Concrete Technology
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Lecture - 39
Special Concrete: Fiber Concrete

Welcome to module nine, lecture four. And, we will continue with fiber concrete, which we just introduced in the last lecture.

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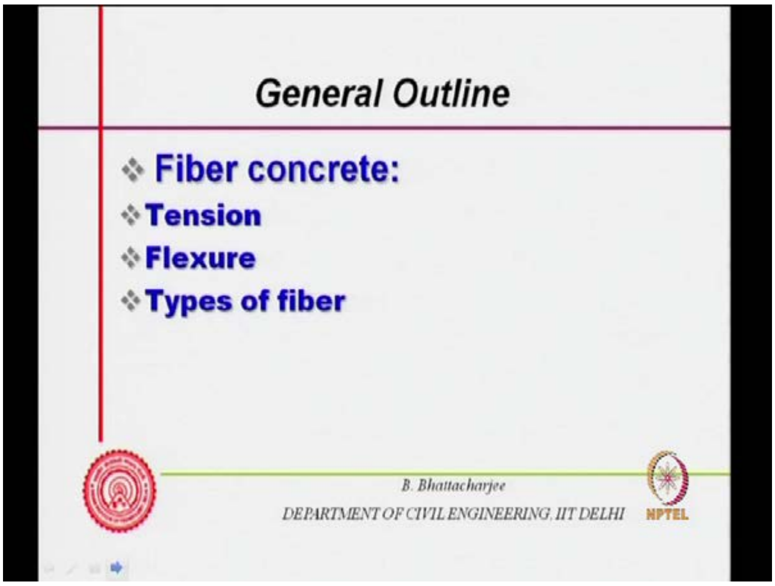

Now, we will look into issues like how it behaves in tension, because that is understanding this behavior will allow us to understand the crack characterization property, also pseudo ductility and various properties of concrete. Then it is advantageous to use fiber concrete in flexure. So, we look into flexure. Then we will look into types of fiber. But before I just go to the fundamental behavior of fiber concrete under tension, I like to mention two points that this concrete is not a recent development; in the sense, the use of fiber is age-old. Second thing – one must understand that, fiber enforcement is not a replacement for conventional reinforcement – steel reinforcement or rebirth, which we use.

In case of such reinforced concrete, which is also a composite, the concrete as a material and steel reinforcement as another material – the act separately; they are not in the… steel is not… reinforcement is not so well-distributed into the matrix of the concrete to enhance the properties of the matrix itself. So, it does not enhance the properties of the matrix itself, rather acts independently and enhance the properties of the section or construction.

While this type of fiber reinforcement – even though I might add steel fiber, it actually enhances the properties of the matrix itself. So, obviously, we are talking in terms of… Supposing if it improves the tensile load carrying capacity little bit, which it does not really; most of it does not; but, flexural load carrying capacity – it increases. But by using fiber, I cannot replace the conventional (( )) reinforcement. So, that should be clear right in the beginning, because these fibers essentially improve the flexural load carrying capacity of the matrix itself. In other words, what it talked about earlier modulus of rupture – it would improve that property. So, that must be understood.

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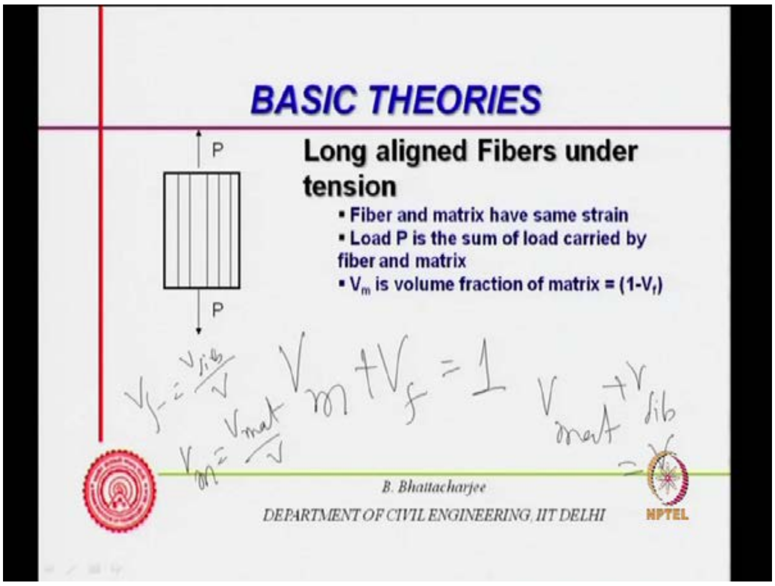

So, let us go back to what we looked in the last class right in the beginning. We said that, let us take an element – small element of fiber concrete. So, I have got fiber, where I apply the load along the direction of the fiber. And the fibers are all continuous. P is the load I am applying. Long aligned fibers under tension that we can look into. So, delta l let us say is the length – small element. Fiber and matrix will have the same strain, because we are assuming that, there is no pull; there is a right kind of bond between the fiber and matrix. Therefore, fiber and matrix have the same strain. Therefore, load P is the sum of load carried by fiber and matrix; part of the load will be carried by the fiber; part will be carried by the matrix.

Let us say V m is the volume fraction of the matrix and it is equal to 1 minus V m, because volume fraction – if I look at it; total volume, let us say volume fraction I am calling off; total volume is V, a part of it is a fiber and a part of it is matrix; sum total again makes it V. Therefore, if I talk of fractional volume, sum total of the fractional volume is equal to 1. In other words, I can say V m plus V f equals to 1. Or, if I was calling it volume of matrix plus volume of fiber is equal to total volume; and, V m is equal to nothing but V mat by V; and, V f is equal to nothing but V fiber by V. So, they are fractions. And, that is why 1 minus V f.

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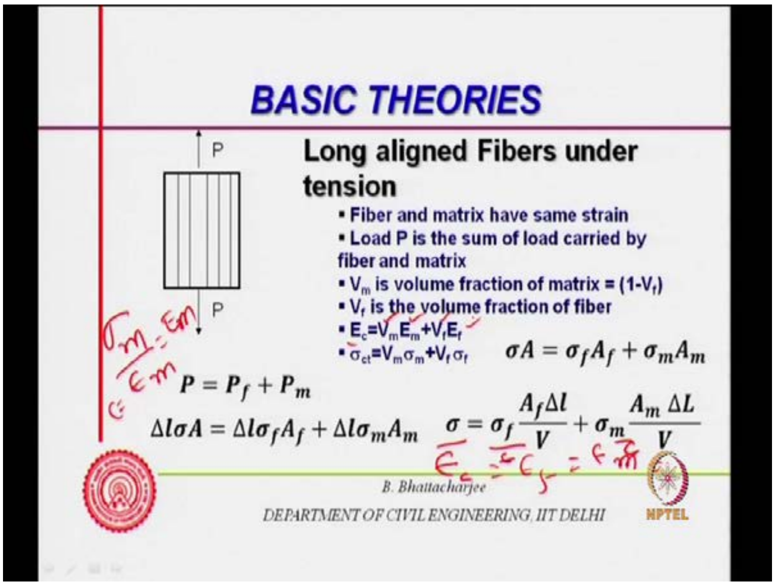

So, V f is the volume fraction of the fiber. Now, I will just… Since, we will look into this. Last class also, we just mentioned. Total force is P equals to P f plus P m – the load carried by the fiber and the load carried by the matrix. If the force P is the load applied, matrix carries P m. Therefore, this is nothing but sigma – stress in the matrix; I mean composite stress into area of composite, which is the sum total. So, P can be written as sigma A. And, P f is equal to sigma f – the stress in the fiber; and, A f is the area fraction of the fiber; and, this portion is the matrix stress – stress in the matrix; and, this is the area of the matrix. So, I can write it like this – if the length of this element is delta l; let us say this length is delta l; just I am taking it as delta l. So, delta l sigma A – multiply everything by delta l; length is delta l. So, fiber length is delta l; matrix length is also delta l. So, I have delta l everywhere multiplied. Therefore, I get A f delta l sigma A f delta l by V. So, that is the fiber volume divided by overall volume V. And thus, matrix volume by overall volume V. Therefore, this follows that, sigma composite; stress in the composite can be written as the volume fraction of the matrix into stress in the matrix plus volume fraction of fiber into stress in the fiber itself.

So, if I simply now go further a little bit more; if I simply now go a little bit more, this stress divided by strain is same; strain is same everywhere. Epsilon is equal to epsilon fiber is equal to epsilon matrix and epsilon composite. So, everything is same – all intention, because I have applied tension. So, epsilon is same. Now, divide this by epsilon; this also by epsilon; and, this also by epsilon. So, stress – sigma m by epsilon m, which is equals to epsilon is the same, is nothing but E m. Therefore, it follows that, E composite is equal to V m E m plus V f E f under the condition when load is aligned along the direction of the fiber; load and fibers are all aligned. And this is how it is. That is what we just looked into last class.

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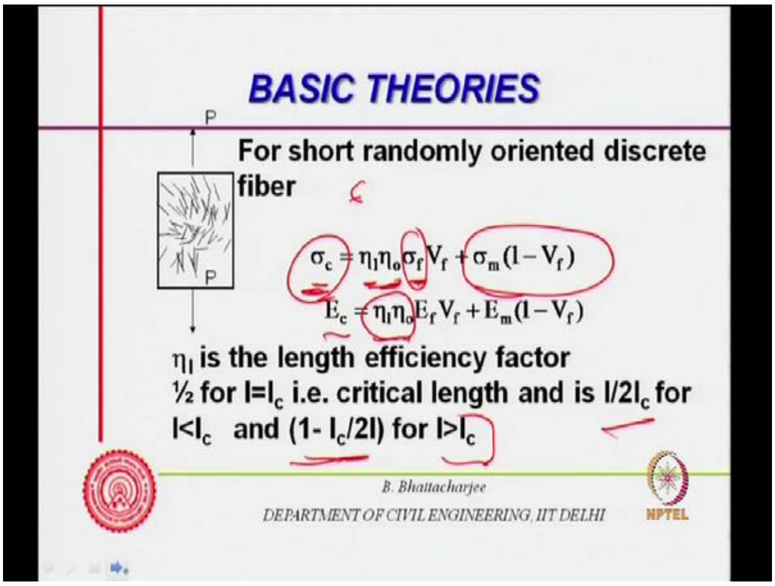

Then, this is the fibers; they are not aligned; they are all random. And, if I apply now load in the same manner as I did earlier, then for short randomly oriented discrete fiber – that is what the fibers are in concrete; they are like this – short randomly oriented and they are discrete. I can add two factors. What will happen, because the length is no longer the full length, I must multiply the load carried by the fiber or the stress in the fiber by n – length efficient factor, which will be a fraction. And, since they are oriented, if this is like this, a part of the fiber will be… Load will come like this. So, part of the fiber will be effective in carrying the load. Therefore, I put what is called orientation efficiency factor. Therefore, efficiency factor and orientation efficiency factor. Therefore, stress now will be orientation of efficiency factor; and, length efficiency factor, orientation efficiency factor multiplied by sigma f V f.

