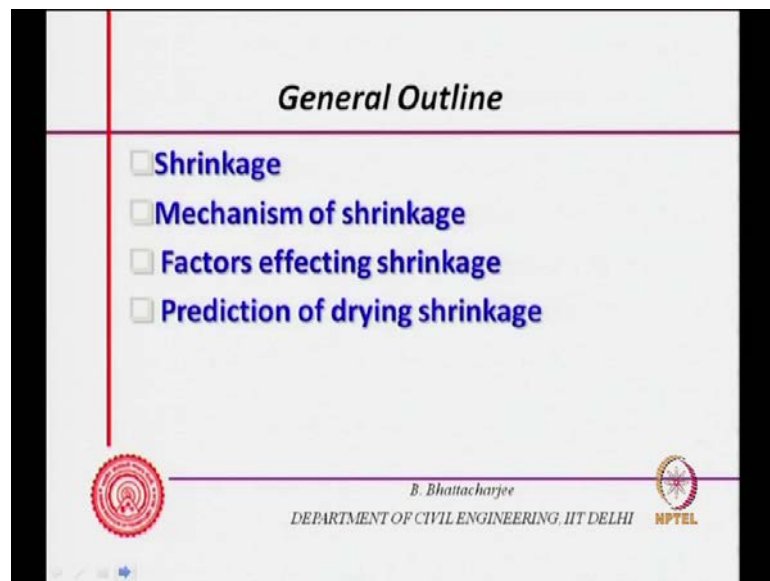


**Concrete Technology**  
**Prof. B. Bhattacharjee**  
**Department of Civil Engineering**  
**Indian Institute of Technology, Delhi**

**Lecture - 29**  
**Shrinkage of Concrete**

Welcome to module 7 lecture 3. In the lecture two of this module we just introduced ourselves to shrinkage, shrinkage of concrete, the different types of shrinkage, namely we talked of chemical shrinkage, plastic shrinkage, autogenous shrinkage and drying shrinkage. So, we just you know terminology is associated with them and what exactly they mean we have defined them, and then we just stated the factors influencing the shrinkage.

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Today first we will look into mechanisms of shrinkage, and then we will look into factor affecting shrinkage, and then prediction of drying shrinkage. You know this is the most important one amongst all them particularly normal concrete. So, we will look into this.

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### Mechanism of Shrinkage

- It is generally related to adsorbed or interlayer water (gel water).
- The movement of interlayer water out of the gel results in reduction in volume of C-S-H.
- Capillary tension is setup by increasing curvature of the menisci.
- Induced balancing compression in the C-S-H skeleton causes volume change.

*B. Bhattacharjee*  
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

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So, mechanisms of shrinkage like creep; it is again generally related to adsorbed or interlayer water like creep it is. So, that is why they are dealt together usually shrinkage and creep and they have similar sort of related mechanism with respect to micro structure of concrete. So, this is related to adsorbed or interlayer water. Movement of this water out of the gel results in reduction in volume of C-S-H; movement of this water the gel water, the interlayer water out of the gel results in reduction of volume of C-S-H. The capillary you know calcium silicate hydrates. So, it actually results in reduction in the volume of C-S-H gel.

Now when something is drying when let us say capillary you know drying is taking drying is occurring, capillary tension is set up because there is an increase in the curvature of the menisci. You know capillary tension is set up because of increase in the curvature of the menisci. So, when actually water is moved out when water moves out the menisci curvature increases and this sets up a kind of tension; this sets up a kind of tension, increases the tension in a way, it increases the tension, and this is balanced by a compression. This is balanced by a compression in the system in the C-S-H skeleton and this induced balancing compression actually causes volume change.

So, this is one of the theories that you know capillary menisci curvature increases when water is moving out this is well understood, there is no problem about that, change in the menisci because of you know withdrawal of water. So, as the water is absorbed from the

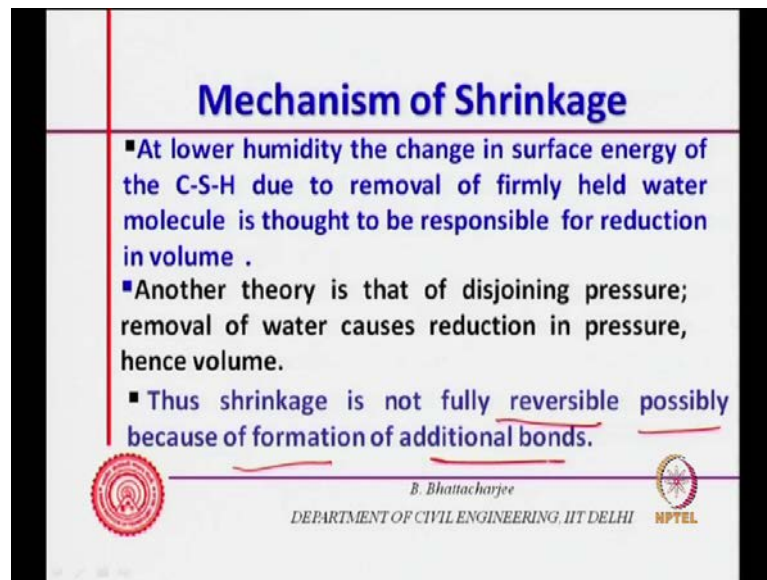
pores capillary menisci curvature of the capillary menisci will increase inversely proportional to actually  $r$ . So, smaller the  $r$  the larger is the curvature is proportional to that. So, it increases larger is the curvature and that is what happens. So, that induces that is tension increases and it has to be balanced. So, there is a kind of compression is induced in the skeleton of C-S-H results in volume change or shrinkage.

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At lower humidity of course change in surface energy of the C-S-H due to removal of firmly held water molecule adsorbed water molecule is thought to be responsible for reduction of volume. So, you had adsorbed water molecule water layers adsorbed water layer and as you remove this, if you remove this one, remove adsorbed water. So, removal of adsorbed water results in change in surface energy because you know the adsorbed water was there very much and the moment water goes away there is a change in surface energy. Now this results in actually forces causing kind of pull within the material and therefore reduction in volume.



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**Mechanism of Shrinkage**

- At lower humidity the change in surface energy of the C-S-H due to removal of firmly held water molecule is thought to be responsible for reduction in volume .
- Another theory is that of disjoining pressure; removal of water causes reduction in pressure, hence volume.
- Thus shrinkage is not fully reversible possibly because of formation of additional bonds.

*B. Bhattacharjee*  
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI



So, this is another reason there is collapse mechanism sort of. So, this results in adsorbed water removal can cause actually change in the volume or shrinkage. So, this is another theory.

So, first one was capillary tension theory as we said; second one is surface energy as we are removing the water the surface energy changes occur, and that results in surface energy changes will occur, and that results in actually kind of change in volume or reduction in volume, and that is why shrinkage occurs. Then another theory is that disjoining pressure that of disjoining pressure. So, removal of water causes reduction in pressure and hence the volume. So, if you remove water the disjoining pressure you know pressure would be reduced pressure which was you know it would reduce the pressure would get reduced; hence, the volume will reduce. So, pressure which was pushing them out now it has got reduced. So, there will be you know reduction in volume.

