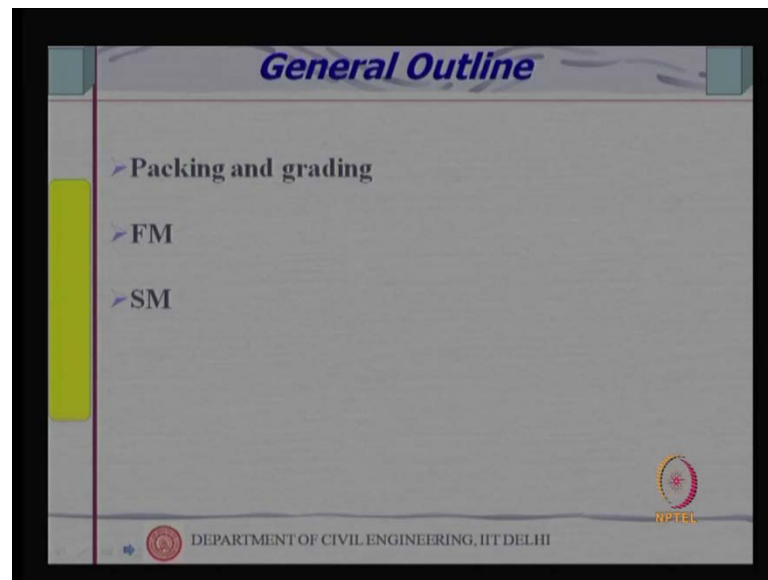


Concrete Technology
Prof. B. Bhattacharjee
Department of Civil Engineering
Indian Institute of Technology, Delhi

Lecture - 07
Aggregates (Packing, FM, SM)

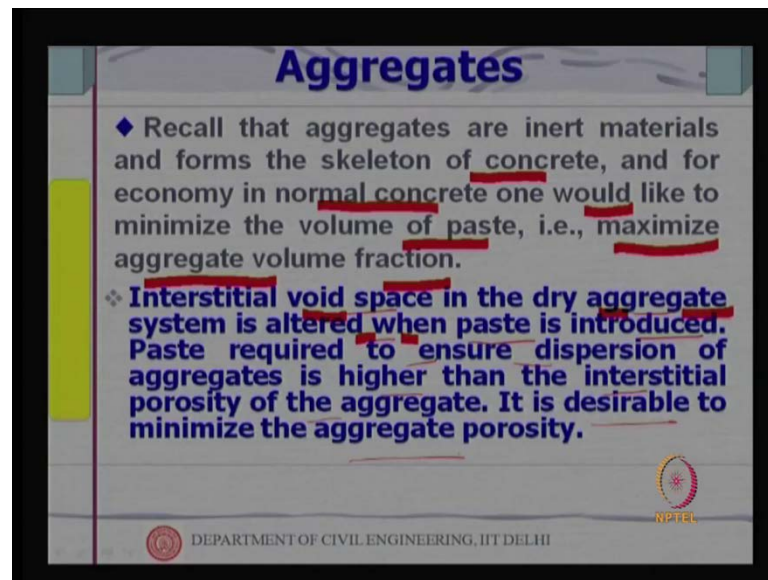
Welcome to module 2 lecture 2, and we shall be continuing with aggregates. If you recall last time we talked about shape and sizes of aggregate, and now in this one we shall be looking into how this sizes affect the packing characteristics of aggregates.

(Refer Slide Time: 01:03)



So, continuing from the same place where we did last class we looked into packing characteristics due to shape, how shape improves the packing, and this time we look into packing and grading, fineness modulus and we shall also define a term called surface modulus.

(Refer Slide Time: 01:25)



Now recall that aggregates are inert material, aggregates are inert material; you know it is there inert material supposed to form the skeleton of concrete and for economy in normal concrete one would like to minimize the volume of paste; in other words maximize aggregate volume fraction. So, paste content cement and that is the costliest material in the concrete. Therefore, one would like to minimize the use of paste from that point of view from the cost point of view; of course, there are other issues related to shrinkage, etcetera, but from sheer cost point of view one would like to minimize the use of paste. So, maximize the amount of aggregate in the system.

Now where does paste go? Paste actually goes into the interstitial void space of aggregate; interstitial void space in the dry aggregate system contains paste and when one puts this paste inside or mixes them together there is you know alteration, alteration of the space, this void space in the aggregate is altered; right, void space in aggregate is altered when paste is introduced. So, paste required to ensure dispersion of aggregates is higher than the interstitial porosity of the aggregate, and it is desirable to minimize the aggregate porosity or void content. So, when you add paste into the aggregate system in fact, paste goes into the interstices of the aggregate.

Now if you pack the aggregates alone you get a void content or void space which you can define in terms of porosity. When you actually introduce paste and mix them together the interstitial space within the aggregate actually is changed because of the

presence of paste it is not same as the original porosity without the paste, right, and this paste causes dispersion of the aggregates; they separate the aggregate fill in the space and separate the aggregate, right.

(Refer Slide Time: 04:13)

Aggregate Packing density

– Packing density is the volume of solids in unit bulk volume i.e. is $1-p$, p is porosity.

Handwritten derivations:

$$\frac{V_p}{V} = p$$

$$\frac{V_s}{V} + \frac{V_p}{V} = 1$$

$$1 = \frac{V_s}{V} + p$$

$$\frac{V_s}{V} = 1 - p$$

Diagram of a unit cell (cube) with volume V . The volume of solids is V_s and the volume of pores is V_p . The total volume is $V = V_s + V_p$.

DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

(Refer Slide Time: 05:24)

Aggregate Packing density

- Packing density is the volume of solids in unit bulk volume i.e. is $1-p$, p is porosity.
- Packing density of single sized particle (say Spheres) is relatively low and depends on packing arrangement .
- For example packing density of simple cubic, body centered cubic and face centered cubic arrangements are: 0.52, 0.68 and 0.74 .
- Actual packing density of single sized spheres will lie some where in between.

DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

Now packing density we define as the volume of solids in unit bulk volume. So, if I have total volume V and volume of solids let us say is V_s , this is volume of pores, then packing density will be V_s divided by V ; this is total is V which is equals to V_s plus V_p . So, you can see that if p is the porosity then V_p over V is p . So, V_s by V is equals to

$1 - \frac{V_p}{V}$. So, therefore, $\frac{V_s}{V} + \frac{V_p}{V}$ is equal to 1 and that is actually volume of pores. So, $1 - p$, p is the porosity.

