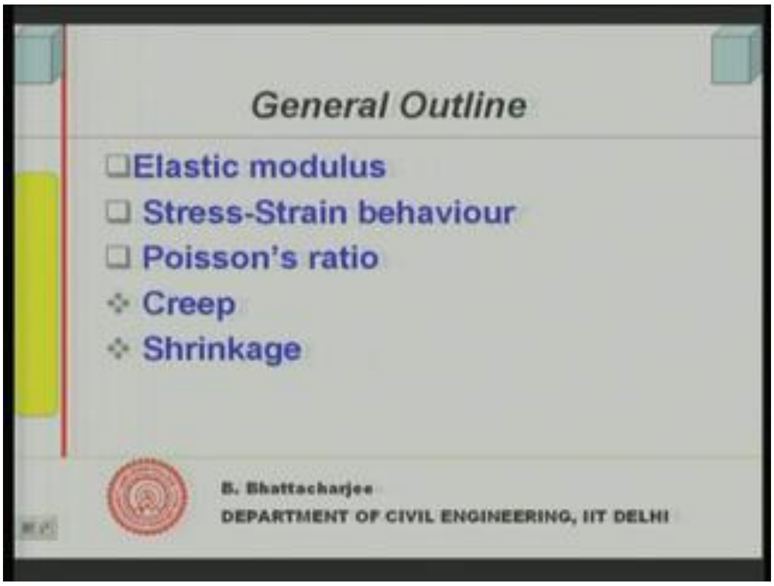
Building Materials and Construction
Dr. B. Bhattacharjee
Department of Civil Engineering
Indian Institute of Technology, Delhi

Module - 6
Lecture - 5
Mechanical Properties of Concrete: Elastic Modulus, Poisson's Ratio

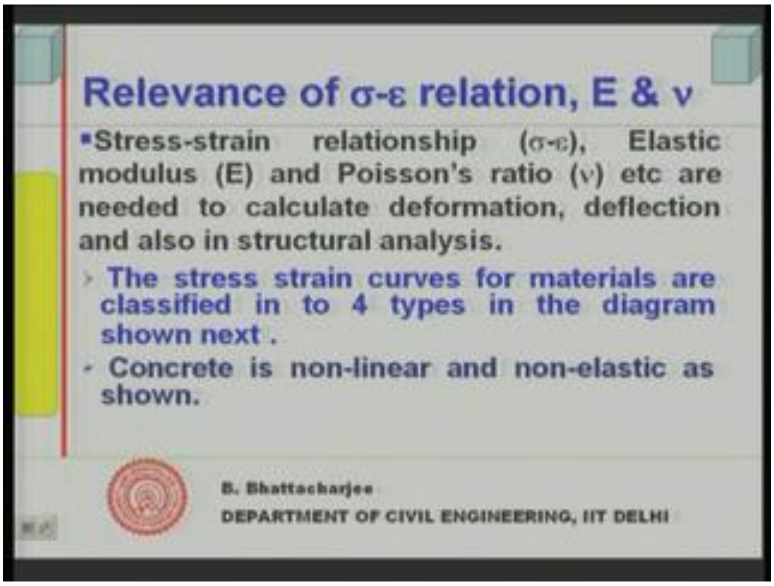
In this lecture, we will look into some other mechanical properties namely elastic modulus Poisson's ratio, etcetera.

(Refer Slide Time: 01:24)



So, general outline of our discussion today would be, first we will look into elastic modulus, then we will look into stress strain behavior of concrete; followed by Poisson's ratio. And we will just introduce the subject matter of creep and shrinkage. A detail of the creep shrinkage is outside the scope of this lecture studies however, we will just introduce this.

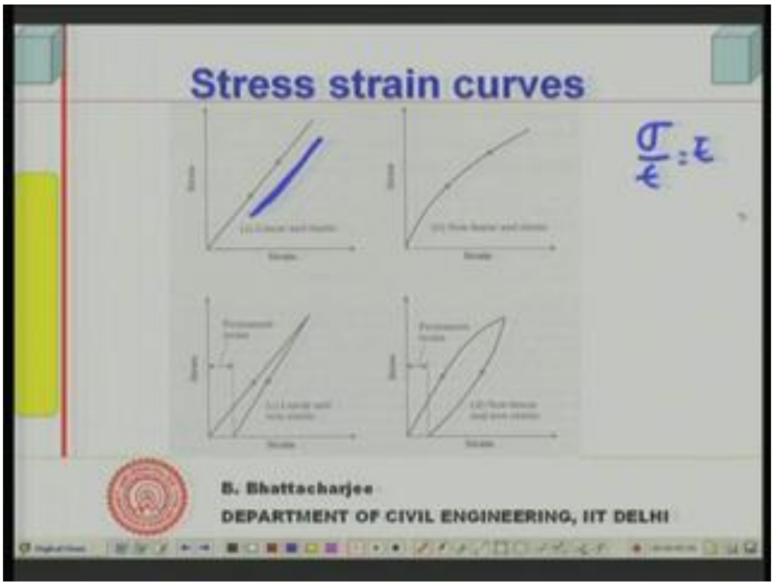
(Refer Slide Time: 01:56)



So, what is the relevance of stress strain relationship modulus of elasticity and Poisson's ratio stress strain relationship that is; what we are denoting by sigma and epsilon, elastic modulus and Poisson's ratio are needed to calculate deformation deflection and also in structural analysis. You want to calculate out forces; in indeterminate system some of these properties will be required.

If you want to do 3 dimensional solutions of 3 dimensional elasticity problems then all these properties are required. So, that is why the relevance of these properties is and they are often required in structural analysis and design. Now, to define the modulus of elasticity of concrete; let us see what kind of stress strain relationships exist, in case of different types of material; 1 can distinguish 4 types of stress strain relationship and out of this, we will see that concrete is non-linear and non elastic as we shall see this in the next slide.

(Refer Slide Time: 03:10)



Now, if you see the stress strain curve there can be varieties 4 varieties we have mentioned. The first variety is linear elastic you know. So, while stressing strain is here stress is along this y direction as you know and it is simply varies linearly and you know the stress divided by strain that is stress is our sigma by epsilon is equals to modulus of elasticity Hoop's law. We know that Young's modulus. So, that is stress strain relationship you know this is linear this is linear and elastic means; it returns back to the same path and there is no residual stress.

So, this type of stress strain curve is defined as linear elastic linear elastic you know and steel up to this point follows this sort of behavior you know exhibit this sort of behavior. Now, you come to non-linear and elastic it means that, it is not straight line any more, but it is varying. It is varying it is not linear it is straight line, but its curved, but it follows the same path back it follows the same path back certain plastic would show this kind of behavior certain plastic would show this kind of behavior.

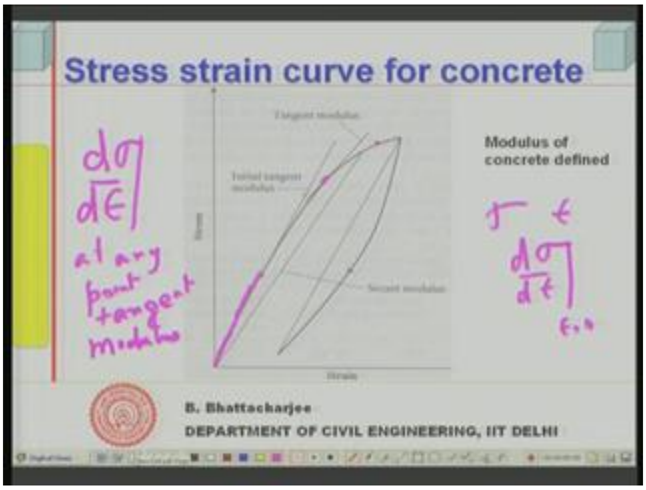
It will follow this path, but again come back by this path even some timbers might show some wood might show this sort of behavior. So, its linear its non-linear, but elastic; that means, whatever stress there is no storage of energy whatever strain energy was actually absorbed, by the material it just releases the strain energy. So, it follows the same path back. As opposed to this to in both this cases you will see that there is some amount of you knows it does not follow the same path back and there is some amount of residual strain remaining.

So, this 1 is again linear, but non elastic; that means, it does not come back to its original position back there is some amount of residual strain. This is non elastic you know some thermal strain is always there in this 1; some glass might show this kind of behavior. So, this is this you know this non-linear linear, but non elastic. The fourth variety is non-linear and again non elastic non-linear non elastic. So, in non elastic you have some permanent deformation. There is some plastic deformation and which is not recoverable and the non-linear is this curve stress strain curve is non-linear.

So, this is non-linear and again non elastic does not follow same path, there is some amount of energy lost. There is some amount of hysteresis as you call it and concrete belongs to this category complete stress strain curve is non-linear and it is again elastic, it does not come back to the original 0 strain level. There is some amount of residual permanent strain after you release the stress. So, it is non-linear and non elastic. Now, in such material when concrete you know after about 1 third of its ultimate load caring ultimate stress strain, you know the strength.

The strength that, it has the stress that it can bear one third of that beyond that its starts behaving non-linearly. So, it is linear over a very short range as again we shall see and then it becomes non-linear. So, how do we define this? This you can define simply it says that: stress is proportional to strain and the constant of proportionality is modulus of elasticity. In this case you cannot define it like that. In fact, it is varying from point to point. So, therefore, we have we define various kind of elastic modulus, in case of you know concrete not 1, but several, but use the particular 1.

(Refer Slide Time: 07:00)



So, let us define modulus of elasticity of concrete you see, I said it is non-linear and non elastic non-linear non elastic right concrete is non-linear non elastic. So, first of all you can define you know, in this portion of course, initially it is somewhat linear and beyond this point it becomes non-linear the curvature, you know its curve and when we release it comes back to does not back to the original position, but comes back to some other position with some residual strain.

