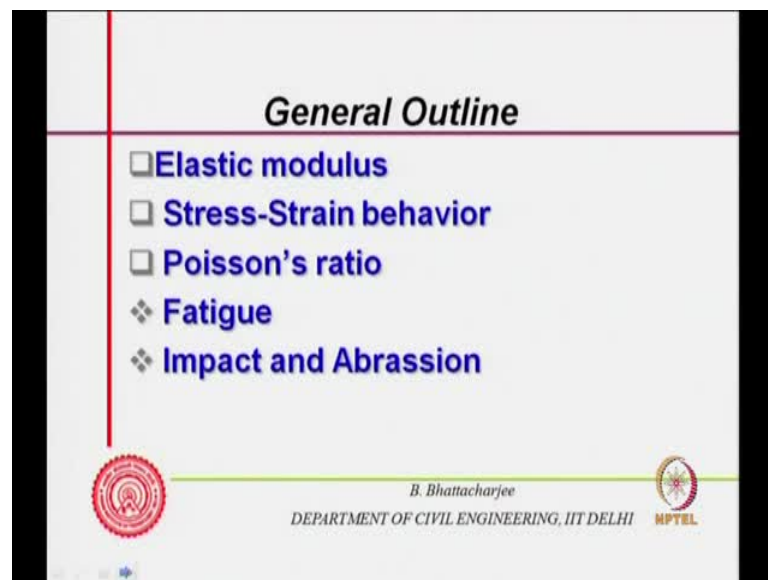


Concrete Technology
Prof. B. Bhattacharjee
Department of Civil Engineering
Indian Institute of Technology, Delhi

Lecture - 26
Mechanical Properties of Concrete: Elastic Modules,
Poissons' Ratio, Fatigue,
Impact

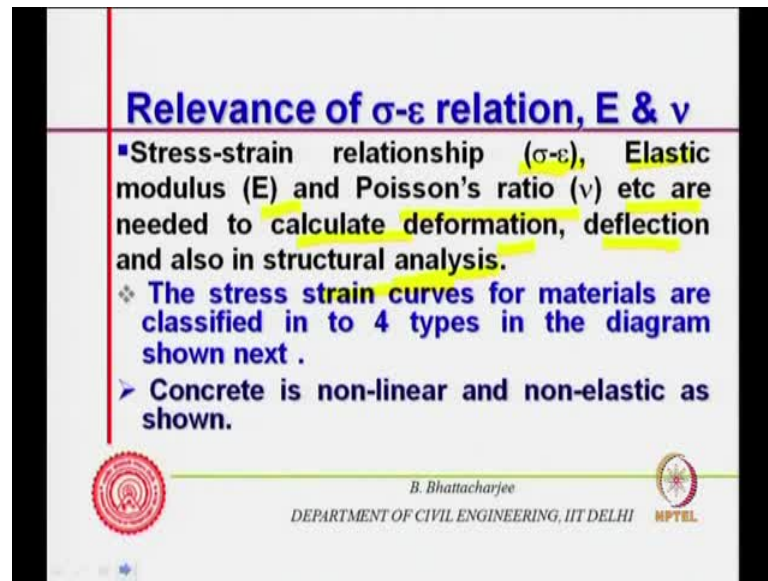
Welcome to module 6 lecture 4. And the last 3 lectures of this module, we looked into mechanical properties of concrete namely composite strength and tensile strength. Also we looked into what are the factors which effect the strengths particularly compressive strength and including the test factors. Now, compressive strength is the most important property of concrete being a material, which can with stand more compressive strength as a brittle material. So it can (()) a lot of compressive forces compared to you know compressive stress compared to tensile out pull tensile stresses. But then other important mechanical properties are elastic modules, Poisson's ratio fatigue, abrasion resistance and impact resistance. So, today we will be looking into this importance important properties,

(Refer Slide Time: 01:42)





So, first we will look into elastic modulus followed by stress strain behavior poissions ratio fatigue impact and abrassion.

(Refer Slide Time: 01:58)



Relevance of σ - ϵ relation, E & ν

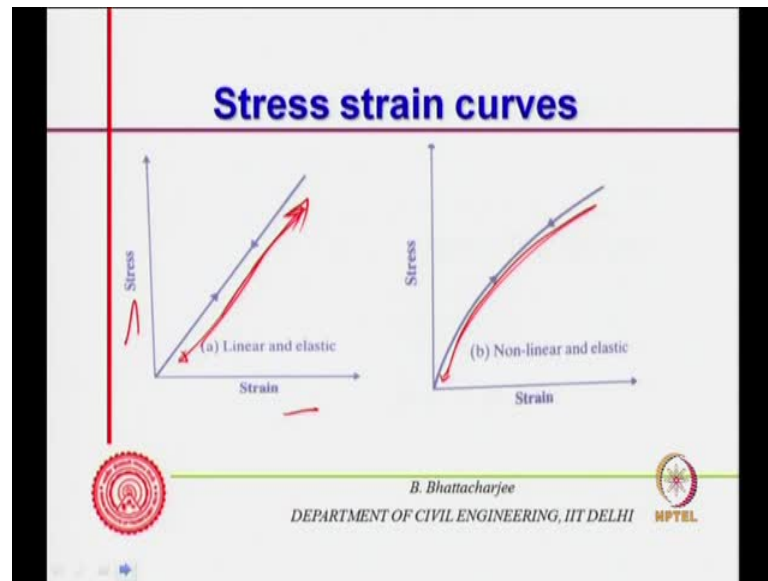
- Stress-strain relationship (σ - ϵ), Elastic modulus (E) and Poisson's ratio (ν) etc are needed to calculate deformation, deflection and also in structural analysis.
- ❖ The stress strain curves for materials are classified in to 4 types in the diagram shown next .
- Concrete is non-linear and non-elastic as shown.

 *B. Bhattacharjee*
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI 

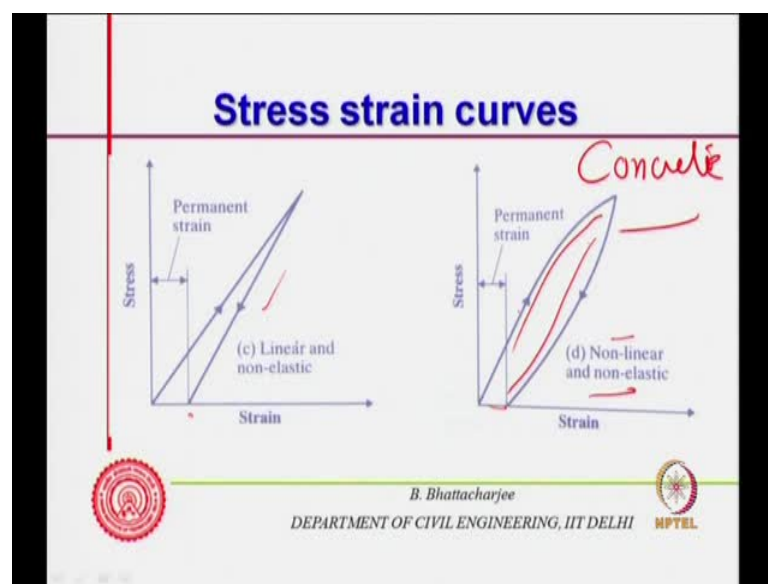
So, let us look at relevance of elastic I mean stress strain relationship and elastic modulus and Poisson's ratio, stress strain relationship. You know sigma I how we denote stress an epsilon. Elastic modulus and Poisson's ratio are needed to calculate deformation, deflection. And you know it is necessary to determine also forces some time in indeterminate structures. So, structure analysis would need this modulus of elasticity particularly and deflection etcetera are dependent on all this. So, these properties are relevant and the stress strain curves for material is important. Because in structural design you know they have their relevance. Now, stress strain curves of materials we can classify into 4 types for all materials in general. Because, we have left out elastomass but, typically these are the 4. So we will see that concrete is a of course, non-linear and inelastic or non elastic material.

Now, simple linear and elastic material linear inelastic material strains versus stress it will follow this stress strain curve during loading. And when you unload it comes back just back to the original with no residual strain remaining this is non-linear and elastic. So, in elastic material will come back to its original state when load is withdrawn. But, this is not linear it is non-linear.

(Refer Slide Time: 03:25)



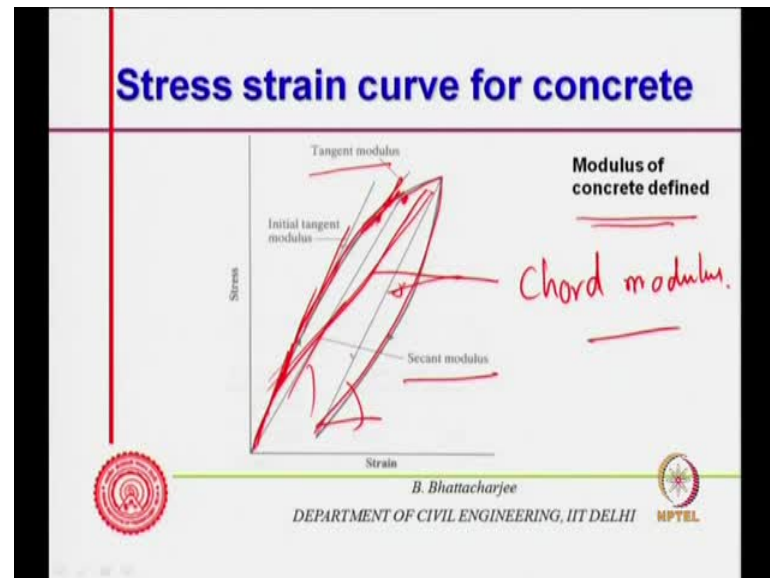
(Refer Slide Time: 04:01)



And if, you follow it up further this is off-course linear but, in elastic or non elastic you know it does not come back. So, there is a residual permanent strain and this is non-linear and non elastic. So there is permanent strain it goes like this at some level of stress you go and when you come back it does not close in. So there is some kind of a hysteresis and it is non-linear non elastic. And concrete belongs to this so concrete belongs to this material concrete cement paste or similar material they belong to this. So, it is actually non-linear as well as in elastic. So, when you release load beyond of course,

certain point not in the beginning is at least 1 by 3. If you go on beyond that there will be some permanent strain when you release the load.

(Refer Slide Time: 04:55)



Well, since it is non linear we have to define the modulus of concrete since non-linear. So, its behavior is something like this and then it follows something like this. So what do we define something called initial tangent modulus? Initial tangent modulus you know this is what it is. So the tangent if you draw here that is we call as initial tangent modulus. But, I can draw tangent anywhere that is called tangent modulus in that particular load. But, I can join the origin with this point and I get another modulus the slope of this line and that we call as secant modulus. And while returning I can also calculate out the same secant modulus from this point the slope of this line. So, for concrete modulus can be defined in different ways initial tangent modulus tangent modulus at any stress or strain level. And secant modulus again at any stress or strain level in fact we can have a chord modulus, joining between 2 points. You can have a chord modulus. So joining between 1 point and another point so that is how we define the elastic modulus for concrete.

(Refer Slide Time: 06:29)

Stress-strain behaviour

- Non-linearity in concrete at usual stresses is attributed to creep.
- ❖ Two ways of measurements are static and dynamic modulus.

$V \propto \sqrt{\frac{E}{\rho}}$

B. Bhattacharjee
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

NPTEL

Now if, you look at stress strain behavior it is non-linear that is what we have shown. And this is attributed to something called creep that is the deformation under the sustain loading we will come to that sometime its 1 way of look at it. But, there is something more we will look into. We can measure this under static condition another reason is under dynamic condition. That means static condition means you apply the load monotonically in steps. And then see how the deformation or strain changes. So stress at steps and strain at major corresponding stress. And from stress strain can find out the modulus of elasticity.



But, you see from you know concepts of mechanical wave travelling in solid media. We know the velocity in the media is a function of velocity in the media is function of E . You know it is a function of E root E over ρ actually you know. So for velocity is proportional to E over ρ . So therefore, modulus of velocity has something to do with this velocity something to do with the mechanical wave or sonic velocities, pulse velocities. So, when wave is travelling through the solid you know there is a compression and rare fraction longitudinal wave travelling through it. Or any wave travelling through it there are deformations. And therefore, the modulus of elasticity plays a role. But, what kind of modulus of elasticity that, will be bulk modulus. So, you can relate this 2 anyway this can relate this 2 velocity. So if you have actually obtained velocity in the velocity of wave travelling through this medium you can actually estimate modulus of elasticity.

(Refer Slide Time: 08:33)

Stress-strain behaviour

- Non-linearity in concrete at usual stresses is attributed to creep.
- ❖ Two ways of measurements are static and dynamic modulus.

