Seismic Analysis of Structures
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Lecture – 19
Frequency Domain Spectral Analysis (Contd.)

In the last lecture, we are talking about the modal spectral analysis. The modal spectral analysis is performed talking help of the mode shapes and frequencies of the structures as we do in the case of the deterministic analysis. The whole idea again is to convert the problem into a set of uncoupled single degree of freedom system. And in that case one can write down the power spectral density function relationship for a single degree freedom system. And then the responses obtained from there are then combined with the help of the relationship that exist between the displacements in the structural coordinate system and the generalized displacements through the mode shape matrix.

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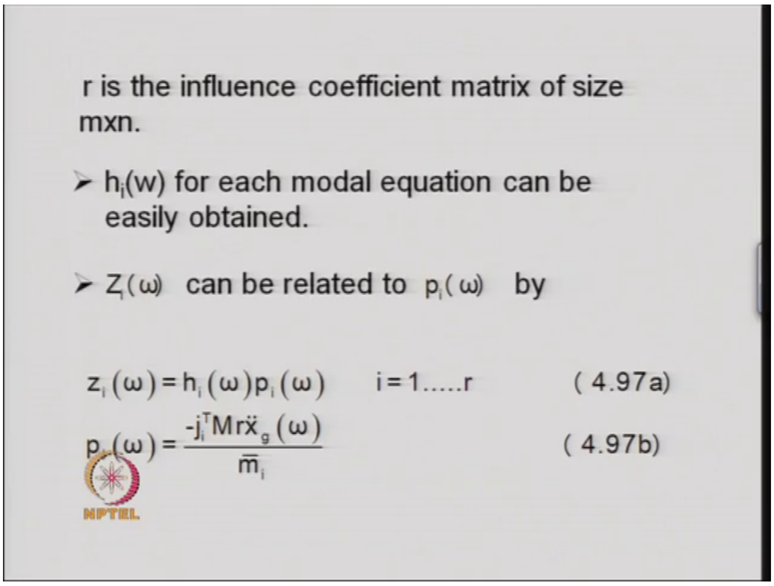

So, the essence of the modal spectral analysis is that first we decouple the equation of motion. And once we decouple the equation of motion then we obtain for each decoupled equation of motion the frequency domain equation which is shown in equation 4.97a and 4.97b, 4.97a and then from there one can write down and pi omega is written in this way, and if r if it is multi supported excitation case then r is the matrix of the coefficients, which is used and jiT is the ith mode shape transpose of that m bar i is the ith modal mass.

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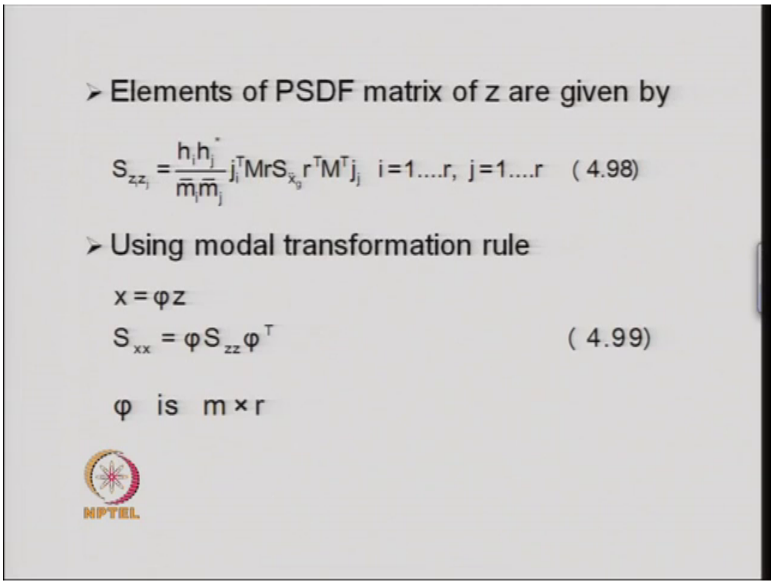

So, once we are able to describe pi omega that is the ith generalized load then using that one can write down S z i z j that is the cross power spectral density function between the 2 modal displacements zi z j as h i into h j star then jiT M r S x double dot g r T M T and j j and both i and j vary from 1 to r, where r is the number of mode shapes that we consider. So, using this expression one can write down the cross power spectral density function between any 2 modal coordinates and then with the help of this we can make the matrix S zz and the diagonal terms of course, will be S zi, zi and I varying from 1 to r.

Then after we have obtained S zz then we can obtain the power spectral density function matrix of displacements in the structural coordinate using this formula, it is coming from the above relationship that is x is equal to phi z and if it is holds good then S x x will be equal to phi S zz phi T. Many a time we do not take all the modes into consideration for example, here we are considering the modes 1 to r; r is less than the number of degrees of freedom. So, in that case the phi matrix will be equal to m into r where m is the number of degrees of freedom and r is the mode shapes that are considered.

So, using this matrix this truncated matrix of the mode shape, one can obtain the power spectral density function of the responses in structural coordinate by considering not all modes, but only in limited number of modes.

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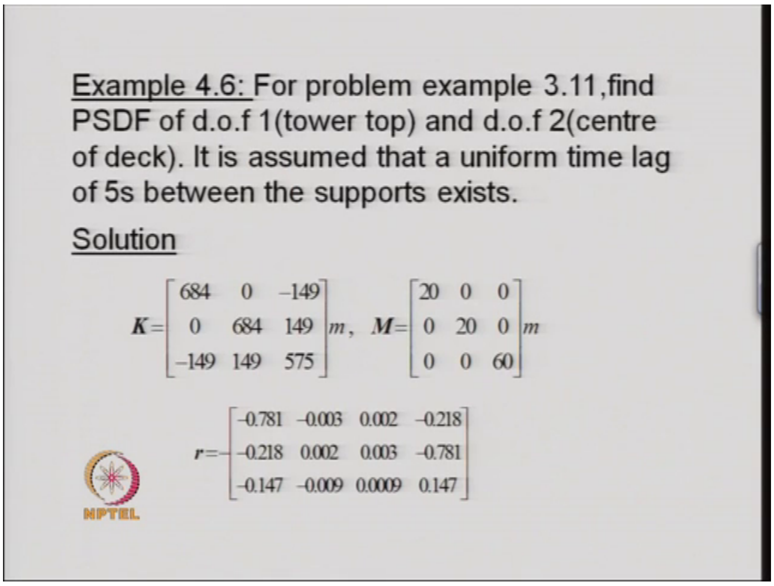