So, packing density is the volume of solid in the whole system and it is given by $1 - p$; it is given by $1 - p$. Packing density of single sized particle is relatively low and depends on packing arrangement; packing density of single sized particles is relatively low, right, and you can see that you can understand that very simply we can understand that very simply this will be because supposing I have got all single sized particle and they are packed like this. So, this is called simple cubic packing, and you can consider in the other direction also other direction of the same packing you know other direction three dimensional same sort of packing, right, just one below the other, this is called simple cubic packing.

Then the void content I can find out; I can find out the void content or porosity because two particles all particles are touching each other. So, therefore, if I find out this volume, so volume of solid will be this and total volume will be. So, if the diameter of the particle is D , D^3 will be my total V and V_s will be equal to $\frac{\pi D^3}{6}$. So, porosity would be given by $1 - \frac{V_s}{V}$ that is equal to p , and this will be given as $1 - \frac{\pi D^3}{6 D^3}$ so which will be simply $1 - \frac{\pi}{6}$, and this will be somewhere close to 0.48. So, for simple cubic packing the porosity is simply 48 percent. So, for single sized particle porosity is fixed and depends upon type of packing, alright; packing density of simple cubic body centered cubic and face centered cubic arrangements are 0.52, 0.68 and 0.74.

So, simple cubic packing's were like this, each particle is just by the side of it, and we have seen the p is equal to 0.48. Hence packing density is 0.52. Now consider this kind of arrangement, let us say another arrangement where this particles are sitting inside; okay, let me erase the previous one out and just see supposing my packing is now like this, this, this, and the next layer it is coming in between. So, actually packing density would change down. The void should be relatively less; void should be relatively less and you can show that for this arrangement packing density will be higher than 0.52. So, depending upon the type of packing for example for face centered cubic arrangement and body centered cubic arrangement, you know body centered cubic arrangement it is like this, face centered cubic arrangement it is 0.74.

So, it depends upon the packing and independent of the size, because if you recall the size b cube got canceled; therefore, it is independent of the size and depends upon the packing characteristics. Well actual packing density of single sized sphere will lie somewhere in between I mean it will lie between this two. We can have best packing or the worst packing; random packing can have something anywhere in between. That is why if you recall we defined something like for rounded aggregate the packing density was defined as 0.67 when we try to in relation to when we are talking of shape. So, to defined shape we used angularity you know the some angularity index we used where we use the packing density as 0.67.

(Refer Slide Time: 10:11)

Aggregate Packing density

- Multi sized mixtures have higher packing density. Thus to have higher packing density aggregate of different sizes are used.
- Proportions of different size fractions shall be appropriate for maximization of packing density.
- Considering a binary mixture it can be easily seen that packing density increases with addition of another sized material.
- The degree of improvement depends upon size ratio.

B. Bhattacharjee
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

WPTSL

However multi sized mixtures have higher packing density. Thus to have higher packing density aggregates of different sizes are used. How is it? Let us first look at it simply supposing this is my particles and they are packed let us say again simple cubic packing. Now let me put another aggregate; let me put another aggregate final in between. In that case packing density of the aggregate will be reduced; in that case packing density of the aggregate will be reduced, and let me then put still another aggregate somewhere there still final you know I can put still final one, right, still final one I can put in very fine; I can put in still finer one somewhere there. So, you can see if I use more than one size multi sized mixtures have higher packing density.

Now the solutes are more; the words have become less. Thus to have higher packing density aggregate of different sizes are used. So, you do not use single sized aggregate essentially, because single sized aggregate will always pack to a low packing density or high porosity. When you use more than one size packing density is improved and thereby void space or porosity is reduced. Hence your paste content will be less; not only that proportions of different size fractions shall be appropriate for maximization of packing density. So, size ratio and one thing is size ratio, size ratio is size ratio; this is an important issue and proportions are important issue. Size ratio means maximum size let us say is 75, my minimum size let us say is 75 micron, then next size let us say is X.

So, one size is X next size is Y; ratio of this Y by X this is size ratio and we said that this quotient generally we use as two. So, this size ratio is important; it is two is good enough, but if it is one let us say then it is with the same size. So, it is not going to have any effect. So, size ratio has some effect; two is good enough and that is why we use the quotient size quotient to define the size as two. Recall that we are talking about the square shape size and the ratio of sizes we actually size quotient to use as two, we use two.

So, size ratio is important, proportion is also important; it is very important have the proportions. So, proportions would make it you know the maximization of the packing density is possible depending upon the proportions of each size fraction. Now this can be explained very easily considering a binary mixture. Let us see you know how packing density it is with addition of another size material, we can understand this much easily; obviously, degree of improvement depends upon sized ratio I have already mentioned and let us look into how does it improve.

(Refer Slide Time: 14:09)

Packing density of binary mixture

❖ **Packing of binary system can be easily understood in terms of void ratio.**

◆ Consider two aggregates A & B, B coarse & A finer. (spherical for simplicity)



Void ratio, $e = \frac{V_p}{V_s} = \frac{V - V_s}{V_s} = \frac{V}{V_s} - 1$;

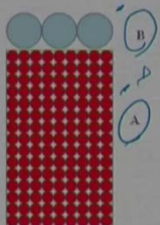
V_p, V_s are volumes of pores & solid

DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

Packing of binary system is very easy to understand. Let us say I have two aggregates A and B, right, let us say B is a coarse size, A is finer, and we are assuming spherical for simplicity; we are considering each factor independently. First we looked into shape and we said that rounded shape packs better. The void ratio increases with you know going from rounded to angular shape. So, therefore, spherical one packs better and we are just now considering the effect of different sizes and their proportions on the packing density. So, we are considering spherical particle again not changing the shape. So, it will be something like this.

Supposing you have got all fine materials this is let us say is A and this is B. So, if you add small amount of B to A; if you add small amount of B to A, right, this is A, and this is B, total volume will increase. Total volume will increase by this quantity, total volume will increase; I have just put them inside, and I can find out how the void ratio would change. What is void ratio? Volume of voids divided by volume of solid or volume of pores divided by volume of solid. So, we defined void ratio as volume of pores divided by volume of solid and which is V minus V_s is the total volume. So, V minus divided by V_s . So, V by V_s , so we can write void ratio as V divided by volume of solid minus 1, right. So, this is what we can write for void ratio and let us see how it would change.