B. Bhattacharjee
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI





Now, that is called dynamic modulus so dynamic modulus is related to dynamic modulus is related to wave propagation within the material itself. Because velocity of wave propagation or characteristics you know wave propagation is related to elastic modulus and one can determine that wave.

(Refer Slide Time: 08:47)

Stress-strain behaviour

- Non-linearity in concrete at usual stresses is attributed to creep.
- ❖ Two ways of measurements are static and dynamic modulus.
- Dynamic modulus corresponds to initial tangent modulus.

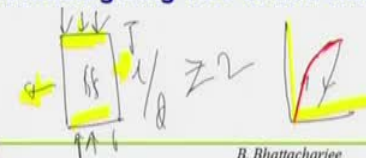
B. Bhattacharjee
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI



(Refer Slide Time: 09:00)

Stress-strain behaviour

- Non-linearity in concrete at usual stresses is attributed to creep.
- Physically, micro cracking of concrete at usual load results in apparently permanent deformation giving rise to non-linearity.



B. Bhattacharjee
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

Now, physically off-course, micro cracking of concrete at usual load results in apparently permanent deformation and that is why it gives rise to non-linearity. I mean creep is 1 which but, why finally. So, micro cracking of concrete at usual load results in apparently permanent deformation, giving rise to non-linearity. That means you know whenever, you are actually for example, if I have a composite specimen applied load like this. Now if it is sufficient l by d is sufficiently large let us, say l by d is sufficiently large l by d is equals to 2 or something of that kind. So, whenever I am applying load remember that we said the cracks will form here there will be cracking micro cracks will develop.

Because of the tension along this direction you know you recall we discuss there will be tension of this direction. And therefore, micro cracks will develop. Now, after side inferior of time suppose we draw the load this you know at this results in deformation along this direction to. So reduction is a length micro crack will form and there will be reduction in the length. Now, when you release this load this micro cracks are not going to fill up they remain as permanent features in the material. And therefore, the it will not go back to its original state. So, once the micro cracks have formed it will not go to its original state. So, there is some kind of this will actually you know manifests themselves this cracks manifest in terms of the deformation permanent deformation.

So therefore, permanent deformation this permanent deformation and also this gives rise to non-linearity. So, permanent deformation non-linearity both comes to this. Because suddenly there is an non-linear depends at given stress suddenly there is an increase the modulus of velocity stiffness reduces. Because the cracks has occurred, so stiffness would reduce. And therefore, the material would tend to show load modulus of elasticity or lower slop. So material shows a kind of non-linear behavior to start with it is like this to start with it is like this. You know it goes like this but, then there is suddenly there are lot of crack formation. So, modulus changes and this if this continues if this modulus is changing when it will show a kind of non-linear behavior. And if this crack is on close then it will show permanent deformation. When it is coming back it will show permanent deformation. So, that is the idea that is the basic you know explanation to why it shows non-linear behavior.

(Refer Slide Time: 11:51)

Stress-strain behaviour

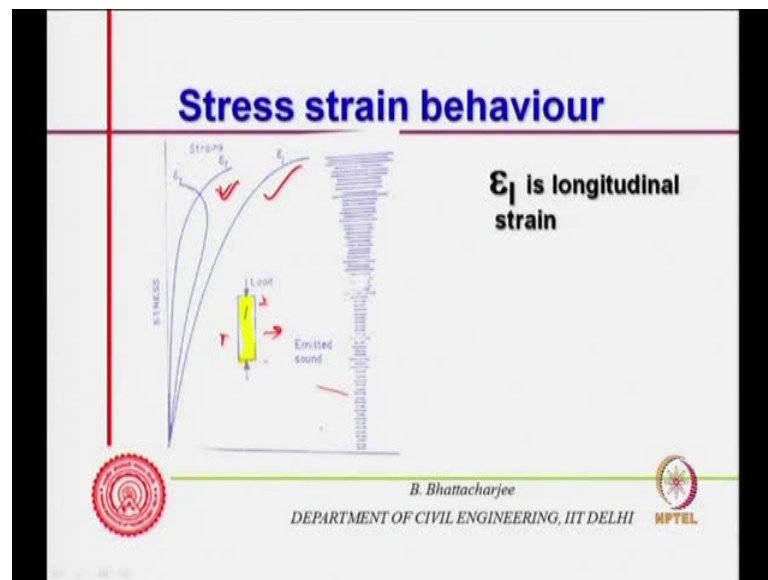
- Non-linearity in concrete at usual stresses is attributed to creep.
- Physically, micro cracking of concrete at usual load results in apparently permanent deformation giving rise to non-linearity.
- Acoustic emission & Ultra sonic pulse velocity test confirms the micro-cracking.

B. Bhattacharjee
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

NPTEL

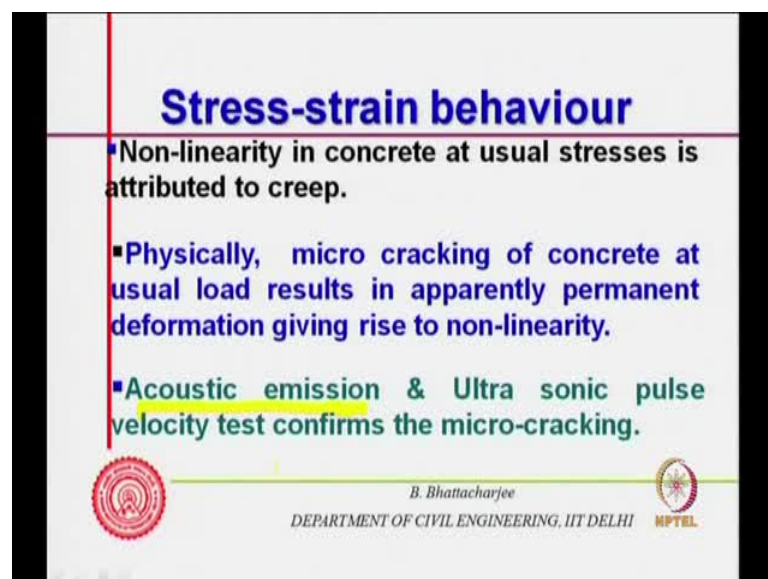
Now, this has been confirmed by test such as acoustic emission and ultrasonic pulse velocity test. So micro cracking is generally confirmed by this. Now, let us see let us look at this, so you see if I look at if I applied load like this. You know if I apply load like this whenever the specimen is here, if I apply load like this specimen is here if I apply load like this. Now, this is longitudinal strain that means strain along this direction, the strain along this direction dimension change.

(Refer Slide Time: 12:04)



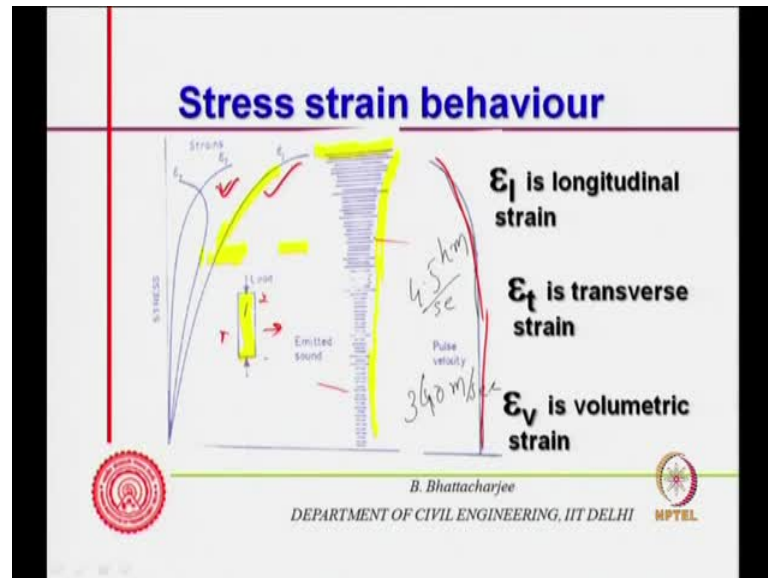
So strain along this direction that is longitudinal strain. Then this is transverse strain that means if I measure the strain along this direction that is transverse strain. And volumetric strain because, volume changes original volume change in volume divided by the original volume. Now, volume actually can be reduction in the volume change because this is reduction. This side there is an expansion and net effect initially there is an expansion then there can be a reduction in the volume as such. Now what is acoustic emission?

(Refer Slide Time: 13:05)



So, this is Acoustic emission we just mentioned about acoustic emission emanate. Before we mentioned that you know you can we, you do acoustic emission acoustic emission and ultrasonic pulse velocity confirms micro cracking.

(Refer Slide Time: 13:18)



So, what is acoustic emission? We, just quickly look in to this for our purpose as much as required for our purpose what is the acoustic emission. You see if you take let us say piece of timber or bamboo simply and try to break it. You will find that the sound comes out once its starts cracking once it starts breaking. Now, this sound is you know some of the strain energy is converted into acoustic or sound energy. You know, some of the strain energy is converted. So even we apply strain energy part of the strain energy is converted into surface energy for creating crack. But, part might go as some sort of acoustic energy because you here sound.

So, when you break a timber piece you here sound. So similarly, a bamboo pieces you hear sound that is basically a kind of when it cracks. So this crack at the time of cracking there's some sound emission. So, what has been observed is so that is actually acoustic emission in a systematic manner one can actually trace this find it out. So, emitted sound as I apply the load, as I increase the load, initially emitted sound is very less and they go on increasing and the sound starts. You know lot more sound emitted sound becomes more as you know loading progresses. In other words permanent deformations or non-linearity is related to all these are related to somewhere. It is lot of cracking and this is

confirmed by a ultrasonic pulse velocity. Also you know sonic pulse velocity. Now, what is pulse velocity in a solid material is more compared to crack material.

Because, velocity of sonic or mechanical waves in solid concrete various solid concrete could be of the order of around 4 point 5 kilo meter per second. So, they it could be of the order you know solid material it could be of the order of around 4 point 5 kilometer per seconds. But, in here we know it is around 340 meter per second. So if you have lot of cracking occurring cracks have occurred then pulse will velocity will reduce. So, pulse velocity starts reducing initial reduction is relatively slow. But, then it stars reducing as soon as lot of cracks starts appearing. So therefore, the cracking of concrete is confirmed by this kind of testing. And therefore, if I am trying to plot the stress strain curve you know stress as stress increases longitudinal strain. There it shows that the non-linearity of the stress strain curve is largely related to micro cracking of concrete.

(Refer Slide Time: 16:11)

Modulus of Elasticity test

- Two type of tests a) Static test b) Dynamic modulus test.
- Static test involves several cycles of loading and unloading prior to actual test to reduce the effect of creep and Hysteresis such that curvature in stress-strain curve is small.
- In the testing machine, stress cannot be reduced to zero, So Chord modulus is determined as $E = (f_a - f_b) / (\epsilon_a - \epsilon_b)$. $f_b = 0.5 \text{ MPa}$; $f_a = 1/3 f_c$

B. Bhattacharjee
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI
NPTEL

And so, is that case is permanent deformation permanent deformation is also related to micro cracking. So, this is the reason why you get non-linear behavior as well as you know non closure of the stress strain curve theirs is a hysteresis. Now, I mentioned that I can determine it you know I can have static modulus of elasticity or dynamic modulus of elasticity. So, we go to do test in 2 different ways, static test is done in a, you know monotonic loading. But, what do we do is, we actually initially apply a several cycles of

loading and unloading at 1 third of the estimated cylinder strength. You know we normally test on cylinder we do test on cylinders so, central 1 third is used metering.

This is the portion we use for metering, this portion we use for metering. So, you put gauges actually in good old days we had actually lamb's extension meter it is called lamb's extension meter. You know it is called lamb's extension meter that is what we are using. So, we will put the actually the gauge there gauge you know there so in the central now this will be 300 mm and central 100 and 50 mm. So, you push put the gauge here. But, the deformation is relatively small, So how do you measure this deformation? It has to be amplified in some manner it was actually amplified optically during those, days earlier days today. Of course, you can put in several types. You know you can put in transducer of the kind of strain gauges various kind of sensing devices. You can put in as to measure this you know deformation. So, several actually deformation measuring devices are there and one can of strain gauges could be there to measure this.

So, earlier this lamb extension meter was used so no I am not going into that translucent part of this at the moment it is not necessary. But, if you are doing a laboratory test then you will be using any one of these, this is the old traditional way. And then, you measure the deformation there the deformation there within that small range. But, before doing that what you do is, you crush the cylinder to find out to the FCY cylinder failure strength. And then load it up to 1 third of FCY that is cylinder strength and then unload it.