This sigma f has to be multiplied. Sigma m – there is no problem; it is same. But, this part – I have to multiply it by… All the fibers will not effective. So, multiply it by the orientation efficiency factor and length efficiency factor to get the composite stress. Similarly, the modulus of elasticity now can be related to this. Modulus of elasticity will be also related to this, because this epsilon is still the strain. Divide everything by the same strain and you will get by this same efficiency factor. We are assuming strain in the matrix and the fiber is still same; there is no slippage; they have perfect bond between the two. So, n l is called nu l, is called length efficiency factor; it is half for l equals to l c, which is called critical length. And, is equal to l divided by 2 l c for l less than l c and 1 minus l c divided by 2l for l greater than l c. So, we must look at what is l c and how this new values are coming. That is what we look into the next slide.

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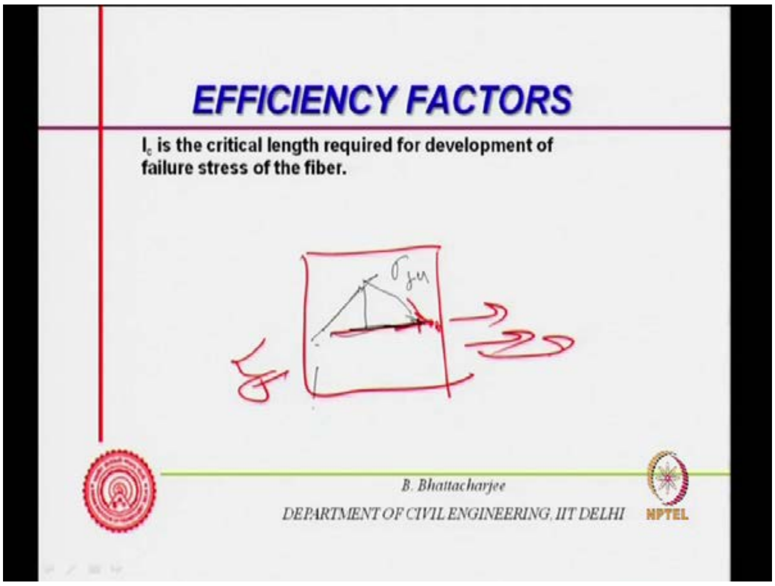

Now, first of all, l c is the critical length and this is required for development of failure stress of the fiber. This is required for development of failure stress of the fiber, because I have the fibers inside the matrix. So, when I apply a pull at the boundary at the end of the fiber at end of the… because they are discrete; they are not continuous; they are discrete. So, if I have, this is the matrix, the fiber starts from here somewhere there inside. So, the stress here – it will be all matrices will be taking the stress.

And at this point, if I have a pull like this; at this point, some of stress will be transferred to the fiber and we assume a linear stress variation through the bond; it will be transferred through the bond. So, when I am trying to pull this, when I am trying to pull this, when I am trying to pull this, when I am trying to pull this, this will also try to pull this material. And, at this point, here if the matrix is alone taking the pull as I go, where there are fibers – this is the fiber. So, fiber will also experience that pull, because there is a bond. And, through the bond, some stress will be transferred to the fiber and fiber stress will go on increasing. But it will go to a maximum value possible, is sigma f u, which is the ultimate fiber stress or ultimate strength of the fiber.

And after that, on the other side also it will experience similar sort of the situation. So, it is a kind of development – the length required for development of the full failure stress – ultimate stress in the fiber. So, that is what I am calling as l c or it is…

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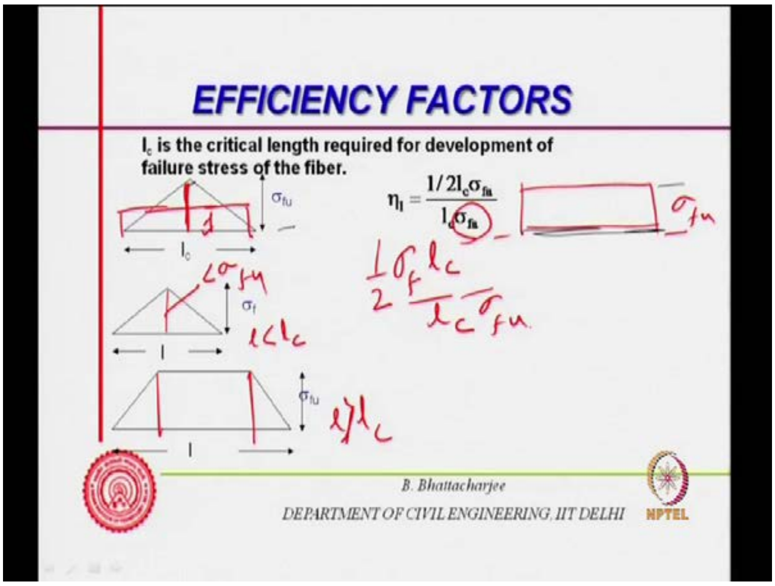

Just let us see; we are calling it as l c. This diagram will show you; this is the l c. So, this is sigma f u. So, this stress will be sigma f u; this stress will be sigma f u; this stress is sigma f u. So, this stress is sigma f u and this… A situation could be there – the fiber length is less than this, less than l c. So, l is less than l c. In that case, full stress will not develop; it will develop some less amount of stress. This will not be sigma f u. It is less than sigma f u. And, in this case, sigma f u will develop; and, beyond that, l is greater than l c. Sigma f u will develop; but, after that, fiber cannot carry anymore load or anymore stress. It would have failed. Therefore, if it is longer than that length, fiber would still carry sigma f u and rest of the stress has to be carried by the matrix or whatever it is. So, this is… If l is greater than this, this is the scenario – l is greater than this – the stress is on this axis; this axis is stress. So, we can look at this algebra now. So, efficiency fiber, efficiency factor is…

Supposing it was continuous fiber, then the load that it would have been able to carry – so, it is the ratio of the actual stress that it is carrying, average stress the fiber is having to the one if it was a continuous fiber. So, if it was a continuous fiber, the stress in the whole of the fiber would have been sigma f u. For continuous fiber, it would have been simply sigma f u. Continuous fiber – for continuous fiber… This is continuous fiber. This stress would have been simply sigma f u. So, this would be sigma… Throughout it would have been sigma f u. But here this is not a continuous fiber. Therefore, the stress is less. Now, how much is the average stress? It is sigma f u at this point; 0 there. And, therefore, average will be somewhere here, which will be this triangle divided by the base. So, what is this triangle? Half sigma f u half into l c; half sigma f u divided by l c divided by l c. That is the average. So, half sigma f u into l c – this divided by l c is the average stress; that is, this value. And this divided by sigma f u, which would have been there throughout. It will give me the length efficiency factor. So, that is what I have done.

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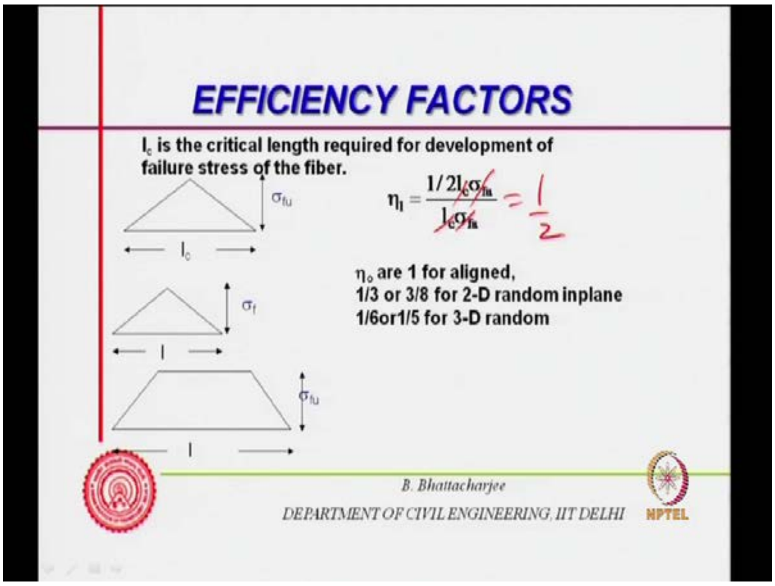

So, this expression shows us this one. Therefore, it gives you, simply this will cancel out, this will cancel out, this will cancel out; this equals to half. So, length efficiency factor is half for l equals to l c. So that is what we just mentioned earlier in the previous slide. Length is half for l equals to l c – critical length; and if it is l divided by l c for l less than l c, and 1 minus l c divided by 2l for l greater than l c.