So, pressure reduces volume would also reduce. It is that idea pushing out pressure is reducing. So, there is a kind of change in the volume. So, this is all; this is another theory for shrinkage, and this is the kind of theory that has been put forward to shrinkage in nut shell. These are kind of possible theories that have been put up, but one thing is very clear that it is the removal of water from the interlayer that causes shrinkage. So, removal of water may be even just internal removal from gel to capillary pores, because the

capillary pores itself was got emptied or change in the reduction in volume because of the one of course was relative to chemical shrinkage which is somewhat different; chemical shrinkage was somewhat different because the reaction products occupies less volume than the original volume reactants volume.



So, this is chemical shrinkage but other cases can be associated with removal of water from the interlayer situations. So, particularly drying or similar shrinkage can be related to you know removal of water. So, since all these are happening you know these processes are not reversible. For example, if you add water some of the capillaries might change by this time or some of the pores might have changed by this time because of the collapse mechanism because of the shrinkage, the dimensions might have changed or whatever there may be changes and then formation of additional breaking of some bonds formation of additional bonds that is possible, and because this process as phenomena you know if you add water back the pressure will not be same as what it was earlier. Similarly the surface energy restoration if you do by adding water back it may not be same exactly as before. So, putting all this together the shrinkage is not fully reversible, and this is possible because of change broken breaking of the bond and possibly formation of additional bonds.

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**Mechanism of Shrinkage**

- Drying shrinkage is more prominent than other shrinkages in normal strength concrete.
- Autogenous shrinkage occur in sealed specimen because of creation of empty spaces from water filled space by hydration and self desiccation .
- In low W/C concrete autogenous shrinkage may be more than drying shrinkage .

*B. Bhattacharjee*  
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

So, that is what we understand about shrinkage it is mechanism sort of, but one thing is very clear drying shrinkage is more prominent than other shrinkage as in normal

concrete. So, this what we talked was related to drying shrinkage and this prominent than other shrinkages in normal strength concrete; autogenous shrinkage which we talked about in the previous lecture this occurs in sealed specimen because of creation of empty spaces, see originally water filled spaces hydration reaction is occurring. So, originally water filled spaces you know. So, this was water, this is cement. So, hydration reaction is occurring; this water gets consumed. Now this is sealed; let us say this is this is all sealed, this is all sealed specimen.

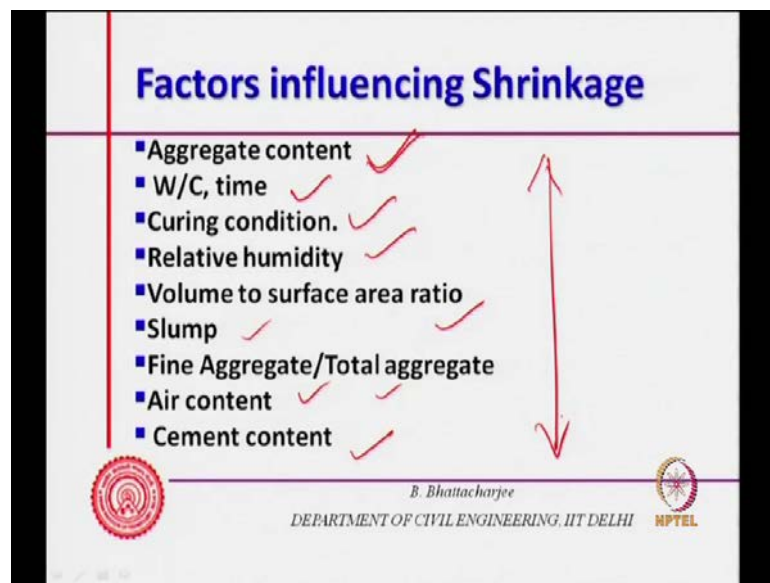
So, this water actually reacts and gets consumed what is called as self desiccation, but this leaves some voids in the space because water filled space has gone out, the volume of the hydration product is also is less, and this space is not being filled up fully and therefore, it can result in kind of empty spaces and that can due to self desiccation process even if you have sealed it no water you are replenishing; there will be water loss in the after the skeleton has been formed after initial setting has set has occurred. So, empty space remains even in sealed specimen because the water has got lost.

Whatever water is still remaining was there in the sealed specimen besides that some empty pores are created, and in this space are actually thought to be causing self desiccation, alright. So, this self desiccation or autogenous shrinkage self desiccation and then autogenous shrinkage, and this autogenous shrinkage is considered to be a more prominent in low water cement ratio concrete; that means your high strength concrete moderned high strength concrete, autogenous shrinkage may be more prominent than drying shrinkage. Drying shrinkage because high water cement ratio concrete there are lot of space or interconnected permeable pore system would be there through which water can evaporate out.

So, drying is more prominent there while in autogenous while in lower water cement ratio concrete you know the permeability is not very high. So, therefore, self desiccation would cause consumption of the water and evaporation would not be as dominant as in normal strength concrete, but self desiccation will result in actually creation of new empty pores, and that is thought to be causing a kind of shrinkage as we have seen; we have defined earlier there is a macro level dimensional change occurring in sealed specimen, and that is what we call it as autogenous shrinkage.

So, they can be more in high strength concrete or low water cement ratio concrete compared to drying shrinkage. So, they are very important currently for high strength. We will see them might get somewhat some better insight into this mechanisms, etcetera, when we look into also the measurement process because measurement separating the shrinkage as different kind of shrinkages and measuring them we will see how we can do that. So, we will see it in the next lecture; so at the moment up to this only.

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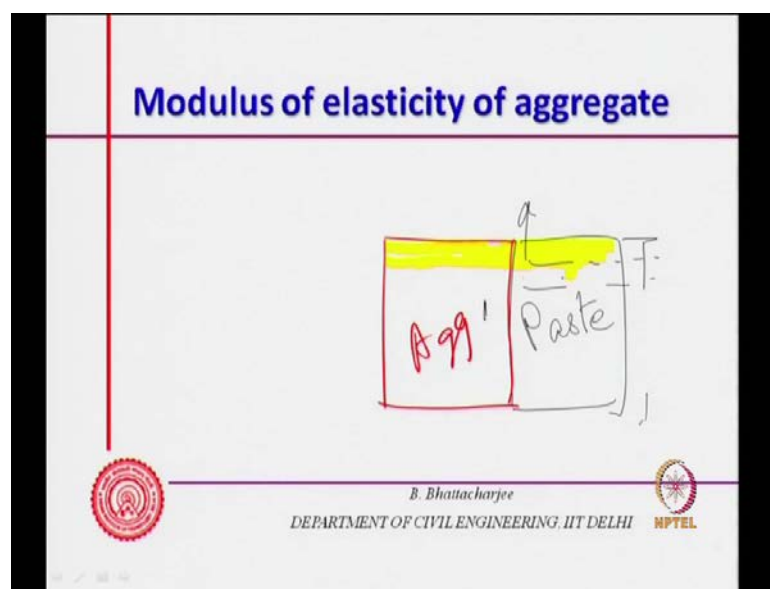
Now then what are the factors influenced you know actors those influences shrinkage. Now we just looked at them and we said that aggregate content and aggregate properties. This will be one of the major thing water cement ratio and time, curing condition, relative humidity, volume to surface area ratio, slump, fine aggregate to total aggregate, air content and cement content. So, all these are related to you know all these are the factors; so many factors actually influence the shrinkage.