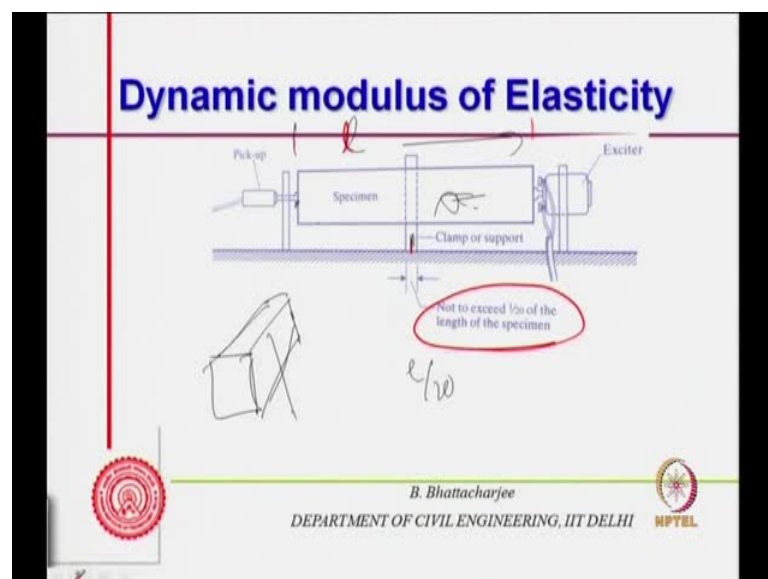
So, this cycles are repeated to see that at least first of all the you know there is no in there will be no deformation in the beginning. Because of plate settling down and things like that number 1 number 2 this is prior to micro cracking. So, what about a small crack that occurs should occur. And then after that actually the actual test will be done. So, this will reduced down the effect of creep some micro cracking would occur and then hysteresis also will be reduced. Because of such micro cracking and permanent deformation. So besides that there's another important issue why we do it this way.

Actually never the load is never applied 1 time in a structure you know we for example, if you consider the room. You are sitting and if it a, reinforce concrete slab on which you are sitting the before prior to actually, even loading. There will be several loading unloading that would occur actually during construction phase or even later.

On several loading and unloading that occurs. So therefore, determining the stress strain curve after some loading and unloading that what. So, what do you do we actually do about 15 times loading and unloading. You know the codes most of the codes would tell you how to do error so roughly about 15 times I think most of the codes would say fifteen cycling 15 times. You know loading and unloading and then you measure the actual strain versus stress. So, load is then applied monotonically in steps and corresponding step to corresponding strain are measured and it is plotted. So, in the testing machine of course, stress cannot be reduced to 0. So, what you do is we find out chords modulus.

Finally after this kind is obtained we find out chords modulus as given by this formula chords modulus. So we find out a chords formulas so fb. So, we actually find out chords modulus like this you know because this is difficult to reduce it to 0. So corresponding 2 point 5 MPa this is epsilon this is sigma point 5 MPa. So, fb so corresponding to minimum stress of point 5 MPa then up to 1 third fc its within the you know kind of savior micro cracking that to savior micro cracking. So, fa minus fb epsilon a minus epsilon b. So, you find out this is your epsilon b this is epsilon a, and sigma or f or whatever you call it. So, you find out a chords modulus according to this or you know sop the code suggest how much where you have to go to test. So, fc 1 third fc is the elastic limit and often you might find it there itself. So, this is how we determine the static modulus of elasticity.

(Refer Slide Time: 22:13)

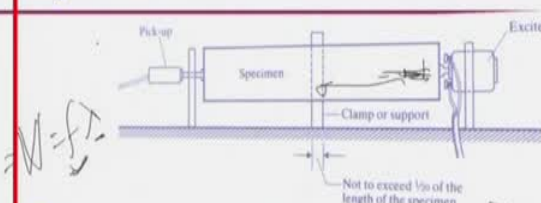


Dynamic modulus of elasticity is determined in this prospective view of the diagram. That you see, you have a clamp support which should not exceed $\frac{1}{20}$ of the length of the specimen. So, this is length of the specimen this is length of the specimen this is l the length of the specimen this is l and this should not exceed l by 20. So, it should be less than l by 20 there is a clamp of the support. So, a specimen rectangular specimen generally rectangular cross section. You support it rectangular cross section actually you know clamp it somewhere in between like this. And then you have an exciter that actually imparts acoustic imparts actually, acoustic pulses or acoustic you know wave basically there.

So, there is a longitudinal wave will be travelling along this direction they will be travelling along this direction and there is pickups which will pick up the, you know pulse transmission. So, signal that is transmitted through it so this picks up and at as you go on changing the frequency, of this sonic wave. That is tem travelling along this direction that is exciting this at time will come, at certain frequency there will be resonance occurring there is resonance occurring. Resonance should be occurring when actually this dimension of this one matches with the wavelength I mean some sort of matching with the wavelength. Because the mode would depend upon because, it is a central support your trying to vibrate it. And it will be related to length of it is related to the frequency resonance frequency is related to the length of the specimen itself.

(Refer Slide Time: 24:14)

Dynamic modulus of Elasticity



■ Longitudinal vibration is propagated (100-10000Hz) to obtain fundamental resonance frequency, when $L = \lambda/2$; $E = 4n^2 L^2 \rho \times 10^{-15}$; n = resonant frequency in Hz, L = length, ρ = density in kg/m^3 .

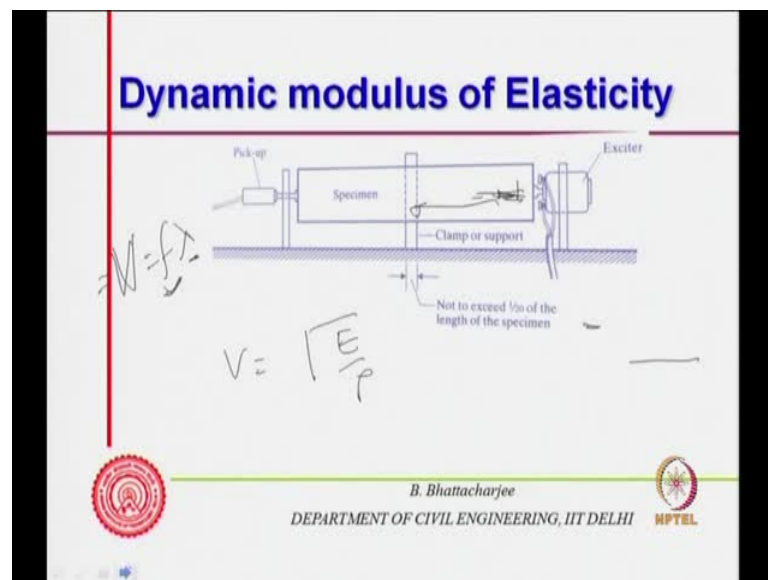
B. Bhattacharjee
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

NPTEL

Now, how it is related, it is related like this. Longitudinal vibration is propagated you know. So, you will have vibration along this direction, longitudinal vibration. So it would be actually it would be actually exciting in this mode. So longitudinal vibration so a compression and rare fraction will be travelling along this direction. And, from 100 to about 10000 hertz at 100 to 10000 hertz. This range you actually make it to propagate and you will get fundamental resonance frequency. When λ is equals to in a $2L$. So, when wavelength is equals to $2l$ where L is the length. So, or λ is equals to l is equals to λ by 2 at that time resonance would occur.

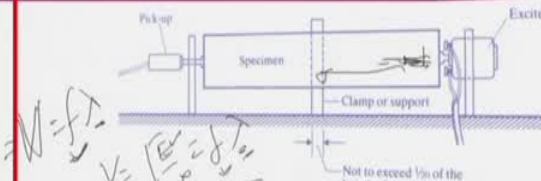
So, you find out the frequency at which resonance is occurring and you know V is equals to $f \lambda$. So the frequency you have determined therefore, λ is equal to length. You know λ is equals to $2L$ and V is actually just I mentioned a few minutes before. V is a function of E by ρ therefore, you can actually relate e actually you can relate to V is a function of E by ρ . And is also equals to f by λ and λ is actually at resonance frequency is known λ is given by this equation l is equals to. Since you know l So, λ is $2L$, in other words you can find out E . Because f is known l is known and this is.

(Refer Slide Time: 25:22)



(Refer Slide Time: 25:31)

Dynamic modulus of Elasticity

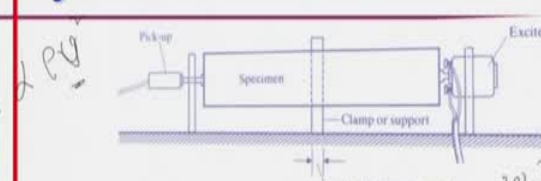


■ Longitudinal vibration is propagated (100-10000Hz) to obtain fundamental resonance frequency, when $L = \lambda/2$; $E = 4n^2 L^2 \rho \times 10^{-15}$; n = resonant frequency in Hz, L = length, ρ = density in kg/m^3 .

B. Bhattacharjee
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

(Refer Slide Time: 26:14)

Dynamic modulus of Elasticity



■ Longitudinal vibration is propagated (100-10000Hz) to obtain fundamental resonance frequency, when $L = \lambda/2$; $E = 4n^2 L^2 \rho \times 10^{-15}$; n = resonant frequency in Hz, L = length, ρ = density in kg/m^3 .

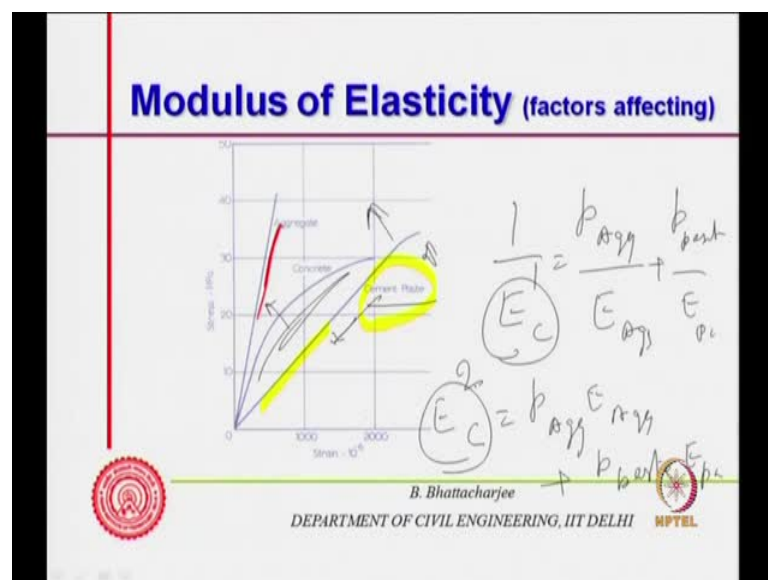
B. Bhattacharjee
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

This is $2L$ is known this is $2L$ or you know. So, you can actually find out as $4n^2 L^2 \rho$ into the to the power 15. Where n is the resonant frequency you know f that I was saying n is the resonant frequency. So, you determine resonant frequency. So resonant frequency this is λ you know by 2 therefore, velocity and then velocity. And then the ρ comes into picture because, velocity is under root E by ρ . So, if you want to find out the E it is proportional to b^2 so ρb^2 . So E is actually E will be proportional to ρV^2 alright so ρV^2 . And therefore, you can find out V is given by $n \lambda$ and

from that you can find out. Therefore, knowing the density of the material so you have a assumed density or measured density of the material. From that, you can find out the modulus of the elasticity.

So, dynamic modulus can be determined in this manner 2 a first what you do is you actually impart longitudinal vibration or mechanical wave you know. So, excited through acoustic waves or ultrasonic or sonic waves from 100 to 10000 kilo hertz because the k 10 100 to 10 k alright. So, it is actually sound because sound is from 20 to 20 kilo hertz. So, it is not ultrasonic this is kilo hertz. So this is not ultrasonic, this is actually sonic excitation you can provide and then find out from this modulus of elasticity. So, this is how we find out the modulus dynamic modulus elasticity.

(Refer Slide Time: 27:56)



And we said that dynamic modulus of elasticity is initial tangent modulus. Because no cracks have occurred nothing has occurred in an un cracked specimen. Is that actually your trying to find out the modulus of the velocity usually these values are higher. So, now let us look at modulus of elasticity in terms of that, What are the factors that affect? So, factors affecting modulus of elasticity first one. You see here, is your cement paste, this is for cement paste. If you see this is for cement paste, this is stress strain type of cement paste. This is the stress strain curve of aggregate, generally would show something like that and concrete shows a stress strain behavior of this kind. So,

aggregate stress strain behavior is of course, fixed. You know that is fixed by I mean you have taken the rock and the wave here will be the fixed.