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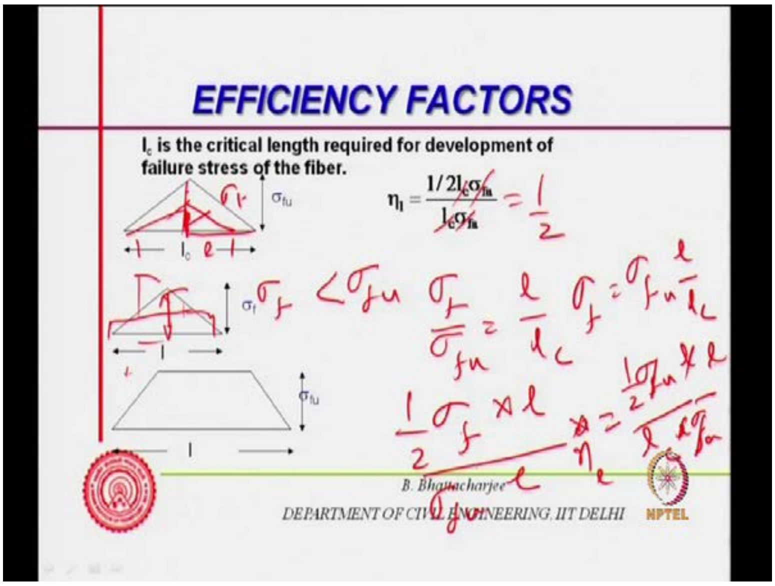

Let us see for the second case. Second case what will happen let us see. In this case, the stress here is sigma f, which is less than sigma f u. How much less? The sigma f by sigma f u will be equal to l divided by l c. If it was half l c, then you would have got it, because if it was l c; if this was half l c; then, the stress in this case – it is smaller than this. So, actually, this is l; let us say this is l. So, the stress developed would have been sigma f here. So, sigma f by sigma f u will be simply l divided by l c. So, sigma f is equal to sigma f u into l divided by l c; sigma f u… This would be… Sigma f would have been input. So, what will be the average stress here? Average stress will be half sigma f into l divided by l, is the average stress multiplied by… Not multiplied, but sigma f u is what? Sigma f u is equal to half sigma f u into l divided by l c into l. And this divided by sigma f u gives me the length efficiency factor.

So, this l will cancel out; sigma f u is cancel out; sigma f u will cancel out. Sigma f u l by l c. So, there has to be some l; half sigma f u l. The average stress will be divided by this; and, this divided by sigma f u simply. But sigma f is equal to sigma f u l divided by l c. So, there is another l anyway here. So, this one l will cancel out. But, there will be sigma f; when you replace, there is l divided by l c. So, you get this as – in this case, you get the efficiency factor as l divided by 2 l c.

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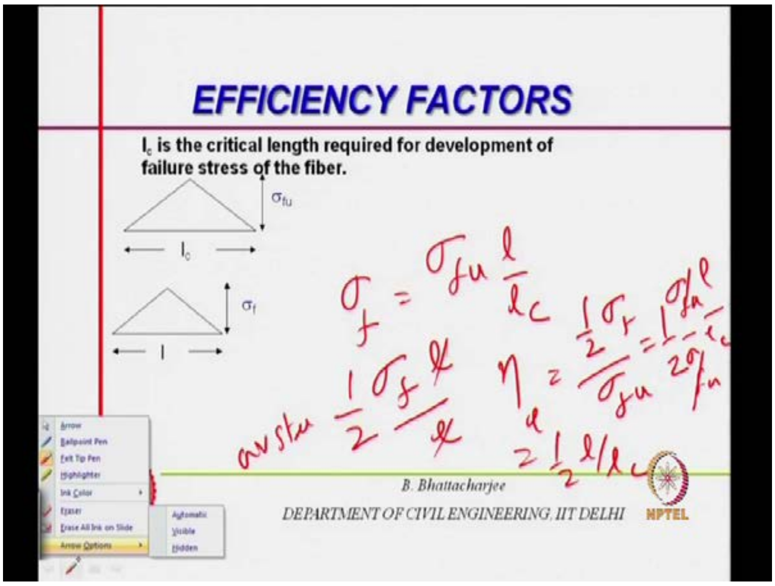

In the second case, you get l divided by 2 l c. Just let us do it here right now, because sigma f is equal to sigma f u l by l c; average stress is half sigma f u into l divided by l; average stress. And nu l will be equal to half sigma f – l by l will cancel out – divided by sigma f u; and, half remains… Sigma f is nothing but sigma f u l by l c; and, this divided by another sigma f u, because this is sigma f u. So, this will cancel out; I will be left with half l by l c. So, if it is smaller, I will be left with this. But if it is longer than l c, let us see what will happen. Just erase this out.

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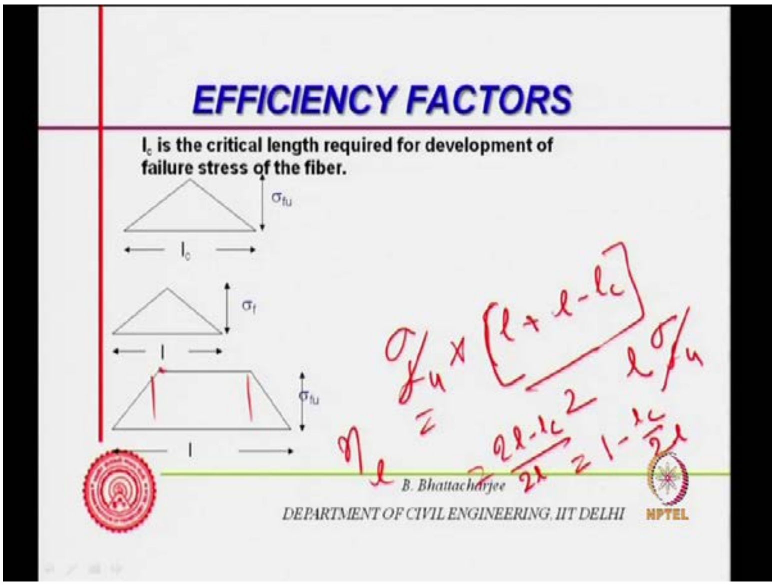

If it is longer than this, now, what is the… This is sigma f u. So, what is the average stress? Is a trapezium. So, sigma f u into l is the total l. So, this plus this. This plus this we will have to… Sigma f u and multiplied by this plus this divided by 2. So, this is l plus l minus l c. This distance is l minus l c. This is l minus l c – this divided by 2. That is the stress. Average stress will be divided by… This is the stress. So, this is the area of this triangle divided by l, will give us the average stress. And, this divided by sigma f u will give us the efficiency factor. So, this will cancel out. So, what I am left with – twice l minus l c divided by twice l, which will give me 1 minus l c divided by 2l. So, that is the length efficiency factor. That is what we have seen earlier in the previous slide. We have seen that 1 minus l c divided by 2l if l is greater than l c. So, if l is greater than l c, we see that it is 1 minus l c divided by 2l; it is 1 minus l c divided by 2l.

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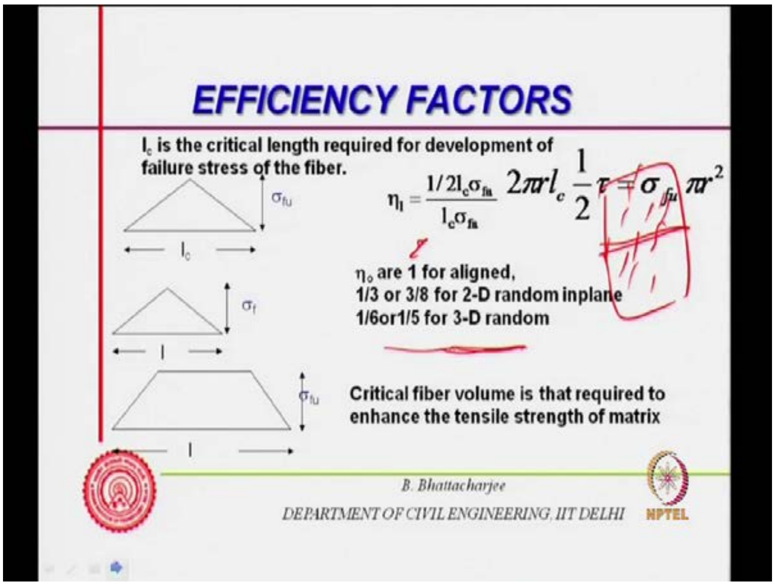

So, here we have efficiency factor – length efficiency factor. But, the orientation efficiency factor have been suggested by people is equal to 1 for aligned and one-third to three-eighth for 2-D random in plane, because it can be spreaded in all three directions. So, 1 by 3. And, 1 by 6 or 1 by 5 for 3-D random. So, 1 by 6 or 1 by 5 for 3-D random, because there are 6 directions. So, orientation factors based on simple statistic plus experimental work. So, this has been arrived at.

So, next we will look into critical fiber volume. And, that is defined as the fiber volume required to enhance the tensile strength of the matrix. For example, if you put few pieces of fiber in a matrix – stray pieces of fiber in a matrix 1 or 2, they will not be able to bridge the crack, because crack simply may pass in between. So, I need a minimum volume of fiber that will ensure that, cracks are bridged crack by the fiber; I need minimum – some minimum quantity of fiber; even a sufficient quantity of fiber. At least some fiber will be bridging the crack.

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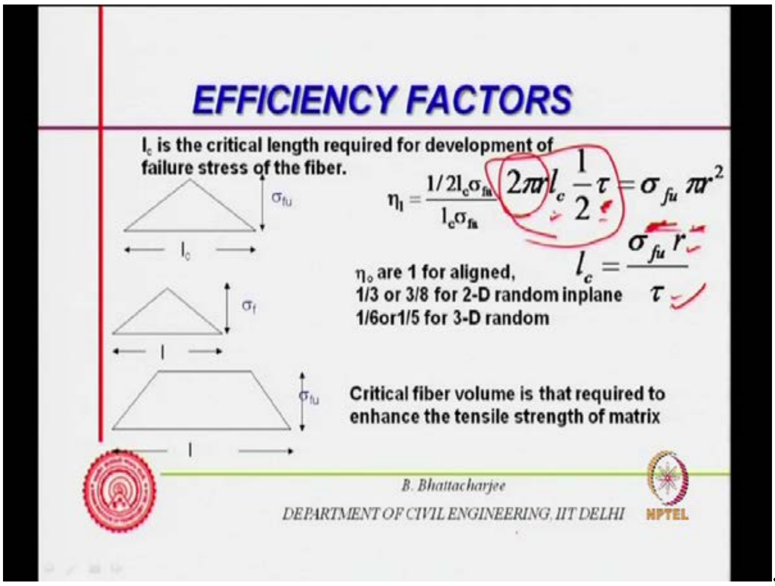

Now, to understand that, we will have to look into how l c is related. l c is the critical length. That is what we said. And, l c is actually related to sigma fu r by tau. How? Let us just see, because the stress from the fiber to the matrix is transferred through bond. So, if the tau is the bond strength, l c by 2 through which sigma f u is developed. So, sigma f u pi r square – that is the stress unto the fiber. I am talking only of one fiber, is good enough for the purpose at the moment. One fiber – single piece of fiber in case of aligned fiber. So, this is the force into the fiber. And, that must be transferred through the length of l c by 2 through bond. So, if the bond strength is tau, twice pi r is the surface perimeter multiplied by l c by 2. That is the area of through which actually the bond stress is transferred, bond is transferred into the bond strength – tau. So, tau multiplied by this one must be equal to sigma f u by pi r square.

