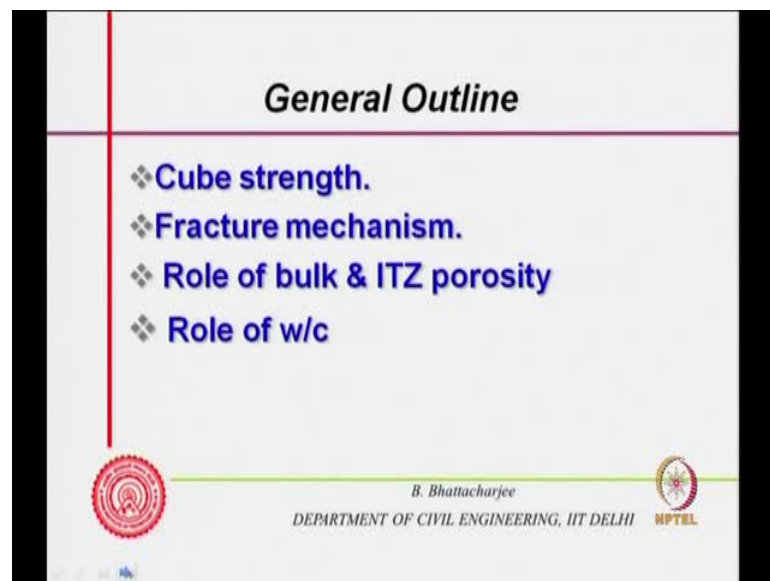


**Concrete Technology**  
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**Indian Institute of Technology, Delhi**

**Lecture - 23**  
**Strength of Concrete: Factors Affecting**

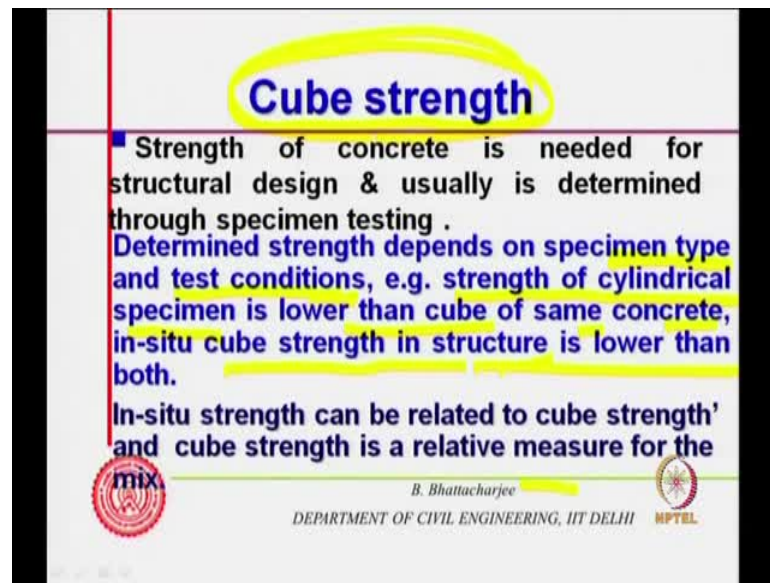
Welcome, to module 6 lecture 1. In this module we should be looking into strength of concrete. And as we know that a concrete is a particulate system bonded particulate system. So here, particles are there which are bonded largely mechanical bonds. And therefore it is not very strong in tension you can pull it a part easily. While if you push it is relatively stronger it can take lot more load. So therefore, when you talk of strength of concrete our main thrust or emphasize is on compressive strength of concrete. However, we discussing about tensile strength of concrete as well.

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Tensile similar are the properties related strength properties as well. So therefore, general outline is of course, cube strength I will mention why cube strength quickly, again fracture mechanism. And therefore what effects the strength? The mix parameters largely or basic mechanics mechanism of failure all that we can do in this lecture.

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**Cube strength**

- Strength of concrete is needed for structural design & usually is determined through specimen testing .
- Determined strength depends on specimen type and test conditions, e.g. strength of cylindrical specimen is lower than cube of same concrete, in-situ cube strength in structure is lower than both.
- In-situ strength can be related to cube strength' and cube strength is a relative measure for the mix.

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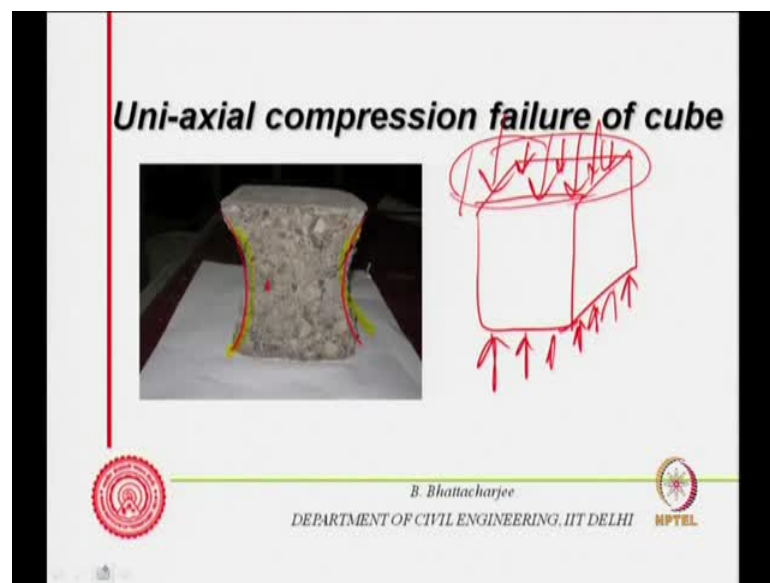
Strength of concrete is needed for structural design and usually we determine through specimen testing. Since, concrete is structural material and you make structure out of it. Therefore it is required for structural design strength of concrete is required. Because safety is a primary issue. So, we determine strength of concrete by specimen testing and determine strength depends on specimen type. It depends on specimen type you know it depends on specimen type and of course also on test conditions.