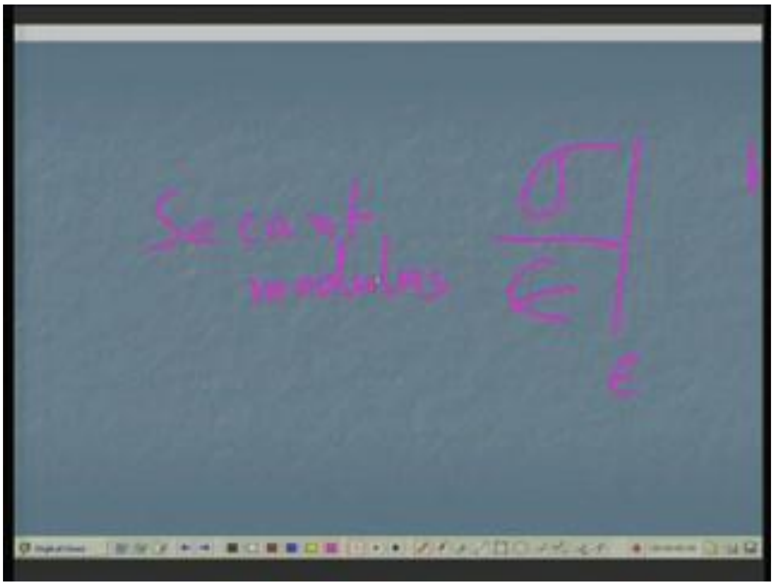
So, how you define modulus of elasticity, in some material such material while 1 way I can define is called initial tangent modulus, which is the slope of this curve in the beginning at 0 strain. So, slope of this curve stress strain curve, in the beginning that is 0 strain. So, this is initial tangent modulus initial tangent modulus; that means, you know if you know the relationship between sigma and epsilon. So, d sigma, d epsilon at epsilon equals to that would give me the initial tangent modulus. Now, if you go further at any point I can define a tangent modulus.

So, a tangent modulus is nothing, but tends to this curve at any point. So, this is tangent modulus. Then, I can define a secant modulus joining this point, with any point. So, this secant modulus will depend upon which point you are joining; that means, like a chord starting from 0 to any point you join this point and this point you get a secant modulus at that particular point.

So, it will vary from point to point tangent modulus, also varies from point to point. So, tangent modulus itself, will be a function of tangent modulus will be you know this simply tangent modulus is d sigma, d epsilon at any point that is tangent modulus. So, that would be the tangent modulus, you know at any point at any point that is tangent modulus point is tangent modulus you know tangent modulus right tangent modulus. So, that is tangent modulus.

Now, secant modulus, I joined origin and any point, in the curve and that gives me secant modulus. So, this is nothing, but this you know this stress divided by this strain, stress at any point divided by strain at that point that is what is the secant modulus. So, I can I can define it something like this I can define it something like this.

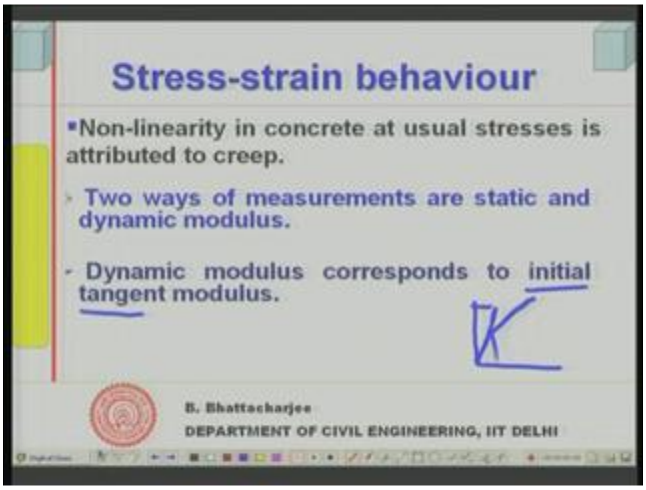
(Refer Slide Time: 09:46)



Stress at any point stress at any point divided by epsilon at any point you know some epsilon equals to some value of epsilon. Now, that is equals to secant modulus. Now, that is how I can define. In addition to that, I can define something called chord modulus something called a chord modulus; I can define something called chord modulus for something called, I can define something called chord modulus for concrete as well; that means, join any point in between, you join any point you know any point to point that will be a chord modulus. So, this is the chord modulus between these 2 points.

That is how we can define the modulus of elasticity of concrete, but most often what we use is nearly a second modulus or a chord modulus at very fine very lower strain to the maximum. Strain or one third strain or whatever one third stress whatever, stress it is. So, this is how we define the modulus of elasticity of concrete, we define modulus of elasticity of concrete well.

(Refer Slide Time: 10:53)



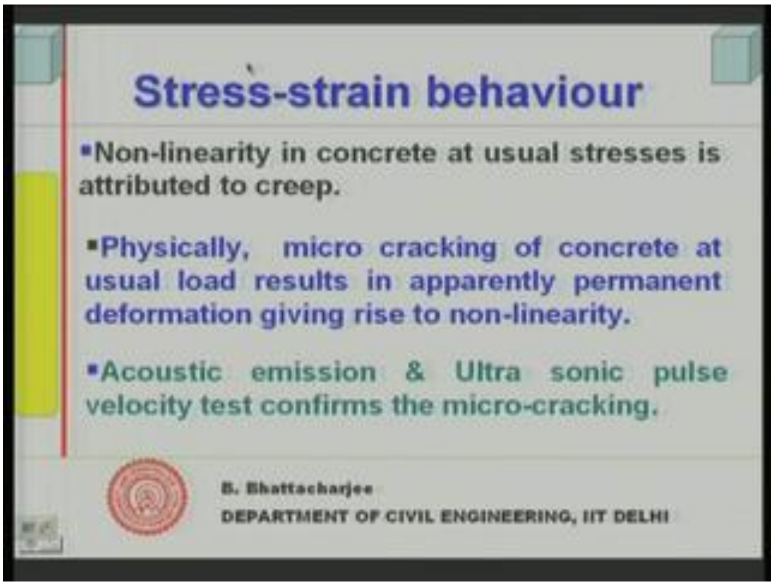
Non-linearity we have seen right now, in case of concrete stress strain behavior is attributed to creep and you remember we discussed about creep somewhat earlier somewhere earlier. There is a time dependent deformation time, if you keep the load sustain the load for certain period of time, you will find that there is some amount of deformation taking place even though you have not increased the load. So, that is time dependent deformation and that is what creep is. So, this non-linear behavior that is a given stress there is some plastic deformation.

It is not straight line it is attributed to sometime attribute to creep right. We will discuss this a little more while there is other possibility of you know, I mean; why physically why non-linear behavior is observed, in case of stress strain curve of concrete we will just look at it again somewhat, but let us see how we measure the modulus of elasticity of concrete. There are 2 ways of measurements and we call them 1 as static another is called dynamic and correspondingly, we call it static modulus of elasticity and dynamic modulus of elasticity.

The reason being they do not give same results static modulus and dynamic modulus. they are not same and therefore, we define measure them differently and define them also as static modulus and dynamic modulus right. Now, see what those ways are. Usually dynamic modulus corresponds to tangent modulus and you can see the initial tangent modulus it corresponds to initial tangent modulus. Dynamic modulus corresponds to initial tangent modulus, as you can see it corresponds to initial tangent modulus and if you remember you know the it is like this stress strain is something like this curve.

So, this slope is more than any other slope. Therefore initial tangent modulus is higher than let us say, any tangent modulus or secant modulus etcetera. It will be you know the slope would be, because this would be maximum minimum. So, sigma by E will be larger. Sigma by E will be d sigma E epsilon is maximum epsilon equals to 0 and that is why dynamic modulus generally, gives you higher value dynamic modulus value gives you higher value higher than static modulus.

(Refer Slide Time: 13:31)



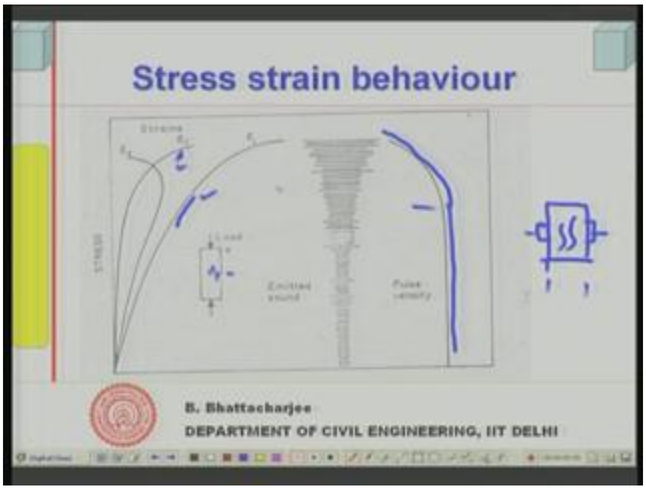
Let us look at the now, how do we measure them other explanation some more explanation towards stress strain behavior. If you look at it physically, when you apply load you know we have discussed about strength behavior you know of concrete failure and things like that. So, when you apply uni axial compressive load to concrete then; if you remember that we said that the: micro cracking stress at the aggregate paste interface may be the maximum size aggregate and mortar interface.

So, the cracks start propagating from the now, if these cracks are forming then these cracks on increasing the load will not collapse. So, once a crack form when you release the load, the strain energy that has been absorbed the crack for creating new surfaces will not be reverted back. So, they will not close down by a large they will remain open somewhat remain open. And if you have large number of such cracks they would contribute to plastic deformation.