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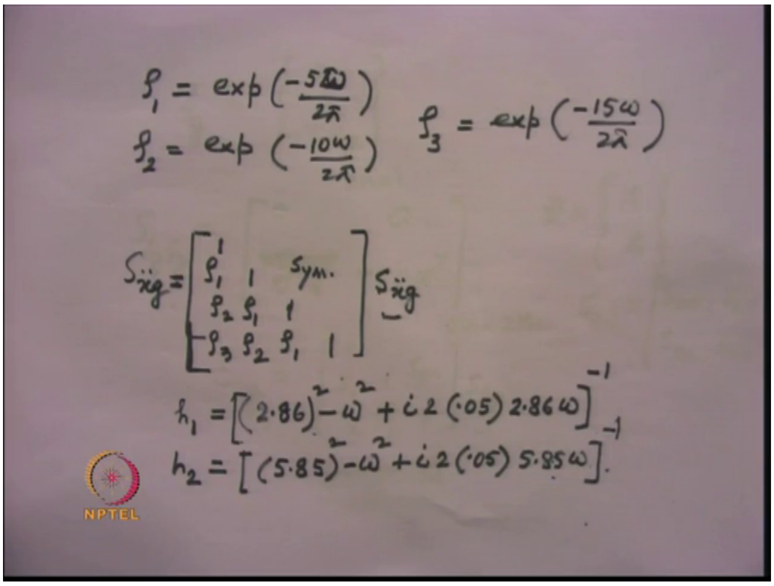

An example problem is solved here to illustrate the modal spectral analysis technique. This problem is the problem that we had solved for the cable stayed bridge, where we want the power spectral density function of the degree of freedom 1, that is the let and tower top and degree of freedom it will not be 2 it will be 3 that is the center of deck.

And it is assumed that a uniform time lag of 5 second between the supports exist that is it is a multi support excitation case.

So, for this problem the K matrix is a 3 by 3 K matrix that is the 2 corresponding to the 2 displacements at the tower top and a vertical displacement at the center of the day. And this is the mass matrix corresponding to these degrees of freedom and this is the r matrix that is the coefficient matrix that that is the displacements that occur at the nonsupport degrees of freedom due to unit displacement at the supports at the 4 supports, that constitutes the r matrix. So, with the help of that we try to obtain the power spectral density function matrix of the degree of freedom 1 and degree of freedom not 2 3. And we proceed with the equations that we described before, now in the case of this multi support excitation, what we will require is the definition of the power spectral density function S x double dot g and the power spectral density function S x double dot g will be written in terms of the coherence function or the cross correlation function between 2 supports. So, between the first and the second support it will be minus 5 5 omega divided by 2 pi because the time lag is 5 second. Between the first and the third it will be minus 10 omega divided by 2 pi because the time lag is 10 second and between the first and the third fourth support it will be minus 15 omega by 2 pi.

Defining row 1 row 2 row 3 this particular fashion, one can write down the power spectral density function matrix for the excitation as in this particular form that is one by 1 row 1 row 2 row 3 so on and they successively because between 2 and 3 support it will be the row 1 that will come into picture. So once we have written down this power spectral density function, then we can use this for finding out the power spectral density function of the load vector that will see little later, S x double dot g over here is a single it is not a matrix is a single quantity the power spectral density function of say here we have taken the elcentro earth quake. So, this is the power spectral density function of the elcentro earthquake say. So, S x double dot g in fact, is multiplied with all of them. So, therefore, we have taken it out.

The h 1 and h 2 is written with the help of the first frequency and the second frequency first frequency is 2.86 therefore, it will be omega n square minus omega square that is the first term, and the second term is i 2 eta omega n into omega. So, eta is 5 percent therefore, it is 0.05 and omega 1 is 2.86 therefore, we in place of omega n we write down 2.86 and we have the compress frequency response function for the first mode.

Similarly, one can write down the complex frequency response function for the second mode. Only thing that we do is that we replace the first frequency by the second frequency 5.85 and we get the frequency response function for the second mode similarly one can write down the frequency response functions for the third mode and taking the third frequency.

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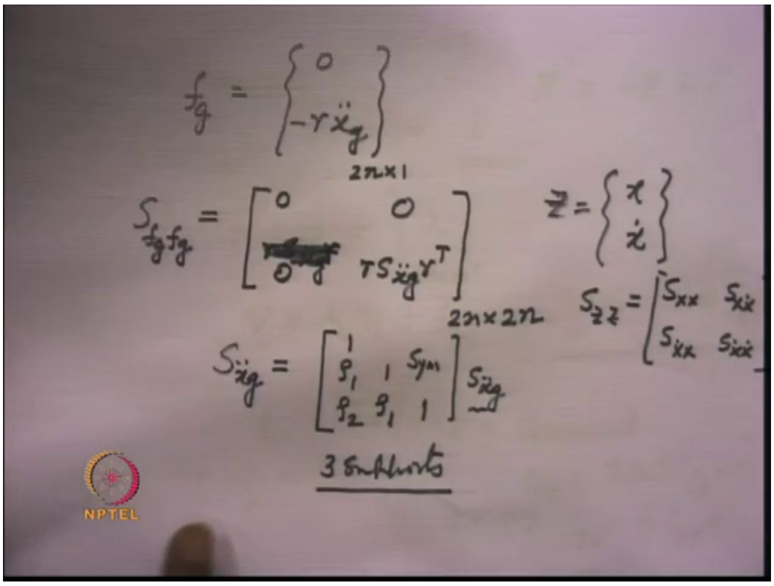

Once we have obtained these h 1 h 2 h 3, then one can plug in these frequency response function into these equation that is h i and h j star that now is known. For any 2 modes and the jiT and j j, they are also known that is the mode shapes for any 2 modes and S x double dot g matrix has been now defined. So, we use we put here the S x double dot j matrix r matrix, I have shown you already. So, you have the r matrix and the m matrix is known therefore, s z i z j that is the cross power spectral density function between any 2 modal coordinates can be obtained with the help of the quantities that we have just now shown.

Now, when all the elements of the power spectral density function of S zz are obtained in this particular fashion, then we construct S zz matrix and then obtain the power spectral density function of S x x.

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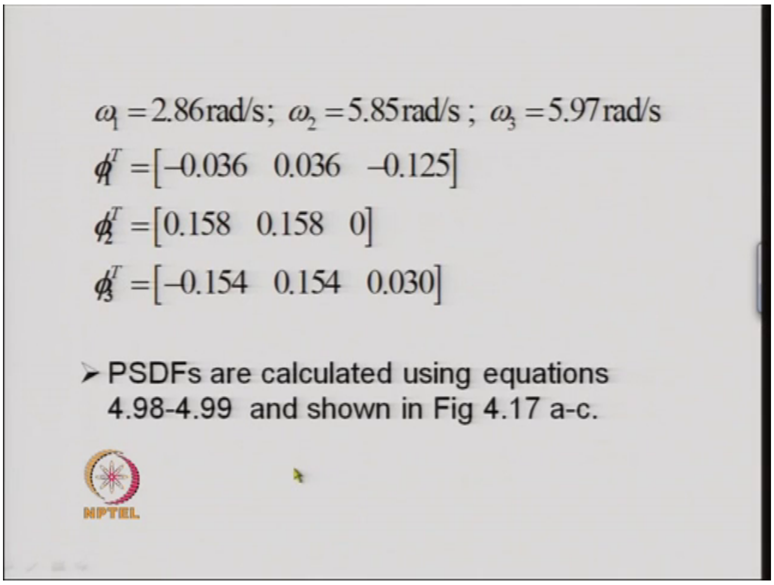

So, using this particular technique.

