left( \frac{\partial^2 C}{\partial z^2} \right)$$

Solution of the above equation is:

$$C(z,t) = C_o \cdot \operatorname{erfc} \left[ \frac{z}{(4 \cdot D \cdot t)^{0.5}} \right]$$

where  $C(z,t)$  = solute concentrations at depth  $z$  and time  $t$ , mg/cm<sup>3</sup>;  
 $z$  = depth, cm; and  
 $t$  = time, sec;  
  $\operatorname{erfc}$  = complimentary error function.

So, the Fick's second law takes this form. For time dependent unsteady transport of non reactive solutes and soil, but diffusion Fick's second law says  $\frac{\partial C}{\partial t}$  is  $D$  dash  $\frac{\partial^2 C}{\partial z^2}$ . Luckily there is a solution for this equation and the solution is the concentration at any depth and at any time is related to the original concentration at the top by this complementary error function  $\frac{z}{4 D t^{0.5}}$ . Now you will have to take this formula is I given. You are not going to do this derivation, but what is it that this formula gives me? You can all read up about this, it is not very complicated, but what does it give me, it tells me that as the solute or the contaminant travels the concentration is changing. So, it is an unsteady case right.

So, if you go into the soil at any depth, at any time. So, I have let me say I have a 90 centimeter thick clay. I want to find out at 45 centimeters, middle what is the concentration? If I say the time at 0 time after 1 year, after 2 years then  $z$  is 45 centimeters,  $t$  is the time, I can put  $z$  here,  $D$  dash is given to me, time is known. I can compute this value right, once you compute this value you can find  $C$   $z$  as a function of  $C$  naught.

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### Complimentary Error Function

$$\operatorname{erf}(\beta) = \frac{2}{\sqrt{\pi}} \int_0^\beta e^{-t^2} dt$$

$$\operatorname{erf}(-\beta) = -\operatorname{erf}(\beta)$$

$$\operatorname{erfc}(\beta) = 1 - \operatorname{erf}(\beta)$$

$\beta$	$\operatorname{erf}(\beta)$	$\operatorname{erfc}(\beta)$	$\beta$	$\operatorname{erf}(\beta)$	$\operatorname{erfc}(\beta)$
0	0	1.0	1.1	0.880205	0.119795
0.05	0.056372	0.943628	1.2	0.910314	0.089686
0.1	0.112463	0.887537	1.3	0.934008	0.065992
0.15	0.167996	0.832004	1.4	0.952285	0.047715
0.2	0.222703	0.777297	1.5	0.966105	0.033895
0.25	0.276326	0.723674	1.6	0.976348	0.023652
0.3	0.328627	0.671373	1.7	0.983790	0.016210
0.35	0.379382	0.620618	1.8	0.989091	0.010909
0.4	0.428392	0.571608	1.9	0.992790	0.007210
0.45	0.475482	0.524518	2.0	0.995322	0.004678
0.5	0.520500	0.479500	2.1	0.997021	0.002979
0.55	0.563323	0.436677	2.2	0.998137	0.001863
0.6	0.603856	0.396144	2.3	0.998857	0.001143
0.65	0.642029	0.357971	2.4	0.999311	0.000689
0.7	0.677801	0.322199	2.5	0.999593	0.000407
0.75	0.711156	0.288844	2.6	0.999764	0.000236
0.8	0.742101	0.257899	2.7	0.999866	0.000134
0.85	0.770668	0.229332	2.8	0.999925	0.000075
0.9	0.796908	0.203092	2.9	0.999959	0.000041
0.95	0.820891	0.179109	3.0	0.999978	0.000022
1.0	0.842701	0.157299			

Now what is this error function complementary? Standard tables are available, do not worry. If you know beta you can find erf c. We will have to go back and look up your maths a little bit.

But if you find this is beta, if you find this value then you can find this error function and quite clearly C cannot be greater than C naught. Can it be greater than C naught? So, it can not be greater than C naught. So, you will get these values varying from 0 to 1 all the time; 1 to 0.

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### Example 8: CCL and GCL – Diffusive Breakthrough Time

For Examples 7 compute the time taken for exit concentration to become 10% of concentration at the top of the liner for time dependent (unsteady) diffusion of non-reactive chloride solute in the leachate.

$$\frac{C(z,t)}{C_o} = \operatorname{erfc} \left[ \frac{z}{(4 \cdot D' \cdot t)^{0.5}} \right]$$

given that  $\operatorname{erfc}(\beta) = 0.1$  when  $\beta = 1.17$ .