So, you will measure them as plastic deformation, because they will be horizontal you know in the transverse direction. There will be cracks opening up, you will have transverse movement corresponding to this, when you release the load all this cracks that has formed will not close down. So, you have vertical deformation under compressive load, there will be some permanent deformation. So, this has been also attributed to micro cracking of concrete and this is been confirmed by certain we shall see that.

So, actually micro cracking of concrete takes place about one third of its ultimate load or ultimate strength the load carrying capacity stress, ultimate stress which we can ultimate strength and therefore, it apparently results in permanent deformation giving rise to non-linearity experiments like acoustic emission and ultra sonic pulse velocity test confirms this micro cracking phenomena we shall see them. We shall just look at this. This diagram this will make it clear.

(Refer Slide Time: 15:39)



We will not, we will not discuss this curves right now; except that this is longitudinal loading load has been applied like this. And this is the longitudinal deformation, this is of course, transverse strains, longitudinal strains this is transverse strain not deformation strain and this is volume strain. Now, we will discuss these phenomena again in correction with Poisson’s ratio, but right now.

Let us see, this longitudinal strain means; the strain along this direction shortening and this is the transverse strain means; the elongation along this direction extension along this direction and you can see that initially, this is increasing in a proportional manner and then suddenly starts increasing at a very fast rate. Here also it is the transverse strain that is elongation along you know strain along this transverse direction. It is increasing proportionally and beyond some point it starts increasing at a very fast rate which means; that there are large cracks formation would have taken place here large cracks formation would have taken place here and therefore, it is bulged.

So, it bulges heavily along this direction what about the volumetric strain well volumetric strain this is actually this is compression. Volumetric strain initially it is compression we are taking say you know compression. So, volumetric strain it actually increases then it reverses back. In other words you know initially it is contracting. There is a contraction because we are applying load like this. So, there is a volume contraction of the whole thing. So, initially it contracts and then suddenly starts expanding.

Now, my point is important point here, is that it suddenly starts expanding from some where there which means; that there must be lot of crack formation taking place. So, stress strain if you look at the strain versus, the stress for you know this is linear strain transverse strain and volumetric strain, we will see that there are sudden expansion takes place. So, which means that there is some large amount of micro cracking and that is what is resulting, in the volume expansion, because in the transverse direction there are significant amount of strain.

So, its expanding largely along this direction and this expansion is such that, it is possibly more than the contraction that is taking place in the this vertically downward direction or the longitudinal direction. So, as a result there is an overall expansion of the whole thing. This can happen when there is large amount of micro cracking. Now, acoustic emission is again gives you the similar kind of understanding you know similar kind of understanding. Acoustic you see when you apply load to any specimen some of the energy when strain energy is lost in cracking some of the strain energy goes in form of acoustic energy sound.

This is our common day experience you try to break something you here the crack sound you know cracking sound. So, this is nothing, but acoustic emission. So, when you subject concrete to concrete to under load and then measure the emitted sound, you will find that the sound emitted is more or less same, but beyond this point, it starts increasing and significantly, becomes very high quantum of the sound the intensity of the sound or the total energy dissipated acoustic energy dissipated become very high. Now, that means, that large amount of crack formation taking place along this zone.

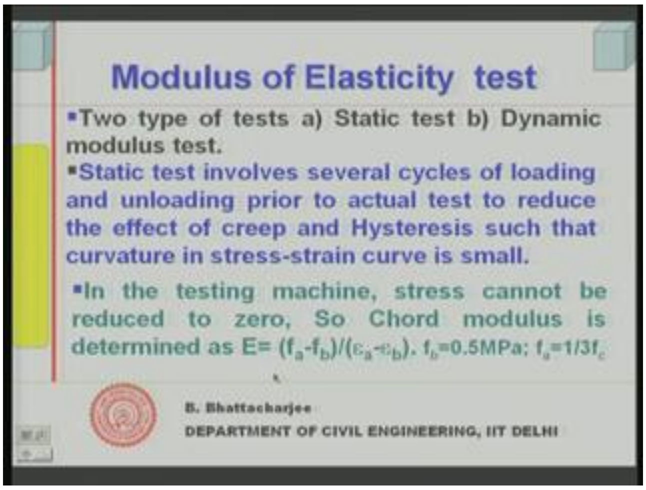
So, from this point onwards large amount of crack and here, the cracks are too many and that is what is confirmed by this expansion as well as you know lateral strain increases etcetera. Similarly, if you look at this point this 1 which is pulse ultrasonic pulse velocity sounds mechanical waves. So, mechanical waves are is you know that is very high it is relatively high in solid compared to that in air. In air it is about 340 meter per second in solids it could be order of kilometer per second.

In concrete of course, good concrete very good concrete will be around 4 and above kilometer per second. Now, if you have air bodies coming into it then the velocity or time of transmission increases through the specimen you know time of like you are passing you are passing the ultrasound. We shall see that sometime later on or sound signal from here to here and then measure the time of transmission of the sound.

Since, you know this length you can find out the velocity and this velocity. If there is lot of cracks actually apparent time of transmission of sound from this point to this point will increase. Therefore, velocity which is same l divided by the time would, now reduce significantly and this is what precisely happens you see it is more or less constant throughout this line, but beyond this point the sound velocity pulse velocity reduces acoustic velocity reduces significantly, showing that there must have been large crack formation in the system.

So, this non-linear behavior significant non-linear behavior of concrete under stress, you know in form of its stress strain behavior non-linear stress strain behavior is due to large amount of micro cracking that goes on in concrete while being stressed and non-linearity can be attributed to non-linearity as well as non elastic behavior, can be attributed to this sort of crack formation because this crack remains as permanent crack. So, they do not close down and therefore, this can be attributed to this sort of micro cracking.

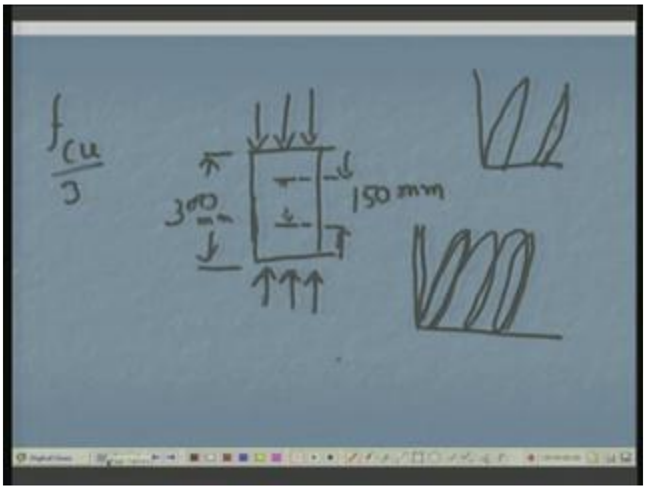
(Refer Slide Time: 21:13)



Let us see the test we said 2 types of tests static test and dynamic modulus test. We all understood the behavior why it behaves non-linearly. Now, let us look at the tests static test and dynamic modulus, test static modulus test and dynamic modulus test. Now, static modulus test we do; how do we do it actually. We simply do it like this you know we have this specimen cylindrical specimen.

So, we do it on cylindrical specimen that classical lamps extensometers were used; I have not described extensometer, but let us just say this is 300 millimeter. We know our cylinder standard cylinder size is 300 millimeter by 150 millimeter. So, what we do is we apply compressive load into over okay.

(Refer Slide Time: 22:10)



Let us look into lets, we will look into slightly more detail it is something like this. We apply a load like this. We have a cylindrical specimen have a cylindrical specimen and I apply load here and load here load here and load here right. Compressive load and measure the deformation. This deformation here by some sort of either strain gauges or lamps extensometers was used earlier where 2 clamps will be fixed, 1 fixed 1 supported and rolled. So, as it compresses the roller allows the you know you know 0.2 and this is the actually the movement on this roller is transmitted into a rotation of a meter and this meters rotate rotation from the meters rotation 1 can find out how much deformation is taking very classical way.

But, any way I am not describing the method, but let us understand similarly, we can have actually other kind of gauges several kind of gauges: strain gauges we can have you know or displacement transducer. We can introduce between these 2 points and we can measure the strain, but usually it is the central 150 millimeter where central 150 where, you measure the strain gauge length is this much this is 300, this is 300 millimeter, this is 300 millimeter and gauge length is central 150 millimeter, in classical test or in otherwise you can use gauge length.

Then measure the deformation you know deformation, I mean; deformation of this space this central 150 taking care of that you are not actually, bothering with the platen effect and things like that. This is free these are central coefficient which is away from the platen effect no restrain from the horizontal restrain and then required to the load. But, since we know there is a hysteresis loss in case of concrete. So, we know the stress strain of concrete initially would be something like this and if I release it, it is something like this.

So, there will be micro cracking. So, what we do is we first of course, fill this cube, I mean; cylinder fill this cylinder under standard static loading and find out what is the f c u you know f c u that is the cylinder ultimate load and take one third of that load and do the cycling. So, what we do we do several cycles several cycles of loading and unloading loading and unloading? Now, in the process what will happen the hysteresis that which was occurring you know the first cycle, will have large hysteresis the second cycle hysteresis reduces and as we go after about fifteen such cycles the hysteresis is minimized.

So, that this effect of creep or micro cracking has been minimized you know and you get non-linearity also, will be reduced and you can now obtain the regular stress strain curve of concrete, you know stress strain curve of concrete free of some sort of initial micro cracking and things like that. So, that is what is done and then load is measured the deformation at the central 150 millimeter is measured and that is what is done. So, static test involves several cycles of loading as you can see it, involves several cycles of loading and unloading prior to actual test to reduce. The effect of creep and hysteresis such that curvature in stress strain curve is small.

So, we now try to get it nearly you know less curvature non-linearity is reduces hysteresis is reduced. So, that would be behavior in fact, it represents the true behavior in field also, because we do not know to apply load once you know there is even during construction. There will be some load application release off this load and so on, so forth. So, over the period of time there will be several loading and unloading at lower load level and that is what we do.