Cement is off-course, is under your control. So if you improve this cement paste modulus of elasticity concrete modulus of elasticity also will increase. So, it depends upon both the paste modulus of elasticity and modulus of elasticity of failure. You know aggregate I mean 1 can think in terms of the range will of course, be something like 1 by E of concrete should be equals to proportion of the aggregate.

$$P_{\text{aggregate}} \text{ or proportion of the aggregate, divided by } u \text{ of aggregate plus } 1 \text{ minus } P_{\text{aggregate}}$$
 that is your paste or proportion of paste divided by E of paste. Or it met might also be written as $P_{\text{aggregate}} \text{ proportion of aggregate multiplied by } E_{\text{aggregate}} \text{ plus proportion of paste multiplied by } E_{\text{paste}}$. So, this 2 will give you actual and extreme boundary of you knows the concrete modulus of elasticity. Assuming series and parallel model I am not going to go to the derivation of this. But, you know this so of course, this he the range this boundary this EC. You have calculated from this or E C calculated from these they give you 2 different values. So, this gives you EC 1 let me call it this is EC 2. This will actually give you the wide range of the modulus of elasticity. But, at the moment it is more important for us to understand the modulus of elasticity of concrete is a function of modulus of elasticity of the paste. And modulus of elasticity of the aggregate. So, high modulus aggregate will tend to give you higher modulus elasticity for concrete and high model. Obviously paste modulus if you improve by increasing their strength. Then you can actually get higher modulus of for concrete as well.

(Refer Slide Time: 30:44)

Modulus of Elasticity (factors affecting)

- Wet specimen exhibit higher modulus.

Handwritten notes:
 $E_{\text{Water}} > E_{\text{air}}$
Pores are filled with water

B. Bhattacharjee
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

So, first thing is this other factors which specimen exhibit higher modulus why because water has got E for E for water for water is more than E for here both are fluid by the way. So, wet specimen the pores are saturated with water. For wet specimen pores are saturated with water pores are filled with water they replace the air. So therefore, modulus of elasticity it is likely to be higher.

(Refer Slide Time: 31:31)

Modulus of Elasticity (factors affecting)

- Wet specimen exhibit higher modulus.
- Modulus of elasticity of aggregate strongly affects the modulus of elasticity of concrete, higher modulus aggregate exhibiting higher modulus for concrete. Volume fraction of aggregate also plays a role.

Handwritten notes:
 $p_{\text{agg}} = \text{proportion of agg}$
 $p_{\text{paste}} = 1 - p_{\text{agg}}$

B. Bhattacharjee
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

Because now, it is this pores are filled up with water instead of air modulus of elasticity of aggregate strongly affects the modulus of elasticity of concrete. Higher modulus

aggregate exhibiting higher modulus of concrete volume of fraction of aggregate will also play role. That is what we have seen for our previous formula, I said P aggregate that is proportion of aggregate and P paste.

(Refer Slide Time: 32:24)

Modulus of Elasticity (factors affecting)

- Wet specimen exhibit higher modulus.
- Modulus of elasticity of aggregate strongly affects the modulus of elasticity of concrete, higher modulus aggregate exhibiting higher modulus for concrete. Volume fraction of aggregate also plays a role.
- E can be related to f_c . E dynamic > E static.
- General relationship of the form $E = k(f_c)^n$ is used. $K = 5000$ & $n = 0.5$ in IS456.

Handwritten notes: $E = 5000 \sqrt{f_c}$

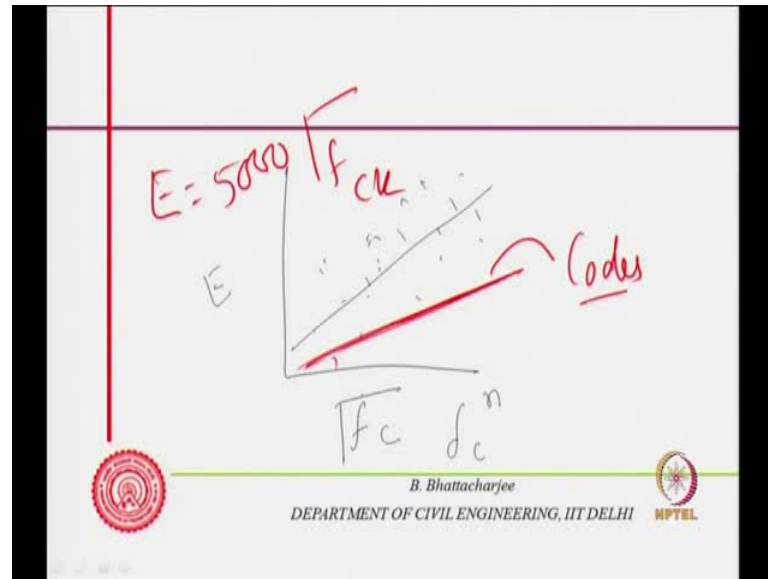
Logos: IIT Delhi, NPTEL

Text: B. Bhattacharjee, DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

Let us, say paste which will be equals to 1 minus P aggregate minus what so whatever there. So basically proportion will play again a big role because whichever bounds. You take upper or the lower bound both cases is function of those 2. You know the series of parallel model which gives you bounds both cases it will be related to that. Then mostly E can be related to mode FC well compressive strength. Now, E can be related to compressive strength you know it can be related to compressive strength. In fact we, have seen that modulus of elasticity governs the strength. Therefore, we can argue it out that a, strength would be also related to modulus of elasticity. Dynamic modulus is somewhat close to initial tangent modulus.

Therefore, it is more than e static. And general relationship empirical relationship people have found out for modulus of elasticity, is E is a function of k into come cube compressive strength to the power n. And you know IS 4562 1000 for design, uses this f_c as if f_{ck} characteristic strength and n as point 5. So, E is written can be written as 5 thousand under root f_{ck} . It is meant for design this actually conservative value in fact it will be something like this. You know if I just plot it here, before this I will just plot it here.

(Refer Slide Time: 33:44)

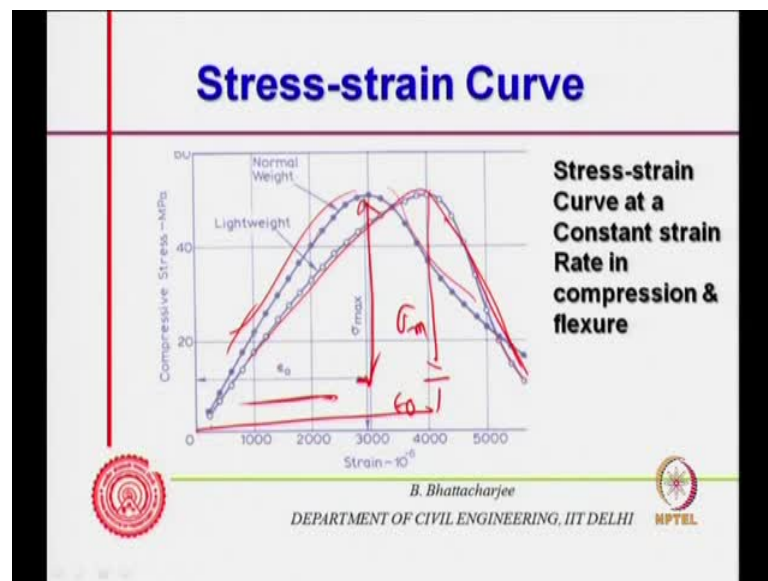


It will be something like this, you know if I plot f_c concrete strength and E then or under root f_c . Then you get all sorts of scattered results and you can plot a curve. So, this is what you are trying to do linear curve, what some people put you the some $1/c$ to the power n may be point 6. Or whatever, values are and $1/c$ can because this is you know scatters are there. Now, code would use a value of modulus of elasticity the code you know the actual values will be there. And the relationship that code tend to use is somewhere there conservative value of E it takes lower value of E . Because, affects of creep long term behavior or several other things comes into picture. So codes takes this sort of stand actually, actual values could be different.

Now, since this is you know under root of c I said it can be f_c to the power something of, so different codes may use different one, but I s 456 off course, uses E is equals to 5000 under root f_{ck} . So, you know the line is somewhere the slop is 5000 and under root f_c that is what is used somewhere down there. Because, it takes long time behavior and other things into account and therefore, try to use a conservative value. So that is related to what code uses and it can be related to composite strength. Now, next look into stress strain curve if you look at stress strain curve at a constant strain rate. Now last class last lecture if you remember we talked of constant rate of loading. Now, you can do 2 tests by constant loading rate of loading, that means you apply load at a particular phase 1 for 14 m p a per second.

So, you apply load at a constant rate and we have seen rate of loading actually changes the behavior you know like behavior changes. So, supposing I apply now I can apply in terms of strain at certain micro strain per second. Some strain rate so if I apply load at fixed strain rate, and I told you also that is difficult to apply. This you need a machine which is capable of applying the server control you know it is should capable of applying the maintaining the strain rate, at a fixed rate whenever required and that would require a control or in a server control system and so on so forth.

(Refer Slide Time: 36:23)



So, if you are applying a load at a constant rate you will find that they are depending upon normal weight concrete you know normal weight concrete behaves in this manner, while light weight concrete shows this kind of behavior. Because, you are able to now since strain rate is there so this portion of the current curve becomes available and this behavior is similar in compression as well as in flexure some sort of behavior like this. So, this is the ϵ_{max} or rather f_{max} or σ_{max} , maximum stress and corresponding to this we call as ϵ_0 . That is the strain at maximum stress strain at maximum for light weight this would be the maximum stress σ_{max} and ϵ_0 would be this much. So ϵ_0 and σ_{max} this is the 2 ones and you see this kind of behavior. So people have tried to fit it an approximate equations to this so empirical equations to this.

(Refer Slide Time: 37:24)

Stress-strain relationship

- A simple expression representing the stress strain relationship is as follows.

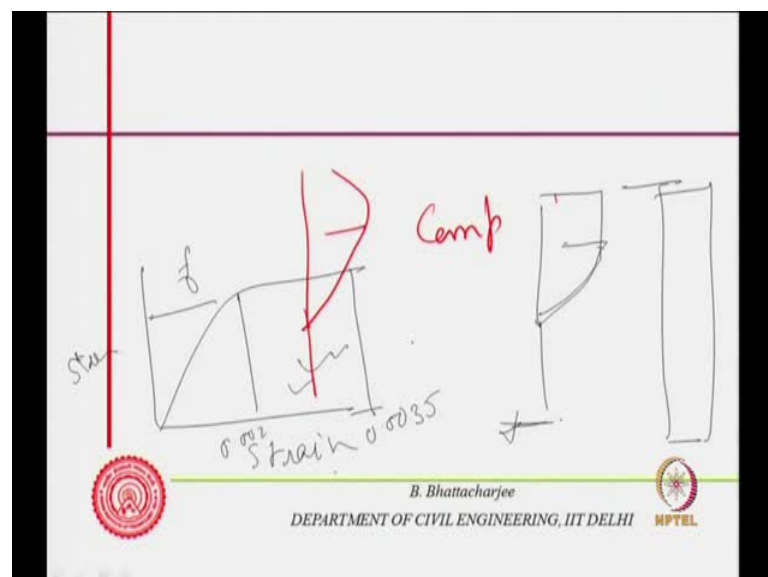
$$\sigma = \frac{E\varepsilon}{1 + \left(\frac{\varepsilon}{\varepsilon_0}\right)^2}$$

- E initial tangent modulus $= 2\sigma_{\max}/\varepsilon_0$, σ is the stress, ε is the strain and ε_0 is the strain at maximum stress.

B. Bhattacharjee
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

And these empirical equations that have you know representing this is the simplest there are maybe many more complex relationships. But one of the simplest relationship is of this form, Where E is the initial tangent modulus and you know, it is which is twice sigma max divided by epsilon 0 and sigma is a stress at any point, epsilon is a stress at any point, epsilon 0 is the strain at maximum stress. So, epsilon 0 is strain at the maximum stress. So this sort of formula is available through which you can define the Stress strain curve and there are complex formula also available. So this was stress strain curve of concrete. Now, we actually if you I just before I go to this in flexural design.