(Refer Slide Time: 16:04)



Void ratio of A alone, $e_A = \frac{V}{V_{sA}} - 1$;
 V_A, V_{sA} are overall volume & solid volume of A

Void ratio of B alone, $e_B = \frac{V}{V_{sB}} - 1$;
 V_B, V_{sB} are overall volume & solid volume of B

❖ Consider small amount of coarse B added to fine A. Packing of fines remaining undisturbed, total volume is sum of bulk volume of A and solid volume of B, & solid volume is sum of solid volume of A & B

DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

Now void ratio of A alone will be volume of overall volume you know it is. So, let us say if it is the total volume alone divided by volume of solids of A minus 1, because we have defined the void ratio earlier by this equation; void ratio we have defined by this equation V by V_s minus 1. This is the total volume, the volume of solid minus 1. So, that is the void ratio. So, void ratio of A alone will be v divided by $V_s A$, that is the solid volume in total v minus 1, right. So, V a V_s they are overall volume and solid volume of A in the system, void ratio of V is v divided by $V_s v_b V_s b$ minus 1; that means, if I fill in the whole system with B alone then the volume of solid that would occupy is given by this.

Now, just look at. So, this was the thing; just consider as small amount of B is added to fine A. This is my A, and I have just added small amount of B; packing of fines remain the remaining undisturbed, total volume is some of bulk volume of A and solid volume of B. Supposing I just add you know this is just added this, right. So, total volume will i just added this let us say even if I mix it properly we are assuming that this volume packing of this one will not be disturbed largely, packing of fines remaining undisturbed I just add from top, total volume of some of bulk volume of A and solid volume of B. So, total volume will be this bulk volume of A plus solid volume of B, and solid volume is the sum of solid volume of A plus solid volume of B. So, total volume is the sum of bulk volume of A plus the solid volume of B and volume of solid is the solid volume of A plus solid volume of B.

(Refer Slide Time: 18:31)

Packing density of binary mixture

Volume fraction of B in total solid $x = \frac{V_B}{V_A + V_B}$;
 V_A, V_B are solid volume of A & B

$e(x) = \frac{V + V_B}{V_{sa} + V_B} - 1 = \frac{V}{V_{sa}} \left(\frac{V_{sa}}{V_{sa} + V_B} \right) + \frac{V_{sb}}{V_{sa} + V_B} - 1$

$e(x) = \frac{V}{V_{sa}} \left(1 - \frac{V_{sb}}{V_{sa} + V_B} \right) + x - 1 = \frac{V}{V_{sa}} (1 - x) + x - 1$

$= (1 - x) \left(\frac{V}{V_{sa}} - 1 \right) = e_a (1 - x)$

Handwritten notes: $x = \frac{V_{sb}}{V_{sa} + V_B}$, $V_{sa} = 1 - \frac{V_{sb}}{V_B}$

DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

So, using this if I use this algebra to find out the void ratio volume fraction of V in total solid of course will be V_b divided by V_a plus V_b , right, let me call define this as a variable X. So, if X is the variable V_b by V_a plus V_b . So, that x is the volume fraction b in total solid, V_a V_b are solid volume of a and b as we have understood. Then $e(x)$ that is the void as a function of this can be written as V plus V_b , now what is this V plus V_b ? V was the original volume. So, this is V ; this volume is V plus the V_b that is the volume of total volume increase in total volume. So, this is the new total volume because we said volume of total volume will be the increase in volume of bulk volume of a plus the solid volume of V. So, this is the total volume; this is total volume, and these are solid volume because volume of solid of a plus volume. So, if I add a small quantity of coarse aggregate void size fraction aggregate to bulk finer size aggregate then what it would do? It will increase the bulk volume by the amount that I have added the solid volume that I have added.

For example V_b is the solid volume of the coarse aggregate B I have added and V was the total volume that is volume of solid plus volume of pores of A. So, if I add this it will just increase the total volume without increasing the pores. Let us say without disturbing the porosity of the fine aggregate, in that case the total volume would be V plus V_b and volume of solid will be this. So, my void ratio by definition would be V plus V_b V_{sa} plus V_b because we said in the V volume V_{sa} is the amount of solid minus 1, and I can write it like this V by V_{sa} , right. If I can write it like this V by two component I am

separating out because V can be written as $V_s + V_b$. So, I can write it out like this
 you know multiply both side by this term V by $V_s + V_b$ can be written as V by $V_s + V_b$ multiplied by $V_s + V_b$.

So, what is this actually? This is the total volume divided by volume of solid, and this is the packing density of the whole thing. So, anyway I will just separate it out so that I can take something common. So, let us see how the algebra works. Let us just see how this algebra works. So, this is what I have done modified this term this divided by this term and the next would be V by V_s , this term remains same, and this I can write as $1 - \frac{V_b}{V_s + V_b}$ because V_s is equals to $1 - \frac{V_b}{V_s + V_b}$, alright. So, $1 - \frac{V_b}{V_s + V_b}$, so this is what I can write; you know I can write $1 - \frac{V_b}{V_s + V_b}$ because V_s is nothing but $V_s + V_b$ is equals to 1; that is what we are assuming total volume, right. V is this volume and total volume is. So, following fraction this is volume fraction in the final mix is this.

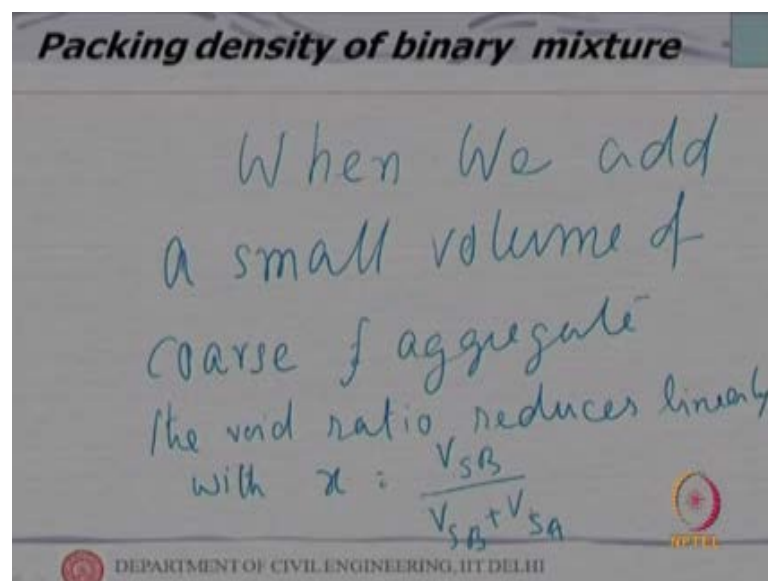
So, I am writing it like this, and if I define x what was the x ? Definition of x was V_b divided by $V_s + V_b$. So, this was the total volume that is fraction of solid volume of b in the total volume; that is what we defined as x . So, this will be this is x and this is also x because by definition and minus 1. So, I can write as V by $V_s + V_b$ $1 - x$ plus x minus 1 and I can take it take common $1 - x$ I can take common. So, if I take $1 - x$ common I will get V by $V_s + V_b$ minus 1. So, this is by definition e_a . So, this is by definition the voids ratio of A alone, this is by definition void ratio of A alone