So, this would give us l c equal to sigma f u r by tau. So, if you want to find out now the length efficiency factor for aligned fiber, you got to know that bar diameter, you got know the bond strength – bond stress that can develop into sigma f u. There was strength in the strength of the fiber – ultimate strength of the fiber. So, l c can be calculated in this manner.

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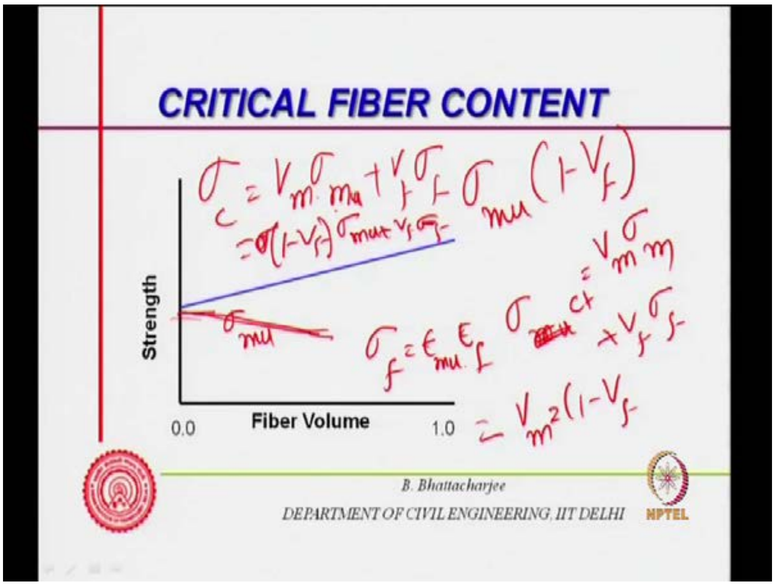

And, we can now look into critical fiber volume, which is required in order to have strength enhancement. So, just consider this diagram. This fiber volume varies from 0 to 1. Hence, I plot between strength and fiber volume. Now, you can easily understand that, when fiber volume is 0, this is nothing but sigma m u. This will be sigma m u. Now, when I have some fiber volume, I will have actually sigma – this line – sigma m u. It is… Sigma m u is a fiber volume, because we said that, sigma composite is equal to V m sigma m plus v f sigma f. So, I can write this as V m 1 minus V f, because I am plotting against fiber volume – fiber volume fraction actually. You can see it varies from 0 to 1. So, as I go on increasing the fiber volume, V m can be written as 1 minus V f. So. what I will write – sigma m u into 1 minus V f at failure, because if the fiber is stronger…

Let me just again (( )) I am talking of situation, where fiber is stronger than the matrix. There can be situation, where fiber is weaker than the matrix. Then, of course, strength will reduce down – sigma m u. And, you will have reduction in strength. But most of the time, we use the fiber, which will have strength higher than the (( )) because you are replacing part of the material. This relationship still remains valid that, sigma composite is equal to V m sigma m plus V f sigma f. So, if this V f is equal to 0, it is simply sigma m u into… And then, V becomes 1 straight away. But supposing the fiber is weak, then what will happen, this will actually be lower value. This will reduce down the strength. So, as I go on increasing the fiber, you are replacing a stronger material by weaker material. So, actually strength will reduce down. But normally, we use fiber, which is stronger. Therefore, strength will get enhanced.

Now, this sigma m can be written as… V m can be written as 1 minus V f into sigma m plus V f sigma f. So, you can see (( )) Now, supposing the matrix fails earlier, this will be sigma m u. Actually sigma f will be equal to – at failure, epsilon m u into e of the fiber, because the fiber will not fail; the strain in the fiber and the matrix is same as long as… So, the load carrying capacity when matrix is just failing will increase in this manner in a form of a straight line.

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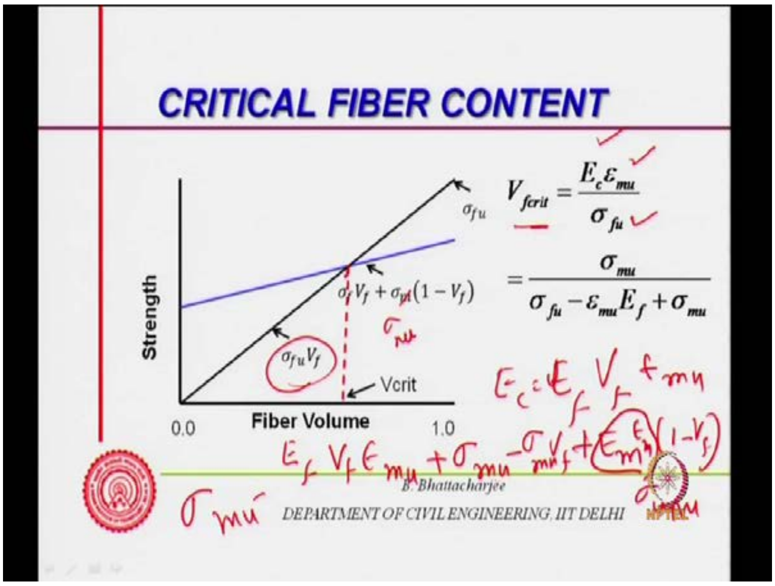

So, that is what it is. Therefore, sigma f V f plus sigma m 1 minus V f. That is the line. And, if it is ultimate, then it will be sigma m u. Now, as I increase the fiber, strength will increase. Now, just look at the simply fiber. This is for the fiber – sigma f u V f; sigma f u is the ultimate load carrying capacity or ultimate strength of the fiber into V f. So, that would increase in this manner when you have hundreds of this corresponds to actually sigma f u. This will correspond to sigma f u as we shall see, because 100 percent. Depending upon… So, this is the contribution of the fiber. This is the contribution of the fiber, which will increase maximum contribution of the fiber. This is the maximum contribution of the fiber and we will increase as I increase the volume fraction. Now, this could… I can write this as sigma m u V f. If this is sigma m u failure; and, I can write this as sigma m u V f. Now, as the fiber content increases, a time comes when fiber alone can take the load even if the matrix fails. So, that would require this much fiber content. So, this we call as weak rate.

What is weak rate? Weak rate is the fiber content. At that fiber content, even after matrix has failed, strength will be enhanced. So, critical fiber content is that fiber content at and beyond which even after matrix fails, the fiber will… This should be sigma m u – actually sigma m u. Even after that when matrix fails, the fiber will be able to carry that load. So, matrix cracking – once matrix cracks, the fiber should be able to carry the load. But it does not matter; we will see that, some cases it does not; most or many cases it (( )) because V critical can be such large that, you may not able to add them. There are practical difficulties of workability. So, you may not be able to add that.

We will see that later on. But even if you do not add, still you get certain properties enhancement. We will see that. But, the critical fiber content will be defined like this. It is the E of the composite – modulus of elasticity of the composite into epsilon m u – ultimate strength in the matrix divided by sigma f u, because at this point, sigma f u V f must be equal to the strength of the composite, which is actually… The strain you see – epsilon f u; and, modulus of elasticity of the composite is E c. So, this E c… So, V crit epsilon m u. Now, epsilon m u… E c can be written as… You can show that as… E c can be… This can be written as E c is nothing but E of the fiber into V f plus E of the matrix into 1 minus V f. You can write it in this manner. And, epsilon m u can be multiplied here – all of them – epsilon m u; and, epsilon m u.

Now, this gives you sigma m u. So, you will get a relationship of this kind – E c in this part. You will get… Just I am writing this; it will be E f V f into epsilon m u plus sigma m u minus sigma m u into V f. So, if I take out V f common from this, I will get a relationship of sigma m u minus… So, all these have to be divided by sigma f u. So, if I separate this; if I write it… I think I will do it in this manner; maybe I have a slide next.