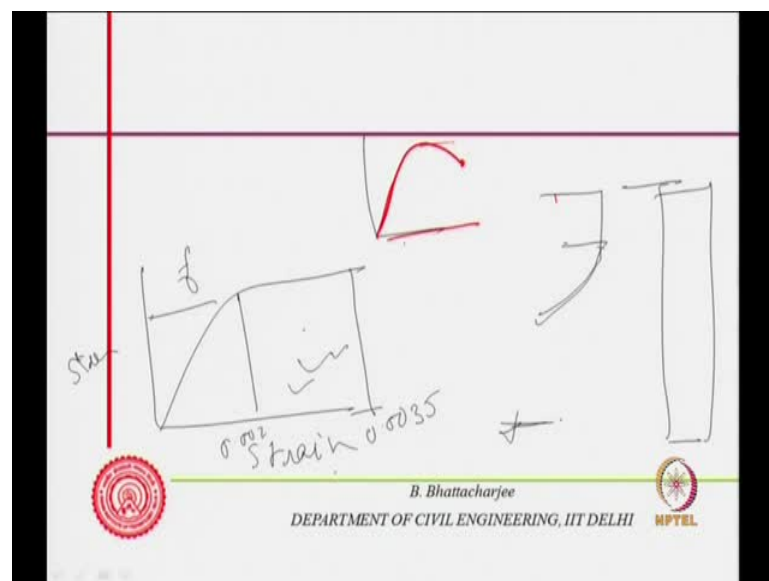
(Refer Slide Time: 38:15)



Therefore, one should be using this sort of curve and compression you know. So, this is if you remember flexural design flexural stress strain curve given in the code would be something like this. You know stress concrete stress strain curve of concrete if you recollect this is strain inflection, you know if, it is a beam section of a beam if I am looking at a section of a beam this is a reinforcement this is the concrete, anyway.

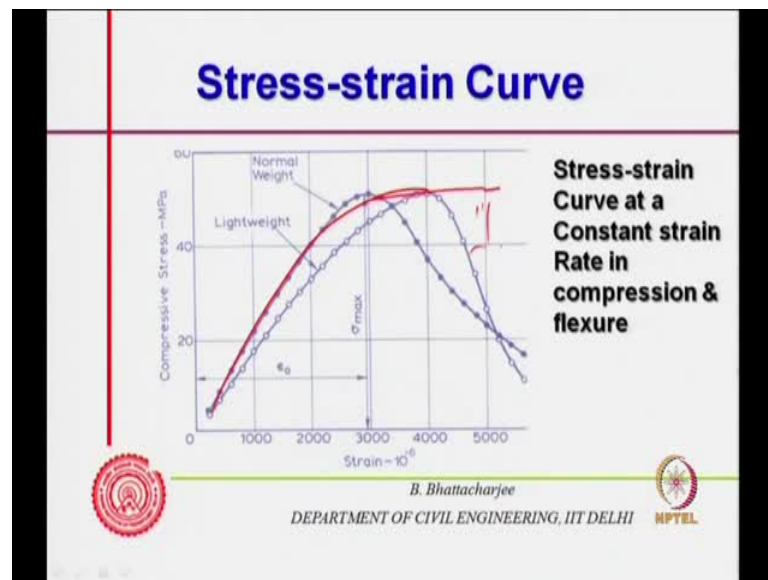
So, stress strain curve will be like this stress versus strain curve if you see in idealized stress strain curve it will be 0.002 here, And 0.0035 stress strain curve. This is actually you know f_c or whatever you call it the stress maximum pillar stress. Now, this is an idealized stress strain curve idealized stress strain curve of concrete in fact stress strain curve of concrete should look like something of this kind you know, some stress strain curve of concrete will look like let me draw it in red color.

(Refer Slide Time: 39:25)



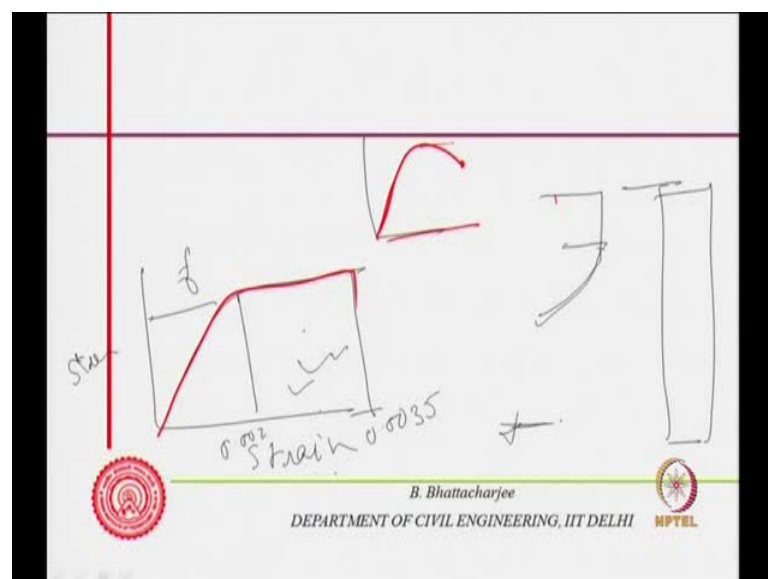
Stress strain of concrete will look like this maximum stress and here. Now, this is idealized stress strain curve beyond that you assume is as if it is straight line. So, actual stress strain curve of concrete will like this and this is important in flexural design. So, this measured stress strain curve that we have seen.

(Refer Slide Time: 39:52)

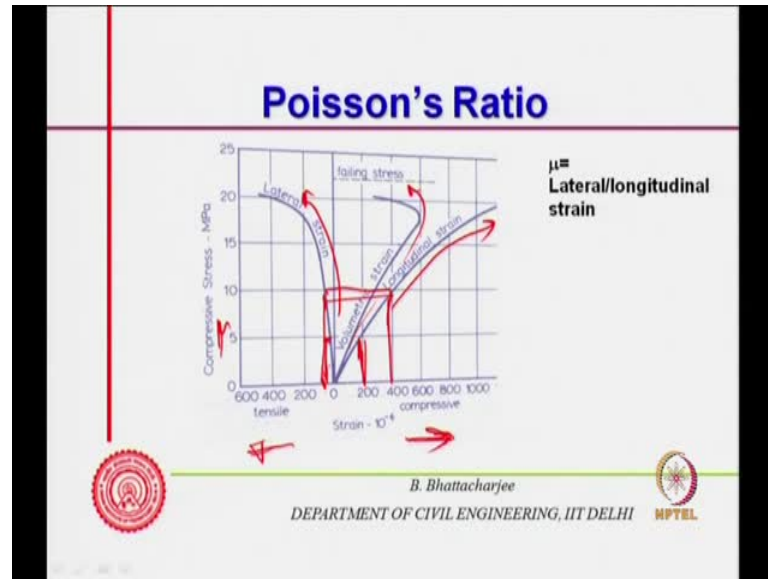


You know whether stress strain curve actual stress strain curve observed as we have seen this is idealized into this sort of a straight line curve. And off course certain factors are applied to bring down the load as well that I am not discussing at the moment.

(Refer Slide Time: 40:06)



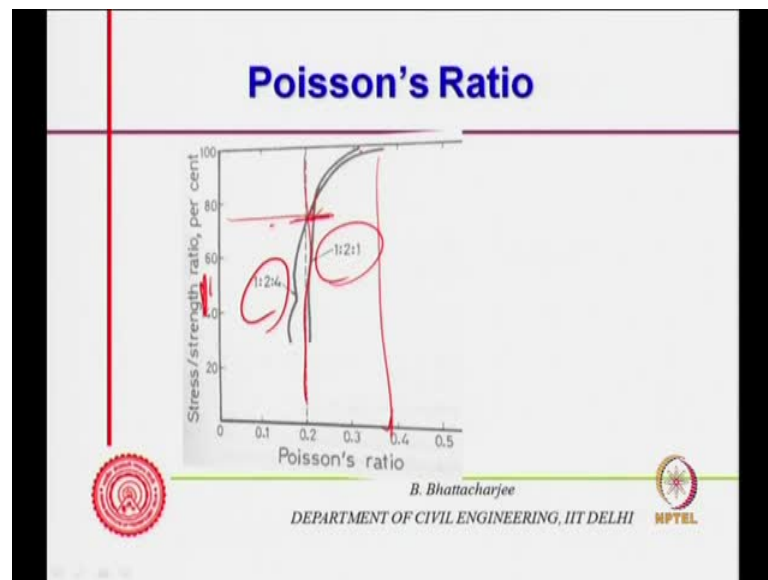
(Refer Slide Time: 40:19)



So, this importance of the stress strain curve is here because in fractural design you got to use this. So idealized curve is something of this kind this is arrived from this real behavior this is arrived from this real behavior. So, there is the importance of the stress strain curve now next to get next property, which is Poisson's ratio. This is important to find out lateral deformation when you apply a longitudinal stress initial situation and so on. So therefore, if you apply load when you apply load you know, you can see this is compressive loading this is tensile the strains are here and this is compressive stress.

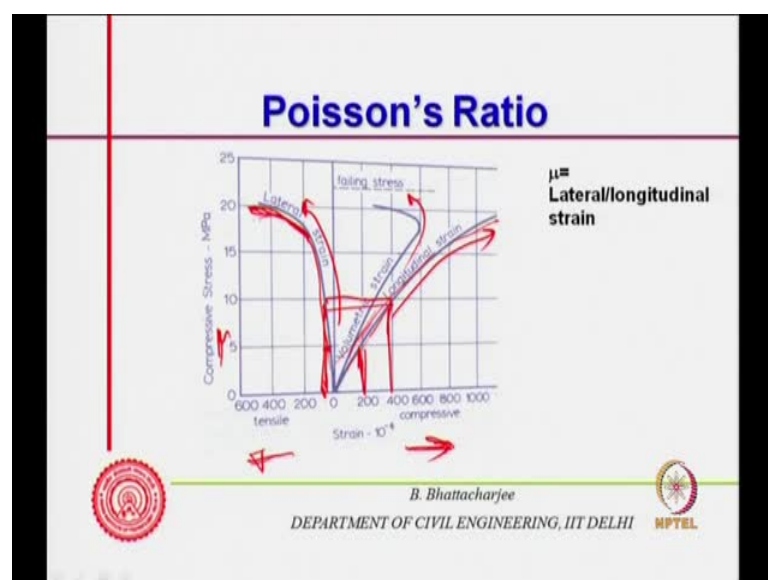
So, what happens is this is a longitudinal strain, this is the volumetric stress that is what we talked about earlier you know, when we talked of after some pulse velocity and acoustic emission and this is the lateral strain which is in the opposite direction it is in the tensile direction. So, μ is defined as lateral by longitudinal strain. So, this is lateral by longitudinal strain μ is defined by this actually so from this if you try to calculate out the mean you know ratio for example, this divided by this ratio, you know this divided this ratio. Or let us say it 400 strain of longitudinal strain corresponding to this may be whatever the value is and similarly, you know so you can actually find out the μ values.

(Refer Slide Time: 41:34)

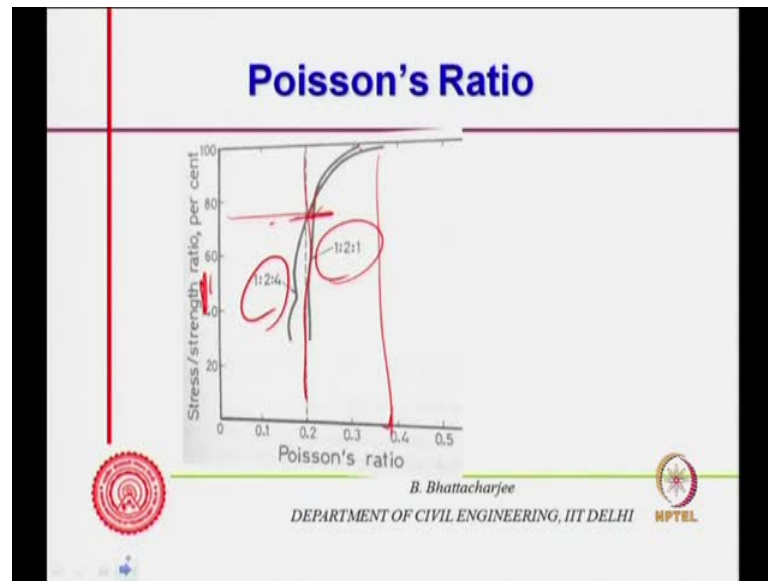


Now, μ values if we calculate out in this manner and then plot, we will see that which is roughly around point 0.2 it is roughly around point 0.2 up to about 70 percent of the load for 2 different concrete good old days people conducted experiments and found out. But, beyond that point actually it starts increasing it starts increasing can go close to about point 0.3 or so on so forth.