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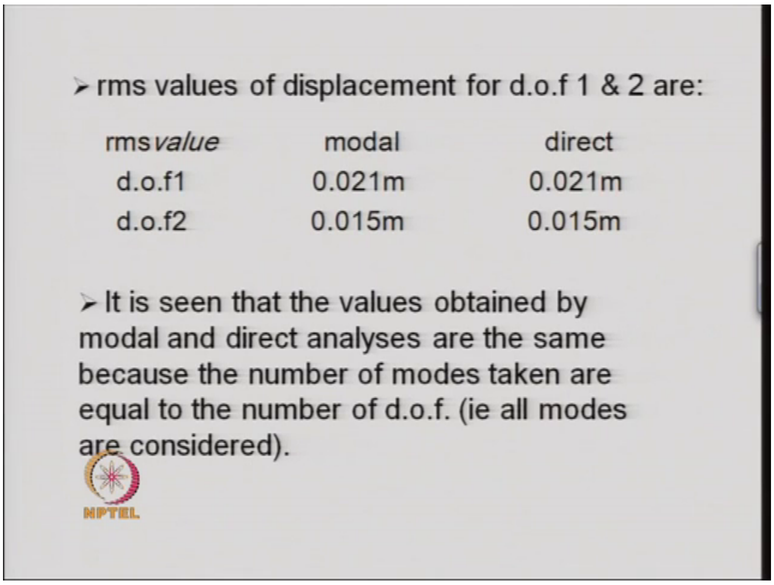

We have obtained the values of the power spectral density function of the first displacement and the third displacement. And the RMS value for the first displacement that is the at the top of the left hand tower that is in the modal analysis it get 0.021 and from the direct analysis we obtain 0.021. So, they are same because we have taken all the 3 modes into consideration. Similarly, for degree of freedom it will not be 2 for degree of freedom 3 the modal analysis get 0.015 and the direct analysis also get 0.015.

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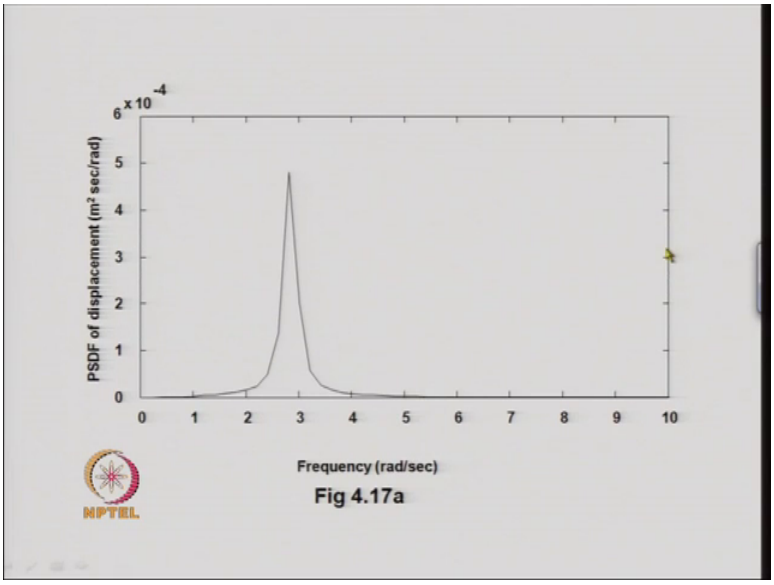

The shape of the power spectral density function for degree of freedom one that is shown over here and we can see that the power spectral density function is picking almost at the first frequency of the structure.

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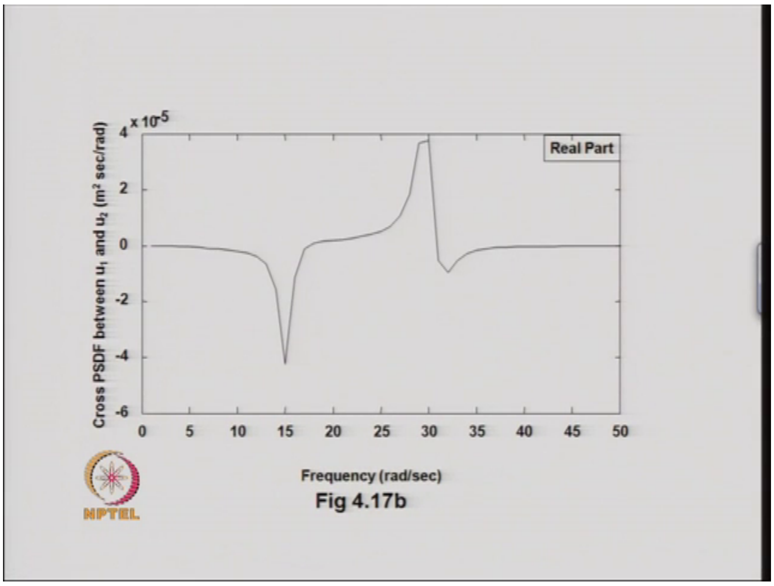

This shows the cross power spectral density function between the any 2 displacements that is between u1 and u 2 and the or rather u 3 it will be u 3. And this cross power spectral density function the g l part is this.

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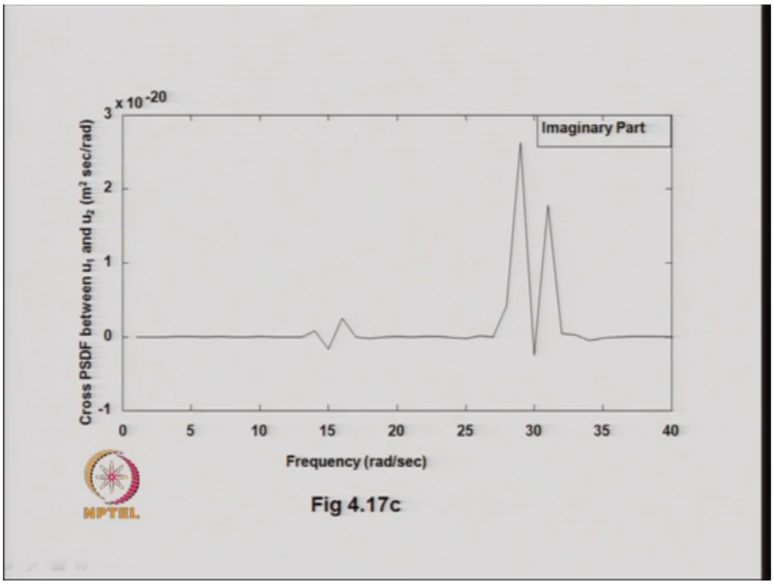

Fig 4.17c

And the imaginary part is this as we know that the cross power spectral density functions is generally complex in character. Therefore, it has a real part and imaginary part.