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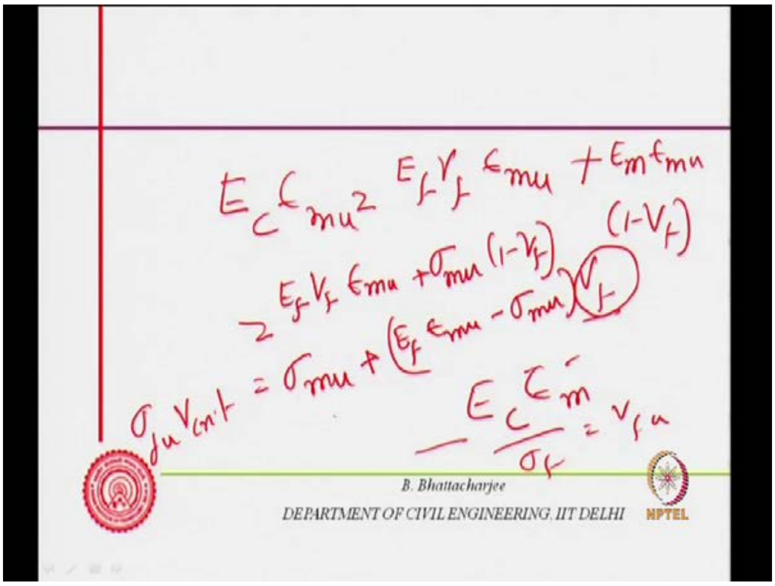

What I am trying to do is I have got E c epsilon m u. And, this I am trying to write as E f V f epsilon m u plus E m epsilon m u into 1 minus V f. So, this I will get as E f V f epsilon m u plus sigma m u into 1 minus V f. So, if I take V f common, I will get sigma m u out plus E f epsilon m u minus sigma m u into V f. So, that is what it will be. And if I divide this by sigma f u; now, sigma f can be again related; actually, sigma f can be related to sigma f u E f. So, V f crit will be this divided by epsilon m u. So, sigma m u, sigma f u, epsilon m u… Let me just see this one; what did I write; This portion – E c into epsilon m u will be given by this. Now, this divided by sigma f u. So, sigma m u divided by this plus E f epsilon m u minus sigma f u. So, I am trying to get an expression for V f. If I am trying to get an expression, this divided by sigma f u should be equal to… Every is… I am trying to get an expression for V f. This must be equal to sigma f u V f crit, because this E c epsilon m u divided by sigma f u was equal to V f crit. Therefore, if I write V f crit, this will be also… So, it will be…

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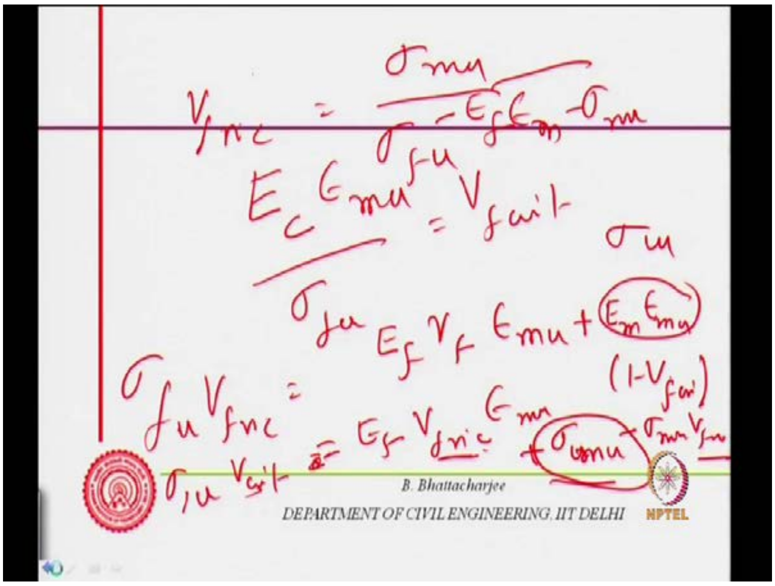

Expression was epsilon c epsilon m u divided by sigma f u is equal to V f crit. Now, this can be expressed in terms of V f crit. And, that is what I did. So, what I write is sigma f u V f crit is equal to… E c was… What was E c? E c was sigma f… E c – I wrote as sigma f multiplied by… What did I do? Actually… Just a minute; let me erase this out. So, E c – I wrote was composite E – was E of the f V f epsilon m u plus E m epsilon m u into 1 minus V f. This will be now V f crit. And, if you write this expression, V f crit you can get on the left-hand side, which will have… This is nothing but sigma m u. So, V f crit will be written as epsilon… This V f crit if I… Just derive it fully. Sigma f u V f crit is equal to E f V f crit epsilon m u plus sigma m u minus sigma m u into V f crit. So, only term, which does not have V f crit is this. Therefore, this I take V f crit common; V f crit common here, common here; and, common from this term. So, I will get sigma f V f crit will be equal to sigma m u divided by sigma f u. This term minus E f epsilon m u. And then, this minus sigma m u. So, sigma f u.

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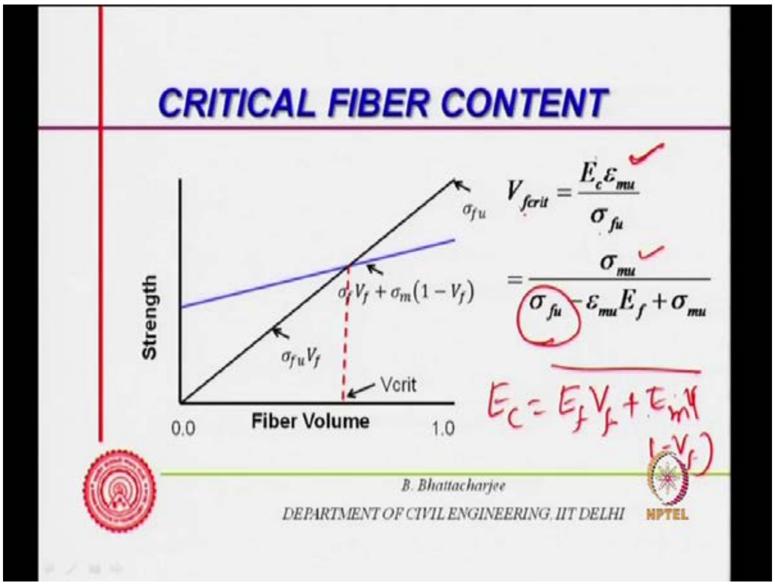

That is what was the expression was – sigma m u divided by sigma f u because sigma f u,V f crit comes on this side; and, V f crit comes on this side also. So, if you take all the terms on the left-hand side, then you will get only one term, which is actually sigma m u; that is, E m into epsilon m u. This will remain here. And, V f crit – we can get an expression of this kind. So, you get an expression of V f crit in this manner, because you express E c is equal to E of the fiber into V f plus E of the matrix into 1 minus V f. So, this you just change this and put here V f crit. The V f crit term will go to the left-hand side here if sigma f u V f crit. And, only term that will be remaining is E m multiplied by epsilon m u, which is sigma m u and this expression will result.

So, this is the amount of fiber you require to have strength enhancement. Now, this is dependent on sigma m u, sigma f u and of course the modulus of elasticity of the fiber. So, more the modulus of elasticity of the fiber, you will require… This term becomes smaller. So, you will require more and so on and so forth. So, one can see this is the function of all those.

So, V f crit one can estimate in this manner. But it is quite difficult in some fibers to add actually V f critical fiber, because of workability. And therefore, strength enhancement you may not get. But, if you get it, well and good; it will depend upon l by d ratio and many other factors, which we will look into.

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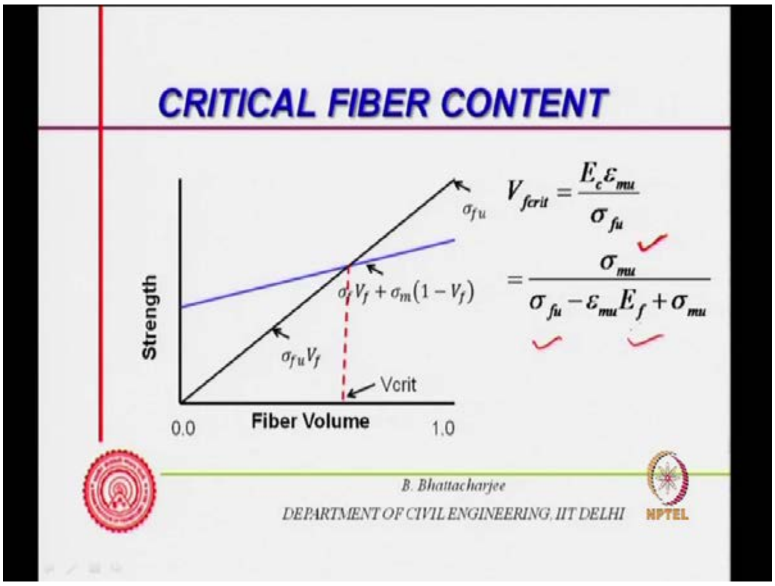

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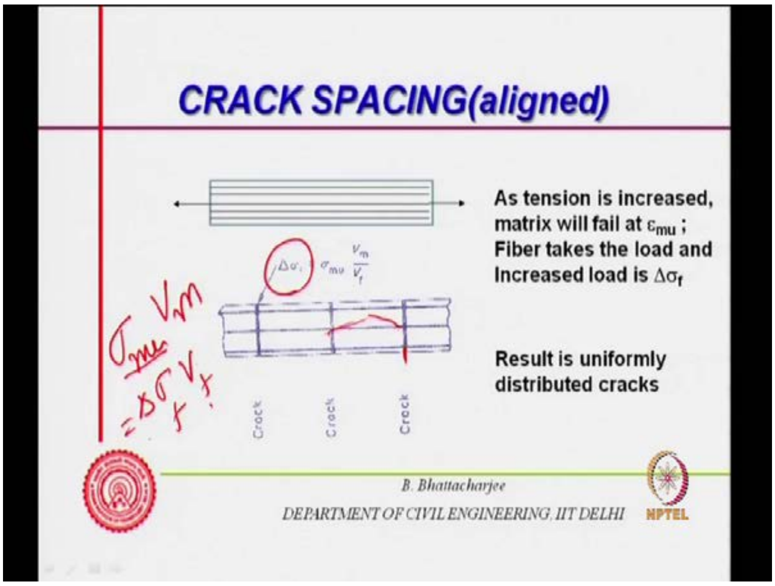

But let us see what happens when you have a strong fiber and weak matrix. So, this is the matrix and these are the fibers. And, I am applying pull along this direction. And, let us say again, it is all aligned. So, as I am applying pull, matrix is failing fast, because I have said, the matrix is weak. So, weak matrix – it will fail first. And therefore, cracks will come at certain regular spacing. In fact, at this point in the crack, there is some slippage also. Fiber will take all the load, because it is still strong; it can take all the load. Fiber will take all the load; and therefore, it will take additional stresses. We will come to this additional stress later on.

We can understand this – additional stress will be the stress that was carried by matrix. So, matrix was carrying actually sigma m u into V m. Now, this must be the additional stress in the fiber. And, that must be equal to delta sigma f into V f, because volume fraction of the fiber. So, this is the volume fraction of the fiber. Then, fiber quantity is still remaining same. That must be equal to… So, additional stress can be simply found out as sigma m u V m divided by V f. That will be the additional stress. So, you will have uniformly distributed crack and not one single crack; and, uniformly spaced cracks; and, that is actually good. Uniformly space cracks – they would be separated almost by uniform distance as long as it is aligned as you are seeing. While if you have even short random fiber, then also, you will have cracks spacing at uniformly distributed… Large number of cracks rather than one brittle local failure with brittle failure. So, you will have large number of cracks. They will not allow failure to occur in one go. Then, sudden failure will be avoided. That is what it makes it ductile. That is why it makes it ductile; pseudo ductile we call it.