(Refer Slide Time: 42:19)



(Refer Slide Time: 42:46)

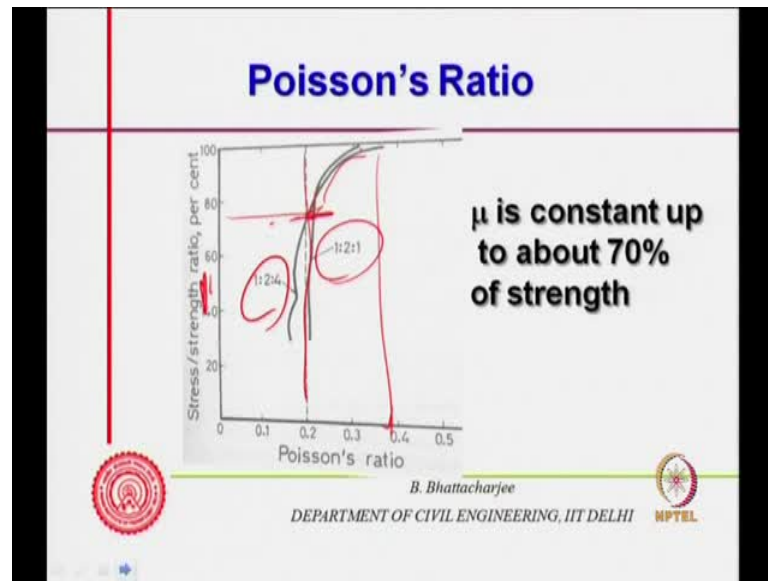


With the you know like if your stress to strength ratios, so when before failure, so before failure Poisson's ratio increases and we understand this. Because, lot of micro cracking would have occurred and that would have caused lateral strain to expand you know increase it at a faster rate than the longitudinal strain.

And this is manifested here also this is manifested in this diagram also. Suddenly there is lateral strains increasing not you know here it was increasing in slow manner. But, it is increase started increasing in the first manner. While, this is increases although there is a faster increase here I mean you know this is not really it is almost close to linear sort of situations like variations are there. But, this increases at a very fast rate this is increases still at a fast rate and this we can understand from the physical scenario.

Because, micro cracking in the wood would result in lot of transverse deformation longitudinal deformation also would be there but, lateral deformation will increase. Significantly, because of the micro cracking so beyond 70 percent of the load actually your Poisson's ratio increases significantly. And you know up to its constant almost constant up to 70 percent of the strength stress 70 percent of the strength.

(Refer Slide Time: 43:03)



(Refer Slide Time: 43:15)

Poisson's Ratio

- Poisson's ratio of concrete range: 0.15-0.20, determined from static modulus test.
- Dynamically Poisson's ratio is determined from Ultra sonic pulse velocity and from fundamental resonant frequency of longitudinal vibration of concrete beam.

$$\left(\frac{V}{2nL} \right)^2 = \frac{1 - \mu}{(1 + \mu)(1 - 2\mu)}$$

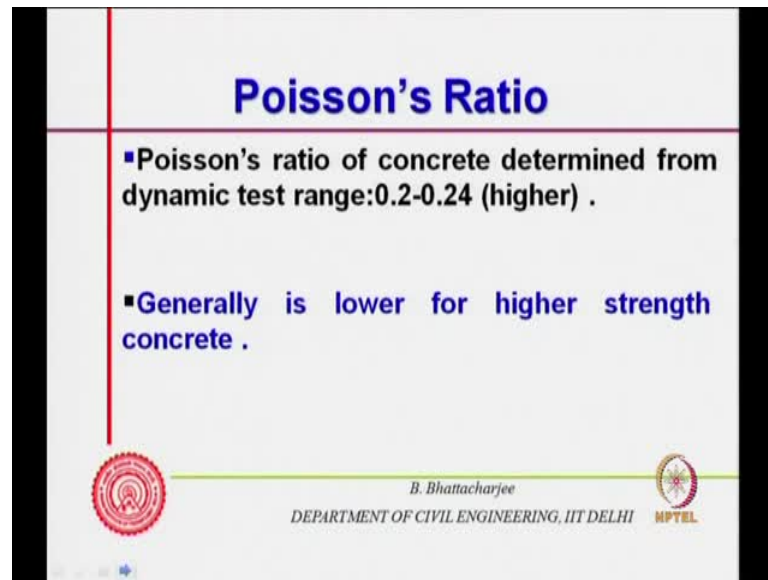
$V = \text{USPV}$
 $n = \text{resonant frequency}$
 $L = \text{beam length}$
 $\mu = \text{Poisson's ratio}$

B. Bhattacharjee
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

Generally, it varies in range of 0.15 to 0.2 that is what is determined from static modulus step test but, you can determine dynamically Poisson's ratio also, from ultrasonic pulse velocity test, from fundamental. Or you know sonic velocity test from fundamental resonant frequency of longitudinal vibration of concrete beam. Similar test that we did from modulus of elasticity. Now, how do you determine this, how do you determine this actually, because the bulk modulus you know it is related to bulk modulus. So, this is the relationship between you know this is involved in the velocity equation. The young's modulus is related to the bulk modulus in this manner when we

use the Poisson's ratio. So, velocity frequency length and is related to modulus of you know Poisson's ratio is related to that. So this is the velocity, pulse velocity, resonant frequency, beam length and Poisson's ratio from that one can find out. So from dynamic testing also you can find out Poisson's ratio.

(Refer Slide Time: 44:29)

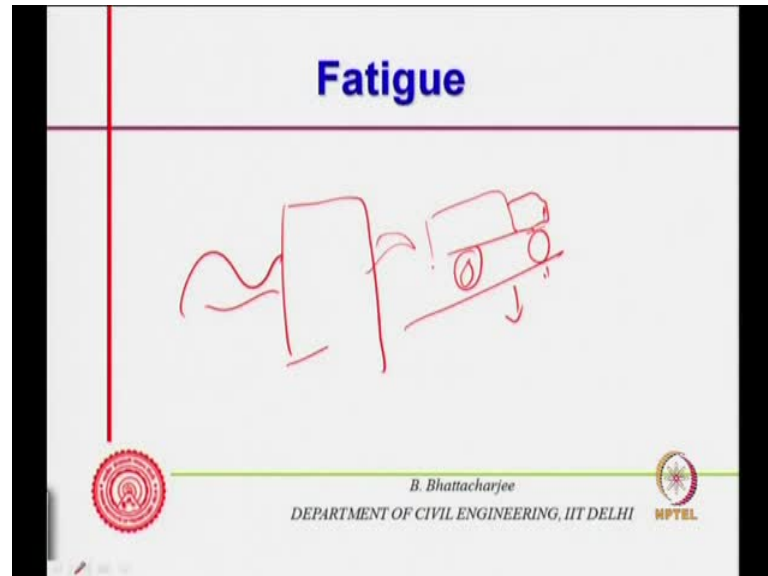


And generally dynamic modulus again gives you higher value of Poisson's ratio. Because, you know higher value of Poisson's ratio usually 0.2 to 0.4. And higher the strength of concrete Poisson's ratio is lower so that is related to Poisson's ratio. Now, let us look at another way mechanical property is called Fatigue. You know fatigue is related to reversal of stresses. And you can understand the importance of this one in many structures. The load that is applied in most of the structure the imposed load there is actually quancy static but, that we do not call it as fatigue. You know like because even in a building where you are sitting let us now, if it is on the first floor or you know in a higher floor rather ground floor.

The slab that you are actually sitting on or your chairs are the load you are coming. And then going out so there is a kind of stresses its quancy static actually what happens is, by enlarge we assume that to be static. And because, major load will come from the permanent features, such as furniture and things like that the impose load, human contribution is would be relatively less. So, we are assuming to be practically you know

static so fatigue is not there but supposing it is a bridge or assume a structure in marine environment.

(Refer Slide Time: 46:04)



Where waves come and hits here, you know waves come and hits. So, it is actually under kind of a reversal of stresses will be occurring. Bridges this vehicular load comes in vehicular load comes on to you know vehicular load comes on to vehicular load comes on to the bridge deck.


And then it moves bridge deck and then it moves therefore, this is there is a reversal of stresses there reversal of stresses you know on to the deck. So therefore, this reversal of stresses is related to what is called Fatigue stress reversal of stresses is related to fatigue strength. So, let us see and in fact the load want the any structure or any structural element can carry under reversal of stresses is much lower than the load it can carry in static conditions.

So, Fatigue is related to repeat loading under repeated loading materials fail at load lower than static strength. The failure load being lower with number of repetition and it is a fatigue failure. A simple example is given, supposing I have a wire small wire, and I try to pull it. It will I can never break it you know binding wire which are used for bonding the steel in concrete while casting concrete.



(Refer Slide Time: 46:51)

Fatigue

➤ Under repeated loading materials fail at load lower than the static strength, the failure load being lower with number of repetition; is the fatigue failure .



B. Bhattacharjee
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI



But, if I try to bent it in this manner that means bent it in this manner you know reverse is the stress. There is a reversal of stresses so you bent first you bent it in this manner then bent in the reverse direction. So, first you bent it in this manner then reverse direction you go on doing this after sometime it will break. So repeated loading you can you know it fails at much lower load and that is basically much lower load and that is basically a fatigue failure therefore.



(Refer Slide Time: 48:11)

Fatigue

➤ Under repeated loading materials fail at load lower than the static strength, the failure load being lower with number of repetition; is the fatigue failure .

- Steel has a fixed endurance limit i.e., repetitions beyond which the fatigue strength remains constant

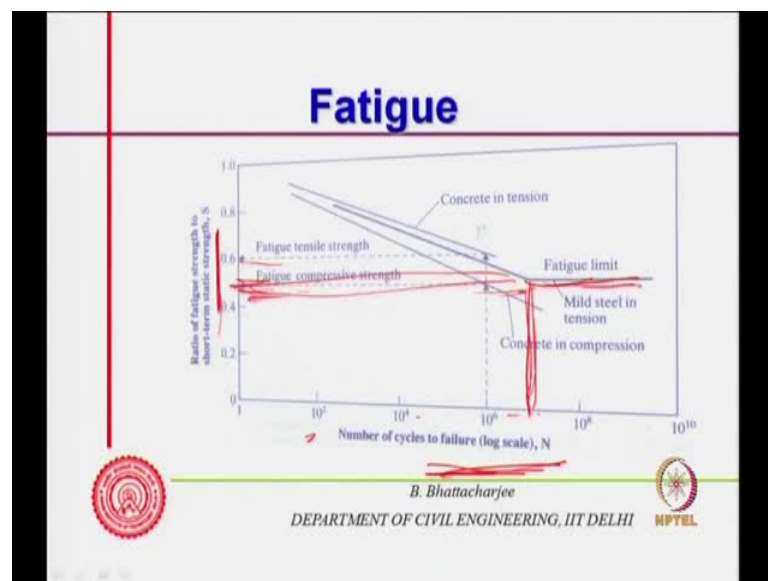
B. Bhattacharjee
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI



Fatigue failure occurs at much lower load so the mechanism you know that is what is the you know that is what we define as Fatigue. And let us see how what happens in case of concrete. Now steel has an interesting behavior we define something called endurance limit. What happens is as you go on number of repetition you know if you apply a load say load to which is 50 percent of the strength and then apply reverse the stresses may be bring it to 0. Some reversal bring it to 0 take it to near 100 percent number of I mean one thing the amplitude.

You know or the maximum stress positive and the negative or maximum reversal range of the stress range in which it is reversing. Let us say we keep it fixed. Now, after certain period of time you will find that it fails. Now if you lower the load, if you lower the range in which you are operating then, it will take longer number more number of cycles. So, the number of cycles is the function of level of the stress, you know reversal where you are reversal of the stress is occurring. Now, endurance limit is defined in this manner it is much better to define this and we come back to this.

(Refer Slide Time: 49:28)





If you look at this diagram on this axis number of cycles drawn in log scale. Because, 10 to the power 2 etcetera this is log scale and ratio of the fatigue strength to short term static strength. So, actually if you are operating at you know fatigue strength you will find that in case of steel, this is mild steel in tension you do reversal of stresses.

(Refer Slide Time: 50:46)

Fatigue

- Under repeated loading materials fail at load lower than the static strength, the failure load being lower with number of repetition; is the fatigue failure .
- Steel has a fixed endurance limit i.e., repetitions beyond which the fatigue strength remains constant
- Concrete does not have fixed endurance limit.