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So, we will see one by

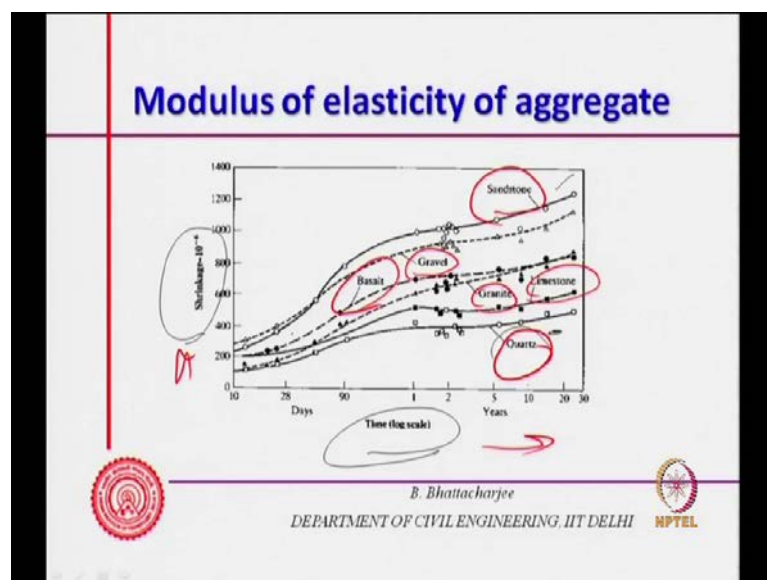




one. Let us see the effect of aggregate, first of all modulus of elasticity of aggregate. Now you can understand this, we can understand this as I said you know the stiffness is trying to shrink, something is trying to shrink, I have one material and the other material. So, I have one material and the other material; this is one material, this is other material, and they are bonded, and they are trying to shrink. They are trying to shrink you know this is trying to shrink. So, let us say this shrinks by this amount, this shrinks by this amount. Now if paste shrinks aggregate do not want to shrink. This is let us say aggregate, and let us say this is paste.

So, this will tend to shrink, this will tend to shrink, this will tend to shrink somewhere, but this will not shrink, because aggregate is inert, and its volume change would not occur like that. So, as this will try to shrink this will also try to bring it together, but this will then exert a net upward force to this; that means shrinkage of the paste will be restrained by aggregate, right. Now aggregate how much it will restrain for same strained the stress would be more if the modulus of elasticity is more. So, if this has high modulus of elasticity this will actually share more load for the same strain, and it will be less loaded besides it will actually provide that restrain more. So, modulus of elasticity has a role. So, aggregate modulus of elasticity is.

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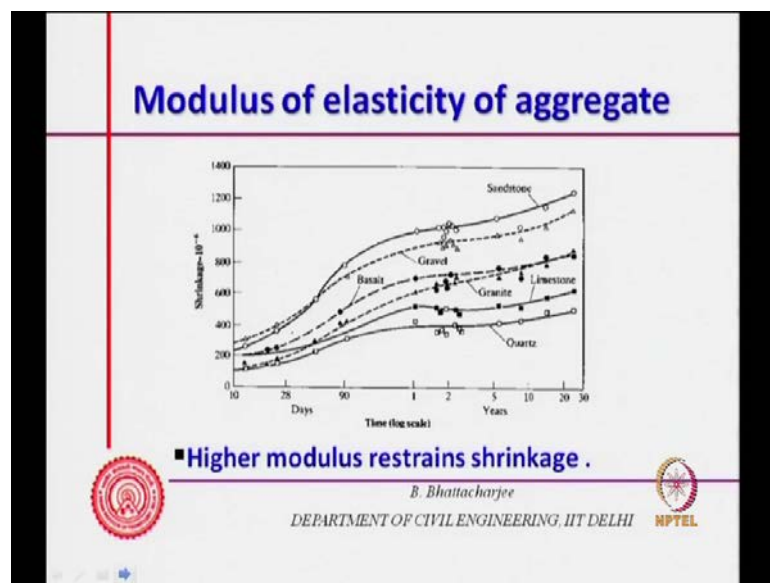


So, if you look at this you know this is sand stone. This is shrinkage along this direction in macro strain, time in log scale quartz. Now quartz is very stiff. So, quartz is here. So,



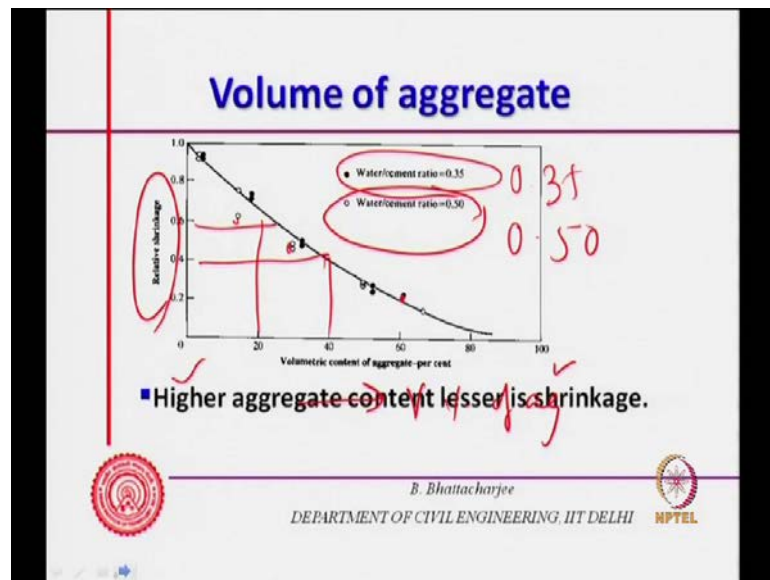
quartz is here actually, quartz is here, here is quartz, this is sand stone, this is lime stone, this is granite, this is some gravel, this is basalt. So, it will depend upon the stiffness of the aggregate itself. So, higher the modulus of velocity it would actually have less you know shrinkage will be less, because it will restrain the shrinkage. So, shrinkage will be less. So, time of time in log scale along this direction, shrinkage along this direction, and different types of stones are same and other things remaining same and at different age shows you know different shrinkage; so stiffer one obviously will show less shrinkage in the concrete, alright.

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So, higher modulus restrains shrinkage; that is what we are saying, higher modulus restrains shrinkage.