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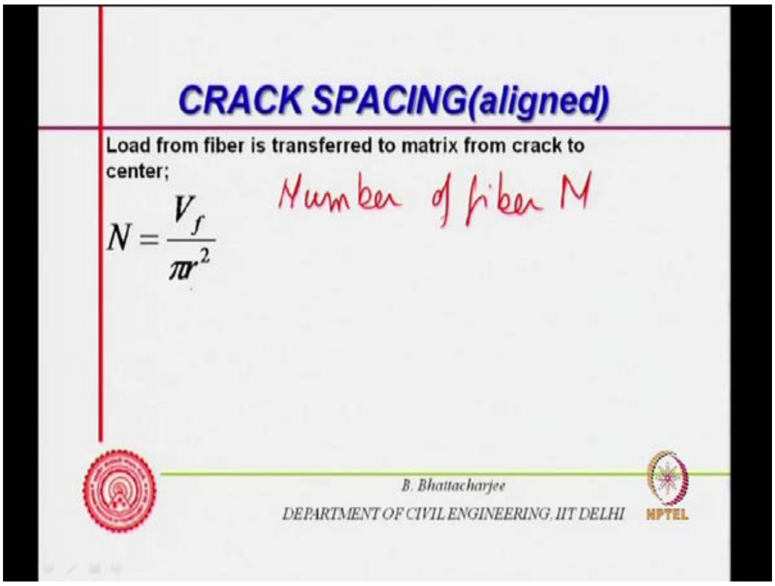

So, load from fiber is transferred. Now, if I look at the crack scenario; at this point, it is the fiber, which is taking the load. From this point, load is transferred from the fiber to the matrix; and of course, again in this point, it is 0. Here load is transferred from matrix to the fiber. Earlier, we said that, just at the end of it, it is transferring from matrix to fiber. Here from matrix to the fiber, because fiber is carrying everything; fiber is carrying everything; fiber is carrying all the load. So, load from fiber is transferred to the matrix, that is, crack at the center; from crack to the matrix from crack to the center.

Now, on an average, number of fiber… I can think of number of fiber N. I can just write it like this – V f divided by phi r square if it is aligned; we are taking of aligned scenario. So, V f divided by pi r square – volume fraction divided by pi r square – that will be the number of fiber. Actually, pi r square into l. So, l is same; length is same. So, it will be simply… This should be written as l into area and this also would be… So, it will be same – pi r square into… So, number of fiber will be written. Average number of fiber will be like this.

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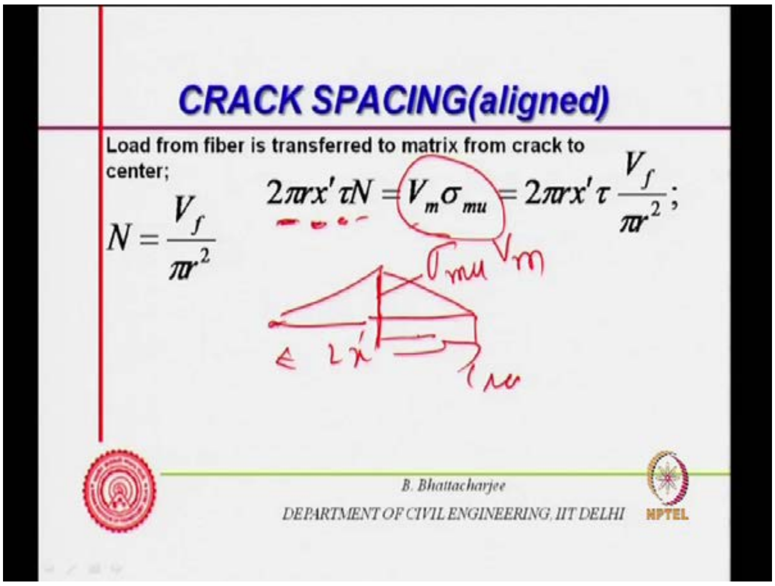

Therefore, single fiber – how much is the crack spacing? We want to find out, because the load is transferred from crack. This is the crack to the fiber. And, maximum stress that can attain in the matrix is sigma m u into V m; maximum stress that would be attained in the center will be sigma m u V m. And, again, it is 0. So, this is transferred through x – 2x just if I call this is a 2x dash. If this distance is 2x dash or x dash, it will be… If I call this as x dash, it will be transferred through x dash. If whole distance is 2x dash, it will be transferred through x dash. So, x dash into twice pi r; where, r is the radius of the… So, this is the parameter into x dash into tau. There is a bond. Bond stress multiplied by number of fiber – that is there per unit volume – total volume. So, volume fraction of this is fiber; fiber volume fraction into sigma m u – that would give me an expression for x dash. That is the crack spacing. So, it will give me an expression for crack spacing.

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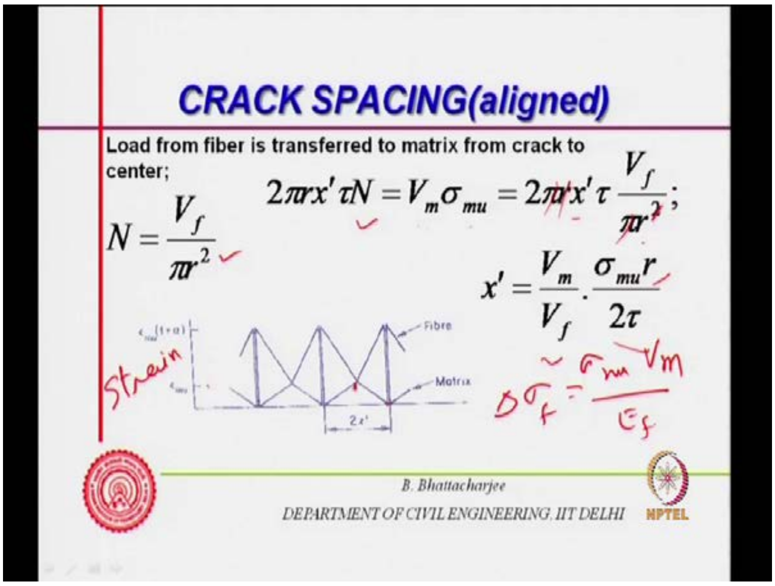

So, x dash therefore… One can write V m sigma m u twice pi r x dash tau. N can be replaced simply – V f by pi r square. And, this will result in a simple expression for x dash, because r cancels out with this, pi cancels out with this. So, I am left with twice tau V f. And, I am interested in x dash; x dash is remaining. So, V m sigma m u. And, r here will come here, r comes here, V f comes here, twice tau comes here. So, that is the crack spacing – half cracks spacing. Crack spacing will be twice x dash.

So, crack spacing will be twice x dash. The strain will actually vary in this manner. How? Here the strain – matrix strain is 0. This is strain. Matrix strain is 0. Then, it reaches to a… At the center, sigma m u or epsilon m u. Here it reaches epsilon m u and then again it comes back to 0. So, actually now, the fibers will be additionally strained or stressed. Additionally stressed – we have seen the additional stress, which was… Actually we said was delta sigma; f was equal to sigma m u V m divided by E f. So, that is what we said was the additional stress that will be taking into account – additional stress. So, this is the additional stress that would be taken by the fiber. Therefore, correspondingly, there will be additional stress strain in the system.

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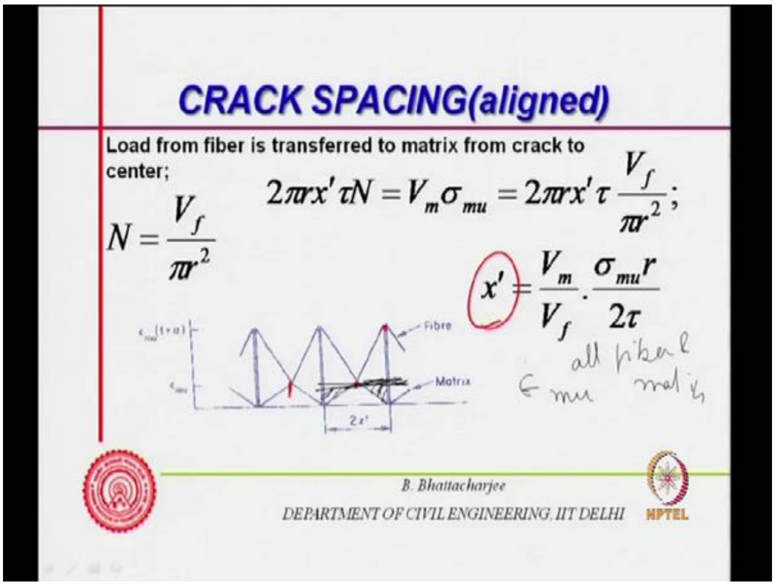

So, fiber takes more strain. And, strain here the perfect… Through the bond, stress has been transferred to the matrix and they are perfectly bonded here. And, therefore, here it will be epsilon m u in both of them, because they are… Strain is same in the fiber and the matrix. So, that will be same. And again, this will be reduced down; strain in the fiber will increase. So, we would like to find out what is the additional strain in the system if you are interested in finding out this x dash, because x dash… x dash is there, but it will find out what is the w, that is, the crack width – increase in crack. How much is the crack – increase in the crack width? If you are interested in finding out that, then we will (( )) So, this is what is happening.

Let us understand this. From here the strain is increased. So, the epsilon m u is here. Originally, matrix was strained up to this throughout. Now, this strain – this amount of strain or these strains have all gone to the fiber. Matrix was equally strained epsilon (( )) just before failure. Now, this strain has gone to the fiber. So, fiber is additionally stressed. And, eventually, fiber was also, before failure, everything was at epsilon m u. So, all fiber as well as matrix and matrix – both were at epsilon m u. Now, this change has occurred.