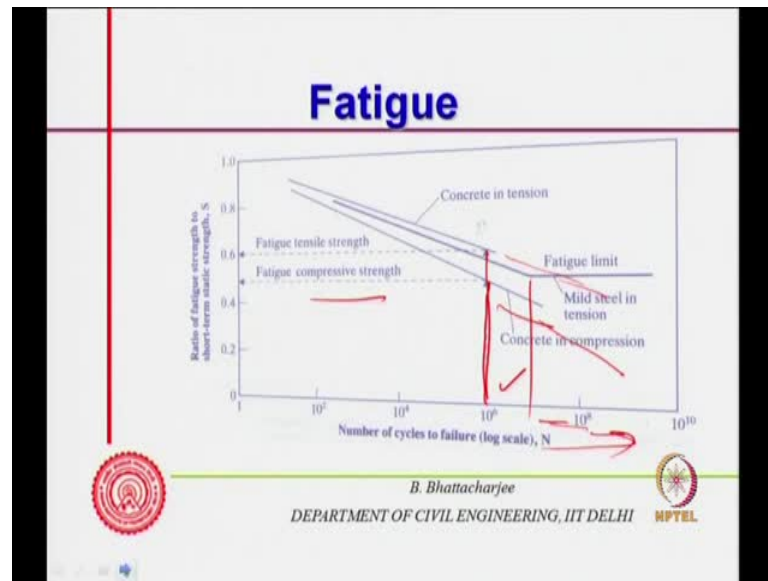
B. Bhattacharjee
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI



But, beyond 10 to the power 7 cycle or some cycles slightly above 10 to the power 6 cycles and 7 cycles you know. The endurance the fatigue the load at which reversal this is occurring at 0.4 or so close to 0.4. So, around you know like 250 mp is the yield strength of mild steel if endurance limit is around 100 close to 100 mpa. So, that means if you apply less then you know 100 mpa and do stress reversal actually it is not going to fail.

Now, endurance limit is defined therefore, endurance limit is defined as fixed endurance limit that is repetition beyond which the fatigue strength remains constant. So, What is fatigue strength? Fatigue strength is the number of repetition or the strength which, you know it can with stand at certain number of repetition. So, it is related to both repetition and the level of this one. Now, endurance limit is that level of the strength beyond which actually, lower than that strength it can with stand large number of repetition. You know because, the fatigue strength because almost constant. Now, concrete does not have a fixed endurance limit concrete does not have any fixed endurance limit concrete does not have any fixed endurance limit.

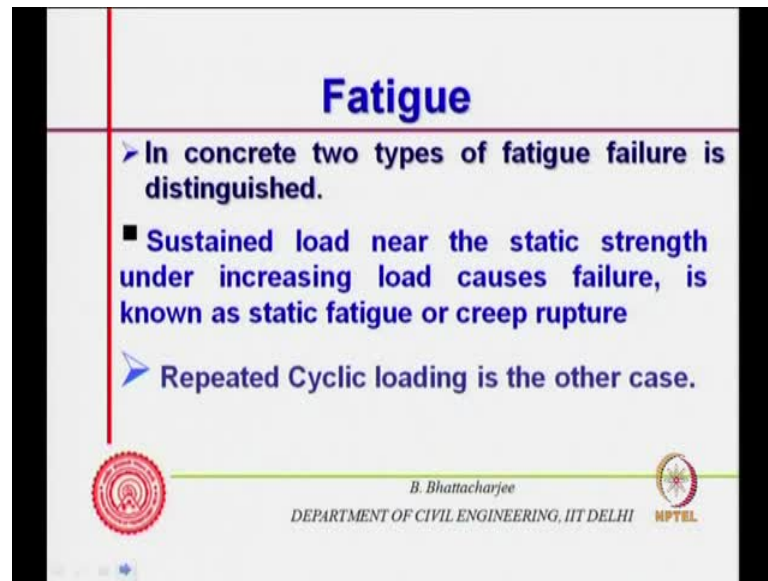
(Refer Slide Time: 51:23)



In tension concrete in tension it shows continuously if you know the fatigue strength will go on reducing as you increase the number of cycles. So, here also concrete in compression same thing. So therefore, how do we define the strength there is no endurance limit how do define this enduring limit. So, this you know steel is advantageous in that sense from usefulness point of view I mean what I say design calculation point of view or understanding point of view because you have got a fixed endurance limit. So therefore, if you know it is likely to come under cyclic load you assume, that it would it can withstand around that endurance limit is you know is which will be the, which you can use in design.

In case of concrete you cannot do that, because it will go on reducing. So, what we will do is we take the fatigue strength corresponding to 10^6 cycles as the fatigue compression strength. Similarly, in this case 10^6 tensile fatigue con tensile strength. So, fatigue compressive strength and tensile strengths are defined with respect to 10^6 cycles because it will go on reducing. For steel of course, you can use the endurance limit, so, it will go on reducing it with number of cycles increased number of cycles.



(Refer Slide Time: 52:49)



Fatigue

- In concrete two types of fatigue failure is distinguished.
 - Sustained load near the static strength under increasing load causes failure, is known as static fatigue or creep rupture
 - Repeated Cyclic loading is the other case.

B. Bhattacharjee
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

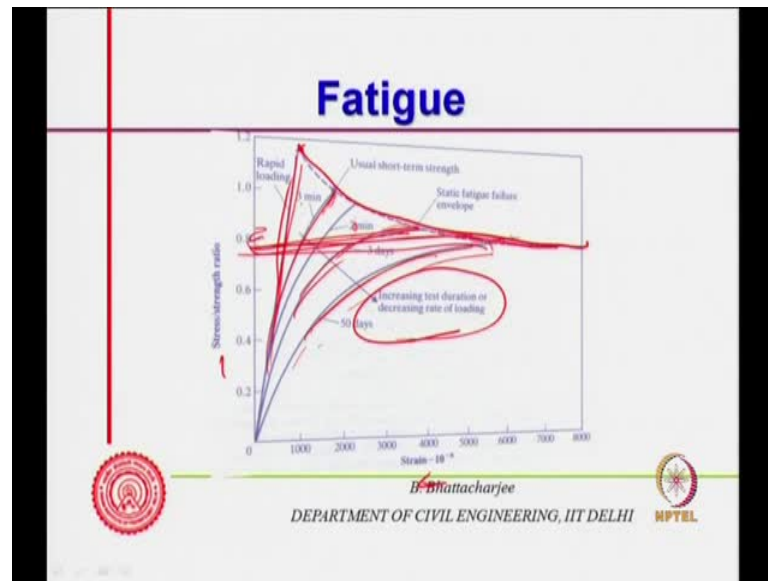
 

So, this is how we define. Now in concrete 2 types of fatigue failure is distinguished the one if a Sustained load near the static strength, under increasing load causes failure. So, its static fatigue or creep ruptures.

Now, concrete shows creep at ordinary temperature that means if you put a load we will discuss this in subsequent module. That means if you keep the load constant and over the time period over the period of time we will find the deformation is increasing. And it can have a failure also depending upon the situation. So, under sustained load concrete exhibits kind of deformation and that is related to phenomenon of creep. So, if you sustain load near the static strength, now fatigue is also nearly sustained because, you are reversing the stresses and near the static strain so you increase the load and it cause the failure.

And this is related to static fatigue or creep rupture you know. So, if you are reversing this stresses close to the static the static strain. Then, there can failure can be by creep rupture because sustain load is sustain very static load is sub strain plus minus something is always occurring you know plus minus something is occurring. So, this sustain load can result in creep rupture. Because, load is sustain very close to the static strain. Repeated sighting loading is off course other case where you are operating at much lower level. But, then it because repeatedly you have done you know reversal of stresses have been done over a large number of cycles so these are the 2 situations.

(Refer Slide Time: 54:36)



And we can see the behavior in this manner. If, you look at this diagram usually you know in this case what you are doing 50 you are sustaining the load for 50 days. 3 minutes stressed and rapid loading stress strain curve is like this is excess is strain this is stress rapid loading stress strain curve is like this. And here it might fail. If, you finish it within 3 minutes it will show you failure like this. I do not have a here I am only controlling the rate of loading. In terms of load part you know pie unit, time not in times of strain control situations. So, what will happen very rapid load loading no you the micro cracks will be formed.

But, at certainly there will be a sudden brittle failure at some higher load. So, it fails somewhere there but, if you are loading under let us say 3 minute failure up to failure. Which could be the case in case of static monotonic loading so it will fail somewhere there not much deformation or such thing it will not show it will show this if you have a strain curve control situation. So, a kind of what you call strain softening would occur you know as the strain increases strength it can load it can carry is lower.

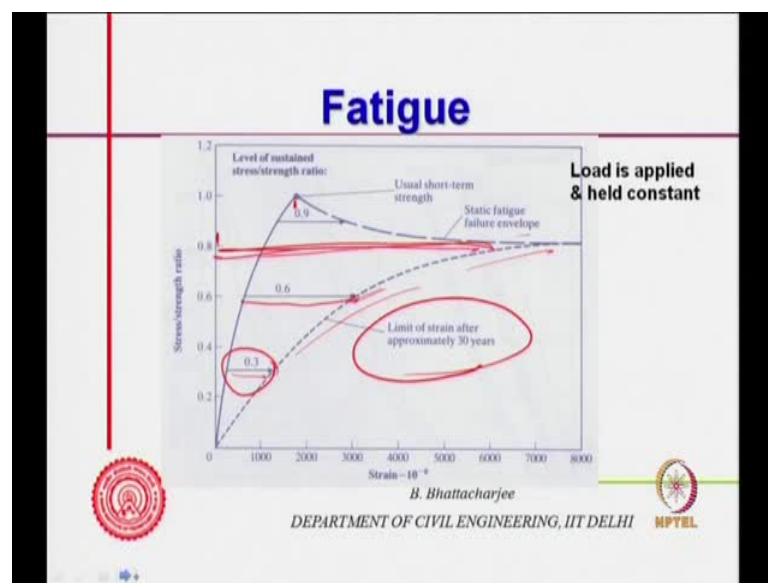
So, if you know if go on increasing this is mono normal stress strain diagram but, if is 3 days this is you know this about 3 days you take if you 3 days if you take then you find that, if the load for 3 days. So, it is 80 percent of the static load, it will actually 80 percent of the static load. It will fail keep the load for 3 days. It might fail somewhere 80 percent I mean there is a notional thing one cannot take it for all the time granted the

values might differ from test to test. And if you keep it for 50 days it is actually fail at somewhere there. So, depending upon this is 20 minutes 3 minutes 2 days 3 days and 50 days. So increasing the test duration then which means that my rate of loading I am decreasing and at lower rate of loading we have seen it fails earlier.

So, 50 days same load you are applying in 50 days you find that the load carrying capacity is much greater than less. So, I can actually have a kind of a envelope. So you can call it static fatigue failure envelope. Because, this is related to micro cracking or creep behavior and so on. So, very close to the load quickly, if you load it fails at higher load you take a longer period of time it will actually, fail after certain period of time. Because, this is sustain for fifty days so it will actually fail. So this so you know this is related to static fatigue failure so, close to this one if you take you know if you do reversal for 50 days it will fail.

So, if you do reversal somewhere here if you do lot of reversal there time frame it will be related to this actually failure would occur even though you would not gone too much of a cycle. For example, for 0 cycles this situation is for 0 cycles. So, even if I have got too much of cycle it will still fail so this is actually related to static failure, static fatigue failures you know so or creep rupture as we are calling it a creep rupture.