Let me repeat this process what I have done. I have added some coarse aggregate into the fine system. If I just add a few of them and may be push it a little bit from the top little bit small amount it is unlikely to disturb the packing characteristics of fine aggregate too much, because fine aggregate quantity is very large; only a small quantity one or two pieces let us say I have added of the coarse aggregate B . So, it will not disturb the overall packing of the fine aggregate minimal disturbance; ideal case it is actually really it does not happen that way but ideal case let us assume. So, in that case total volume will now increase, because I have added some material and that will heap up or increase the total volume by how much? By the solid volume of the coarse aggregate that I have added; solid volume will also increase by the same amount.

So, I can express the void ratio that is the original volume plus the volume of the solid and let us write algebraic expression we are assuming v as the original volume. So, v plus v_b that is I have added volume of solid I have added divided by v_s a plus v_b is the total volume minus 1 is the porosity by definition. Then I am doing a little bit of algebraic manipulation small algebraic manipulation, what is that manipulation? V by I just write take this term out and write it in this manner, multiply top and bottom by V_s a, and this term I keep it as it is. The volume of the solid V_b is because this is the amount I have added, v_b is the amount I have added solid I have added in the overall system. So, V_s b divided by v_s a plus V_b because there is no other solid volume. So, V_b is nothing but V_s b; I have just written it like this. This I could have written V_s b as well all same things because V_b is same as V_s b.

So, I have just written it like this. Now this B this V_s a is 1 minus V_s b because my total volume is 1 , total volume is one and V is the original volume. So, this is original volume and total volume is 1 . So, I am just writing it separately in this manner, right, because original volume V and this original volume I am using to define the V_s a in the same total volume original volume V the e a will be defined as V_s a divided by V a if alone it was packed you know v_s a divided by V minus 1 . So, if alone it was packed if it was packed alone, then the volume that it occupied. So, that is how this is nothing but a . So, a into 1 minus x . So, that is how we come because that will reduce linearly as I increase the volume fraction of B .

(Refer Slide Time: 27:05)



So, from the previous one what we have seen is when we add a small volume of coarse aggregate the void ratio reduces linearly with x which is nothing but volume of total volume you know. So, proportion it reduces with the proportion of solid volume of v . It reduces because the expression is if you recall the expression, expression was e a void ratio reduces 1 minus x and x by definition x is v_s a divided by v_s plus v_b , you know by definition x was something like this. We have defined x as v_b divided by v_a or I can say $v_s b$ divided by $v_s a$ plus $v_s b$. So, that is how I define the volume fraction of coarse solid in the total volume, alright. So, that is what it is. So, we say that it reduces linearly it reduces linearly.

(Refer Slide Time: 28:47)

Packing density of binary mixture

❖ Consider small amount of fine A added to coarse A. Packing of B remaining undisturbed, bulk volume of B is the total volume for solid volumes of A & B



$$e(x) = \frac{V}{V_{sh} + V_A} - 1 = \frac{V}{V_{sh}} \left(\frac{V_{sh}}{V_{sh} + V_A} \right) - 1 = \frac{V}{V_{sh}} x - 1$$

$$e(x) = \frac{V}{V_{sh} + V_A} - 1 = \frac{V}{V_{sh}} \left(\frac{V_{sh}}{V_{sh} + V_A} \right) - 1 = \frac{V}{V_{sh}} x - 1$$

$$\frac{V}{V_{sh}} x - x + x - 1 = x \left(\frac{V}{V_{sh}} - 1 \right) + x - 1 = e_B x + x - 1$$

$$x(1 + e_B) - 1$$

DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

Now let us look at from another angle. Consider a small amount of fine aggregate A fine aggregate A you know added to coarse aggregate B this should be B. Packing of b remaining undisturbed bulk volume of B is the total volume for solid, bulk volume of v is the total volume for solid volume of A and B. Let us just see; the diagram will make this statement very clear. What we are trying to say is supposing I have got now all large particle B and I am trying to adding a small amount of fine particles. So, they will simply go into the interstices of the coarse particle; they will simply go into the interstices, occupy this spaces, simply they will go into the space. This volume is my v original volume v which was packed with coarse particle. So, this packing of this one will not get disturbed because I have put in a few particles; it is not going to disturb this, no way it is

going to disturb. So, it will remain same, right, packing of b will remain same, particle a will all go inside.

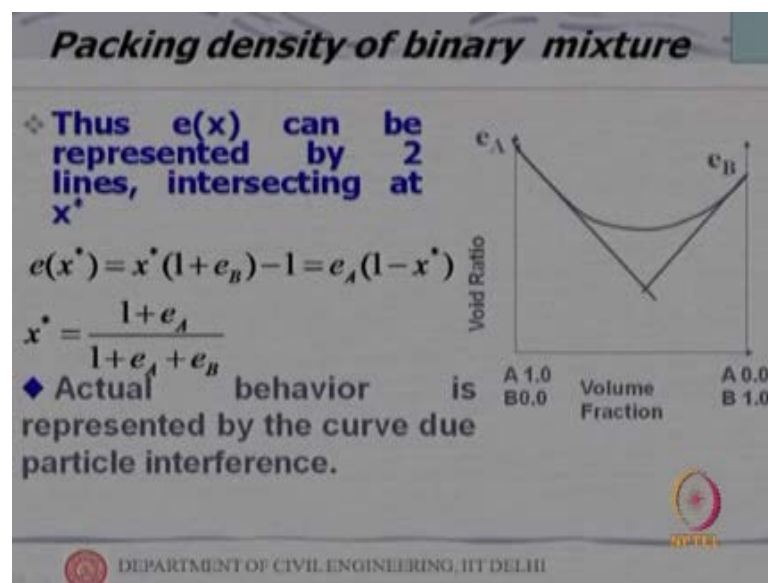
So, what will happen to the total solid volume? Total volume remains same. So, total volume bulk volume total bulk volume will remain same. Previous case bulk volume increased, but here bulk volume will remain same. Volume of solid will increase; volume of solid will be equals to $v_s b$ which was there originally or v_b whatever you call it plus volume of solid that we have added right now these two volume. So, this solid volume will increase by this quantity that you have added. So, therefore, you know e_x that is the porosity of void ratio as a function, v is the bulk volume must remain same; this was the volume of the solid that you have you know the original volume of the solid and the volume of aggregate that you have added; original volume remain same minus 1 because we know we remember this was v_s minus 1.