For example, strength of cylinder cylindrical specimen is lower than cube of same concrete. And in-situ cube strength in structure is lower than both so you see it depends upon types of specimen. You test in Indian scenario we do test cubes and that is why this heading cube strength not only is India, Europe UK. Of -course, from where that we know the concept of cube strengths came to India they do use cube largely North America. And even North and South America and Japan those countries you cylinder. So is the type specimen that is very important we test this specimen.

And test condition also affect the strength that is a past. Now in situ strength can be related to the cube strength. You know whatever the cube strength it can be related to the strength in actual structure in some manner. And therefore, cube strength is relative measure of the mix that I have using. You know we have looked into mix proportioning. And it is making totally the casting the procedure production and you know all that.

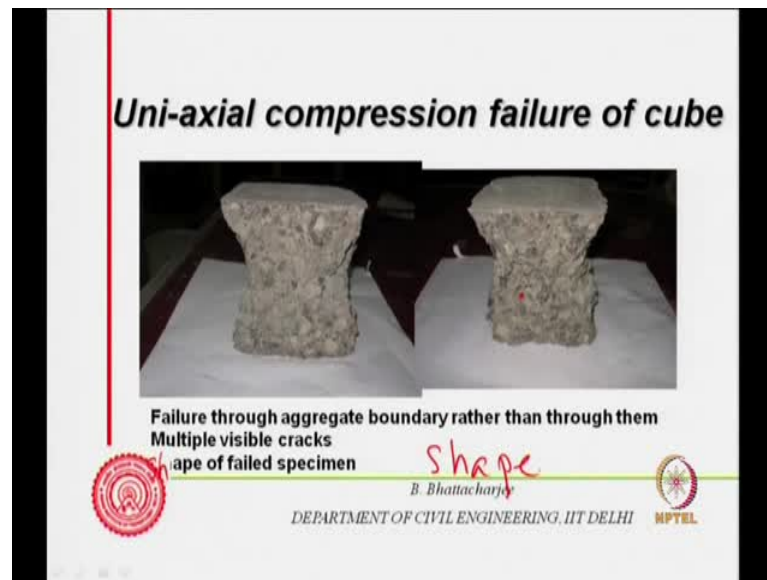
So basically cube strength since, I am preparing it separately not taking from the structure. Cubes are prepared separate separately, specimens are prepared separately and that is not taken from the structure. Therefore cube strength is largely measured the relative strength potential of the mix basically. So it is relative strength potential of the mix proportion only and structure strength in the structure is somewhat different. But, this strength potential or relative strength potential can be related to actual strength. I can expect in structure for the design purposes in some manner. And therefore, it can be use in structural design as such.

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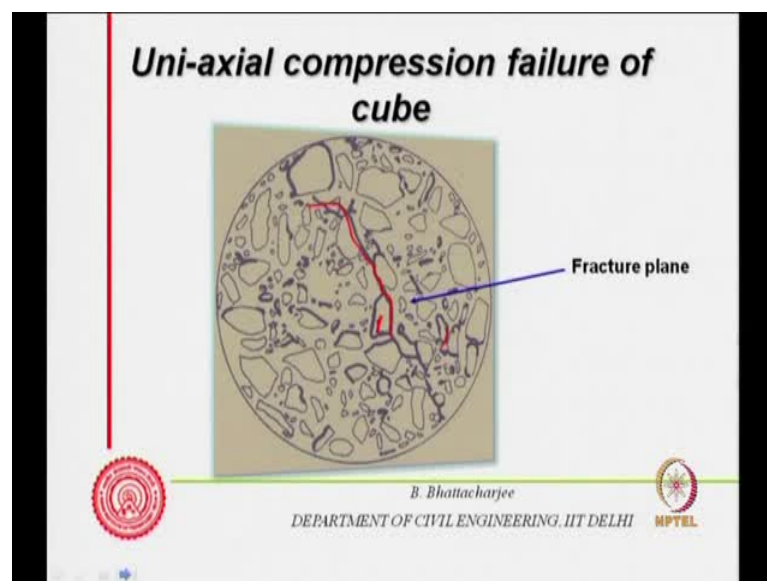
Let see if I look at a cube, How does it fail? If this is the failure pattern of Uni-axial failure pattern. That means I have applied some sorts of load on a cube in a cube specimen this for the cube specimen. So since, in Indian scenario largely you know we test only cubes will be talking more about, the cube strength and many other countries do follow the same. So, in a cube actually you will be applying the load through machine platens may be this is circle is 1. And you will apply load like, that and the reaction comes from the bottom. And once you have applied load and made it to fail it will fail like this, is the kind of failure plane. You can see the typical failure plane taken from the photograph. And you can see this is for normal concrete. So your aggregates are exposed aggregate surface are expose and actually crashing of a aggregates are occurred here. So this is typically the Uni-axial you know failure of cube.