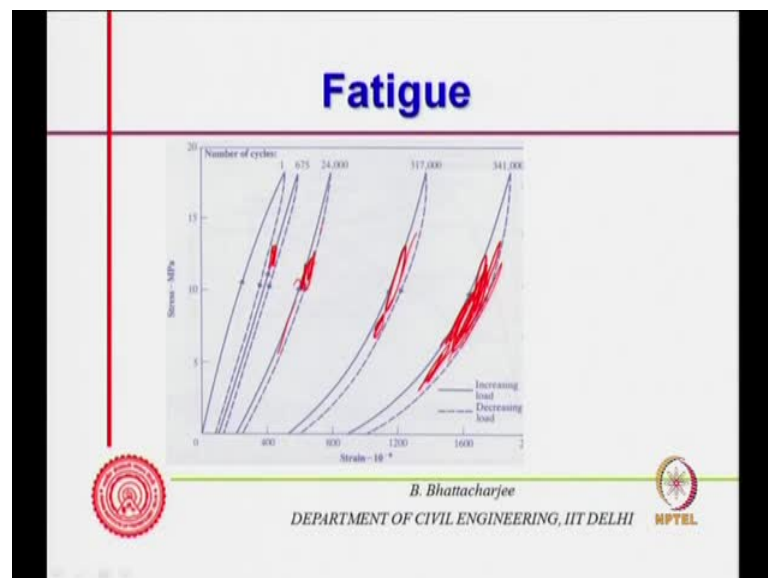
(Refer Slide Time: 58:09)



And this is again strain versus stress graph. So, limit of strain after approximately 30 years. So if I look at it static rupture curve is here maximum strain after 30 years this

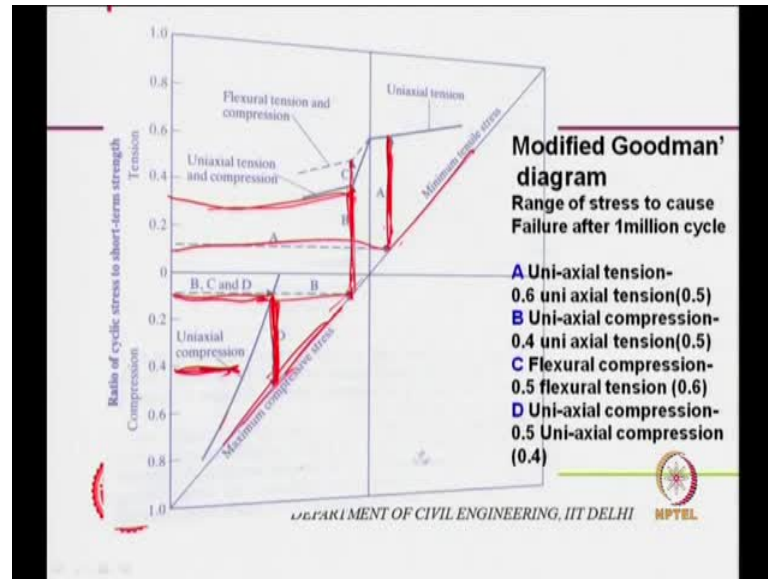
envelope gives you 50 days. We looked into so, 0.6 of the strength 60 percent of the strength if you load it to after 30 years deformation will be somewhere here. You know after 30 years deformation will be somewhere here off course, it will not fail but, if you put it at 80 percent after 30 years it will actually fail. So this is it and this is the usual short term strength which is about 3 minutes or so, 0.3 is less than 30 percent of the strain it will actually after 30 years deformation will increase but, it will still not fail. So, it fails up to something like 80 percent of the static strength or 30 after 30 years and so but, if you have something more than that it will fail. So this is how you know this is related to this fatigue.

(Refer Slide Time: 59:13)



And if you do reversal of stresses one cycle the behavior is like this. See, 1000 and first so that this hysteresis area reduces significantly. And then this area would increase. So, what will do is you know the fatigue we have introduction to fatigue I have given to you. Now, and you can see that after long period of time because of long cycling number of number of you know cycling causes more micro cracks to appear. So therefore, this increases in the beginning there is a reduction but, under long cycling actually no micro cracks formed under fatigue loading. Because, there will be frictional and heat generation as it happens or whatever mechanism are there, so fatigue causes failure in this one where something called modified good mans diagram.

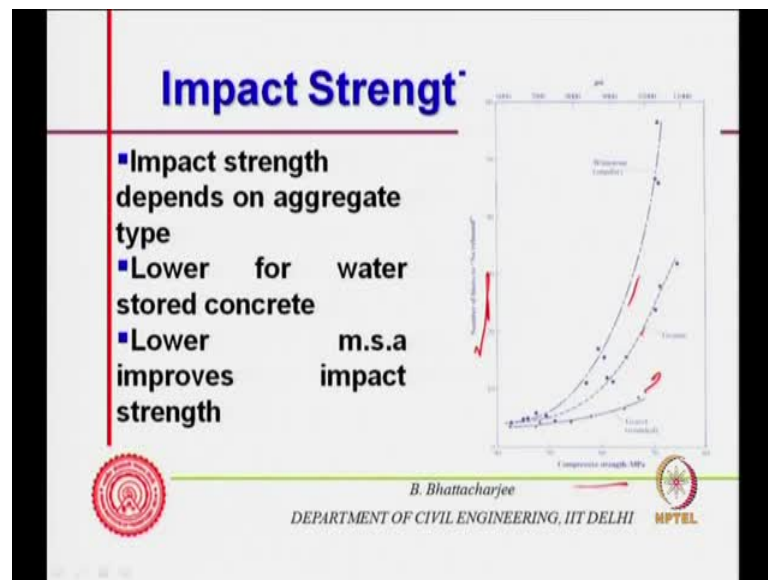
(Refer Slide Time: 60:08)



Here, this diagram helps us in finding out what is the strain level or reversal of stress concrete can with stand. For example, this curve you know this is a minimum compressive stress line this is the minimum tensile stress line. Now, Uni axial compression if you are operating at 40 percent of that strength level. Or let us say 20 percent 10 percent of the stress level you can do reversal up to this and come back in compression mode only. So, failure would occur at 10 to the power 6 cycles. And if you are applying let us say tensile stresses you can go up to 20 percent of the tensile strength 30 percent of the tensile strength.

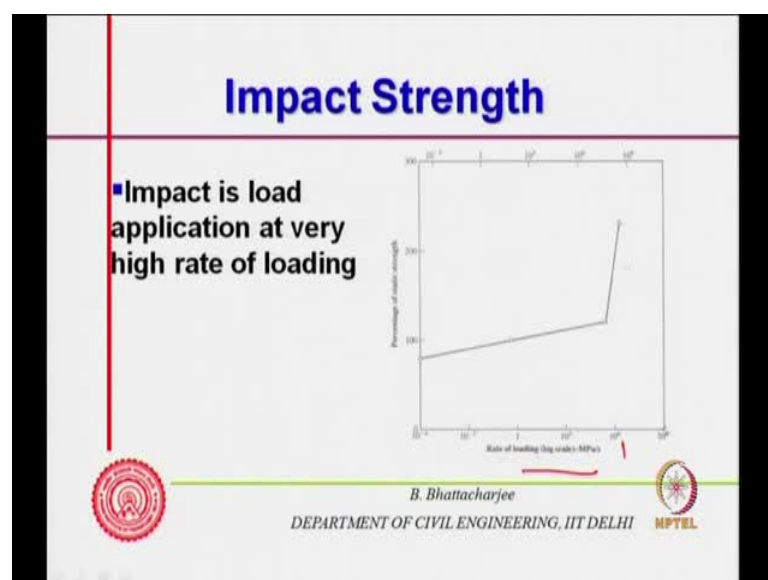
And reversal can occur from 10 percent to the compression to 30 percent of the tensile. If, this in compression Uni axial tension and compression this is in flexure. And if you are doing purely tensile so that 10 percent of the tensile load you can go up to this and come back. So, Modified Goodman diagram can be used to find out what level of stress reversal you can do, what level of stress reversal you can do at what level of static strain static stress level at 10 percent of the stress level you can reverse the stresses given form this diagram. Say if it is compressive stress. If, it is tensile stress one can determine from this diagram so, Modified Goodman diagram can be used to find out what level of ranges of stress you can use.

(Refer Slide Time: 61:31)



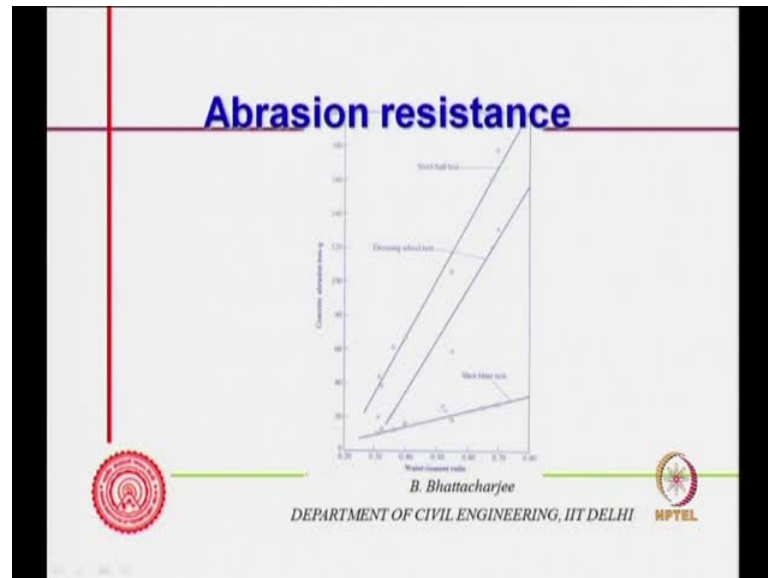
Impact Strength is again can be related to compressive strength and it depends strongly upon aggregate type. Lower for lower water stored concrete and lower m.s.a improves impact strength and that is shown by these diagram different types of aggregate. This is the number of actually impacts actually it can take the weight is measured depending upon compressive strength.

(Refer Slide Time: 61:53)

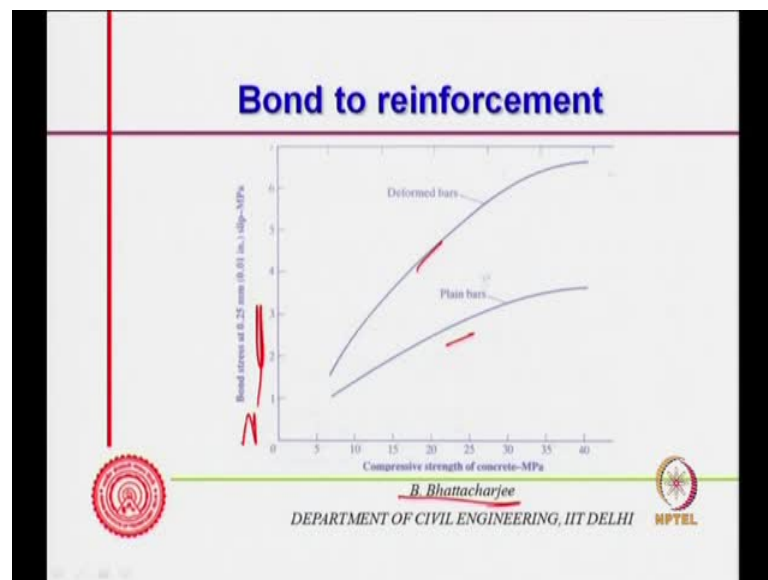


Both impact and both impact strength as well as you knows Abrasion their function of compressive strength. And impact is loading at a very high load you know high rate very fast rate. So, rate of loading is very high impact is related to that.

(Refer Slide Time: 62:13)



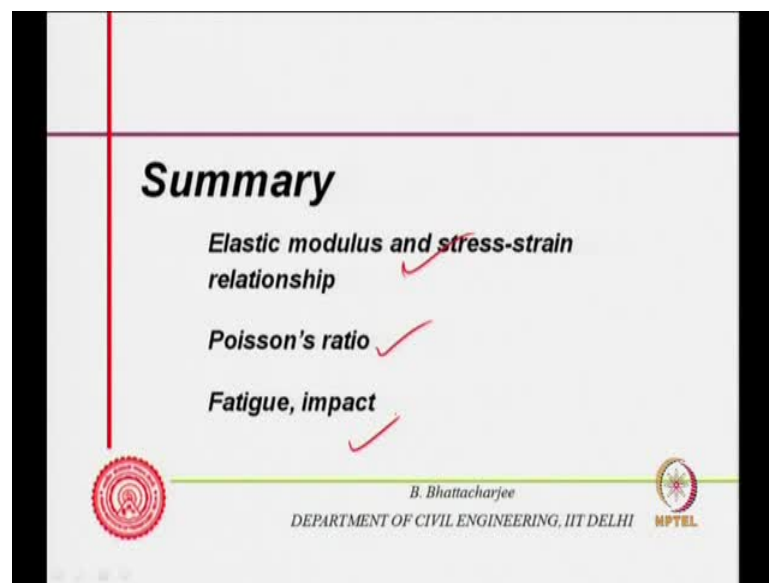
(Refer Slide Time: 62:30)



Abrasion is again related to compressive strength, you know as you were rubbing how much now how many what kind of depends upon different types of test. Actually you can rub it and find out percentage of the material will be generated. And this is also related to compressive strength.

So Bond reinforcement again for plane buds and reformed buds related to compressive strength. So, bond strength you know so bond by pull out test one can find out. So what we find is all these tests are all these strengths are related to compressive strength. Bond fatigue not bond impact and abrasion resistance. These are all related to compressive strength. Essentially, higher the compressive strength of concrete all other properties can be related to that and they improve. So, higher strength means better impact resistance, better abrasion and better you know better abrasion, better impact resistance or impact strength. So, all properties actually higher modulus of elasticity.

(Refer Slide Time: 63:15)



So I think we have discussed in this one Elastic modulus, Poisson's ratio and fatigue and impact with this module will complete. And next 7th module we will look into creep and fatigue. thanks.