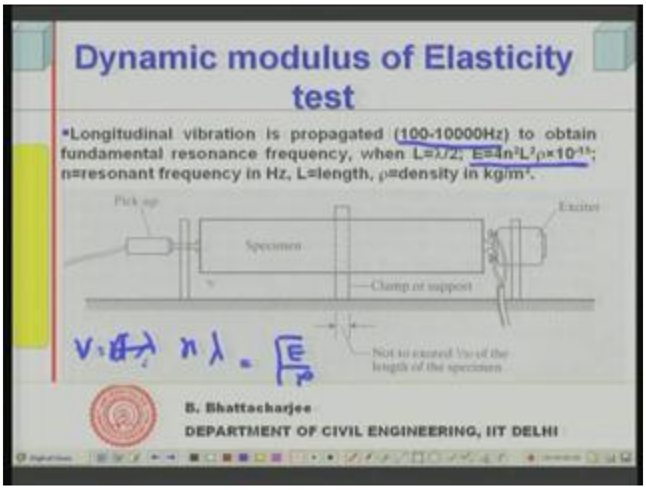
Actually you apply one third of the load that, we should be applying or its failure load and do this cyclic; for about 15 time and sixteenth and sixteenth and seventeenth cycle again we, now note down the P and also the stress. And therefore, the deformation of the central 150 and therefore, the strain there, because for the gauge and 150 millimeter, we know the deformation we can find out or shortening if you know it we can find out the strain. So, stress strain curve can be plotted from it. So, this is how we actually do static stress strain curve you know we do it.

Static stress strain data curve we determine in this manner right. Now, 1 point in the in the testing machine stress cannot be reduced to 0; what happens is when you are loading to have right kind of you know them. So, that it is the load is now, on to the cylinder just 0 is very difficult to maintain. So, what is done you have you have a minimum load always minimum strain always also, there is moisture condition etcetera are maintained same maintained same. If they are already, some cracks drying cracks and things like that that would change the behavior.

So, that is why initial you know initial few mega Pascal is avoided and you actually measure a chords modulus point five MPa and 1 third of the f c. So, or you know like, because secant modulus is what we try to find out and this secant modulus according to the rhylum standard. This is what it is we find out you know since it is you cannot put the machine to 0 stress, you will always have some stress, because you will have to tightened it the load will be little bit load will be transferred to the cylinder. And therefore, 0 stress condition is very difficult to achieve do not try it, what we do is we measure the secant chords modulus, between five MPa between five MPa and between 5 MPa chords modulus, between 5 MPa and the location where I want to measure.

So, it will be five MPa and in that case of course, is one third of the ultimate strength, you know the strength of the concrete cylinder strength of the concrete; which I have found out earlier by doing test and I can measure the E as the stress, at this point that is 1 third of the f c f b which corresponds which is actually 5 MPa, this will be 5 MPa and corresponding strain. So, I find out actually chord modulus. So, this is how we measure the modulus of elasticity of static modulus of elasticity of concrete.

(Refer Slide Time: 28:51)



Let us see we measure how we measure dynamic modulus of elasticity. The dynamic modulus test arrangement looks like this. You see we have the specimen here, which is rectangular in shape and I have an exciter here, which is basically a kind of oscillator which will impart longitudinal vibration to this longitudinal vibration to this right and it is clamped here clamped on the support there is a clamp support here which is here support. So, this specimen is supported here and I have a specimen there.

Now, this longitudinal exciter; which imparts longitudinal vibration is this vibration transmitted through this beam can be sensed here the vibration that is transmitted here its intensity can be sensed here. And there is a pick up or a kind of a sensor which will sense this vibration and then is amplified to see and characterize you know see vibration intensity etcetera. Whole idea is to actually pick up the resonance.

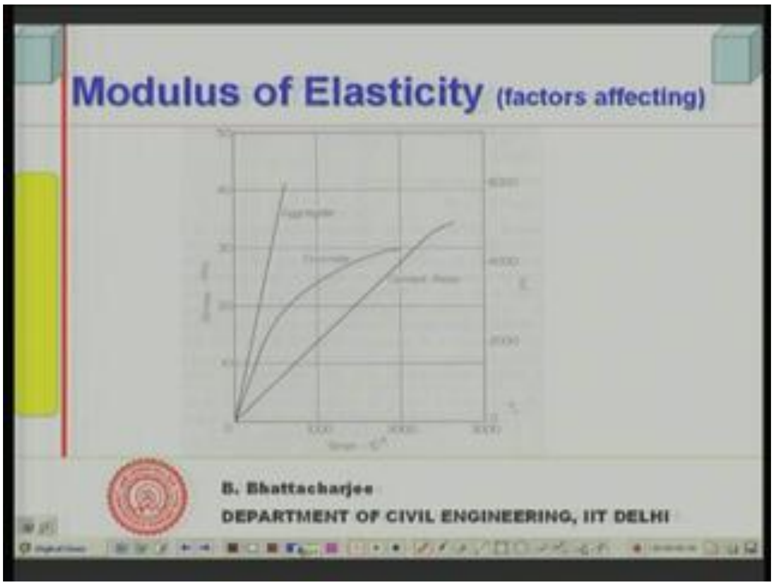
So, what you do you go on increasing the frequency then longitudinal vibration we propagate and from 1 hertz to 10k, 10 kilo hertz, 100 hertz to 10 kilo hertz; 1 can find out what will be the resonant frequency of such a specimen for the given mode of vibration that you will have. So, you would like to find out what is the first fundamental at which natural frequency of the system of such a system 1 can find out experimentally what you do is you go on changing the frequency quickly, obtain the resonance; that means, when this amplitude here would increase very large.

Because, the resonance frequency the amplitude will become amplitude you know the energy here will be energy received here will be very large. So, from this pick up we find out when the resonance has occurred first fundamental and this corresponds to actually L equals to lambda by 2. So, this length if L is the length of the specimen, then L equals to lambda by 2 such that we can calculate out E equals to 4 n square L square ro and 10 to the power of minus 15. Because of the unit combustion when this is in GPa giga Pascal this is GPa giga Pascal length is in millimeter and this is density in kg meter n of course, is in Hertz.

This is how with, this we can find out and this is very complex, because we know the velocity of propagation is nothing, but velocity of propagation or we write, it will be f into lambda. Now, if know the f is sorry, f into lambda which is in our case; we are calling it n you know natural frequency n into lambda. So, correspondingly we know what the lambda the n is the frequency is and V is nothing, but V is nothing, but under root E over ro modulus of elasticity for you know medium in a solid medium elastic medium.

The velocity of propagation is under root E by ro where E is the modulus and ro is a E is the elastic modulus bulk modulus actually, I mean; elastic modulus you can call it we call it dynamic modulus and ro is the density of the material. So, since I know resonance lambda equals to how much lambda equals to L by 2 n is equals to the natural frequency. So, E can be calculated by this formula and since usually, E lambda should be in meter when it will result in Pascal all that unit conversion gives me ten to the power minus 15. So, actually the dynamic modulus is found out from the principle that velocity is equals to under root modulus dynamic modulus divided by density. So, from that I find it out okay. So, that is how I find out dynamic modulus of elasticity that is how I find out dynamic modulus of elasticity, right.

(Refer Slide Time: 33:01)

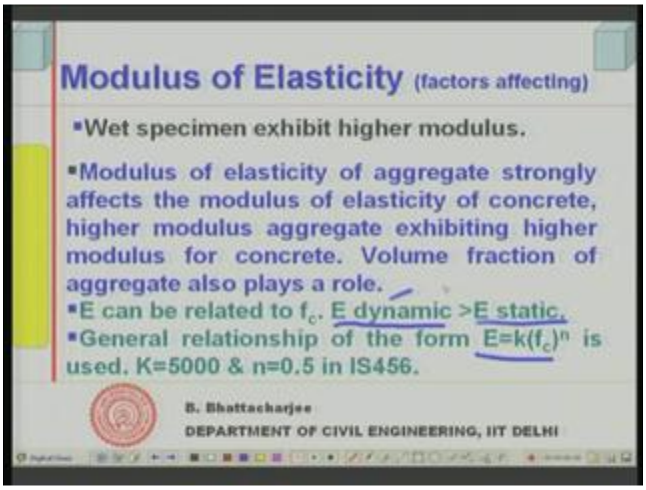


Let us look at factors affecting modulus of elasticity. The most important factor is the aggregate type considering concrete as a 2 phase material. As you can see from this diagram, you can see from this diagram that you see aggregate modulus is here cement paste modulus. You know stress strain diagram is here and concrete stress strain diagram is here.

So, therefore, modulus of elasticity this is slope of this slope of this curve is somewhere between the slope of this 2. So, it is the aggregate cement paste anyway is the modulus of elasticity would be modulus constant it will change of course, depending upon the strength of the concrete, but aggregate in modulus influences the modulus of elasticity of concrete in a big way. In fact, considering it to be a 2 phase material; there are several models available like series model parallel model. There are various bounds through which 1 can predict the modulus of elasticity. However, this is not really important for our purpose, but we must understand that aggregate modulus of elasticity influences the modulus of elasticity of concrete in a big way.

So, it has to be high. For example if you take a material. Let us say weak aggregate light weight aggregate, then the modulus of elasticity of concrete will be lower. So, aggregate affects the modulus of elasticity in a big way right. Let us see, this is number 1 factor then the other factors affecting the modulus of elasticity are moisture content wet specimen exhibit higher modulus right.

(Refer Slide Time: 34:26)



Higher modulus aggregate; obviously, exhibit higher modulus for concrete volume fraction of aggregate also is important, because as I said that is you can model it as a 2 phase material and both volume fraction. If you look at only the simple law of mixture then E can simply be written as E concrete can be written as E of aggregate into volume fraction of aggregate plus E of paste volume fraction of paste. But, this is of course, a simple model in parallel materials, in parallel; in series of course it would be 1 over E c equals to V aggregate over E aggregate plus V paste over E paste.