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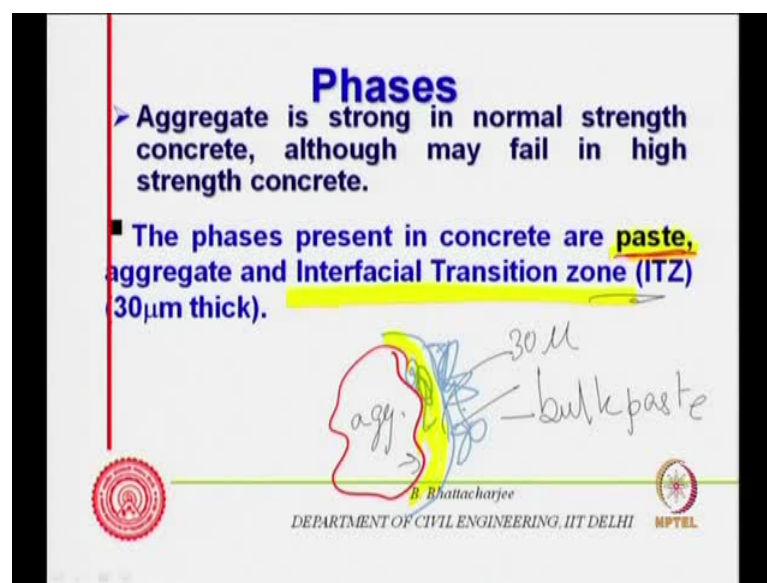
And this is another diagram from another side. And all shows that aggregates are exposed. You know they do not fail in normal strength concrete. So, failure through aggregate boundary rather than through them. And you can have multiple visible crack this is a shape of failed specimen. As we have seen shape as we have seen now, will explain this, Why this shape we get it? But one thing we must like to understand from here, is that a normal strength concrete failure is through the aggregate boundary, rather than through the aggregate themselves.

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Actually fracture plane therefore, would be passing like this the aggregate boundary. And may be passing through some water, and then again through aggregate boundary. So, if you take a portion of that fracture plane in a Uni-axial compression failure. Structure plane in normal strength concrete look like this, these are the aggregates and the mortar around. And you can have multiple course cracks here and there etcetera but, failure plane finally. You know as the fracture occurs it would occur through this kind of a situation it will break through aggregate boundaries. So, thus normal concrete fracture plane passes through boundaries aggregate boundaries.

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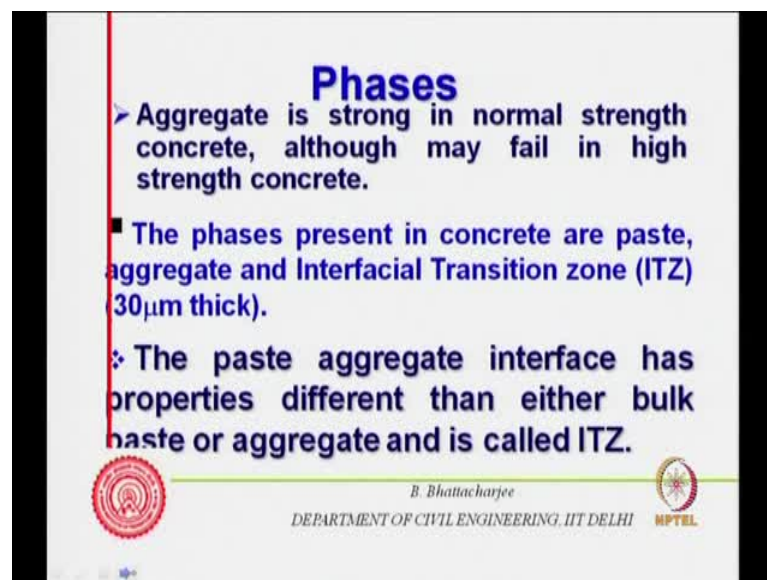
What are the Phases in concrete, so far we have looked in to Aggregate is the one of the phase. It is very strong in normal strength concrete you know. In high strength concrete of course, you try to attempt the phase strength such that it nearly matches with that of the aggregate strength. And thereby or whatever other phases you know all are the phases strength of all other phases almost matches with that of aggregate. Therefore, aggregate may also fail in high strength concrete. But, normal strength concrete up to let us say 60 mp concrete aggregate normally does not fail the failure is through the aggregate boundary.

And the phases other phases present in the concrete is a you know paste phase. So, it is the paste phase is the paste phase. You know this is paste phase so paste aggregate when I say course aggregate as well as fine aggregate, you can. As you seen earlier it is agree

aggregate in mortar. And in the mortar again the paste and the fine aggregate system. So paste can be considered as a phase. But, I actually can identify another phase well it is not a phase but, the transition zone between the aggregate and the paste phase. So, this we call as interfacial transition zone we call as interfacial transition zone. So this is nothing but, your paste boundary if this is your boundary aggregate this boundary. You know boundary of the by boundary of the like you will have some this is your bulk paste let us say bulk paste.

So, this is the boundary surrounding and this could be something different this could be something different. You know this could be something different for example, this could be the bulk paste phase. So this is about, so this I can call this as bulk paste aggregates. And there is transition zone which is about, 30 micron, thick this is called aggregate paste interfacial transition zone. You know and because this is not smooth here aggregate is a bulk paste but, there is transition zone between this 2. And this is apply about of 30 microns so, this is you can you can call it as a phase. And this is very important. So you have got actually aggregate and then you have got paste and then you have got ITZ.



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**Phases**

- Aggregate is strong in normal strength concrete, although may fail in high strength concrete.
- The phases present in concrete are paste, aggregate and Interfacial Transition zone (ITZ) (30μm thick).
- ✧ The paste aggregate interface has properties different than either bulk paste or aggregate and is called ITZ.

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You have got ITZ the paste aggregate interface has properties differ then either bulk paste or aggregate and this called ITZ. So it is different when the bulk paste itself it is different when bulk paste itself or you know aggregate itself. So, it is different in paste and aggregate and it is, that is why we call it defines it almost as a separate paste. Now,



why it is we can be able to actually understand gradually will be able to understand it gradually.

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**Paste strength**

- Strength of paste is attributed cohesion (van der Waals forces) acting between gel particles acting across gel water.
- Large discrepancy exists between theoretically estimated strength calculated from bonding forces & observed fracture strength (10-1000 times less).