So, v_s just increases by this small amount of a that you have added minus 1. Again I do a little bit of algebraic manipulation and what I do; B can be separated out as I mean just multiply it by top and bottom by $v_s b$ both sides. So, I just multiply by this $v_s b$, no separation here; simply multiply this by because I want to get x and this is my x . So, I want to get x . So, what I have done I have just multiplied by this. So, this gives me v by $v_s b x$ minus 1, right. So, x is equals to v by $v_s b x$ minus 1 and then what I do is I just add minus x and plus x . So, I just do a little bit more algebraic manipulation. This I add minus x add x subtract x and if I do then this x gets common. So, I will have b by $v_s b$ minus 1, because this is what I want to get which is nothing but the e_B void ratio of B alone. So, I wanted to get void ratio of b alone and for that I must subtract x out of this. So, x if I take common I get like this and this is x minus 1.

So, what I get is $e_b x$ plus x minus 1 or I can write that x into 1 plus v_b minus 1 that is again linearly related to x as my x reduces, here x will be reducing, because x is the proportion of coarse aggregate in the system. So, as my x reduces we will find that this void ratio smaller the x void ratio will reduce. So, two things we have observed from our discussion so far. Physically you can understand when you add small amount of fine aggregate into otherwise packed system of coarse aggregate, the fine aggregate will simply occupy the interstitial space within the coarse aggregate thereby reduce the void volume; that is what is manifested in the void ratio. It so happens that void ratio reduces linearly; you know algebraically you can show that void ratio will reduce linearly.

Similarly, when you add in a packed system of fine spherical particle I will say or aggregate some large coarse particle very small quantity it will again reduce down the void in the system, because you will add only some solids. Total volume will increase, solid volume will also increase; void volume remaining same as the original that was the void volume in the fines, the void quantity remaining same that was there in the original fines. So, void ratio or porosity will reduce because volume of voids in the total volume has reduced.

(Refer Slide Time: 34:59)



This we can draw in a manner like as a function of x ; for example, void ratio and this is my x actually. The x is increasing along this direction, x is increasing along this direction because we said b fraction is one volume of b this fraction and this is zero here. So, x is zero here and one here; you know the proportion we are saying that x is nothing but volume of coarse aggregate in the total volume. So, here it is one that is all coarse aggregate, and here it is all fine aggregate, and in between I will have different values of x . So, e_A stands for the void ratio of A alone. So, when I have only A this is the void ratio of A ; when I add B , a little bit of B void ratio reduces down, and similarly here it is e_B initially, and as I add a little bit of A in other words reduce down the volume of B the void ratio should reduce in this manner linearly with x .

So, this is my x it reduces. As the x reduces from one to smaller value e_v reduces that is what we seen through those equations. This is what happens if I add a very small

quantity, and if the same thing was happening then ideally I should have minimum voids somewhere there at some proportion. But in reality what happens is actually you get a curve like this; if you calculate the void ratio, experimentally try to determine them at different proportion of two binary mixtures two mixtures only two sizes you have taken spherical particle let us say marble balls playing marbles or small balls you have taken, then you will get this sort of a relationship of void ratio versus volume fraction. This occurs, because of our assumption was that the packing characteristics of the fine aggregate will not be disturbed if I add coarse aggregate; no new pores will be created.

Similarly, when we added fine aggregate to coarse system it will simply reduce down the voids going into the interstices of the coarse aggregate; that is what we assume, but in reality what happens is for a small amount this may remain valid, you can imagine that. For example, if you have a box containing large size spherical balls, you put few more small balls, they will just show go trickle down and sit somewhere inside, but when you start putting more and more a time will come when this particle will push the larger particle away.

In other words it will disturb the packing of the larger particle; not only that they themselves their packing will not be there, ideal packing which there were actually having when they were packed alone. For example, fine aggregate if you pack alone you get a kind of packing you get a voids, but if you put the same fine aggregate in a coarse system interstices of a coarse system the void content will not be same as that rather it will increase. So, because of this kind of behavior that packing get disturbed what you call particle interference; when you put another particle it will interfere with the original system, and that is how the curve never remain straight line rather it goes it becomes curvilinear rather it tends to become curvilinear in this direction, and the void ratio is much higher than what you would have got for an ideal case.

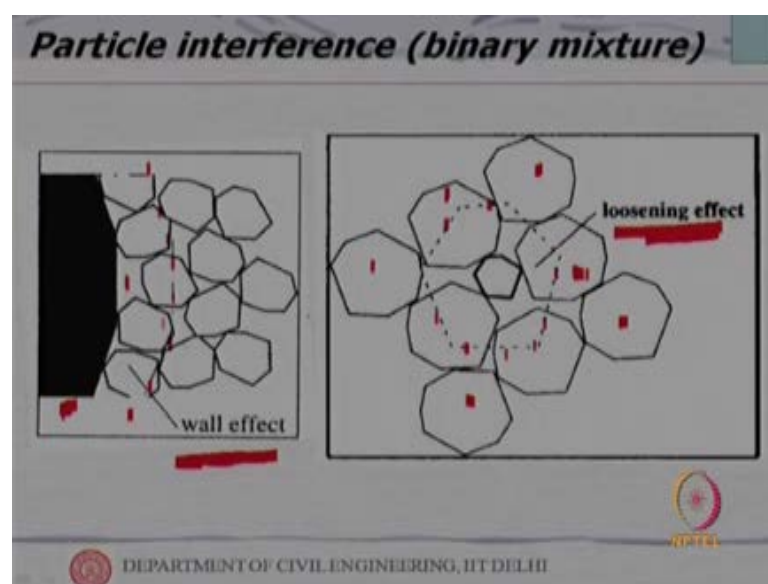
But one thing interesting is by and large at this point you will have minimum, near this point you will have the minimum packing. So, minimum packing will be very close to what you would have found out even in case of ideal case. But the important point here is there is a specific proportion of this volume proportion of b and a where this will be minimum; that means it is not independent of the proportion. The minimum void content is a function of the proportion of b in a or otherwise you can say proportion of a in b; that is e_x can be represented in a two lines intersecting at x_{star} ; that is what we just said, we

have just shown x^* . x^* is the intersection point; this is my x^* , right, x^* and you can calculate out this x^* algebraically because from this side if I calculate out from this side if I calculate out it is given as $x^* = 1 + e_v - 1$.