Now, this models do not give us the correct result anyway, that tells us about the bound, but idea is of course, not to get estimate the modulus of elasticity from paste modulus and modulus of elasticity of aggregate, but what we understand it that aggregate modulus influences the modulus of elasticity of concrete in a big way and wet specimen exhibits higher modulus wet specimen right well. E can be related to E can be related to concrete strength that is what is done practically and people have been using this and that is you see the compressive strength is the 1 major factor, which we measure and also used in quality control that is what we have seen cube compressive strength.

Therefore, since all this you know phenomena is related after all it is related to the deformation stress strain relationship and the modulus of elasticity, as we have discussed. So, far we have seen that the non-linearity comes because of micro cracking the non elastic behavior comes because of the micro cracking we know the strength is also governed by the same thing. Therefore there has to be a relationship between the modulus of elasticity and the compressive strength.

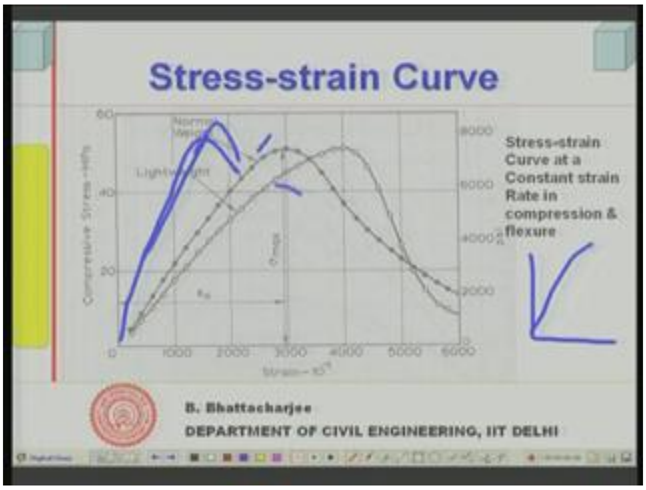
We said compressive strength is a measure, which we determined which we can get very easily from laboratory test and we have actually designated cube also through the compressive strength. Therefore why not relate compressive strength to the modulus of elasticity. You know there is likely a relationship existing between the 2 and it has been observed that there is relationship empirical relationships exist. First of all the general relationships are of this form. General relationship of this form has been identified.

Some E the modulus of elasticity that can be used in design is proportion is a function of some constant into f c to the power n and this K value is taken to be 5000 in IS 456 and this n constant value is taken to be 0.5.

In other words and for f c k of course, since we designate the concrete by a grade is designated by f c k characteristic strength, which we mentioned very long back right sometime in the beginning in one of those lectures. So, therefore, modulus of elasticity of a grade of concrete can be found out using this sort of a relationship for K 5000 n is 0.5. And this can be used for design purpose of course, it would be a little bit conservative value considering the point of design points may be take some aspects of creep into account etcetera.

We have also seen that we have understood why E dynamic is greater than E static, because this measures initial tangent modulus whereas, this static measures chords modulus therefore, this is this is higher. So, this you know gives us this tells us what are the factors which affect the modulus of elasticity of concrete.

(Refer Slide Time: 38:26)



Let us look into stress strain curve of concrete. Stress strain curve of concrete when you do it in a strain control machine you see usually, we will be doing stress strain curve in where we can I can control the rate of loading in terms of stress per unit time. Remember we talked of while testing cube I should apply load at the rate of 140 kg per centimeter square per minute or 14 MPa per minute.

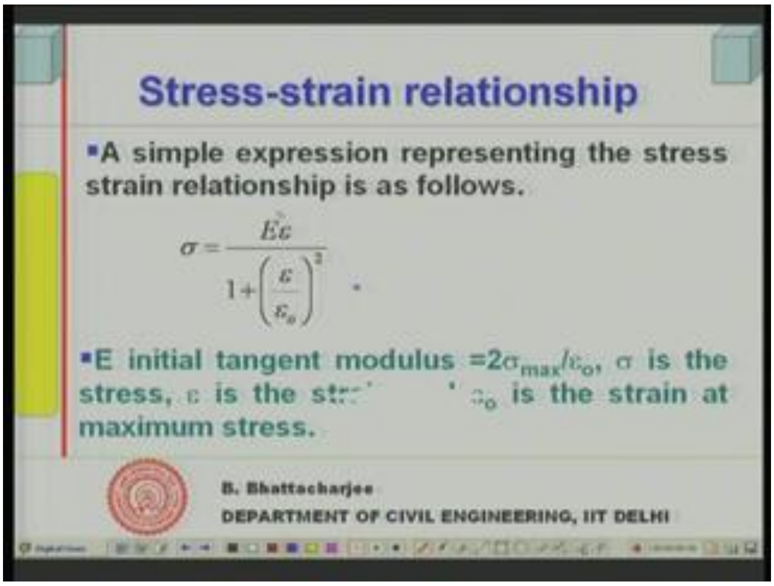
Now, same rate of loading is applied for modulus of elasticity determination also when you apply the rate of loading; however, when you apply rate you know load in with through the machines, which can only control the rate of stress application then you get stress strain curve only up to this much, but this portion will not come you know you will get up to maximum stress the drooping portion as we call it you will not get it. This can be obtained only at strain control machines servo controlled machines, where there is a feedback mechanism and according to the strain the stresses are you know stresses are adjusted.

So, you actually you apply loading at a given strain rate. So, when you do that you get this sort of down coming portion of the curve also. So, strain stress strain curve at a constant strain in compression and flexure both have been seen to do a strain control test you get strain curve like this. So, normal rate concrete behaves like this light weight concrete shows like this, you know light weight concrete which will have light weight aggregate has lower modulus of elasticity, but its ultimate strength is also even; if they are same.

Let us say: if they are same, but after you know this shows droops out like this whereas, normal strength concrete behaves like this. So, this is the natural stress strain concrete of normal strength concrete, but if I look at the stress strength concrete of high strength concrete just for the sake of interest it will be somewhere there, but this drooping portion will be significantly small. This drooping portion also would be significantly small. It will be much higher. In fact, much higher than this; if it is 50 MPa, it will be 100 MPa much beyond the screen.

But drooping portion would also be small high strength concrete gives you higher strength higher modulus of elasticity, but this drooping portion is small well for the reason not discussed at the moment, but less micro cracking etcetera. So, this portion this is what is a stress strain behavior of normal concrete is right. So, that is what it is right.

(Refer Slide Time: 41:01)



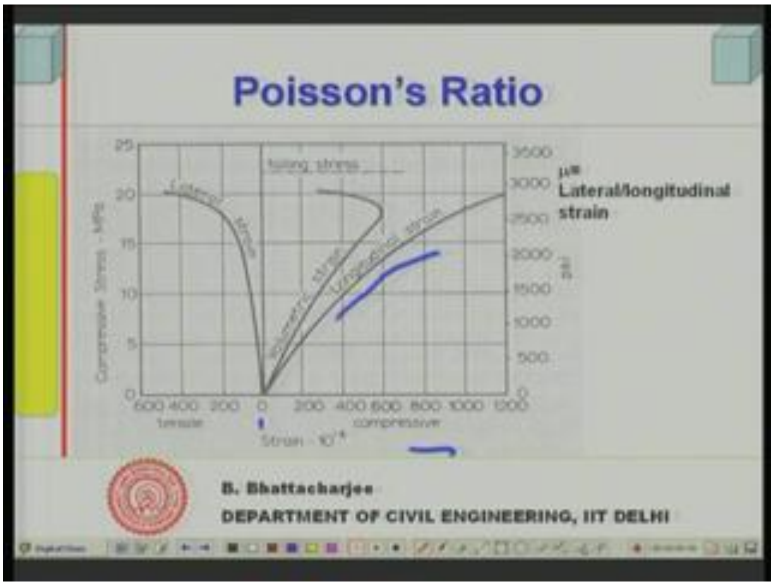
Let us see, we can express this stress strain curve in some sort of equations under simple expression representing this stress strain relationship is given by this, you know simple expression that relates the stress strain curve 1 by given by Desautel is looks something like this where, E is the initial tangent modulus and which is taken as twice sigma max by epsilon 0 you know you remember the last slide where curves like this; where the curves like this and then drooping like this and we said this is our sigma max.

So, this corresponds to sigma max and corresponding strain is E 0, when intial tangent modulus is E is initial tangent modulus and which is assumed to be twice the second modulus at the maximum stress maximum you know the strength ultimate strength sigma max the stress that can it can carry divided by epsilon 0; that will be the secant modulus. So, twice of that that is what is assumed and that is what the E is that is what is you know that is what the E is sigma is the stress at any point and epsilon 0 we have already defined in the previous diagram.

So, you know sigma is a stress and sigma is a stress at and epsilon 0 is strain at maximum stress. So, that is what right and maximum stress is. So, that is what that is what it is. So, so this is the 1 of the simplest 1 of the model there are more complex models, but any way we are interested in them. We know the nature of the curve and from this some standard stress strain curve that is idealized for concrete in design you must be aware of you might have seen in. In case of design structural design it is parabolic up to point 0 strain to strain and then it is taken as a straight line.