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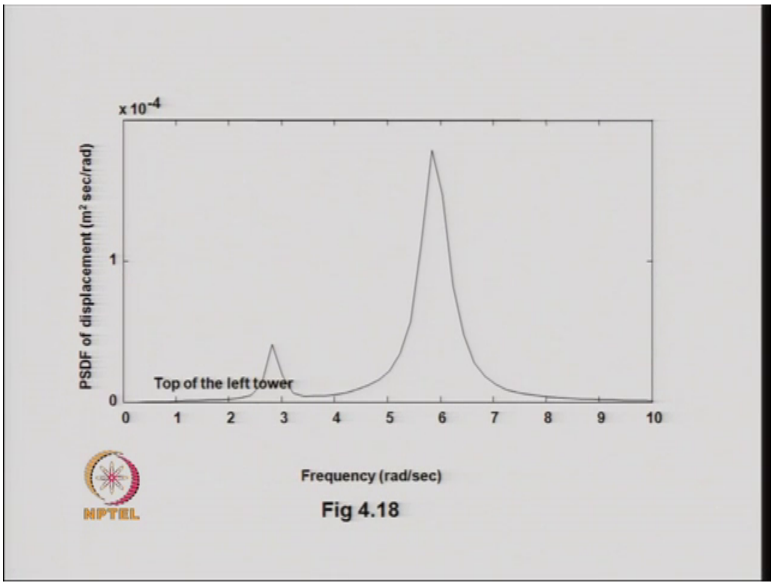

Fig 4.18

And this shows the power spectral density function of the top of the left tower.

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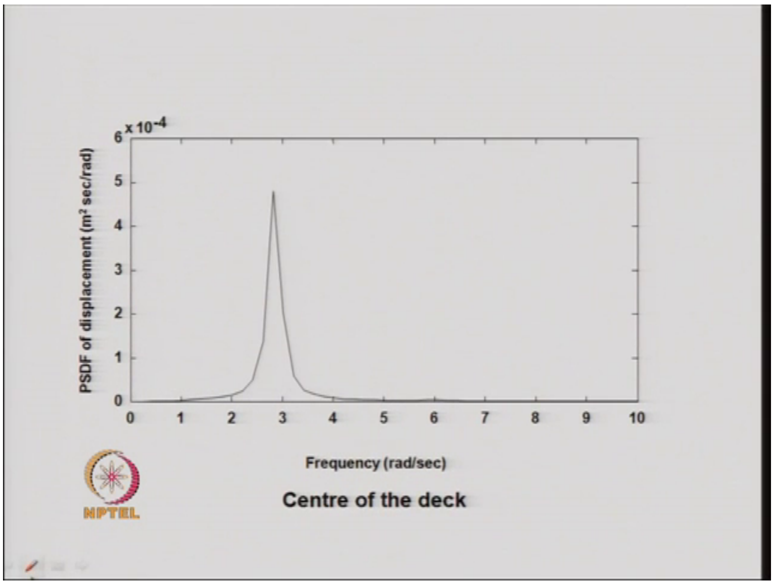

And this shows the power spectral density function at the center of the deck.

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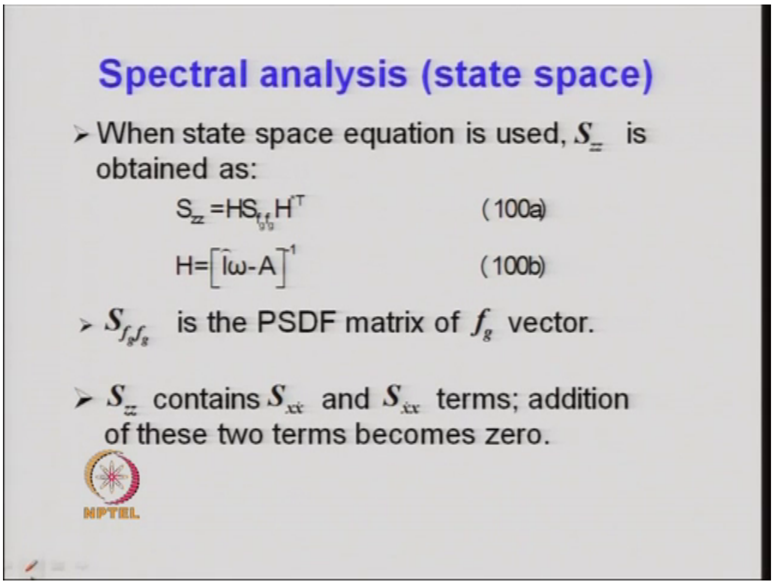

Next we come to the spectral analysis in states space. So, when we perform a state space analysis, then the equation that we use is S zz and S zz is equal to H S f g f g h star t. Now this h is the frequency response function for the state space equation, and you know now the what is the frequency response function for the state space equation. So, it is written as i cap that is a identity matrix of size 2 n by 2 n and omega is the frequency minus a matrix that is again 2 n y 2 n. And if you recall the equation in the state space is equal to z dot is equal to a z plus f g a is a is of dimension 2 n by 2 n z is of dimension 2 n into 1 and f g also is a vector of 2 n by 1, first n values are zeros and second n values second set of n values are r x x double dot j that will I am show you later.

Now, what we have to obtain is that S f g f g matrix and once we are able to find out the S f s f g S f g matrix that is the power spectral density function matrix of vector f g. Then we can plug in that matrix over here, and h omega can be computed from this equation because a matrix is explicitly known and by inverting this matrix we can get the f r f or the frequency response function matrix h. So, let us see how we obtain the S f g S f g. So, this is the f f g in the vector form that is the first n values are zeros and then we have minus r x double dot g.

So, it is of sign 2 n into 1 and z is defined as x and x dot that is the states of the system defined by the displacement and the velocity the S f g S f g matrix; obviously, would be 0 zero 0 r S x double dot g r t, because here this last term is equal to r x double dot g. So, where x x double dot g is again a they power spectral density function matrix of the excitation. Now if this is partially correlated that is the there is a time lag between the excitations, then say for the case of 3 supports you will have the S x double dot g matrix defined as 1 row 1 row 2 1 row 1 and 1 and on this side it be symmetric and S x double dot g will be a single quantity that is the power spectral density function of the elcentro earthquake in this particular case and row 1 and row 2 depends upon the time lag.

So, between the first 2 supports if the time lag is 5 second, then row 1 will be exponential of minus 5 omega by 2 pi that is what we had shown before and between the first and the third support it will be row 2 and it will be equal to minus 10 omega divided by 2 pi exponential of that. So, that is how one can calculate S x double dot g matrix and one and plug in this into this expression and obtain the value of the S f g S f g. And once S f g S f g is obtained then using the equation that you have described before one can get the values of s z s z.

Now, s z s z. In fact, contains S x x S x dot S x dot S x dot x and S x x dot because z is defined like this therefore, from the S zz matrix which will be of 2 n by 2 n size this will be n by n size this will be n by n size. So, will get 2 n by 2 n matrix. So, we can choose the appropriate terms from these matrix to obtain the power spectral density function of any response quantity in the structural coordinate that is x. So, and if we were wanting to know the velocity power spectral density function of velocity, then we can select the appropriate terms from this matrix.