That must be equals to $e_a - 1 - x^*$, because x is same for both and just simply from this you can calculate out eliminate out, I mean you can get an expression for x , x^* you bring it this side and e_b and this side you have got $x^* e_a$ with a negative sign. So, if you bring it to the other side coefficient of x will be $1 + e_a + e_b$, and this term is independent of x^* , so it goes to the other side. So, you will have $e_a + 1$ divided by this. So, x^* can be algebraically evaluated as this, and that is the optimal proportion at which void ratio will be minimum. In other words packing density will be maximum.

Actual behavior is represented by the curve due to particle interference. This is represented by curve because of particle interference, but the point I am trying to make is that proportion is important, size ratio is important and obviously, you should have more than one size. So, you take two sizes; packing density improves that is void ratio will reduce, but the amount of reduction of the void ratio depends upon the proportions. So, proportion is important; obviously, size ratio is also important and shape we have seen earlier.

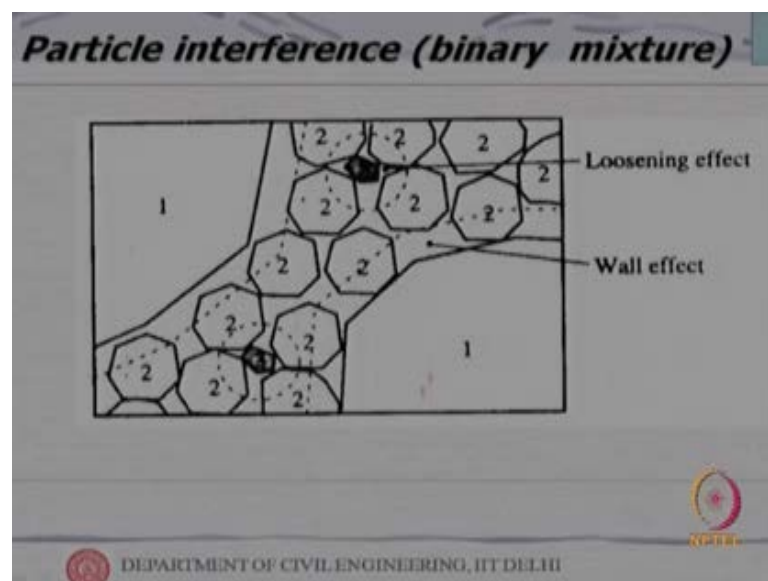
(Refer Slide Time: 41:14)



Now there are two effects which are recognized; one is called wall effect. For example, if you see this is my coarse aggregate; this is the coarse aggregate, and this is the fine aggregate. The packing here is unlikely to be very good. You know it is not packing like bulk packing because in front of a large surface this packing is never good. This we call as wall effect and this is recognized everywhere. So, this dotted line is the boundary where the wall effect actually increases the pores. So, you can see the packing of the fine particle, finer particle here we have not taken spherical, but we are showing some sort of regular particle. Packing of the fine particle itself is going to get disturbed because of presence of the coarse aggregate coarse particle.

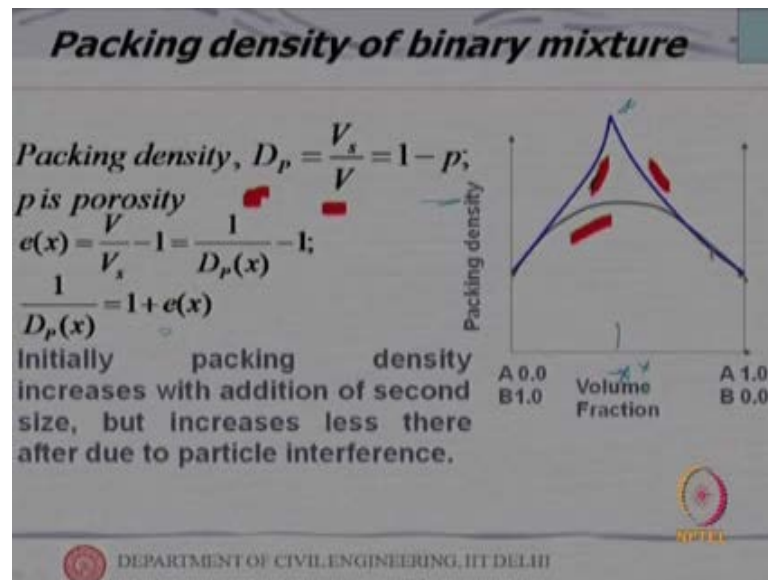
So, coarse particle causing a disturbance; this is called wall effect, and this particle inside will be pushing the particle out. So, sphere of this particle is something like that, and if the voids otherwise were smaller; now this particle depending upon its size it will push out. So, this is called loosening effect. So, wall effect and loosening effect these are the part of the particle interference, and due to this packing density of the both coarse and the finer one gets disturbed. So, loosening effects cause disturbances in the packing characteristics of void content in the coarser particle while wall effect cause increase and cause an increase in the void content of the fine particle which has gone inside the coarse particle. So, that is this two effect causes the curvilinear behavior.

(Refer Slide Time: 42:59)



So, you can see that both together is causing wall effect and loosening effect in a large aggregate. This is your aggregate tools of fine aggregate; these are the additional voids which have come in, additional because of you know additional voids that is coming. So, particle interference is occurring here, etcetera.

(Refer Slide Time: 43:23)