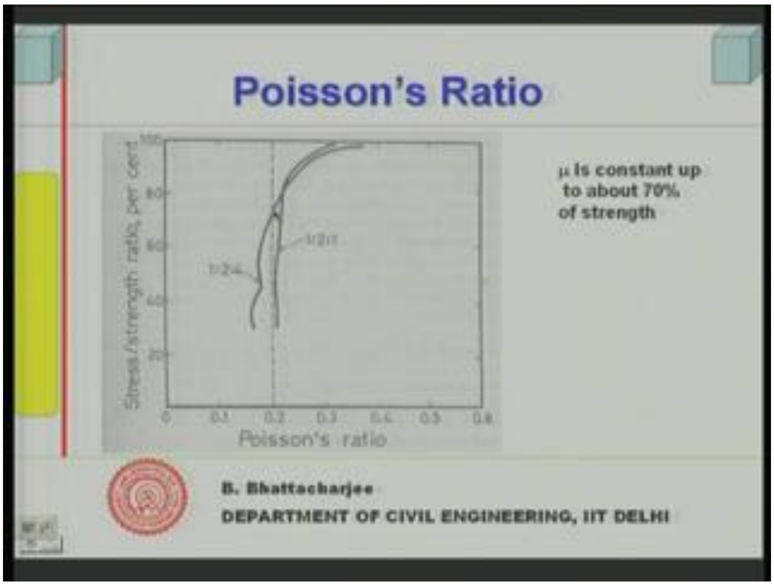
In design we take it something like this 0.002 and then 0.0035; we take this as straight line. That is idealized curve, but anyway that is now right. Now much interest to us that is why the stress strain behavior is.

(Refer Slide Time: 43:08)



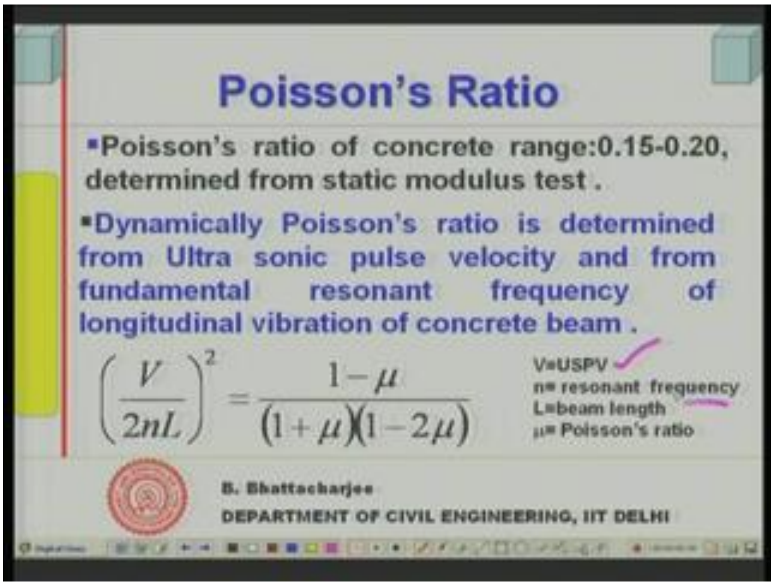
Poisson's ratio you remember this curve we have shown it earlier. So, you can see that you can see that this is my longitudinal strain and this side is the stress. This curve I have explained earlier this is the and this is 0 this is compression and you see the volumetric strain which I denote really go in details earlier. This is compressive then suddenly it starts expanding. So, compressive strain actually reduces. So, it starts expanding.

(Refer Slide Time: 44:03)



So, this is the you know this is what it is and lateral strain is something like this and this we explained this, because of the micro cracking that takes place. Now, this strain the Poisson's ratio is defined to lateral longitudinal strain. Poisson's ratio can be defined in lateral to longitudinal strain and that roughly varies between, 0.15 to 0.15 to 0.2, in case of concrete most of the concrete you know you can see that mu is about its you know mu is constant up to about 70 percent of the strength and that is about 0.2 roughly close to 0.2. So, it is varies between 0.15 16 to about 0.2, when you are measuring it statically and then when you measure it in a dynamic manner you get somewhat different right.

(Refer Slide Time: 44:30)



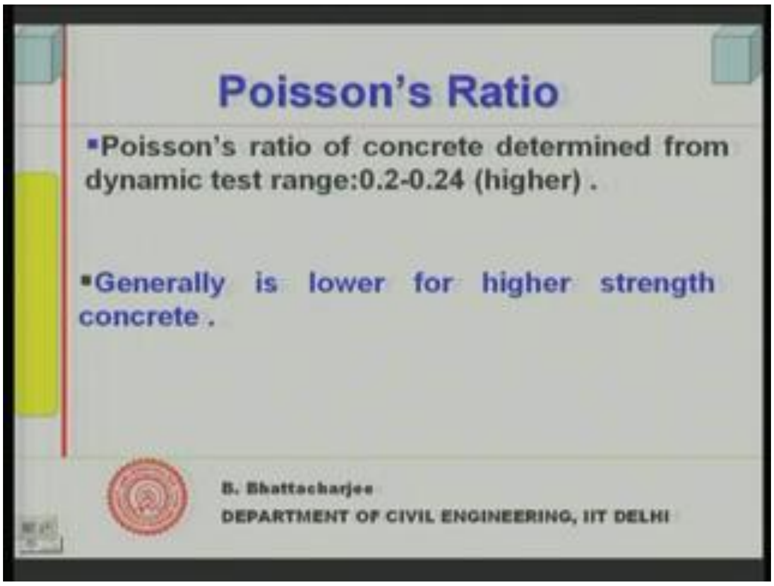
The variation starts stated here. Poisson's ratio is stated here point 1 five to point 2 0 determined from static modulus test, when we I find out the strength from the you know sorry the Poisson's ratio from the modulus of elasticity test is the static modulus of elasticity test. Then I get it within, this range I get it within this range you know I get it within this range, I get it within this range, but when I determine it dynamically then of course, I get somewhat higher value. How can I determine dynamically I can determine from ultra sonic pulse velocity test.

So, I subject the specimen to ultra sonic pulse velocity likewise, I mentioned earlier and find out the velocity and also if find out its natural frequency, which I have found out in case of dynamic modulus test and from both these results, I can find out the Poisson's ratio from the simple formula that is given here, where V stands for the ultra sonic pulse velocity V stands for the ultra sonic pulse velocity. You see it stands for the ultra sonic pulse velocity. N is the resonant frequency that we have seen earlier.

We determined the resonant frequency in case of dynamic modulus determined in case of dynamic modulus right and then L is the length of the specimen and mu is Poisson's ratio. So, from this relationship V divided by twice n L whole square, I can find out the dynamic modulus of you know dynamic from I can find out the Poisson's ratio dynamic Poisson's ratio dynamically Poisson's ratio eventually this dynamic Poisson's ratio values are again somewhat higher.

It is somewhat higher and it gives you the values you get it somewhere between 0.2 to 0.24 .So, they are higher. So, it shows the range is higher. So, in any case Poisson's ratio does not vary for concrete up to 0.7 percent 7.7 or seventy percent of the strength Poisson's ratio remain constant, but after that of course, there is large expansion that is why you see there is an increase in Poisson's ratio that we have seen in one of the previous diagram, but that is not of our interest because we do not go to that low.

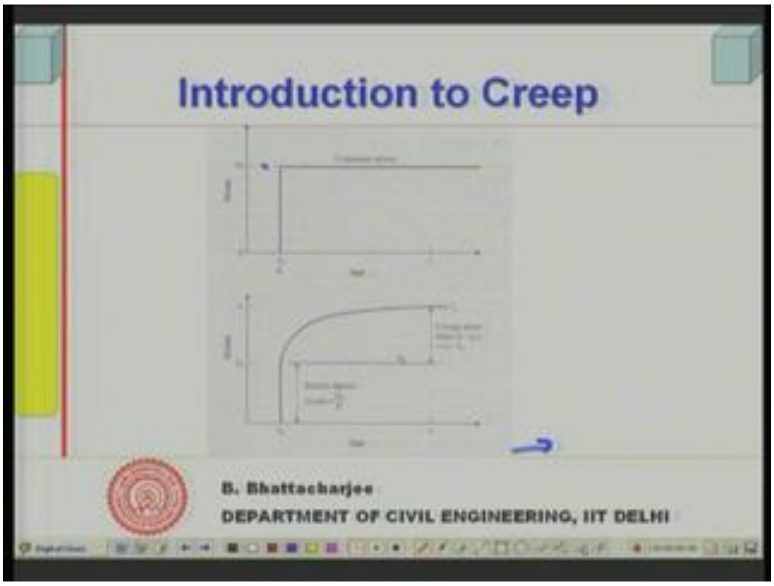
(Refer Slide Time: 47:01)



So, our interest is only up to only up to 30 percent of the strength or ultimate load, it can carry out 40 or 55 percent, because working load will be much less anyway. So, that is about the Poisson's ratio of concrete right.

Generally is lower for higher strength of concrete generally it is lower for higher strength of concrete. There are some sort of relationship between fraction of aggregate and things like that however, it is not you know it is still lot of research need to be done, but we understand that it is generally higher is generally, lower for higher strength of concrete. So, that is about how Poisson's ratio varies you know varies with how Poisson's ratio varies with strength of concrete; other factors which affect is not much known it is not really of much interest. So, roughly one can take it to be 0.180.2, etcetera.