It is to be noted that if I add up S x dot x and x x dot. If I add up this 2 then the addition would be equal to 0, because we know that the cross power spectral density function between the displacement and velocity the summation of this 2 quantities they turn out to be 0 that you have proved before. So, in the state space formulation, one can also obtain the response in this particular way we have solved a problem here.

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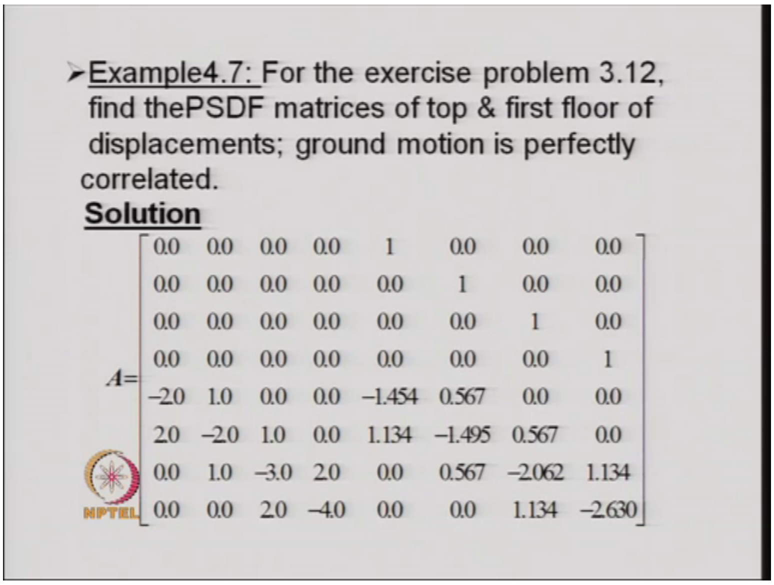

This problem is a the problem in which the top and the first floor this 2 displacements are considered and the power spectral density functions for this 2 displacements are the responses that we look for.

So, here is the a matrix for the system and this a matrix we have generated before while solving the same problem for deterministic ground motion.

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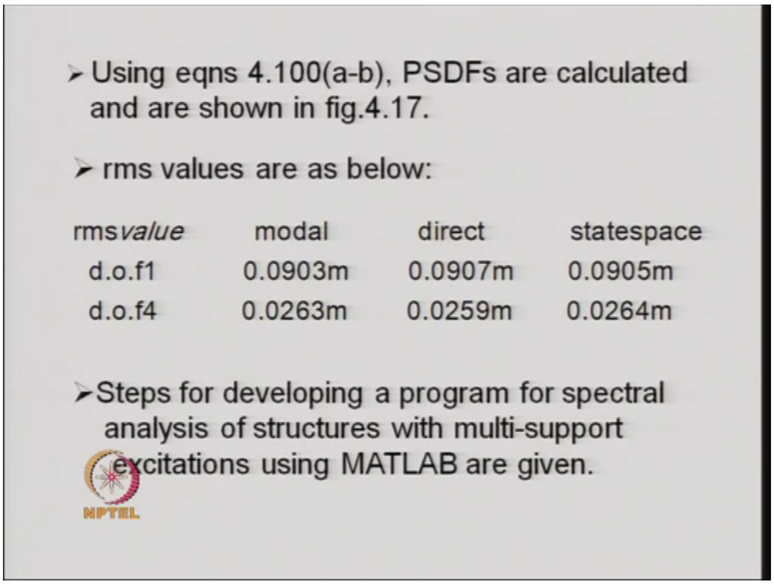

And then using the formulation that we just described we obtained the rms value of the degree of freedom one that is for the top floor displacement and degree of freedom 4 that is the first floor displacements. So, we obtain this by 3 methods in which the this one is the modal spectral analysis, that we just described before this is the direct analysis in which we do not use the mode super position technique, but take the entire k m and c matrix to obtain the responses. And this is the state space solution that we just described and we can see that the rms values obtained by the 3 different methods they are almost the same similarly for degree of freedom 4 that is a first 4 displacement the rms values are again quite comparable they are close to each other.

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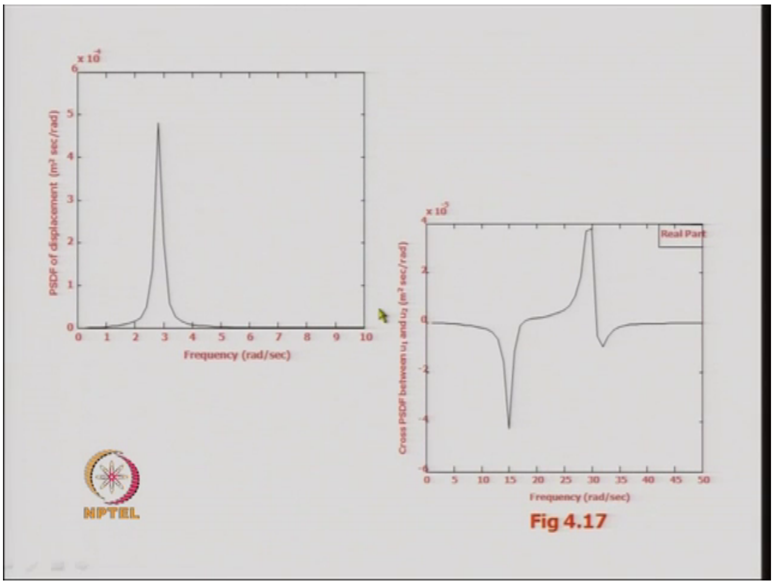

So, thus we can see that one can obtain the power spectral density function of response of a particular structure by a different methods. For example, one can use the direct method using the second order differential equation and there the input the we required is m k and c, and once we know the matrices m k c then from there one can obtain the frequency response function matrix h. And then obtain the power spectral density function matrix of excitation and there it depends on whether the excitations are single support excitation or a multi support excitation if it is a multi support excitation. Then from S x double dot g that is the power spectral density function of the specified earthquake, this power water (Refer Time: 24:32) quantity and the time lag between the supports and using a coherence function, one can construct the power spectral density function matrix of the excitations, and once it is known then one can straight away obtain the power spectral density function matrix of the responses.

In the case of modal analysis in addition, to m c k matrix we must know also the mode shape matrix and using the mode shape matrix. One can decouple the equation of motion into a single degree of freedom equation and then use the relevant expression for finding out the cross power spectral density function between any 2 generalized coordinates and with the help of those cross power spectral density function matrices defined one can obtain the power spectral density function matrix of the generalized coordinate that is S zz matrix. And in terms of the structural coordinate the power spectral density function is obtained by simply multiplying pre multiplying the S zz matrix with 5 and post multiplying with 5 t.

So, that is the modal analysis technique and for the state space analysis, the h matrix that is different than the h matrix that we obtain for the second order differential equation. And once you obtain the matrix then one has to find out S f g S f g matrix how you obtain S f g S f g matrix that I have described and one can obtain the S zz matrix that contains also the power spectral density function of velocity along with the power spectral density function of displacements and depending upon the requirement one can choose from the matrix the relevant terms to obtain the power spectral density function of any response quantity.