*Handwritten notes:*

$\frac{\text{Strength/bond}}{\text{No of bond/area}} \rightarrow \text{No of bond/area}$

$\frac{\text{Strength from theory}}{\text{Strength/bond Van der Waals}} = \text{No of bond/area}$

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Now, Strength of Paste is attributed to cohesion you know paste because gel. So between the gel particles, across gel water, causes forces something the Vander walls forces exist. So you see the strength of the paste the paste if, you want to separate out you has to actually break this cohesion force. So therefore, paste strength could be quite high. Large discrepancy off-course exists. Between the theoretical estimated strength can calculated from bonding forces and observed fracture strength. For example, if you calculate I mean if you get X strength from theoretical strength from theory. Why, how because strength per bond is known you know the Vander walls bond Vander walls kind of bond Vander walls bond.

So, I know strength part van der walls bond. And if I also know number of bond number of bond for unit area. Then I can calculate out the theoretical strength so, I got to know actually number of strength from theory. That would be actually, strength per unit bond. And if I know number of bond per unit area, then I can find out strength per unit area. So I can find out strength per unit area so, that is your X let us say. Now this can be estimated theoretically. What has been observed is that theoretically estimated strength is 10 to thousand times more.

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**Paste strength**

- Strength of paste is attributed cohesion (van der Waals forces) acting between gel particles acting across gel water.
- Large discrepancy exists between theoretically estimated strength calculated from bonding forces & observed fracture strength (10-1000 times less).

$$\sigma_{th} = (10-1000) \sigma_{actual}$$

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So, this  $\sigma_{th}$  theoretical you know or whatever, you call it sigma theoretical sigma theoretical. Let us, say for tensile strength is theoretical is 10 to thousand times more than sigma actual. So, therefore, this is large discrepancy whatever theoretical you calculate does not actually exist so why it is?

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**Paste strength**

- Strength of paste is attributed cohesion (van der Waals forces) acting between gel particles acting across gel water.
- Large discrepancy exists between theoretically estimated strength calculated from bonding forces & observed fracture strength (10-1000 times less).
- ❖ The difference is attributed to existence of cracks and flaws within material even before any load applied.

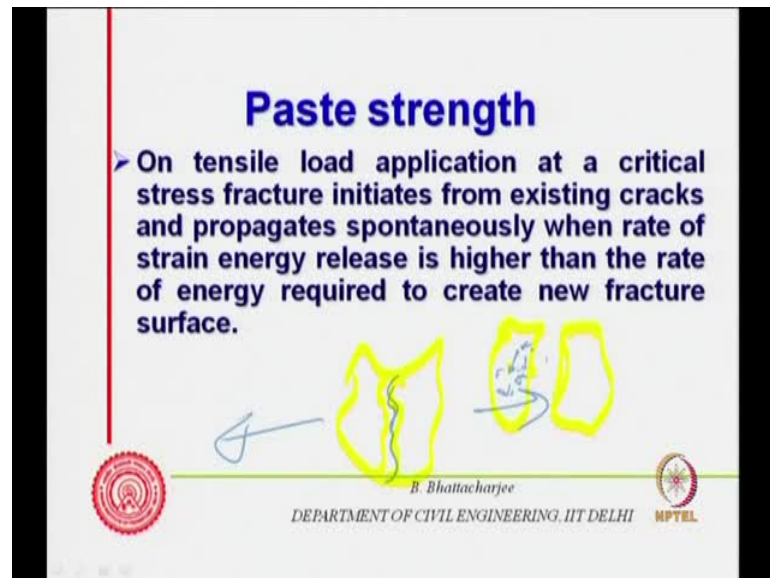
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The difference had been attributed to existence of cracks and flaws within material, even before any load applied. So even before you have load applied the load the material is not absolutely homogeneous. If, you have just the phase even the paste is not also lid that



could be some flaws. If, there not cracks can come from shrinkage and similar kind of thing. But, the other kind flaws should be there which are inherent to the system. For example, capillary force and you know gel force off-course, does not take part is to small to influence and significant. Where the strength but, it can still influence largely it is capillary.

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Flow force which are the force which is in the material. So thus difference that comes in 10 to thousand times, thus essentially attributed to attribute to these essentially attributed to flaws or force within the material. On tensile load application at a critical stress fracture initiates from existing cracks or flaws and propagates spontaneously. When rate of strain energy release is higher. Than rate of energy required to create new fracture surface. What is actually, what is failure? What is fracture? What is failure? Failure means material getting separated out under tensile load let us say.

So, if we have some material l and I am applying pull from this side somewhere the material get separated so this is material is getting separated. Now, this separation means creating new surface so you are creating surface here and creating new surface there. So it is actually now kind of a new surface is been created. You know which is matching off-course, so by pulling them apart they have actually create new surfaces. Now each surface is associated with kind of surface energy there is interface between solid and liquid or solid and air or air or liquid. At the interface always there is some sort of energy

associated with it. Because the material here, will be material here would be pulled. You know it will be interactive with materials all the directions. Where 1 in the surface is it will have it will have corrosive forces which will pull. Which will get it you know that other are the particle on the side on the solid side pull, where there is here there is such material. So, there is a kind of a surface energy existing at the surface. Surface energy exist as the surface and this occurs to any interface.

So since, there is surface energy required the surface to create a new surface, you must provide that surface energy. And where from this energy comes in case of fracture or failure you, are applying load. Which means that you are applying stress this causing a strain before failure. So this half stressing to strain is actually strain energy and when the strain energy is supplied it is high or rate of strain energy is supplied is high. I know release is higher than rate of energy required to create a new fracture surface fracture will propagate spontaneously.