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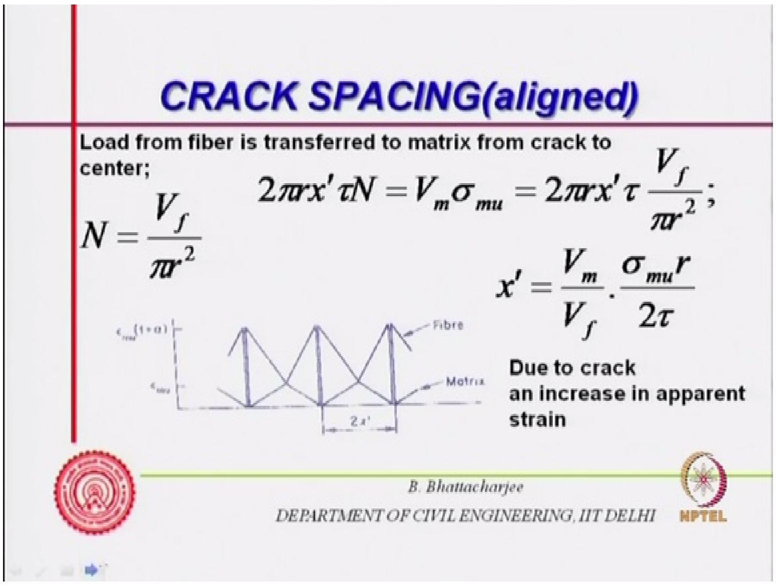

Now, let us find out. So, due to crack, an increase in apparent strain. Overall strain will also increase. And, there is total elongation would have occurred in the composite and that would include the crack width; that will increase the crack width also. So, apparent increase in the strain would be observed.

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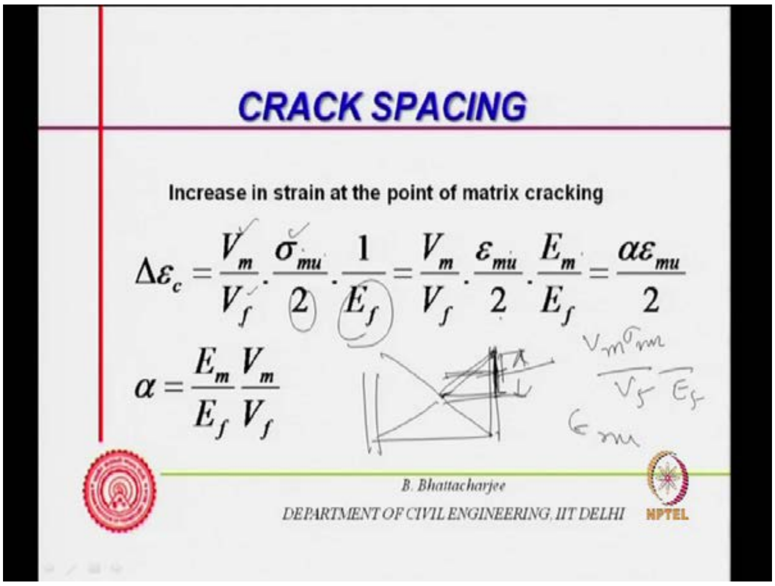

Now, let us see how it is… Increase in strain at the point of matrix cracking let us see. Sigma m u V m divided by V f is the stress; sigma m u V m divided by V m is the additional stress the fiber is taking. This is this term we have already seen. That is actually… This divided by E f is the additional strain at that point. So, additional strain at this point. If you look at, this is the crack and there is another crack and we said… So, this additional strain it is taking – delta epsilon c – this will be – is maximum here and 0 there. Earlier, everybody was taken epsilon m u.

Now, this height would be… Average strain increase would be half of this. So, this height will be bearing this. V m sigma m u divided by V f divided by E f. So, this height will be simply V m sigma m u divided by V f divided by E f, because this was an additional stress. So, this will be the additional strain here. Now, half of this is the average increase in strain in the fiber – average strain increase in the fiber. So, delta epsilon c is the average strain increase in the fiber. Earlier, it was epsilon m u – average strain increase in the fiber. So, this is what it is. If you write this sigma m u, write it as epsilon m u into E m. Then, you can write it as alpha, where alpha is defined as this.

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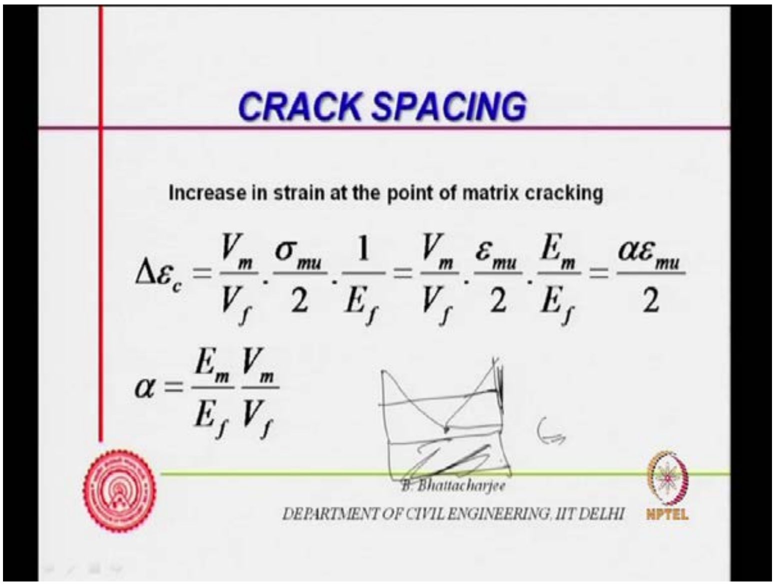

So, this is what we are taking of additional strain in the composite will be, because everything was at epsilon m u earlier. Now, this is the additional increase. So, everything was at epsilon m u was earlier. This strain we have found out and half of this, because it goes from the 0 to this height. So, half of this average would be simply given by this and that is how we are finding out. So, this gives you alpha; where, alpha is defined like E m by E f V m by V f. So, that is the increase.

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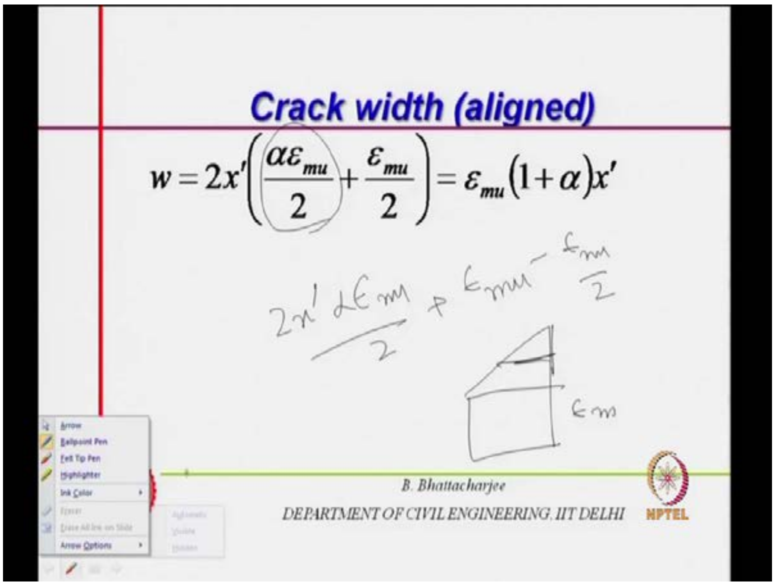

Therefore, originally, the strain was… Originally, this is the average increase in the strain, average increase in the crack, which I am trying to find out. So, this is the strain increase into 2x dash alpha epsilon m u by 2. That is the average increase in the strain; and, plus epsilon m u was the equal strain everywhere. But there is a minus epsilon m u by two as well, because the matrix has reduced, matrix strain is now not epsilon m u; actually, it is… The total strain would be this epsilon m u plus as much as is there. But, matrix has the reduction in the matrix.

So, the total… Earlier was… Earlier, the stresses in all cases were same. So half… This is the increase. This is the additional strain being taken by the composite… There is an increase in the additional strain – average increase in the strain. And, this minus this; this crack width – if I am interested, I should be actually finding out… This is the additional average increase in the strain minus the matrix has actually… There is reduction in the matrix strain. Now, average strain in the matrix is half epsilon m u by 2. So, matrix strain now is half epsilon m u by 2.

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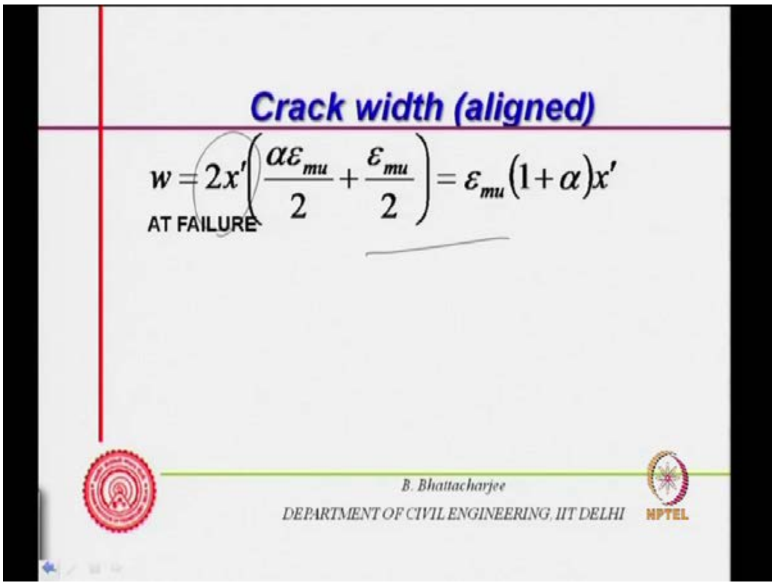

Average strain in the matrix is half epsilon m u by 2. Originally, it was epsilon m u. So, this multiplied by the length – 2x dash – over which it has actually occurred would give me… It will give me the… So, if I bring in 2, I will get it 1 plus alpha epsilon m u. So, this is simple algebra. These two will cancel out. So, that is the strain. And, this diagram shows you that actually. This diagram shows you that. This diagram shows… So, this was alpha epsilon m u. And, this height that increase is epsilon m u 1 plus alpha, but average will be average increase. So, this is 1 plus alpha. That is what we have seen. And, average increase will be half of this. It was 1 plus alpha. So, average… If I am trying to find out the crack width, I can find out by multiplying the length of this one. So, this is the crack width. At failure, sigma f u is equal to… After that, it is the matrix, which will take the strain alone.