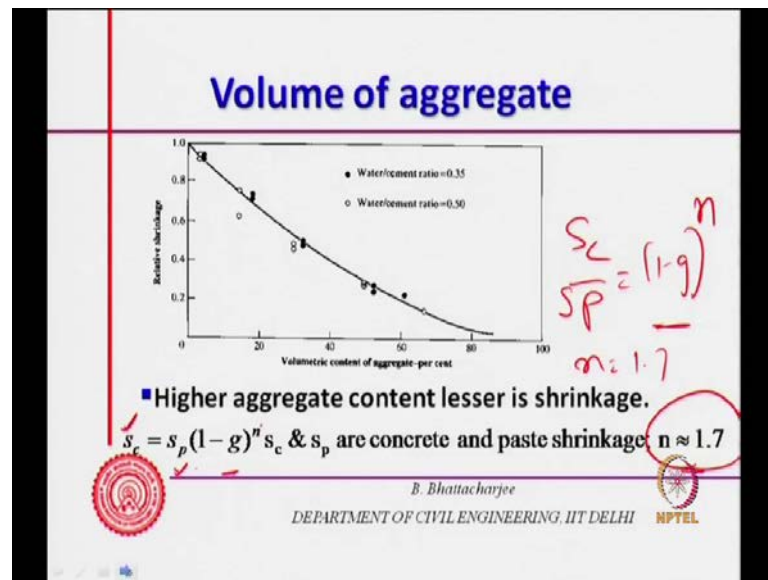
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Volume of aggregate that is quantity obviously now this is very clear, because it is a paste which actually shrinks, aggregate does not. So, therefore, this is shown for two different water cement ratio, water cement ratio 0.35, this is water cement ratio 0.50. So, this is 35, this is of 50 somewhat less, but you see the volumetric content of aggregate percent. So, volume percentage you know of aggregate 100 percent aggregate of this is not the situation 0 percent aggregate. So, if the paste shrinkage is considered to be one this is relative shrinkage; paste shrinkage is considered to be one with 40 percent aggregate, you will get possibly 40 percent; shrinkage will get reduced by 60 percent, at 20 percent it will get about 40 percent reduced.

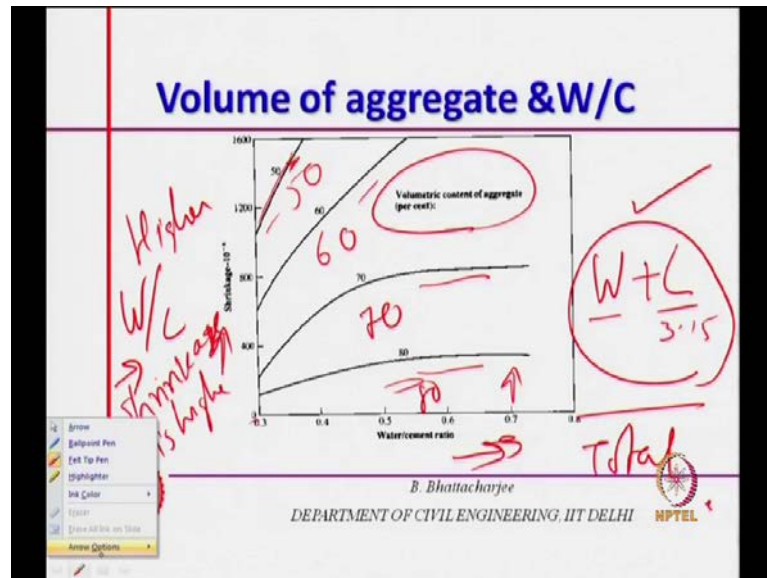
So, depending upon the volume content of the aggregate shrinkage would get reduced; more volume means less shrinkage, more volume means less shrinkage. Of course, when you look into the effect of shrinkage, but this we are talking of the shrinkage in the system itself. There is a case where you have actually very low volume of aggregate shrinkage in the overall system could be lower or shrinkage cracking could be lower; we will talk about that sometime later on. So, volume same, same aspects you know more the volume because aggregate do not shrink; therefore, more the volume less will be the overall shrinkage.

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So, higher aggregate content lesser is shrinkage; higher the aggregate content lesser is the shrinkage, okay. Now simply one can look into a simple empirical equation. This is the shrinkage of concrete; it can be related to shrinkage of paste into 1 minus g to the power n where g stands for aggregate content, n stands for a value of 1.7. So, you know it is a kind of you can say square, because this is try to fit in empirical equation. So, you know sort of  $s_c$  by  $s_p$  is 1 minus g to the power n where g is the aggregate content. This is the non-linear curve; so one can actually relate to this one. So, n is roughly about 1.7, n is equal to roughly 1.7. So, this is an empirical equation. So, we can understand that more the aggregate content lesser will be that because this is aggregate content. So, more the aggregate content lesser is the shrinkage.

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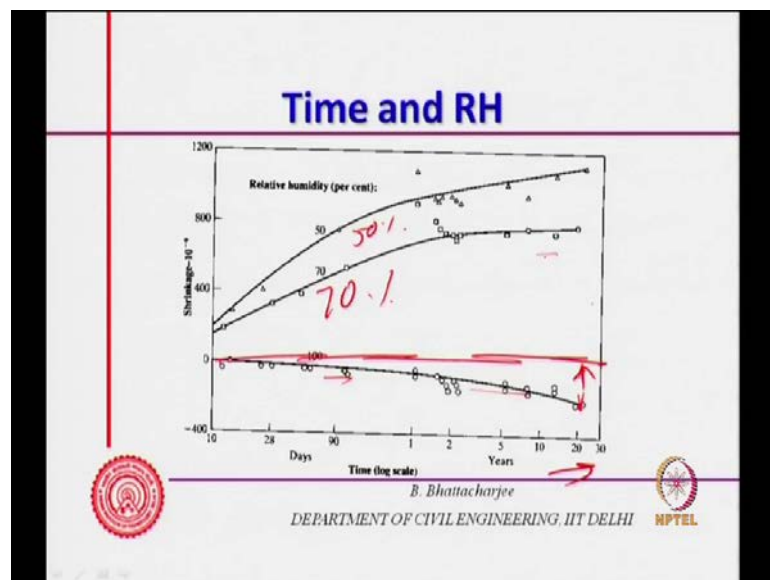
Water cement ratio let us see water cement ratio. So, this is we are plotting against water cement ratio, and this is the shrinkage. Now volumetric content of aggregate percent, this line is for 50 percent aggregate 50, this is for 60, this is 70, and this is 80. So, if you see 80 percent you know aggregate is 80 percent, shrinkage will be lower, but as my water cement ration increases shrinkage increases; water cement ratio increases shrinkage increases. This we are talking mostly of the drying shrinkage. We are talking of normal strength concrete not very low water cement ratio situation. So, we are not going below 0.3; it is only up to 3 we are talking about. So, we are looking at normal strength concrete not very high strength concrete where it is autogenous shrinkage which actually governs which can be more than the drying shrinkage.

So, this is essentially we are looking at drying shrinkage for normal strength concrete. So, you see as the water cement ratio increases shrinkage increases in all cases you know; in all cases shrinkage increases as water cement ratio increases. Now water cement ratio increase in water cement ratio obviously would mean more capillary pores in the system and which is drying out it is related to drying out; it can easily dry out. So, this is higher the water cement ratio it has a tendency to exhibit higher shrinkage, but absolute cement content has also got a role because paste content itself if it is high shrinkage will be more.