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**Paste strength**

- On tensile load application at a critical stress fracture initiates from existing cracks and propagates spontaneously when rate of strain energy release is higher than the rate of energy required to create new fracture surface.
- Decrease in Strain energy for elliptic crack of length  $2c_1$  in plate of unit thickness is considered together with surface energy

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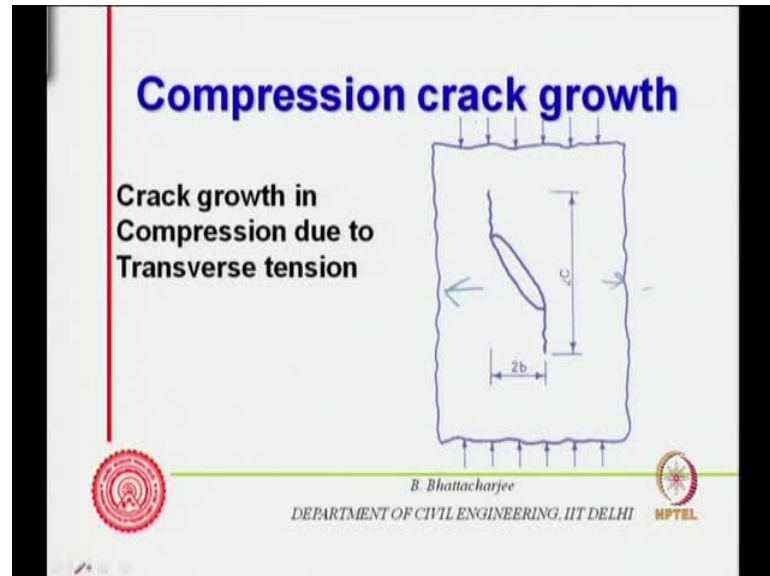
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And this is what was actually postulated by graphics. Somewhere around 19 hundred 18 or so around that time may be second half of the second decade of 20th century.

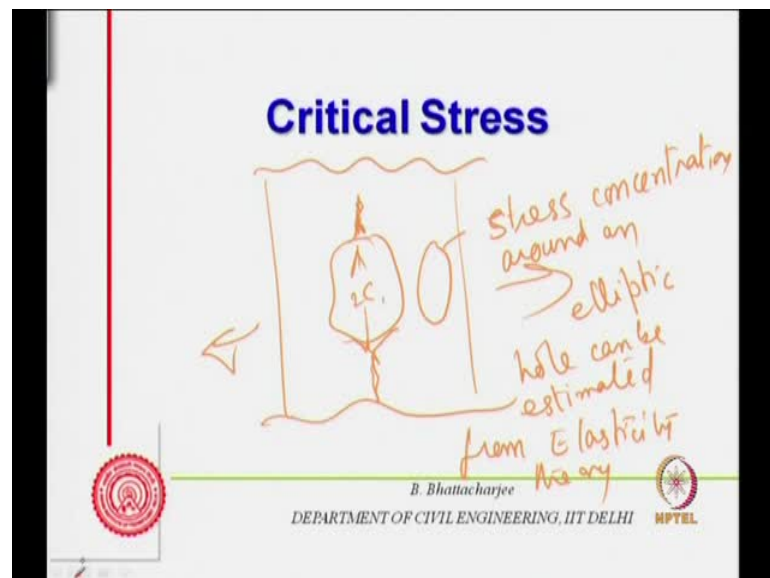
When you was looking at the as failure of glass or such brittle material. So decreasing strain energy for elliptic crack of length of choose you 1 in a plate of unit sigma is consider to whether with surface energy so that is what we did. So let us see, How what it do well? I think I will come back to this diagram. But, let us let me look at 1 more

diagram slightly later. And then I will comeback I will just come back again to this diagram here for example, this one.

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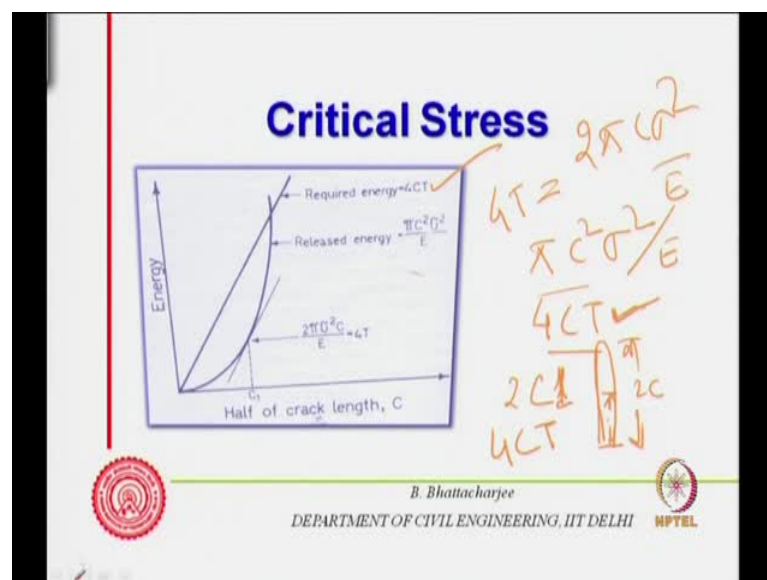


Supposing, I have got a surface something like this, there is hole and this is my material there is a hole. And I am pulling this material from this side, then what will happen, this crack will try to spin. Now if this crack tries to expand or, this hole expand the fracture propagates along this direction. So fracture propagates along this direction that means it creates new surface is here. Now this stress around a hole for an elliptic hole I can

estimate stress concentration around an elliptic hole can be estimated from elasticity theory. And if, you apply more load what would be the further stress that one can find it and correspondingly this strain energy 1 can find out.

Expression for strain energy this are available so lu for available solutions of elastic theory elasticity theory this already available. And then, the rate of strain energy change with this crack with you knows  $2C, 1/2 C^2, CL$  if I call it. This is this was actually determined and from this if you equate that ratio of strain energy in the limiting case rate of strain energy releases equal to, rate of surface energy required. So, if you equate this you get find out what is called critical stress. So, you can find out what is called critical stress so critical stress is that stress at which, the fracture will propagates spontaneously.