So, in this case we are going to give you a problem, simple problem. How do I calculate breakthrough time? Breakthrough time is when the first contaminant comes through right. For the purpose of this problem we are going to say when does the concentration at the bottom of the liner become 10 percent of the concentration at the top. You have to give some value. When you say you know when the first come through it can be 0.000001 percent and that is a difficult to do mathematically. So, it says for the previous example, compute the time taken for the exit concentration to become 10 percent of the concentration at the top of the liner for the time dependent diffusion. So, you recall  $C(z,t)$  is equal to  $C_0 \text{erfc}\left(\frac{z}{2\sqrt{Dt}}\right)$  into this. So, we are saying  $\frac{C(z,t)}{C_0}$  should be equal to 0.1; that means, error function of something should be equal to 0.1 and if you go back to this table, error function of something complementary error function of something should be equal to 0.1; it will come here some somewhere right and that value is 1.17.

So, now I know 1.1, I know  $z$  all have to do is find out the time  $t$ . So, please put  $\frac{z}{2\sqrt{Dt}} = 1.17$  and please compute the  $t$  for me then days, months or years. Do you remember that all processes are couple. Actually in the field advection and diffusion are accruing together. So, what we are doing here is very very simplified, but it still does give us an idea of what is happening fast, what is happening slow, how much time does it take. So, if this is value is 1.17 this becomes 0.1 which is 10 percent. So, please compute the  $t$  for the  $z$ . Please understand that the  $z$  for the compacted clay liner will be the 90 centimeters and for GCL it will be 0.7 centimeters.

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**Example 8: Solution**

For  $\frac{C}{C_o}$  to be 10% (0.1),

$$\frac{z}{(4 \cdot D \cdot t)^{0.5}} = 1.17$$

$$\frac{z^2}{(4 \cdot D \cdot t)} = (1.17)^2 = 1.37$$

$$t = \frac{z^2}{5.48 \times D}$$

CCL,  $z = 0.9$  m

$D = 6 \times 10^{-10}$  m<sup>2</sup>/sec

$$t = \frac{0.9^2}{5.48 \times 6 \times 10^{-10}} = 2.46 \times 10^8 \text{ sec}$$

= 7.75 years


GCL,  $z = 0.007$  m

$D = 2 \times 10^{-10}$  m<sup>2</sup>/sec

$$t = \frac{0.007^2}{5.48 \times 2 \times 10^{-10}} = 4.46 \times 10^4 \text{ sec}$$

= 0.0014 years = 12.4 hours

Diffusive Breakthrough is faster in GCL than CCL



We will have to match over to you your units, but the values which I have got as 7.75 years and 12.4 hours, but I would like you to compute it on your own and tell me what values do you get. Well anybody has some values in seconds? in minutes, in hours, in weeks, in months, in years for compacted clay liner and what it for do the same for the GCL.

Student: 12.4 hours.

Let us see what I got. So, I have got about 7.75 years and i have got 12.4r hours. So, once again breakthrough in CCL is faster than breakthrough in CCLl remember advective mass flux breakthrough time was ten points seven years diffusive mass flux the time is 10.7 years. So, everything is going to happen about after 7.8 years diffusion and advection are going to happen after 7 and half years, but in gcl at advective breakthrough times was advective breakthrough time was 3 to 4 month, 110 days. Diffusive breakthrough time is 12.4t. So, the flux is going to be much much earlier. It will be lower, once the breakthrough takes place through the compacted clay liner the quantum of advective mass flux is going to be larger, but the quantum of diffusive mass flux is going to be smaller.

So, with this we get a good feeling, let us just go back to these examples and the first thing we wanted to know was comparison of the active mass flux and what we found was that compacted clay liners had more about three times about three times that of the

GCL, but if you look at that breakthrough time there was a huge difference is this was 10 years and this was 0.3 years. So, there was a factor of safety of about 30 which was different. Then when we said what about the diffusive mass flux and suddenly there the advective mass flux was three times for the compacted clay liner. Here GCL has 51 one times the advective mass flux. The diffusive mass fluxes 51 times and lastly the breakthrough times again in years for compacted clay liners, but in hours for this years.

So, with this we end our discussion today on the solid examples given you two or three facets; one is this action leakage rate. Why do we talk about a thousand liters per hectare per day? And also what happens by the composite effect? And secondly, the equivalence between compacted clay liners and GCLs. So, we will stop here today and if you have any questions I will be happy to answer. Still I am saying both advective and diffusive have been treated separately here very very simplified actually, it will be much more different.

The compacted clay liner will not be saturated. So, the saturated permeability will not operate till the waiting front moves through it. We have used the saturated permeability, steady state condition, same thing will happens through diffusion. So, we have simplified the problem. So, that we get it cleans in idea of what is happening as the leachate passes through the barrier layer at the bottom. I hope you have better idea than what you had earlier. I mean defiantly the moment you bring in break through time, the whole thing changes and the moment you look at diffusion also which is so much more important when permeability is fall that is important for us for the purpose of design. Have a good day.