So, this has got higher paste content; this has got less paste content. In other words water cement ratio is one issue, but paste content is also an important issue. Now paste content means its sum total of water plus cement. So, water plus cement. So, water plus cement divided by you know 1 and 3.15 specifically. So, this is volume of water plus cement. So, paste volume will be this, paste will be this. We are not talking of pozzolanic cement or anything of that kind; if that is there that will also add to the paste. So, this divided by total volume obviously will give us relative paste content, and if this is high my shrinkage is higher.

Shrinkage is higher because paste is what shrinks, aggregate do not shrink, alright, and in this one again if the water to cement ratio is high water to cement ratio is high. So, higher w by c shrinkage is more, higher, the shrinkage is higher. So, for a given paste content shrinkage increases with water cement ratio; for a given paste content shrinkage increases with water cement ratio because large amount of capillary pores. There is a possibility of more drying, drying is possible, more water may move out and even with larger capillaries the scope of emptying the gel pore or inter layer water movement in the capillary space is now more. So, this is related to those aspects.

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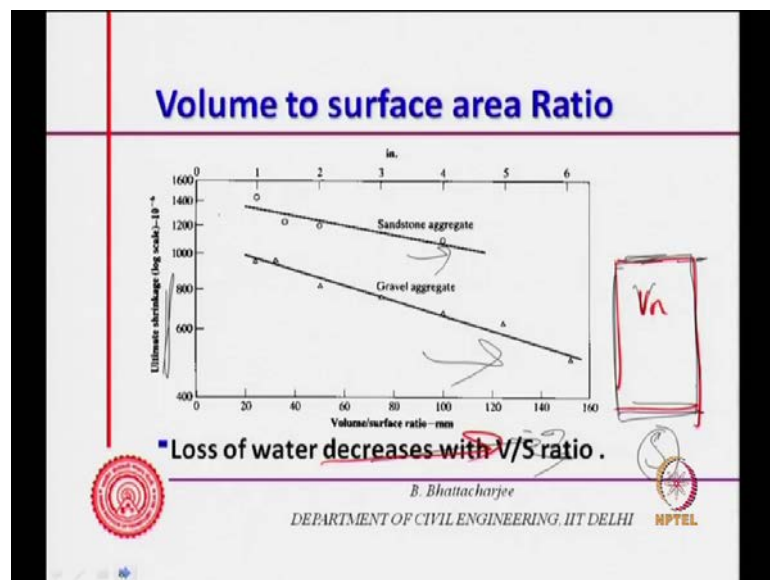


So, this was water cement ratio and of course volume of aggregate, then relative humidity and age. So, if this is age we have seen earlier also with age shrinkage increases, we have seen that earlier also, with age shrinkage increases, right, with age

shrinkage increases and in this case of course there is a reduction if you have 100 percent relative humidity which means that you will have some amount of swelling. So, this is 0 and this is 100 percent relative humidity. In fact, you will have swelling. So, this is swelling actually this will be swelling, but if you have lesser 70 percent, 50 percent. So, if you have lesser shrinkage goes on increasing, but then it tends to be somewhat you know flattening out in the long run; it has a tendency to flatten out in the long run, alright.

So, this is time and relative humidity. So, if it is fully saturated there could be swelling; if the relative humidity is very high close to 100 percent or just you know 90 percent or so shrinkage will be very little, but it is all related to drying. So, one can understand when you have 50 percent relative humidity in the surrounding which means that drying would occur; you know drying would occur by evaporation. So, capillary water will get out by evaporation and as capillary tends to become dry interlayer water will move out from gel pores gel water will move out from gel pores to the capillary pores, and as we have said drying mechanism involves movement of this water out the shrinkage would actually occur. So, this is understandable, alright.

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Now volume to surface area ratio; it is almost similar to the situation of creep again volume to surface about you know volume to surface area ratio. So, if you have higher volume to surface area ratio. So, the volume is here and surface area is the peripheral

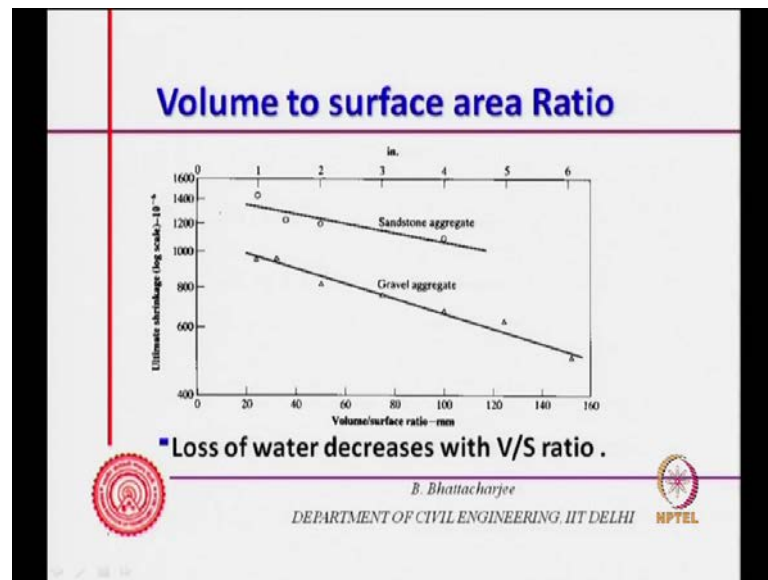
surface area in this case. So, this is the surface area; this is important. Now surface area volume to surface area ratio if it is large which means that I have got lesser surface area per unit volume. So, I will have less evaporation. So, when volume to surface area ratio is small I will have more evaporation. So, you know more evaporation

For example, if this volume to surface area ratio this is  $s$ . So,  $v$  by  $s$  more  $s$  per unit volume means more evaporation. So, less  $s$  per unit volume means less evaporation. In other words as  $v$  by  $s$  increases you know I will have less evaporation. So, therefore, for gravel aggregate and sandstone aggregate shrinkage ultimate shrinkage that is long term shrinkage because you have seen that shrinkage actually is a function of time; therefore, one should talk about shrinkage at different ages. So, ultimate shrinkage is final infinity at infinite time, and it is seen that it depends upon the volume to surface area ratio; we are talking again of drying shrinkage not the other shrinkage which are less dominant. So, we are talking of drying shrinkages at the moment more which is more for normal strength concrete.

We will discuss something about autogenous shrinkage later on when we talk about measurements and so on, so forth. So, drying shrinkage is what we are concentrating on right now, and if you have large surface area less volume per unit volume or more volume per unit surface area then shrinkage will be less. So, evaporation is less. So, this is what it is. So, you can see that it is the function of it is related to evaporation loss, right.