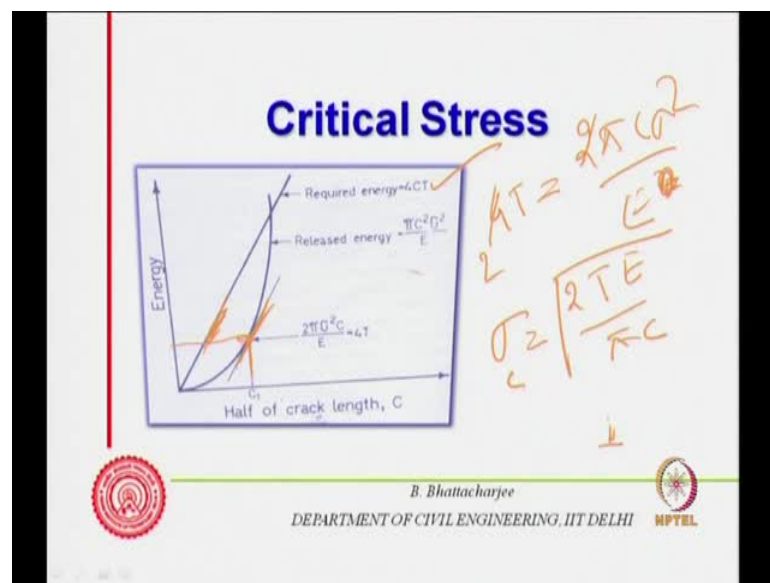
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So, that was the idea, so this is the release energy  $\pi c$  square sigma square and half crack length is. See in this particular case and  $4CT$  is the fracture energy. So you know because while the  $T$  is the  $T$  is a surface energy, and thickness is taken as unity. So, it so you can assume as a plate sort of a thing, which is on the tension with a unit thickness. So the area would be 2 surfaces if, you want create if you half crack with this  $2C$ . So crack with this self or hole itself is  $2C$  it is with will be  $2C$ . So this is your elliptic crack this is  $2C$  into the thickness  $T$ . Which is unity so, into 1 and this side again  $2C$  so,  $4CT$  is a surface energy per unit area.

So,  $4CT$  is the surface energy  $4CT$  is the surface energy. Now required surface energy because  $2C$  is this full length  $C$  is half seat crack with  $C$ .  $C$  is half crack length so,  $4CT$  and correspondingly from the stress concentration concept, from theory of elasticity. I can find out  $\pi C^2 \sigma^2$  by  $E$  is a released energy. So, when rate of this 1 rate of this 1 is same at a given  $C$  at that  $C$  you know the stress for a given  $C$  is known to me so  $\sigma$  corresponding that  $C$  would so would is a critical stress. So let me as differentiate this will come as twice  $\pi C \sigma^2$  by  $E$  and that must be equals to  $4T$ .

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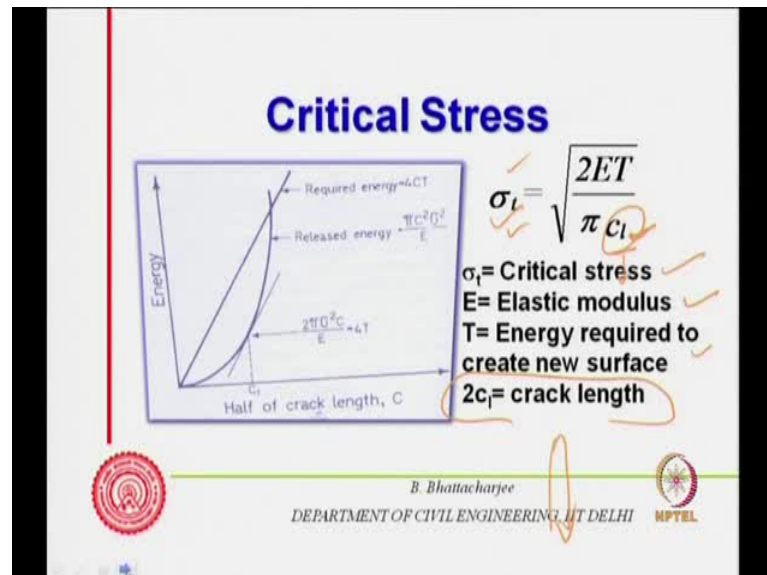


So, if I take this force if I take this rate of strain energy release this is the situation rate of strain energy. So at this stress at the stress corresponding to this point is a critical stress. So therefore, you can actually by  $E$  this is your this is by  $E$  square this is by  $E$ . And therefore, you can find out  $\sigma$  is equals to this will be 2 so  $2TE$  divided by this is cancelled out  $\pi$ .

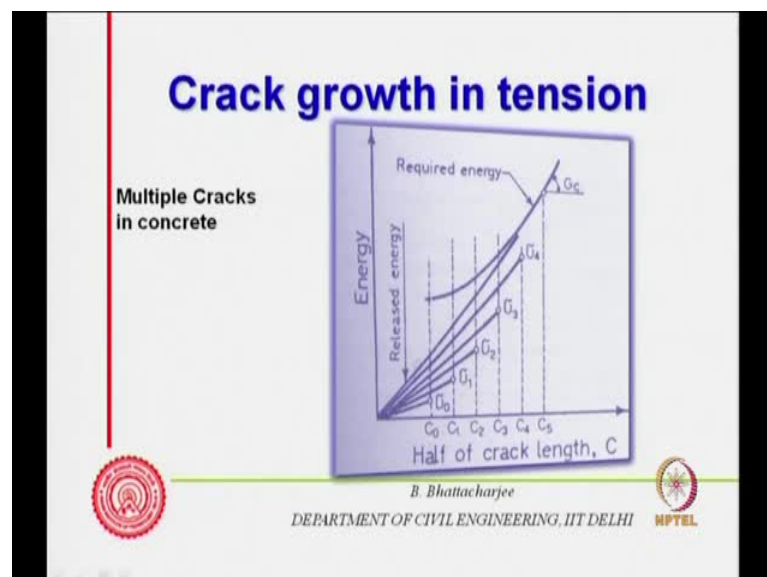
$C$  everything under root so critical stress is  $2TE \pi C$  everything under root. Where  $C$  is the half crack. So you see this is what this is the this corresponds to this strain. Where this strain energy release rate is equals to this slope of this line rate of strain energy release is same as rate of strain energy with respect of  $C$  off-course. So that is critical stress so,  $\sigma_T$  is equals to  $2ET$ . So you can see the critical stress at which fracture