For the previous problem this is the power spectral density function of the response of the top displacement, and this is the cross power spectral density function between the top displacement and the first floor displacement the real part of this cross power spectral density function is shown. Similarly, we have a imaginary part for this cross power spectral density function.

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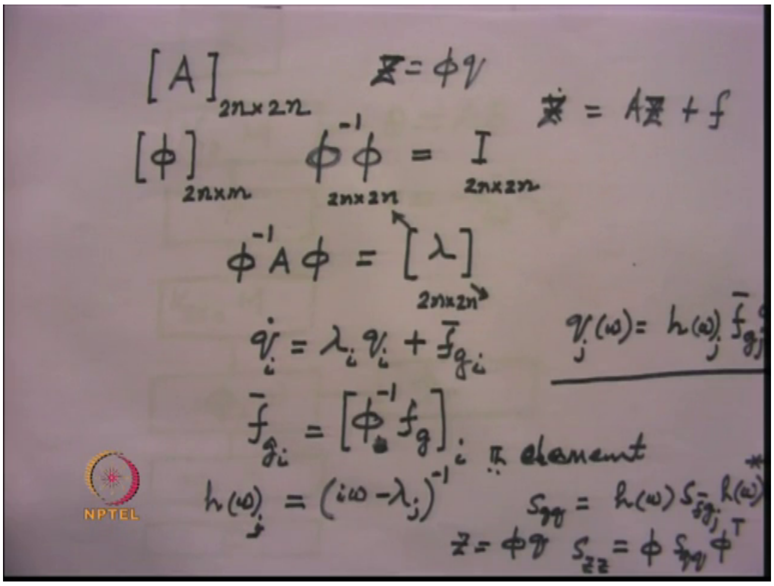

Now the in the state space one can obtain the responses using a again modal analysis. Now for that say this is the a matrix in the state space formulation. So, we write down z to be is equal to 5 into q where is the Eigen values and Eigen or other Eigen vectors of the matrix a. So, this will be of size 2 n by n provided we consider the m number of the modes, but if we consider all the modes then it will be 2 n by 2 n matrix.

In the 5 inverse 5 of course, will be a identity matrix of size 2 n by 2 n. So, 5 inverse a 5 that is equal to a diagonal matrix of size 2 n by 2 n and the diagonal terms are the Eigen values of matrix A. Therefore, using this relationship phi inverse a into phi we can decouple the equation of motion in this particular form, that is qi dot is equal to lambda i qi plus f bar gi, where the lambda i is the ith Eigen value of the matrix A and qi is the generalized Eigen generalized coordinate and f bar gi is the ith generalized load vector or rather load not vector.

So, f bar gi is obtained like this phi inverse f g and ith element of that is a bar gi. And the once we have these relationship that is the phi f bar gi is equal to phi inverse f g, then one can find out the power spectral density function of f bar gi simply by finding out S f g S f g multiply, it pre multiple it by phi in inverse and then post multiply it by phi inverse t. And once we do that we get f bar gi. And once we get the f bar gi then qi omega or q j omega; that means, for a particular mode the generalize coordinate in frequency domain is related 2 the f bar g j through frequency response function for the jth mode and frequency response function in jth mode is given the as this i omega minus lambda j inverse where lambda j is the jth Eigen value of the a matrix.

So, we know h j omega or in other words the frequency response function for any mode. And once you know that then we can use simply for a single degree of freedom equation the power spectral density function relationship between the response and the excitation that is S q q is equal to h omega S f bar g j into h omega star. This h omega is for the jth mode, if this is for the jth mode. And S q q will be the quantity of or power spectral density function of the jth generalized coordinate. And one can obtain the S q y q j by simply taking the ith mode for this frequency response function and jth mode for the this frequency response function the for which we are obtaining the complex conjugate.

So, that is how all the elements of the S q q matrix, that is written over here can be obtained and once we know the S q q matrix then one can obtain the S zz matrix by simply pre and post multiplying by the mode shapes and the transpose of the mode shapes with S q q. So, that is how one can also obtain a modal spectral analysis using the state space formulation.

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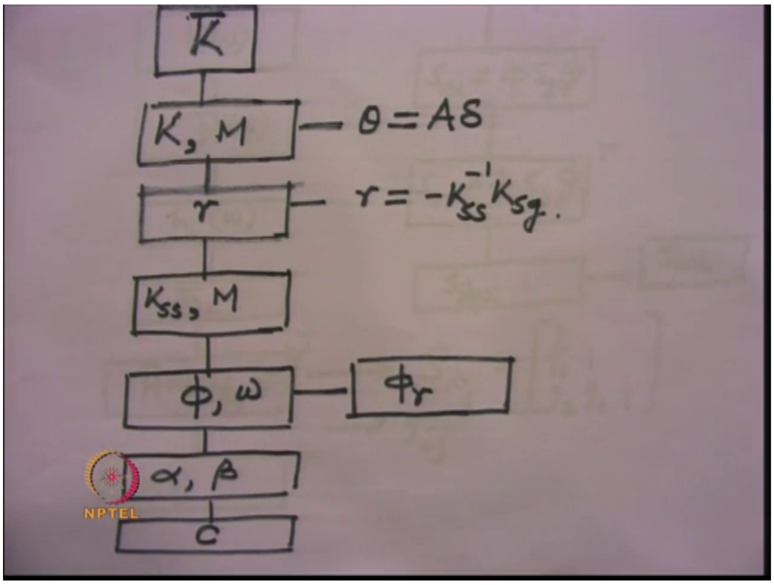

Now, we have seen that there are many ways that we can perform the spectral analysis and these spectral analysis is or can be obtained for a single support excitation and multi support excitation by carefully obtaining the power spectral density function matrix of the excitations. Whenever we have the multi support excitation case then coherence function must be defined. The different methods that you have described can be programmed in mat lab following a particular flowchart and the using certain relevant inputs.

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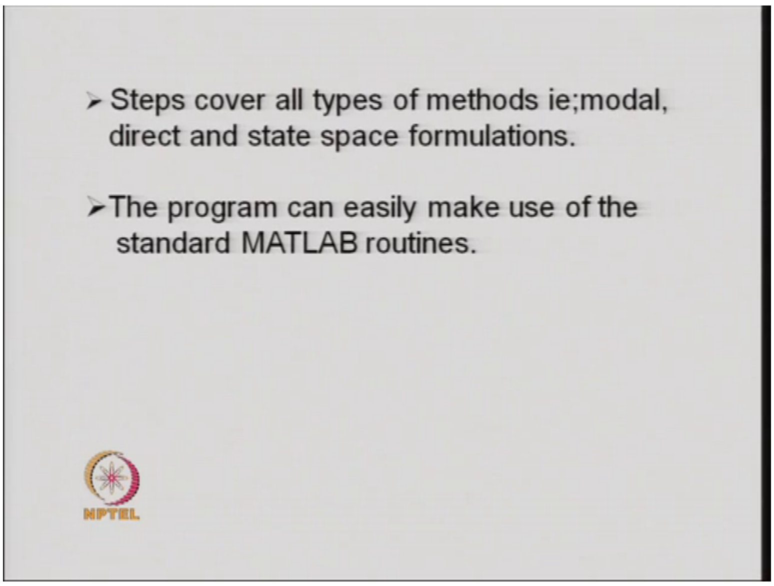

and at the end of the book that from which we have we have made all the slides. In the end of the chapter of the spectral analysis the flowchart is given with the help of which one can obtain a or write down a program for the spectral analysis of structure by using all methods. So, let me explain that flow chart here.