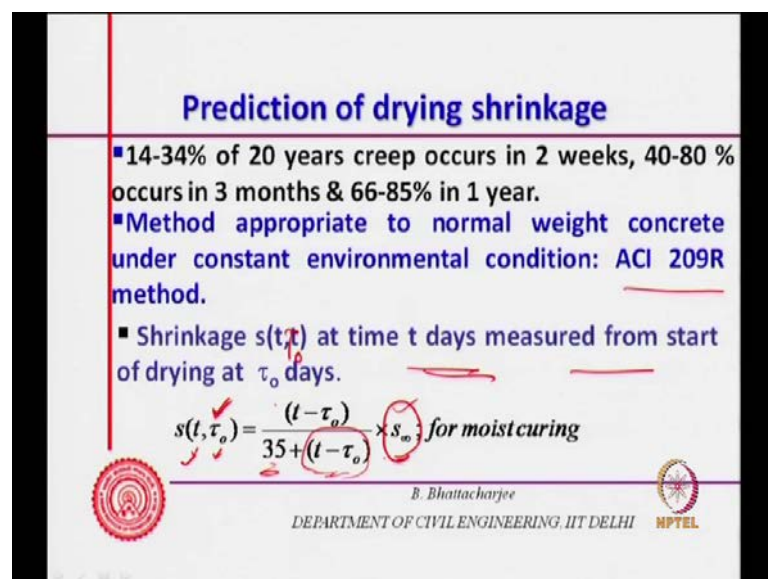


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So, loss of water decreases with volume to surface area ratio with increase in volume to surface area ratio; that is the idea.

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So, we will come to prediction of drying shrinkage, but remember we did not talk of some of the factors namely slump, aggregate fine to coarse aggregate ratio, etcetera, because we did not discuss this; let us just go back to that slides quickly lets, quickly go back to that slide and see what are the things we did not really look into account for. You see we did not talk of slides, slump, fine to total air content, cement content. Now this is

indirectly covered; actually slump is there the factor that formulate is there, flowing concrete or less flowing concrete it is something it is the water content and cement content is something to do with the paste content; air content would create some more voids in the system, and fine aggregate to total aggregate ratio because total aggregate has got a role; it will be shrinking less compared to fine. So, this other factors are there; rest all we have actually discussed.

So, let us now look at the prediction of shrinkage like we did prediction of creep and similarly as a formula the British formula we will look into. So, quarter idea wise 14 to 35 percent of 20 years creep occurs in two weeks, and 40 to 80 percent occurs in three months, and 60 to 85 percent in one year. So, if you look at you know three months or so majority of the actual shrinkage occurs in this. So, all the problem related to shrinkage like cracking if there is any drying shrinkage cracks if they come; they come quite early in the life of the structure within three, four months, maximum six months. You know usually it is two, three months the cracks come, right; you do not see them very longer, so rarely one year or so.

So, most it comes pretty early. So, therefore, this is important. Now method appropriate to normal weight concrete is under this ACI 209R method; you can estimate using this, and that is what we will try to do, and then of course the British VACA and system would be there, and we will discuss that as well. So, what it says you know shrinkage at any time when drying has started from  $t_0$  would be given as  $s_\alpha$  which is the ultimate shrinkage, this is given by this formula. So, shrinkage  $s_{t-t_0}$ , this should be  $\frac{s_\alpha (t-t_0)}{t-t_0}$  at time  $t$  days measured from start of drying at  $t_0$ . So, this is measured you know at time  $t$  days measured from start of drying. So, drying start from  $t_0$  times and shrinkage is given. So, where this is infinity and you know related to the time.

So, it is a kind of hyperbolic equation as you can see that as this time increases this time will increase; this will increase but there is a 35 added there. So, how higher this would tend to actually this will tend to a constant value that is ultimate. So, it gets related to a fraction, right. So, when this is very large this value will be small compared to this, right, this is in days actually; these are all in days. So, this is also in days. So, if you have something like you know large value of this one this will be nearly one. So,  $s_\alpha$  infinity is that is how we are defining. So, ultimate shrinkage for moist curing, but if it any other condition the formula will change.

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**Prediction of drying shrinkage**

$$s(t, \tau_o) = \frac{(t - \tau_o)}{55 + (t - \tau_o)} \times s_{\infty}; \text{ for steam curing}$$

■ Ultimate shrinkage  $s_{\infty}$  is further related to various factors.

$$s_{\infty} = 780 \times 10^{-6} k'_1 k'_2 k'_3 k'_4 k'_5 k'_6 k'_7$$

$k'_1 = 1$  for steam curing 1-3 days & for 7 days moist curing

■ For other cases values are given in Table.

B. Bhattacharjee  
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So, this is for steam curing this value is 55; for steam curing this value is 55, you know this value is 55. In other words the shrinkage is you know initial early days when this is small, shrinkage is relatively lesser. This is small, this is small let us say, this value is small; you know shrinkage will be relatively lesser, because this value is higher, this value is higher. So, for steam curing it is the ultimate shrinkage is related to the shrinkage at any time. So, shrinkage at any time  $t$  when drying has started from  $\tau_0$  is related to ultimate shrinkage, alright, and now ultimate shrinkage you can find out ACI 209 method actually tells you the procedure to find it, alright.

So, yes ultimate shrinkage is given by this formula  $780 \times 10^{-6} k'_1 k'_2 k'_3 k'_4 k'_5 k'_6 k'_7$ . So, like the creep formula creep formula at seven such factors  $k_1, k_2, k_3, k_4, k_5, k_6$ , if you recollect depending upon curing condition then volume to surface area ratio, slump, etcetera etcetera, if you remember aggregate coarse to fine ratio and all that here there are such seven factors; here there are such seven factors, and here there are some seven factors, and each factors are related to the relevant property. For example,  $k_1$  for steam curing one to three days and for seven days for moist curing.



So, for you know  $k_1$  is equals to one for steam curing one to three days. So, it is for related to curing condition  $k_1$  dash prime is related to curing condition. So, it is one for if you have done your steam curing for one to three days and seven days if it is moist

curing. So, it is actually the concrete condition; we are talking of normal concrete again and this is largely related to drying shrinkage prediction. So, if you do steam curing for one to three days or moist curing for seven days as the  $k_1$  could be taken as 1. So, that is the datum.

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Prediction of drying shrinkage	
Period of moist curing	Shrinkage Coefficient $k_1$
1 ✓	1.2
3 ✓	1.1
7 ✓	1.0
14 ✓	0.93
28 ✓	0.86
90 ✓	0.75

*B. Bhattacharjee*  
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI



For other cases you can actually find out from this table. For example, moist curing one to three days you know this is seven day we have said is one, and this factor if you are less curing this value increases. If you have more curing this value reduces; similar values are available for steam curing also. So, this is you can use simply  $k_1$  equals to 1, 1.2 depending upon number of days occurring there. So, part of the table has been reproduced for the purpose of understanding and getting familiar with the procedure.