And, it will be the failure finally, sigma f u sigma composite; c u composite will be sigma f u into V f. And that is what is shown in the previous diagram that we have seen – at failure, it will be sigma f u. This is sigma f u into V f. That would be the V f at (( )) failure occurred if it is failing anywhere alone. So, if it is feeling alone, then it will be sigma f u V f finally at failure.

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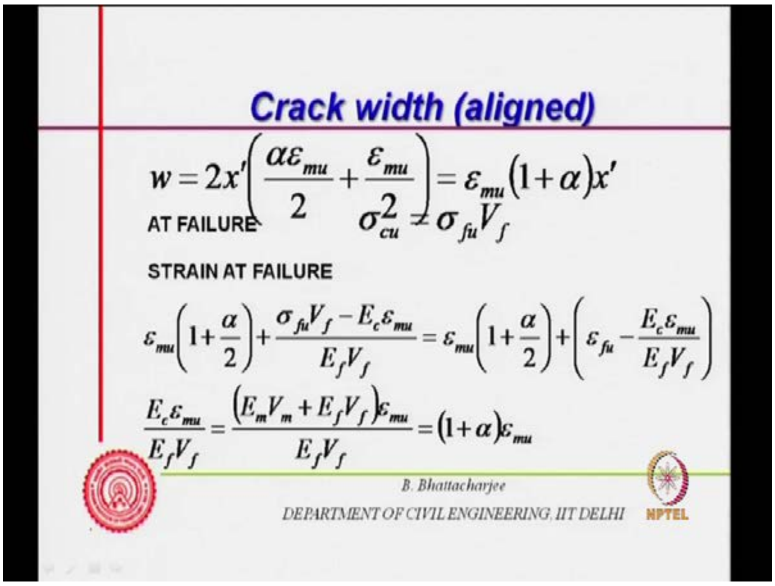

Sigma f u V f at failure. So, sigma f u… Strain at failure – of course, one can find out what will be the strain at failure, because strain at failure will be less than the…

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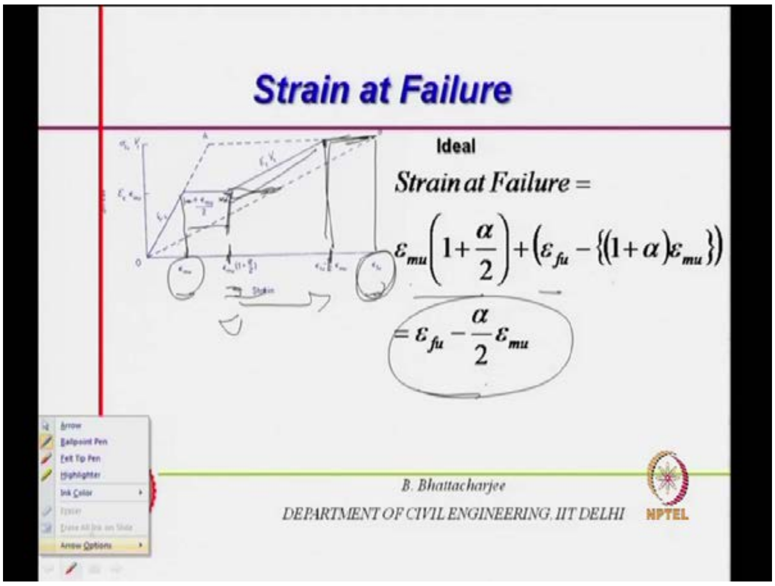

This diagram we will make it clear – this diagram. Strain at failure will be less by sigma f u. Failure would have been here. But, it would be less by some amount. If I look at the stress-strain diagram, what I will see? Up to this, nothing fails. It is epsilon m u. Then, there is a sudden increase of alpha epsilon m u by 2, because it is a crack. So, apparent increase sort of a ductility provided by this. And, after that, it will be E f V f curve, which would follow. And finally, will fail at sigma f u V f. So, this will be sigma f u V f. It will fail at sigma f u V f. So, the strain is somewhat less than epsilon f u. Strain is (( )) Strain at failure is somewhat less than sigma f u. By how much? By geometry, you can find it out actually – from here to here; this length. So, it will be epsilon f u minus 1 plus alpha. This increase that has occurred – 1 plus alpha epsilon m u. So, epsilon m u 1 plus alpha 2 plus… 1 plus alpha 2 is here. After the crack has formed, this is the strain. And this is strain plus sigma f u minus… This is the additional increase between this point – sigma f u minus 1 minus alpha epsilon m u.

So, this totally gives us actually epsilon f u minus alpha by 2 epsilon m u. This gives us this. This is what… This is the strain after the crack has formed. That is what we have seen. Then, sigma f u V f minus E c epsilon m u divided by E f V f. That is from the similar triangle that I was talking about. And, you get it epsilon m u 1 plus alpha by 2. And, if you divide by E f by V f, this will be epsilon f u, because E f divided by sigma f u and E c by E f epsilon m u by V f. So, E c by epsilon m u by E f V f – this term, will be simply E m V m plus E f V f into epsilon m u replacing E c by this. And, this will again give you 1 plus alpha epsilon m u.

Therefore, these two terms put together, I get the strain at failure. And, it comes out to be epsilon f u minus alpha by 2 into epsilon m u, because this length is same as this length; or, this is a… It would have gone here. But, now, it has gone here. That is basically because of this length change. So, it is related to that. And from that, we have derived… From the similarity of triangle, this relationship has been derived. From the similarity of triangle, this relation – epsilon f u V f sigma f u V f minus E c epsilon mu; that is the stress in the matrix. So, from the similar triangle, divide by E f V f. This relationship has been derived.

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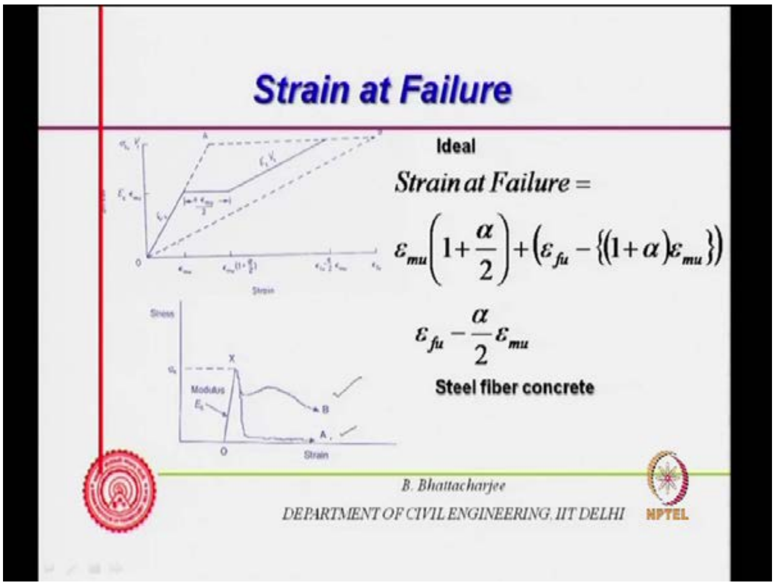

And you will have strain at the failure as this. Anyway that is still fiber. Now, supposing I take practical fiber concrete. We barely can put in the critical fiber. So, this portion we do not get. What we normally get is something of this kind. Depending upon the fiber, some a or b type, a steel fiber concrete percentage of fiber will vary. And, I can have small amount enhancement.

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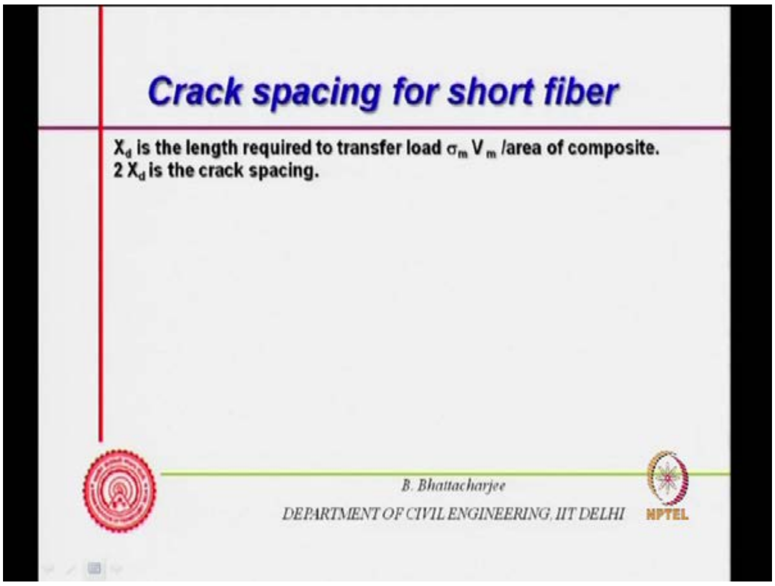

So, with this actually, we can look into crack spacing in short fiber; crack spacing in long fiber we can find out. Now, crack spacing in short fiber we can derive from this. So, I think, this is what we look into in the next class; continue with this short fiber. But, one more thing I would just like to mention here is that, most of the time, short fibers do not fail by failure of the fiber. Snapping of the fiber rarely occurs. So, it is essentially the pull out of the fiber, which actually causes failure of the fiber. So, short fiber – first, we look into what will be the cracks spacing. But then, we will look into the pores required for pull out. And then, force required for pull out. And lastly, then how this… Although I may not be able to get enhanced strength – tensile strength, still I may be able to get enhanced flexural strength. So, we will look into that in the next class in out next lecture.

Thank you very much.