So, first what we do is that, we consider or we take the structure and obtain the stiffness matrix of the structure considering all the degrees of freedom that is the rotations also are considered. And from this matrix we obtain a k matrix which is a condense stiffness matrix corresponding to the dynamic degrees of freedom which are generally the translations. And at that time we store in the program this relationship that exist between the theta degrees of freedom and the displacement degrees of freedom which is related through a matrix A, and this matrix A can be easily obtained from the condensation relationship. And n is the diagonal generally the diagonal mass matrix, but it need not be diagonal, it can be also a matrix which is a coupled matrix depending upon the problem which you have seen before.

Now, from next we obtain once we obtain the stiffness matrix corresponding to all displacements. Then we obtain the r matrix and if it is a multi support excitation case then in this k matrix we consider also the degrees of freedom at the supports that is the translational degrees of freedom at the supports (Refer Time: 36:21) action and once we have that matrix, then from that matrix one can obtain a r matrix which is also stored and this r matrix is of the form of minus k s s inverse, that is the inverse of the nonsupport degrees of freedom the stiffness matrix corresponding to that n k s g that is the coupling matrix between the nonsupport degrees of freedom and the support degrees of freedom.

So, after we have obtained for the multi support excitation case these r and it is stored then we have the nonsupport degrees of freedom and corresponding to that the stiffness matrix and the mass matrix, that we use for the solution we obtain the 5 matrix from this k s s and m and omega the natural frequencies. Then from this also we can obtain phi r that I have discussed before, that is if you these phi is the mode shapes for displacements whereas, phi r could be a mode shape matrix or mode shape vector for the response quantity of interest. And how to obtain that that we have described before that is if I subject the structure 2 a force of m omega square, where m is the matrix and omega I square is the ith frequency. Then we the response that we get displacement response that will correspond to phi and the solution for any response quantity that is bending moment sheer force taken out from that solution that will give the mode shape coefficient for that response quantity.

So, here depending upon the response quantities that we want we can have a phi r matrix and that can be stored. From the frequencies one can obtained alpha and beta coefficient and then construct the required c matrix.

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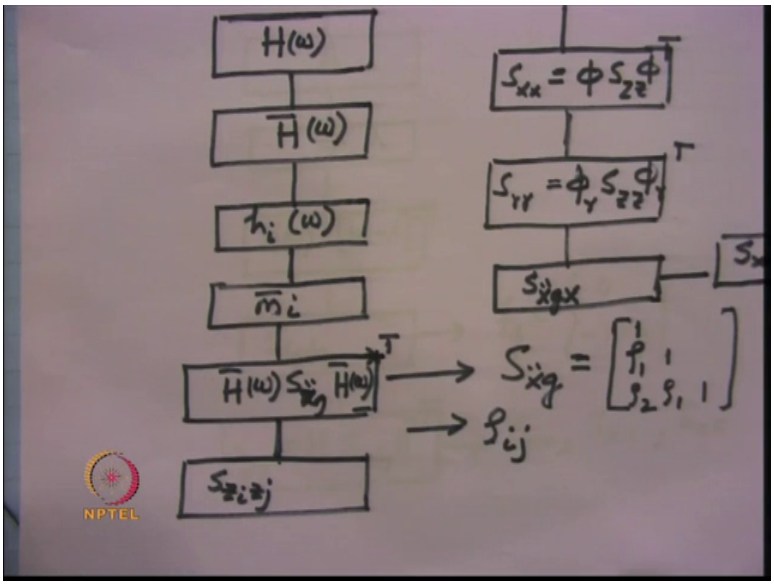

So, once we have that then one can obtain h omega matrix that is required using k m and c. If we are not interested in all the response, then the deduced frequency response (Refer Time: 39:13) matrix h bar omega that can be obtained. Then for each mode one can obtain h i omega, and for that what will require is the modal mass at the at a particular mode. And after we have obtained these 2 quantities, then we can go for different kinds of analysis the first kind of analysis is the direct spectral analysis where h bar omega is the complex frequency response function matrix for the degrees of freedom which are of interest that is we use the reduced compress frequency response function matrix.

S x double dot g matrix that is constructed with the help of the powers given power spectral density function of the ground motion and the coherence functions from which we obtain row 1 row 2 etcetera and that is how S x double dot g matrix is determined. And row i z that must be expressed with the help of an expression for the coherence function taken for from the literature which is appropriate for the particular site. Then one can obtain the s z i z j that is the cross power spectral density function between any 2 generalized displacement. And once we know the s j i z j then one can obtain S zz matrix from that, and if you use that S zz matrix then this particular equation uses the mode shapes to find out the S x x that is the power spectral density function matrix in the structural coordinate.

So, this constituent this one this one and this one sorry this one, they these 2 constitute the modal spectrum analysis part of it. If we interested in finding out the power spectral density function matrix for any other response quantities than the displacement response quantities then that can be also obtained by pre and post multiplying S zz matrix with the phi r matrix phi r matrix is the mode shape matrix for that response quantity that we have stored before.

Then if we are wanting to obtain the power spectral density function matrix for absolute displacement which we again we had discussed before we have 2 kinds of displacement one is a relative displacement with respect to the base. Other is the total displacement by considering the support displacement also. If you are interested in finding out that then what we require is an additional expression which is S x double dot g x that is the cross power spectral density function matrix between the excitations and the displacements. And this is also given in the form of an equation which we have explained before.

So, we obtained that and also keep it in the memory if we are required to obtain the power spectral density function matrix of absolute displacement.

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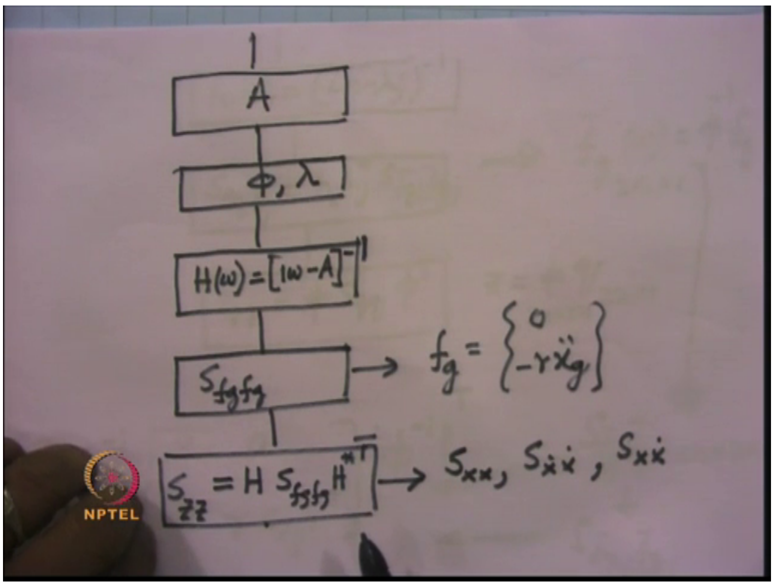

Next if we are wanting to go for the states space analysis, then a matrix must be first generated from the k m and c matrix. Then we obtain the Eigen values and Eigen vectors of matrix A. And obtain the frequency response function matrix using matrix A and then identity matrix i. Then we obtain S f g Sf g matrix that is the power spectral density function matrix of f g vector which consist of 0 and minus r x double dot g and how to construct S f g S f g that I had described just before. So, we obtain that and store it and then go for a direct analysis in which S zz is straight away obtained by using this power your frequency response function matrix and S f g S f g matrix and the transpose of the complex conjugate of h matrix.