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**Prediction of drying shrinkage**

■ Humidity Coefficient  $k'_2$

$$k'_2 = 1.40 - 0.01h \quad (40 \leq h \leq 80\%)$$

$$k'_2 = 3.00 - 0.030h \quad (80 \leq h \leq 100\%)$$

Handwritten calculations:

For  $h = 80$ :

$$3.00 - 0.03 \times 80 = 3.00 - 2.4 = 0.6$$

$$k'_2 = 1.40 - 0.01 \times 80 = 1.40 - 0.8 = 0.6$$

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So,  $k_1$  factor is related to curing; the table for steam curing is separate or you know the way to find out is given and that is given in the committee report SCI 209. I have just introduced the normal moist curing scenario only;  $k_2$  is humidity coefficient, it is exposure condition. So, this is  $1.40 - 0.01h$  where  $h$  stands for relative humidity where your relative humidity is 40 to 80 percent, and this value is different if your humidity is more than 100 percent. In fact, this will be much less as one can see  $k_2$  factor would be you know 82  $h$  relative humidity 82. So, this formula is given this. For example, you take let us say 40 percent relative humidity. So,  $k_2$  would be equal to  $1.40 - 0.01$  into 40.

So, which would be simply  $1.40$  minus how much it would be  $0.4$ , 1 percent of 40 is  $0.4$ ,  $0.04$ ,  $0.4$ , 1 % of 40 would be  $0.4$ . So, this is simply 1, and if it is 80 in this formula let us put it. So,  $3.00 - 0.03$  into let us say 80. So, this would be 80, you can find out from this formula also. So, from this formula if I find out I get  $3 - 80$  into  $0.03$ . So, this would be  $2.4$  which is  $0.6$ , and if I put in this formula  $1.40$  into 80 let us say, just rub this off, rub this off, this I am just rubbing off. So, this is 80 and 80 is you know  $1.4$  minus  $0.8$  which will be  $0.6$  again. So, from both the formula you get it; you get it because here the multiplying factor is more, right. This value is more, but this multiplying factor is more. For example, if you have 100 then this will be simply zero; if you put  $h$  is equal to 100 then it is simply zero. So, actually there is no shrinkage; no shrinkage if it is 100 percent relative humidity. So, this is the factor related to relative humidity.

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### Prediction of drying shrinkage

- Humidity Coefficient  $k'_2$ 

$$k'_2 = 1.40 - 0.01h \quad (40 \leq h \leq 80\%)$$

$$k'_2 = 3.00 - 0.030h \quad (80 \leq h \leq 100\%)$$
- Volume to surface area ratio coefficient

$k'_3$  depends on member volume / Surface area ( $V / S$ )

$v/s(mm)$	$K'_3$
12.5	1.35
25	1.17
37.5	1.00

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Now volume to surface area ratio  $k_3$  depends on the member volume to surface area ratio, and this is the part of the table. If it is 12.5 it is 1.35. You know we said that surface area more means more evaporation. So, as this increases there will be reduction and this reduces. So, 37.5 have been taken as the base case, and if it is less than that then you will have more values. So,  $k_3$  dash depends upon volume to surface area ratio, alright.

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### Prediction of Drying Shrinkage

$$k'_3 = 1.23 - 0.006 \frac{V}{S} \text{ for } 95 > \frac{V}{S} > 37.5 \text{ mm } t - \tau_o < 1 \text{ year}$$

$$= 1.17 - 0.006 \frac{V}{S} \text{ for } 95 > \frac{V}{S} > 37.5 \text{ mm } t - \tau_o \geq 1 \text{ year}$$

$$= 1.2e^{-0.00473 \left( \frac{V}{S} \right)} \text{ for } \frac{V}{S} \geq 95 \text{ mm}$$

$$k'_4 = 0.89 + 0.00264s \text{ s is slump in mm}$$

B. Bhattacharjee  
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So, for any other values other than 37.5 it is given by this kind of formulae. For example, if it is more than 37.5 but less than 95 then the formula is given as 1.23 minus 0.006 v by s and t minus t 0 less than one year. If this is more than one year for this case the formula is given 0.006 is 5, but this is 1.17. So, you know this is basically depending upon this depending upon the age. So, if it is less than one year the value is something like this; if it is more than one year the value is something like this, and if it is more than 95 centimeter this is given by this formula for v s greater than. So, it is 1.2 exponential of 0.00473 v by s.

So, you know the curves that we gave earlier we discussed earlier it is related to those curves, right. So, that is how we find out the volume to surface area ratio. k 4 is related to slump; more the slump k 4 is more which means my ultimate shrinkage will be more. So, s is the slump. So, 0.00264 s slump in mm, so for 100 mm you know or 100 more the slump more or lightly. In fact, it is talking in terms of the water content of the system. So, more the slump you know more is the shrinkage.

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**Prediction of Drying Shrinkage**

$$k'_5 = 0.30 + 0.014 \frac{A_f}{A} \left( \frac{A_f}{A} \leq 50\% \right)$$

$$k'_5 = 0.90 + 0.002 \frac{A_f}{A} \left( \frac{A_f}{A} > 50\% \right)$$

$A_f$  is fine aggregate &  $A$  is total aggregate

$$k'_6 = 0.75 + 0.00061\gamma; \gamma \text{ is cement content kg/m}^3$$

$$k'_7 = 0.95 + 0.008a; a \text{ is air content}\%$$

B. Bhattacharjee  
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI  
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We can look into k 5. k 5 is related to fines to coarse aggregate ratio. So, equations are given again. If the fine to coarse total aggregate content is less than 50 percent; in other words if my fine content in the concrete is less than 50 percent, alright. So, equation is here. If this is more if more fines are there more shrinkage; if lesser fines are there less shrinkage, because coarse aggregate is the one which will actually restrict the shrinkage.



So, more fines in the system you will likely to have more shrinkage, and if this is more than 50 percent fine is very high; this value is relatively less you know 0.90, but this value is higher. So, two empirical formulas is given depending upon the fine content of the system.

So,  $k_5$  is related to cement content, and higher the cement content  $k_6$  is related to cement content, higher the cement content more will be the shrinkage, because paste will increase and this is given in kg per meter cube. And  $k_7$  is related to air content, because air content means more void and therefore, it is related to higher the air content higher will be the drying shrinkage. So, what you have seen. There are seven factors through which actually you can predict the shrinkage. Number one factor  $k_1$  prime was related to curing. So, for the datum is taken as one to three days of steam curing or seven days of moist curing which is a standard practice. So, if you are curing less than that you will have higher value of  $k_1$ ; if it is more than that then you will lesser value of  $k_1$ . So, it will depend upon curing condition.  $k_3$  is related to volume to surface area ratio.

$K_2$  depends upon humidity condition. So,  $k_2$  depends upon humidity condition;  $k_3$  depends upon member volume to surface area ratio, and  $k_4$  depends upon slump,  $k_5$  depends upon fine aggregate content and six depends upon the cement content and seven depends upon the air content of the system. So, there are these seven factors, and it is pretty easy to calculate from this formula knowing all those factors, and you can estimate the ultimate shrinkage and shrinkage at any other state. For example, at any other time when you have actually started drying at  $\tau_0$  can be related by the first formulae that I have given. So, therefore, you predict the shrinkage at any time using this formula that is given in the 290 r.