(Refer Slide Time: 47:52)



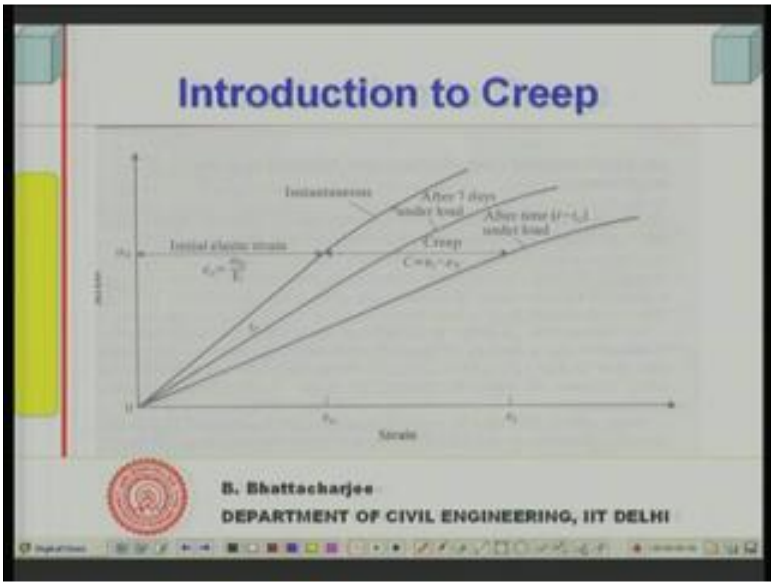
So, so far we have looked into the modulus of elasticity of concrete, we have looked into what factors affect the modulus of elasticity and we have looked into stress strain behavior and also the Poisson's ratio. Now, by and large these are utilized for design purposes, but see 2 phenomena related to elastic properties of concrete are creep and shrinkage and although. We are not going to deep into this matter, we would like to I like to introduce this to introduce you what is creep because we have been referring creep time and again earlier and you have, now with respect to elasticity and so, on.

So, let us define creep and understand little bit just the definition and things like that, I mentioned already that creep is actually time dependent deformation time dependent deformation right; it is time dependent deformation. So, if you look at this 2 curves you know they are in the same this is the age or time this axis is time same scale same axis, what I have done at this point at some point t 0; I have just applied a load or stress and kept it constant.

Now, if I look at the strain immediately, I will have some strain which I call as elastic strain, because I have loaded the specimen when I have loaded the specimen say concrete specimen in compression. So, when I have loaded the concrete specimen in compression there will be some amount of deformation and instantaneously, in that you know immediately as soon as; I have applied the load together with the load, but when I have kept this load constant that is what I have been mentioning that, when I sustain the load for a long period of time what I find that this deformation goes on increasing.

The strain goes on increasing you know it will it will further deform right. And it goes on increasing and this we call as this we call as creep deformation. This we call as creep this deformation we call as creep. This we call as creep this portion, we call as creep. So, this is this increase with time, we call as creep after a given time. So, we see that initially, it increases significantly at a faster rate and then it reduces becomes asymptotic and increases at a slower rate. So, that is what a creep phenomenon of creep is.

(Refer Slide Time: 50:30)

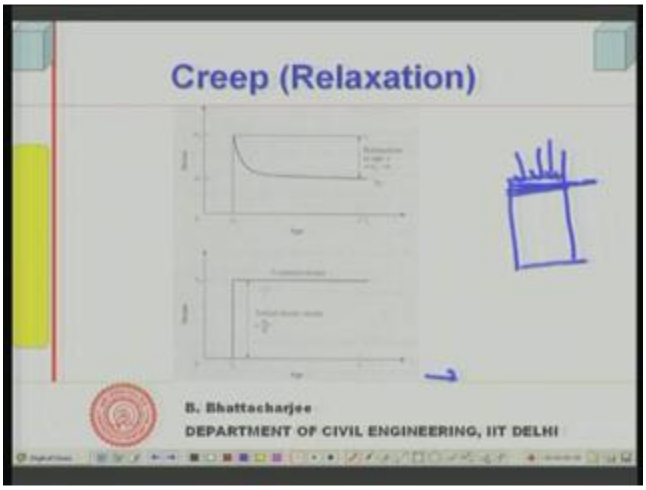


Now, we can look further like this; if you look at these curves strain versus stress at different times. This is instantaneous if I measure stress strain curve not up to the full load, but up to some load I am just showing and we will find that it increases like this. Then keep this load there and you know keep this load there and measure the strains after 7 days like at this stress level measure the strain after 7 days.

At this strain level stress level measure the strain after 7 you know 7 days. At this stress level measure the strain after 7 days. So, when after 7 days when you have measured the strain you find this strain higher not the same. That is understandable. We have already defined that creep. So, this is due to creep and after time let us say some other time hypothetically, you will have you know strain you know stress strain curve would be something like this.

So, what do we see out of all this. This is up to this is elastic initial elastic strain instantaneously, the moment I have loaded I got this beyond this point is the creep any time this is 7 days creep or this total thing is the creep deformation creep strain. So, creep strain is after you have sustained sustain, it for certain period of time the strain that you get is the creep strain. So, stress strain diagram also gets modified if you have sustained the load for certain period of time right. This was what we have seen in case of fatigue also when you are discussing fatigue we have seen this as well. A similar situation can happen slightly differently if you look at.

(Refer Slide Time: 52:14)



Now, I do not apply I do not apply you know I do not apply, I apply the load alright, but the specimen I do not allow it to deform, I restrain. Now, in earlier case the specimen could deform, I did not have a restrain supposing I have a specimen which is restrained from deformation right which is restrained from deformation. So, something like this what I have done I have now same time axis here and strain.

Now, what I have done now, I have applied the strain and there is a restrain provided to it cannot deform further. So, apply the strain kept the strain constant what happens is this is a relaxation this is a relaxation of the stress. This is a relaxation of the stress now this happens I can give an example, you know in case of prestressing what we do. We pull it pull the prestressing wire you know what is prestressing.

We have discussed about this sometime little bit, in case of production of concrete prestressing is you know prestressing is applied to the concrete through steel wires or rods or similar other steel members; what is done is steel which is bounded in some manner either through the anchors, in case of post tension system or in a pre tension system right through the surface. So, in either case what is done is you have a steel which is pulled and then under that stress, it is tied up or bounded together with the concrete system.

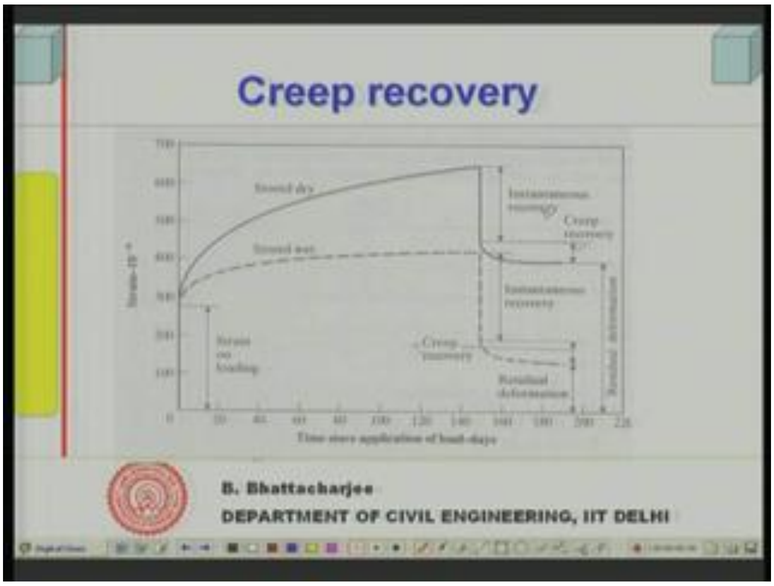
So, the steel tries to shorten and in the, in that in that process it applies a kind of compressive pre compression to the concrete such that under load, it can withstand the tension if there is any. But more important point is. So, when you have actually stress the steel and anchored it the concrete. Now, is under stress, but you know it can of course, will have some elastic deformation, but after that what will happen is actually it is you know the the there is a kind of restrainment. There can be relaxation of the prestressing even even steel shows this sort of phenomena.

So, if there is a restrain there can be relaxation of the relaxation of the stresses. Stress can get reduced since, there is no it cannot deform further if it is restrained from deformation. There will be some amount of relaxation of the stresses. There are many other situations where strain remains; that means, you do not allow to further deform. It is deformed and just held it in position. The stresses remain now stress, because this natural position now is already you knows in case of compression.

Let us say supposing this is your specimen you have applied as strain through some load and kept that load. Now, this cannot further deform you have restrained its movement by some mechanism. So, what will happen now, this length will remain fixed. This is not this is not this is not changing any more, but due to creep it, would have like to deform further since, it is not being allowed to deform effectively the stress here would get reduced because its natural position itself now has relatively shortened.

So, as if there is a you know you can say the counter a pool which is counteracting the compressive force there. So, there is a relaxation. This is relaxation phenomena. So, when would you have restrained specimen, we maintain the strain constant you find the stress that you have applied originally gets reduced and this is due to relaxation. This is we call relaxation and then this actually happens initially at a faster rate and then at a lower rate. So, relaxation is other phenomena associated with creep of concrete.

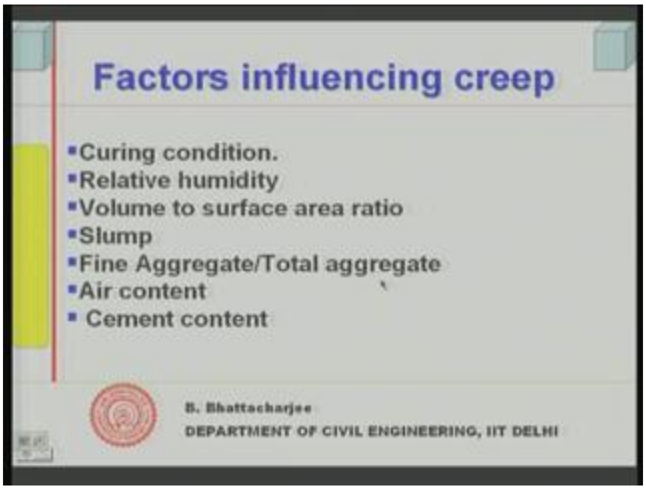
(Refer Slide Time: 56:12)



There is something called creep recovery you see if this is again this side is time since application of the load. So, the creep we have talked about supposing I release the load now then there will be instantaneous recovery. There will be some creep recovery instantaneous recovery and then there will be some creep recovery. So creep recovery also takes place. I have put the load for some certain period of time then suddenly, I withdrawn it after certain period of time, I kept it and withdrawn it.