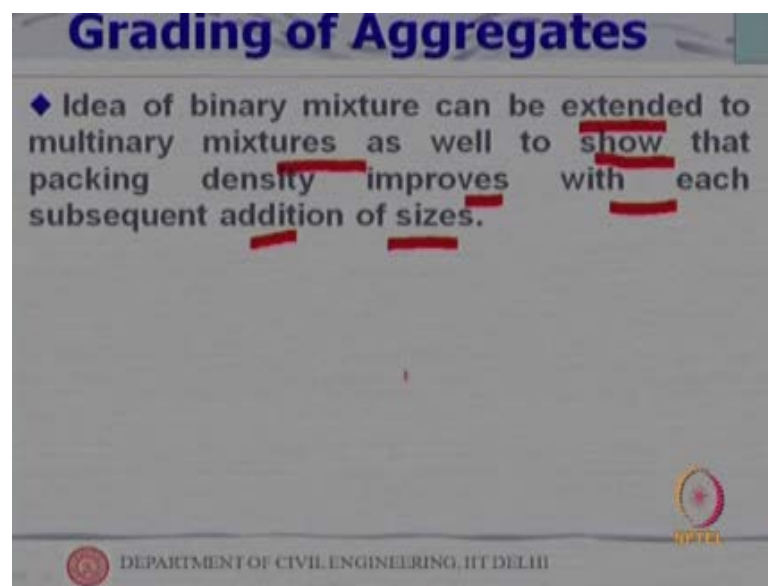
If I calculate packing density of the binary mixture d_p let me call it as packing density and I have defined it like this which is $1 - p$ then e_x is v divided by v_s minus 1. So, this is nothing but the one by packing density, so $d_p \times$ minus 1. In other words I can write 1 by $d_p \times$ is equals to $1 + x$, right, and this graph will now look like this. If I plot with x , e_x is known to you. So, packing density if I try to plot with x because e_x is known you know if I try to plot it with x equations were x were available. This is for $1 - x$ e_a into $1 - x$ if you recall and from this side it will be again e_x expressions were available, and this will give me something like non-linear curve of this kind, and from this side also again if I plot at same x star you will have maximum packing density.

So, initially packing density increases with addition of second size, but increases less after due to some particle interference, and you can see that something like this will occur following this form of a curve. So, this would have been the ideal case, but real case is something like this, because of the particle interference. Ideal case would have been something like this following this equation this side; real case is somewhat different, ideal case coming from this side, real case will be something like this. So,

packing density is something of this kind as x increases. There is a peak; the peak of course remains same at x star which we have evaluated.

So, idea of this binary mixture is that if you mix two particles packing density increases not ideally but in some manner, right, but it increases, and it has got a maximum at some proportion of the coarse fraction in the total or some proportion of you know which is same as fine. Now I can extend this idea of these binary mixtures to ternary, quaternary and so on, so forth.

(Refer Slide Time: 45:48)



So, idea of this can be extended to multinary mixtures as well to show that packing density improves with each subsequent addition of sizes. So, you add two size you add the third size with appropriate size ratio packing density will improve. At a fourth size and packing density will increase further; fifth size packing density will increase further and so on. So, as many size you add packing density increases. Therefore, we do not use single sized particle in concrete aggregate system; we use large number of sizes varying from smallest size of 75 micron because we cannot use silt and clay they have other problems, we will discuss that sometime later on.

So, sand size which are largely inert do not hold onto particles very close to the surface because surface forces are not so dominant in sand; in silt somewhat cement particles might get stick to it, etcetera, etcetera. There are problems which we shall discuss but sand is inert they do not stick to each other neither they will cause any kind of problem.

So, use sand which is 75 micron and above to as many large sizes possible maximum size possible. The use of maximum size is other effect of course, but positive effect is it will improve the packing density; other negative effect we will consider later on. So, optimal fraction also we can identify, you can identify also the optimal fraction and therefore, to ensure dispersion and to take care of practical interference amount of mortar used in the aggregate system is little more than the voids corresponding to maximum packing density in the coarse aggregate.

For normal concrete practical grading curves are proposed in the code to ensure appropriate range of size fraction in overall aggregate system. So, what we do is the grading system is nothing. It tells us what proportion of what size one should use; with the current concepts of packing density perhaps one can determine this algebraically knowing certain properties of the aggregates but earlier when such concepts of particle packing's were not so well developed, people suggested grading which actually states how much proportion of which particle should be used in your mix. Actually grading ranges we are suggested from experience so that minimum proportion of a particular size and the maximum proportion of a particular size in a mix have been specified and this are given in code.

Of course, we have taken care of the aspect of particle packing etcetera, etcetera, simply from experience or empirical observation. For best result amount of mortar should be slightly more because particle interference will be there; therefore, amount of mortar in the aggregate should be slightly more than the void content in the coarse aggregate, and it should be mortar should be slightly more in the system, and in the mortar sand if you look at the packing characteristics of the sand paste should be slightly more than that in the sand itself, okay. This concept we will still use when we look into fresh concrete.

(Refer Slide Time: 49:34)

Maximum size of aggregate

- Lowest size of fine aggregate is $75\ \mu\text{m}$. Thus increasing nominal maximum size of size aggregate implies increasing number of size, say from 20mm to 40 mm means addition of one more size.
- Overall voids in the aggregate would thus reduce by increasing m.s.a.
- The paste required would be less.

DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

So, lowest size which says 75 micron because that is the sand size thus increasing nominal maximum size of aggregate implies increasing number of size by one. So, if you are using maximum size as 20 mm, and now you decide to use 40 mm; it means that you have added one more size which means my packing density will increase and I can use less paste, right. So, addition of one more size; it is very important and therefore, overall voids in the aggregate would thus reduce by increasing m. s. a. So, this we have understood now, and the paste required will be less. So, increase m. s. a. or maximum size of aggregate your paste required will be less.

(Refer Slide Time: 50:26)

Grading of Aggregates

- The grading requirement in the table is for all in aggregate, separate grading requirements are also given in code .
- For fine aggregates 4 grading zones are given (finest is zone IV and zone I is coarsest).
- Fineness modulus (FM) is another common way of defining size of fine aggregate .

DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

The grading requirement in the table is for all aggregates separate grading requirements are also given in code. So, you see this; this is why the code gives you table tabular form suggestion of grading. The grading requirements are suggested that gives you the range of the sizes that should be present in an overall aggregate system. For fine aggregates of course, we define grading zones depending upon in of course in Indian code as well as in BS code the grading zones are defined. For finest zone is the zone IV and zone I is the coarsest because fine aggregate themselves quite often they may be natural sand river sand.

So, they have their grading; there are different proportions of sizes and according to the sizes generally prevails four grades, grading zones have been actually specified or defined and zone fine is the most fine, zone I is the coarsest. Then we define something called fineness modulus. It is another common way of defining size of fine aggregate or even total aggregate. You know we talk in terms of an average size we will just look into this.