will, propagates spontaneously is a function of the half crack width. Now, if already some crack or holes are existing they would behave like that.

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So, in other words this you can think of in terms of if there are you see this is the critical stress  $\sigma_c$   $T$  elastic modulus energy required to create a new surface. And this is crack length you can say this is a crack length of the elliptic crack. So, it is under tension this is under tensile force these all it will behave in other words this place a major role. So, if

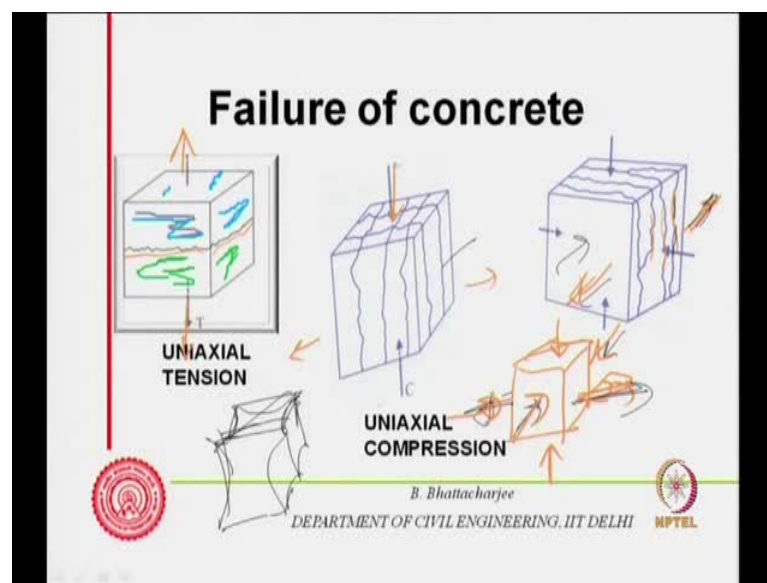


already some holes are existing and if already some holes are existing and their dimensions. So, larger the dimensions less will be the critical stress. So they will start actually fracture will start propagating from them at some stress that is  $\sigma_T$ .

Now, in concrete you have lot of capillary force. And therefore, multiple cracks should actually generate you know multiple cracks will generate. So, this is your required energy line here it will not be a single not will be straight line. Because it will be some total of number of straight line. Therefore, because there are number of holes and corresponding to for example, at some  $\sigma_0$  at this C0 cracks will start. Propagating at some size other  $\sigma_1$  you know cracks will propagating from a smaller sized voids or holes of force, that are existing in force.

So, that is for the actually multiple cracks in concretes so multiple holes in concrete and from their actually cracks start propagating. So, largest hole for the crack will start propagating for largest hole. Even if there is, shrinkage cracks if there is shrinkage cracks from there it can propagate. Whatever is the largest holes or flow in the material from the crack would start propagating. And then possibly joint the motor cracks in the motor. Which are also might start you know which I been moral you know other cracks somewhere else. And once the join together the failure or fracture occurs.

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So thus have the mechanism of failure concrete and this you can understand. Supposing, I applying tensile load to concrete it should ideally fail Uni-axial tensile strength ideally

you should say like this. And this load you know directly if, I have been to able to apply this is because material. So it is weak in tension because particle bonds are not very strong. So this is weak in tension and if you look at uniaxial compression applying load like this from this side to this. And this side to this Uni axial compression it might develop multiple fracture ideal Uni-axial compression. Because, material is strong in compression. Just go to the next diagram then we will come back again. So suppose I applying bi axial both this sides, force from this side.

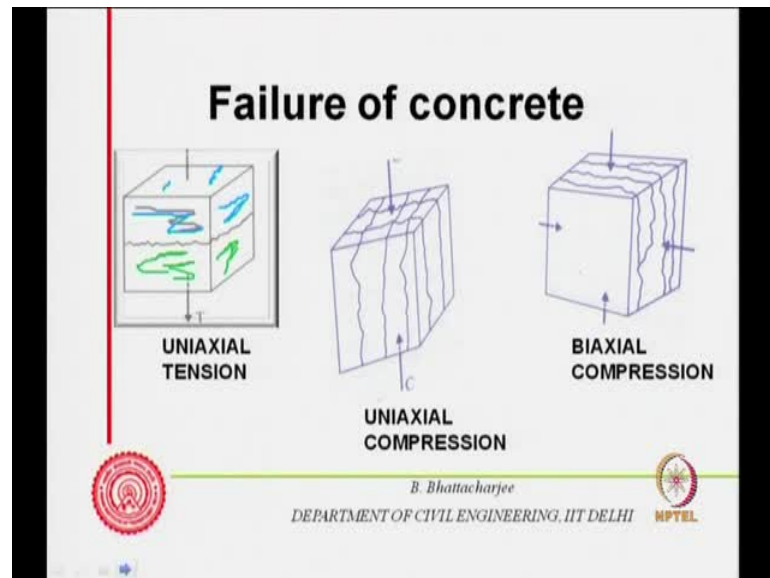
Now, what will happen when I force apply force from this side it will actually try to actual try to move away in this direction. And when I push it because, from this side also material try to move towards this direction. Therefore, you have cracks like this. So in case of cubes whenever you applying you load, the material has a tendency to move along this direction. Because of persons effect you know material has a material has a tendency to move along this direction. Because of persons effect, if it is biaxial applying load like this this will be further moving.