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**Prediction of Drying Shrinkage**

- BS method gives 6 months and ultimate (30 years) shrinkage and swelling.
- In British method shrinkage is determined from figure for 190 lit/m<sup>3</sup> of water. For other water contents the proportional shrinkage values are calculated. Equations are available for estimation of swelling and shrinkage

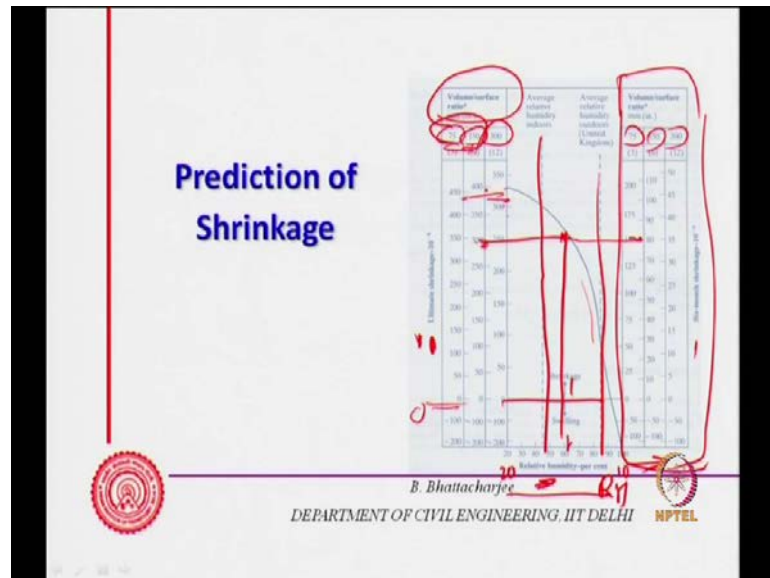
$$s(t, \tau_o) = s(28, \tau_o) + 100[3.61 \ln(t - \tau_o) - 12.05]^{1/2}$$

B. Bhattacharjee  
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI  
NPTEL

British method of course gives two cases 6 months and ultimate that is 30 years shrinkage and swelling, and it gives you in a kind of a nomogram like we had one. So, in British method what you do? Actually it is determined from figure of course a nomogram for 190 liter per meter cube of water. For other water contents the proportional shrinkage values are calculated; equations are available for estimation of swelling and shrinkage. So, like we talked about the creep you know in case of creep there was a nomogram; here also there is a nomogram, and you can estimate the swelling and shrinkage from the 6 month or 30 years, 30 years you know ultimate shrinkage which will see from the nomogram you can obtain it.

But you know this method of course is referring to a nomogram. It is also slightly less elaborate, because we can see the factors that it takes into account while the ACI method of course is definitely more elaborate taking as many factors as possible seven factors as we have seen, and it is easy to implement in Microsoft excel or some such sort of a spreadsheet or similar sort of thing. So, it is easy to actually utilize the method. So, their first formula is  $s(t, \tau_o)$  and  $t$  is in  $\tau_o$  is  $s(28, \tau_o) + 100[3.61 \ln(t - \tau_o) - 12.05]^{1/2}$ . So, what you want to know at anytime you want to find out with drying starting from  $\tau_o$ , anytime you want to find out drying starting from this date, first you have to find out  $s(28, \tau_o)$ , and then you can relate this. Now  $s(28, \tau_o)$  can be obtained from six months or ultimate shrinkage by formulae which are available.

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Let us see the case of this nomogram. This nomogram as you can see it will be relative humidity  $r_h$  along this axis. So, it is very strong from 20 to 100, and this is the average humidity in united king condition, this is the average relative humidity inside in u k, but elsewhere it could be different. So, this side is shrinkage, this is swelling. So, the value of shrinkage air is zero; you know y axis gives you ultimate shrinkage, right, and this gives you that is 30 year shrinkage, and this side this portion gives you six month shrinkage and which can be related to 28 days shrinkage, right.

So, it is related to relative humidity one factor that is taken into account and age factor there are two ages that is given. So, ultimate and basically ultimate and 6 months shrinkage whichever you what, and they can be related to 28, and then at any age you can find out. Now this is volume to surface area ratio in millimeter or in inch in bracket; so for 75 millimeter volume to surface area ratio and 150 and 300 and here again 75, 150 and 300. So, what is happening? This 300 is this scale. So, supposing I have relative humidity let us say is 60 percent. So, I go here and I know my volume to surface area ratio is 150.

So, I will come to this scale, and it will be close to 300 into 10 to the power minus 6 for six months, and if I am to get for ultimate shrinkage for the same relative humidity then I would go for this is for 150 you know this is volume to surface area ratio. So, this is ultimate 300; six months will be somewhere here which is about 80. So, for 60 percent

relative humidity if I have a volume to surface area ratio is close to 150, I will have 300 micro strains. After 30 years while I will have about 80 micro strains after six months.

Now anything anywhere in between you know you can for example, this is scale this is graduated. So, if you have volume to surface area ratio, say, 145; sorry this is graduate from micro strain 145 then you have to actually interpolate between these two. You can determine for 75 and determine for 150 and then possibly as you know and then for 300 also, but they are not linearly varying while this is 450, this is 400, anyway along this line this will be 400 less than 400, and this will be much less than 350. So, one can even plot a curve with three parts and then interpolate.

So, some kind of interpolation is possible, but this you can see is relatively less elaborate besides actually you know graphs are really cumbersome these days. So, what you have on this scale is relative humidity 20 to 100. This curve is there which you can use in this nomogram; here the ultimate creep three volume to surface area ratios are given and from this curve you can arrive at the ultimate shrinkage knowing the volume to surface area ratio that you are working on for six months shrinkage you are on this side.

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So, therefore, what we see is that both this method can predict the shrinkage. So, let us summarize this then. To summarize this essentially what we have looked into? We have looked into mechanisms of shrinkage today, and we have explained fully that how it is related to interlayer movement of interlayer water, and the theories those are available

we have put them forward, and we have understood that it cannot be reversible, but you see this when you have more than one theories it is not very easy to come out with a generalized mathematical formulation. So, therefore, you have empirical formulae or predication of this one.

So, we have seen on what does it vary, what does it vary actually? What are the factors on which it varies? We looked into mechanism, then we looked into factors on what it varies, and then the predication as per various course that is what varies. So, factors major factors that we have seen of course is a relative humidity; relative humidity curing condition, relative humidity condition, volume to surface area ratio, and fine aggregate content in the total aggregate system, slump, air content, etcetera. Now some of those factors and then aggregate of course the aggregate type we have seen. So, what are basically factors which effect that that we have seen, and then we seen the predicting formulae, and we have also understood that it is possibly easier to predict through the SCI method rather than the British method which is existing, right.

So, the next thing that we would be discussing in the next class is the measurement of shrinkage. Now one you know the Le Chatelier methods were used for measurement of shrinkage of paste conventionally and other is the chemical shrinkage. So, some modern methods are there; that is what we look into, and then potential of shrinkage cracks that can occur. That is what we shall be looking into the next class, right. So, with that actually we will conclude our discussion today.

Thank you.