(Refer Slide Time: 51:42)

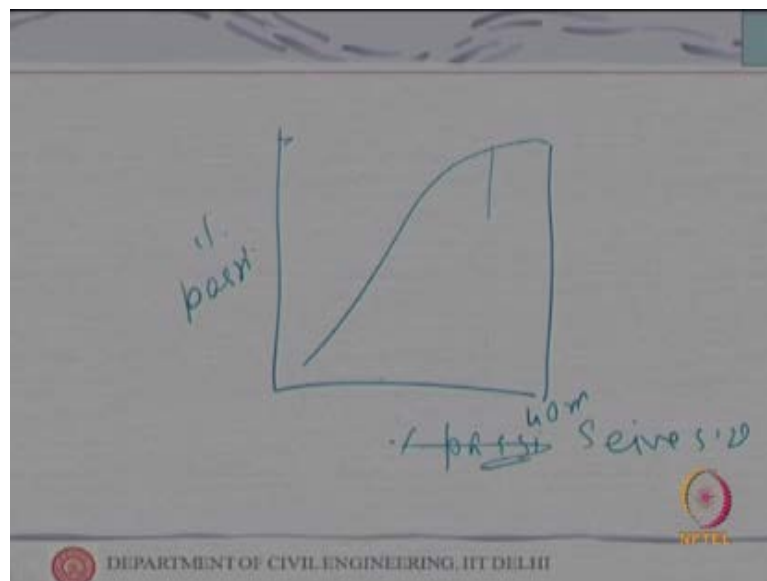
Grading of Aggregates		
Sieve Designation (mm)	Percentage Passing	
	40 mm m.s.a	20 mm m.s.a
80	100	
40	95-100	100
20	45-75	95-100
4.75	25-45	30-50
0.6	8-30	10-35
0.15	0-6	0-6

This is typical grading requirements you know. So, if you have sieves designations are given if it is a 20 mm aggregate m. s. a 40 mm then 100 percent must be passing through this, 95 to 100 percent you know grading requirement given in code. So, this is the grading requirement given in code a tabular form. So, it says that if it is a 40 mm m. s. a.

best result you will get when forty mm 100 percent should pass through 80 mm; in 40 mm 90 to 100, and 20 mm 100 percent should pass through it, etcetera, etcetera.

So, for 40 mm m. s. a. this is the grading requirement; for 20 mm m. s. a. this is the grading requirement. So, such grading requirements in terms of the range of sizes are given in code and that should be used in order to get the best possible packing. This were actually determined empirically, right, from experience people I have actually suggested, but as I said modern concepts of particle packing are also available. Let us look at typical grading curve. Well I think I must be having a typical grading curve just let me have a look at typical grading curve if I have one, may not have right now. So, we will see it in the next class may be; we have a typical grading curve, typical grading curve it looks something like this; it will look something like this.

(Refer Slide Time: 53:27)

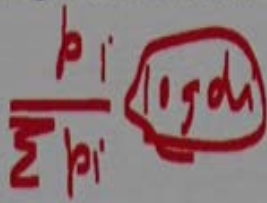


Sieve size, size on this side percentage passing. So, if you have 100 percent 20 mm 40 mm m. s. a. let us say 100 percent passing something like this, etcetera, etcetera. So, these are typical you know grading curves are plotted like this. I must be having somewhere; I will just come to that sometime later on, right. So, now what we have discussed so far is you have looked into the effect of sizes proportions of different sizes, particle packing, etcetera, etcetera. We will just look into fineness modulus one more concept; we will start this and continue with this in the next class together with the surface modulus which I mentioned.

(Refer Slide Time: 54:24)

Fineness modulus

- Since size varies logarithmically, weighted average can be obtained as $\sum p_i \log d_i / \sum p_i$
- Instead one can obtain average sieve number as:

$$\frac{\sum p_i}{\sum p_i \log d_i}$$


DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

So, fineness modulus you see since size varies logarithmically weighted average can be obtained as $p_i \log d$ size, I cannot take average size. So, I take weighted average size, this is the proportion of total aggregate. So, I talk in terms of p_i divided by $\sum p_i \log d_i$. This is the weightage function. So, $\log$ of diameter. So, this is what one can use, right. So, average if I want to calculate out I should use here weighted average; one could have possibly averaged it in form of you know in geometric mean in fact that is what we are trying to do. So, weighted average or geometric mean you can look into and that is what we are trying to do and instead you know. So, averages we get the averages sieve number.

(Refer Slide Time: 55:32)

Fineness modulus

- Since size varies logarithmically, weighted average can be obtained as $\sum p_i \log d_i / \sum p_i$
- Instead one can obtain average sieve number as:
 $0p_0 + 1p_1 + 2p_2 + 3p_3 + \dots$; p_1, p_2, p_3 are mass fraction of particles of 150, 300, 600 micron sizes etc. This is FM = $(\sum \text{Cumulative \% retained})/100$.
- Thus FM represent weighted average sieve number.

DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

So, this fineness modulus concept I will define; I mean I will explain in the next class, because this would take a little bit of time, and we define fineness modulus as the weighted averages sieve number it is fineness module as gives us weighted average sieve number. It is the fineness modulus gives us weighted average sieve number, right. What we do is we give the sieve number this size as zero, then one, sieve number two, three, four, etcetera, etcetera, and this numbers are you know this numbers So, if I want to find out an average weighted average sieve number then the waiting functions are nothing but p_1, p_2, p_3 , where p_1, p_2, p_3 are nothing but the proportions of that sized aggregate in the total aggregate, right. So, this concept we will discuss in the next class in the beginning.

(Refer Slide Time: 56:40)

Surface modulus

- Specific surface can be related to surface modulus
- Specific surface for sphere is $6/d_p$
- For 2 sieve sizes d_i & d_{i+1} specific surface is $6/\text{geomtric mean size} = 6/(d_i \times d_{i+1})^{1/2}$.
- Since $d_{i+1} = 2d_i = 4d_{i-1}$; as the size increases specific surface will reduce by a factor of 2.
- Considering the contribution of size fractions

Surface Modulus SM is

$$SM = \left(p_0 + \frac{p_1}{2} + \frac{p_2}{4} + \dots + \frac{p_n}{2n} \right)$$

DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

And we will discuss one more concept called surface modulus and surface modulus is in terms of specific surface. So, specific surface can be related to what is called surface modulus, and this concept is for a sphere we know the specific surface is $6/d_p$. Here we are defining the specific surface as the surface area divided by volume of surface area per unit volume. So, specific surface for each particle can be defined, and then we can define the surface modulus again in terms of a kind of weighted function, and we will look into this in the next class.

(Refer Slide Time: 57:26)

Summary

Packing

FM

SM

DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

So, therefore, we can summarize today's discussion in terms of what we have discussed is essentially packing. Fineness modulus and surface modulus we have just started, and in next class we will look into this, and that is what it is. Now while looking at packing we have seen the principles of the packing density how it changes in a binary mixtures and extended this idea; one can extend this idea to ternary mixtures. We have also discussed concepts like loosening effect and wall effect which leads to particle interference.

Thank you very much.