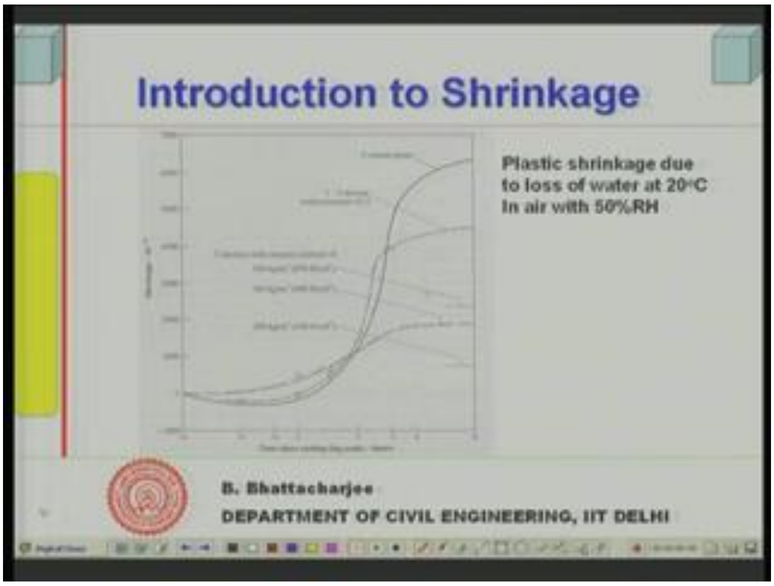
Now, some of the stress comes back the creep stress that gets released, but some remains. So, there is some creep instantaneous recovery and gradually that recover, but of course, it recovers nearly fully, but you know. So, more or less, but total elastic stress is not recovered. There is some residual deformation of this remaining okay. if it is stored wet you can say creep is different and stored dry. I think we leave at this stage because we will not go details the mechanism of creep etcetera.

(Refer Slide Time: 57:08)



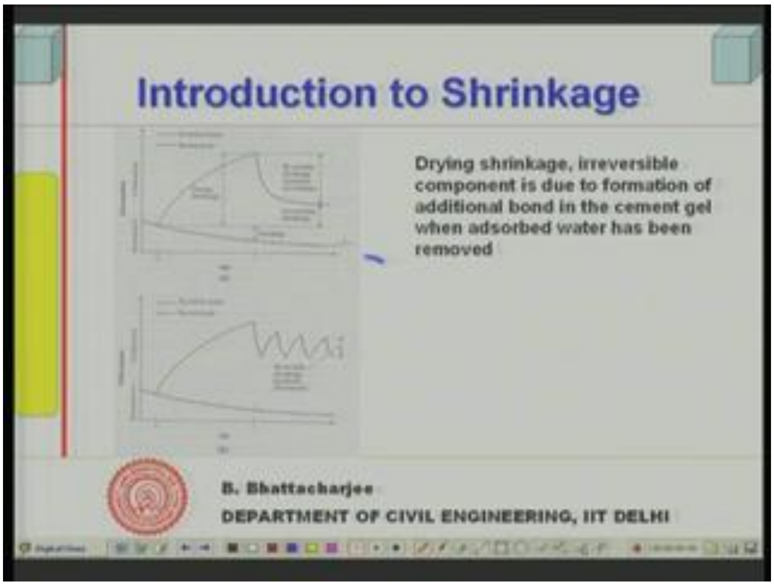
Let us see, what the factors are affecting influencing creep curing condition, relative humidity volume to surface area ratio, because its related to you know evaporation taking place from the surface. The slump of the concrete fine aggregate to total aggregate and air content, all this factors affect the creep of concrete.

(Refer Slide Time: 57:36)



Similarly, other important factor is the shrinkage of concrete. Plastic shrinkage takes place, you know there are 3 types of shrinkages plastic shrinkage takes place due to loss of water at twenty degree centigrade in air at fifty percent relative humidity this graph is. So, plastic shrinkage takes place when evaporation loss takes place and you can see it is maximum for the paste mortar somewhat less concrete somewhat less. So, plastic shrinkage takes place right, in the beginning when it is drying immediately, in the plastic state the volume change takes place, because water may evaporate, even if water does not evaporate. There is some amount of you know water, will come up and some amount of due to due to water desiccation self desiccation some amount of water lost is there. So, overall there is a volume change takes place and that is what shrinkage is…

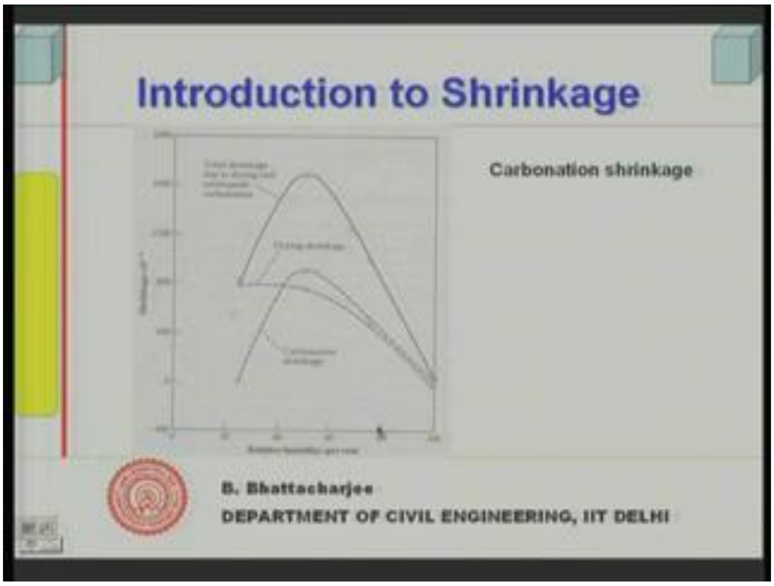
(Refer Slide Time: 58:25)



Then, there is another shrinkage which is called drying shrinkage and drying shrinkage you know, which takes place over somewhat longer period of time not in a plastic state. It has become solidified, but then water can evaporate and drying shrinkage can takes place. It has got some irreversible component, if you can look at this it is something like this. You see if I keep it constantly stored in water, I will have this curve which is which is swelling you know which is swelling and if I keep it stored in water is this portion allow it to dry then store in water this is the phenomena.

So, it is actually, there is contraction taking place shrinkage taking place, because I have allowed it to dry then I wet it some amount is required, but full amount is not required. So, there is some amount reversible some amount is not irreversible right not this irreversibility is been actually, attributed to addition of bond in the cement gel when adsorbed water has been removed. If you remove the adsorbed water some amount of bonding you know addition of bond would form, in the cement and that is what it is related to right.

(Refer Slide Time: 59:38)



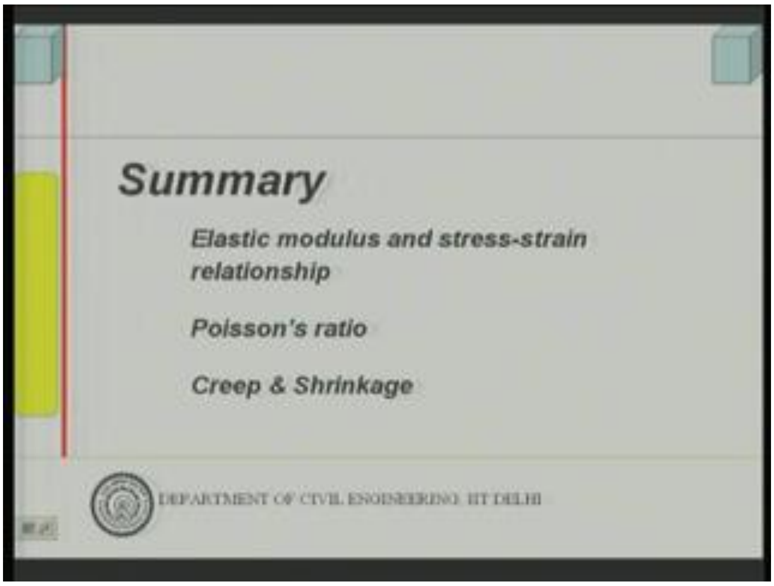
So, that is what is drying shrinkage and the last kind of shrinkage you call it carbonation shrinkage you know carbon dioxide in the atmosphere can relate, with the calcium hydroxide in the cement and calcium carbonate can be formed. We will look into this somewhat later on. The carbonation process, but at the moment we must understand this carbonation process can introduce shrinkage and this is called carbonation shrinkage.

If you look at it carbonation shrinkage of course, is a function of relative humidity and its maximum at certain relative humidity at very high relative humidity the carbonation is actually reduced, because carbonation takes place in presence of moisture. In saturated concrete carbonation cannot take place. The reason being the reason being carbon dioxide absence of carbon dioxide minimum amount of air should be present to have carbonation.

So, carbonation shrinkage goes in like this and if this is your you know it goes on like this. This is the carbonation shrinkage this is the carbonation shrinkage this is the carbonation shrinkage. This is the drying shrinkage total shrinkage will look something like this.

So, these are the 3 types of shrinkage again, we have introduced the shrinkage right now and let us see, what are the factors influencing the shrinkage, curing condition, relative humidity volume to surface ratio slump fine aggregate to total aggregate air content and most important is the cement content say high cement content always needs more shrinkage. We have seen in the beginning you know plastic shrinkage. So, that is are the factors.

(Refer Slide Time: 01:01:04)



So, with this we can summarize our discussion, elastic modulus and stress strain relationship, we have discussed. Then, we have discussed Poisson's ratio and we have introduced actually creep and shrinkage to you. I think this concludes our discussion of this lecture.