Note that S zz matrix contains S x x, S x dot x dot and the cross (Refer Time: 45:11) density function between displacement and velocity and velocity and displacement. Sum of them is equal to 0 and the depending upon whether you require the power spectral density function of displacement or velocity we can choose the appropriate quantities from these matrices. So, that is the direct states space analysis that is direct state space power spectral density function approach.

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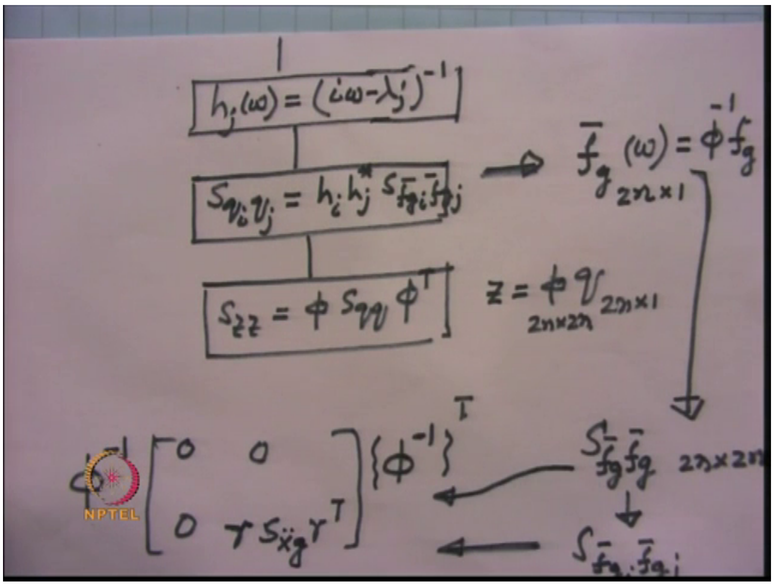

If you are wanting to find out the or use the modal state space analysis in the spectral analysis, then we obtain for each mode the complex frequency response function. And these will require the knowledge of the Eigen value of the particular mode. These Eigen value is generally a complex quantity. So once we obtain this complex frequency response function for the jth mode then s qi q j that is the cross power spectral density function between any 2 generalized coordinate can be written in this particular fashion that is h i into h j star and S f bar gi f bar g j.

Now, this f bar gi and f bar g j is obtained in this particular fashion say f bar g is equal to phi inverse f g. And S f bar g S f bar g matrix would be given by phi inverse 0 0 0 r x double dot g r t. And how to obtain S x double dot g for a multi support excitation that I have described before in which you have the cross correlation function terms row 1 row 2 etcetera. And from this one can choose this f bar g f bar g will be a 2 n by 2 n matrix and from this matrix one can choose any cross power spectral density function term between 2 generalized load f bar gi and f bar g j. So, that is how one can obtain these term over here. And this is of course, stored before and the these matrix also is stored. And once we have the knowledge of s qi q j, then we can form the S q q matrix that is the power spectral density function matrix in the generalized coordinate and obtain the power spectral density function matrix in the state space form that is S zz which would be equal to phi S q q and phi t and this S zz matrix will contain both x and x dot.

So, from that one can obtain all the power spectral density function of displacement velocity and any response quantity of interest. So, we see that the frequency domain spectral analysis can be carried out in different ways and the method that one uses depends upon the response quantity of interest. And the way one wants to solve the problem it can be obtained in the direct form in which either one can use a state space formulation or one can take the second order differential equation in both the cases the c matrix is to be constructed by obtaining alpha and beta using true modes of the structure. And once we obtain the c matrix then one can perform a direct spectral analysis.

If we are wanting to obtain the spectral analysis in modal coordinates or we what we call as the modal spectral analysis, then we have to find out the mode shapes and frequencies of the structure. And this is usually done for systems which has many degrees of freedom therefore, inversion of the h matrix and construction of the h matrix that becomes somewhat tedious. And therefore, one can go for a modal spectral analysis and we can take only a limited number of modes that is may be the first 5 or 10 modes of the system. And then obtain the first the power spectral density function matrix in generalized coordinate using the complex frequency response function for each mode. And constructing a matrix of the generalized forces and for that what one has to obtain is the

S x double dot g matrix that is the matrix of excitations if it is a multi support excitation. And once we have obtained that then one can find out the power spectral density function or cross power spectral density function between any 2 supports in the generalized system. And with the help of that one can obtain the cross power spectral density function between any 2 generalized ordinates and from that one can obtain the S q q matrix. And once the S q q matrix that is the matrix of the power spectral density function matrix of the generalized coordinates that is known then one can obtain the corresponding power spectral density function matrix in the structural coordinate by using the modal transformation rule that is S x x will be equal to 5 into S zz into phi t.

The state space formulation can be used in many cases where we may be interested in the velocity, that is the power spectral density functions of the velocity of the degree of freedom. Then the state space formulation is advantageous and again in that case one can go for a direct state space analysis. And when we obtain the state space analysis in the direct form for the spectral analysis then a h omega matrix that is the complex frequency response function matrix, that is of different form then the complex frequency function matrix that you use for the second order differential equation. Here it uses simply the a matrix for obtaining the complex frequency response function matrix for the system. Once it is known then use we again use the same formulation to obtain the S zz matrix that is the psdf matrix of the state space of the state of the system x and x dot. So, in the S zz matrix, we get S x x as well as S x dot x dot and the pending upon the requirement one can choose any term from these matrices to obtain the power spectral density function of the required response.

In a obtaining when doing this particular formulation, one has to obtain a power spectral density function matrix of f bar g. That is the generalized load which consists of the first n terms of that will be 0 and second n terms will be equal to minus r x double dot g and how we can obtain the S f bar g S f bar g that we have describe before. Finally, one can also obtain a modal analysis in state space performing the spectral analysis and in that case we use the mode shapes and frequencies of the matrix A and with the help of that one can obtain the responses in the generalized coordinate q. And or in or in other words you obtain the cross power spectral density function between any 2 generalized coordinate from that we obtain S q q matrix and then we can obtain S zz matrix.