So, in biaxial case you can see the cracks are particle that means material is trying to going along to this direction. So, tensile action is along this direction and cracks comes because of the direction in this 1 Uni-axial. Therefore, crack can come in both the direction I mean Uni-axial compression crack can come in both the direction. And if you know recall this situations where you had to actually seen from both the side. We have seen the failure plane looking like this both the sides.

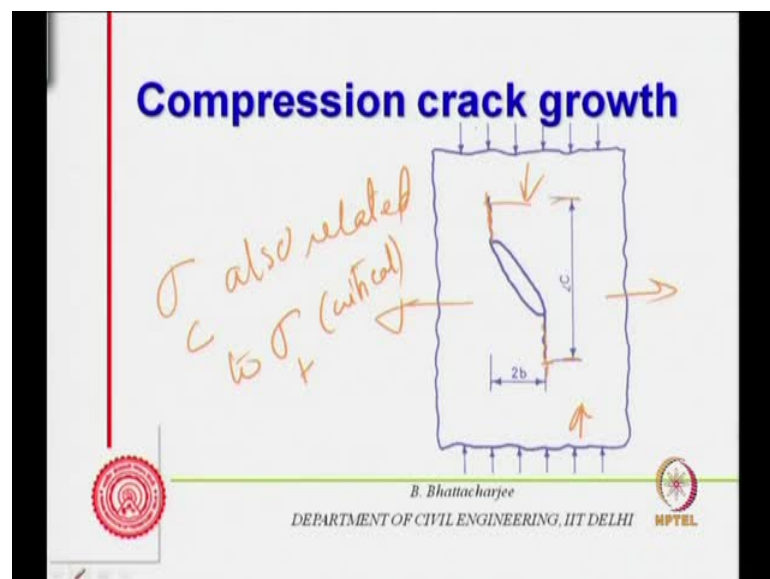
You know both the sides coming like this you had something like this kind. If you remember the failure plane that we have seen earlier were something of this kind you know something of this kind failure plane. We saw in case of concrete are both, because under Uni-axial compression material trying to both the transfers direction. And the concrete is weak in tension. Therefore, it will fail while if you will apply from all 3 direction compression you can really do nothing it will not fail to nothing. You know if you apply compression from all 3 directions this side as well. As you know compression from all the sides crack axial situation it can extend good lot of load. Physically you can understand what will happen if you try to compress it from all the sides, simply the voids will get you know they will get reduced.

Possibly all particles will come close to each other. Now, after that after that you know crushing them at atomic level is not possible. So it can take actually enormous amount of load under tri axial situation, biaxial off-course. Because there is no tension so any that is separate issue that can take much larger load under tri-axial situation. Biaxial failure will be like this Uni-axial failure will be like this.

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So failure of concrete therefore, we understand. Even the compression failure essentially is occurring not really by compression but, because of the tension in the transverse

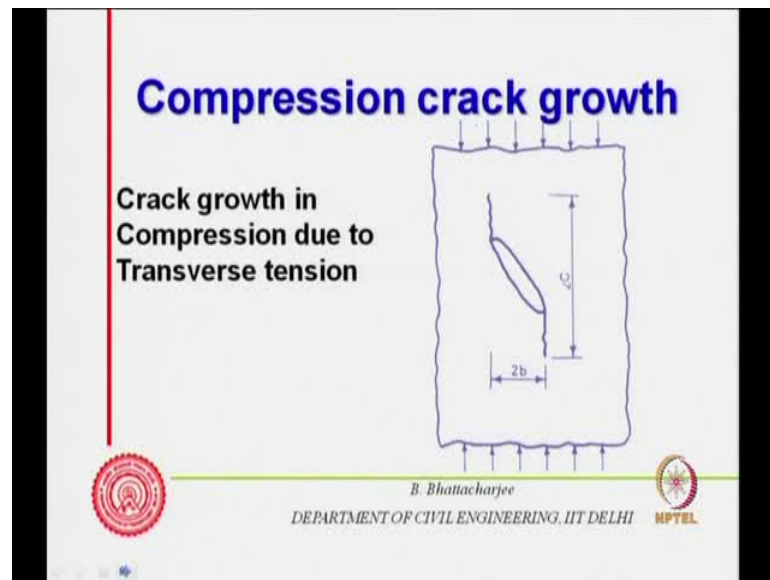
direction. Let us, see something more tension in the transverse direction transverse of this is what it is. Suppose you are applying compression like this they will be tension along this direction. So, if you have a capillary pole large pole or large fisher or kind of a, you know crack. Or whatever it is actually the crack will propagating this manner. Even under compression and this we once the cracks propagating up to this or this crack. You know this is the 2 c. So you can see the compressive strength is also related to  $\sigma_T$  critical that we talked about.

So compressive Uni-axial compressive strength will be related to critical a stress that we have talked about. Wherever some more other it is not easy to model them but, it will be somehow related to the  $\sigma_T$  that relationship perhaps is not so easily understood. It requires more result so that is it. Therefore, What you have understood so far? Just quickly trying to recall look at it. Failure of concrete takes place through aggregate interface in normal strength concrete high strength concrete it can occurred there. So, you will come back to that interface that we have already know an interface exist. Which is transition from aggregate phase to the paste phase around thirty Micron. if you look at the failure of concrete the failure occurs of the aggregate boundary.

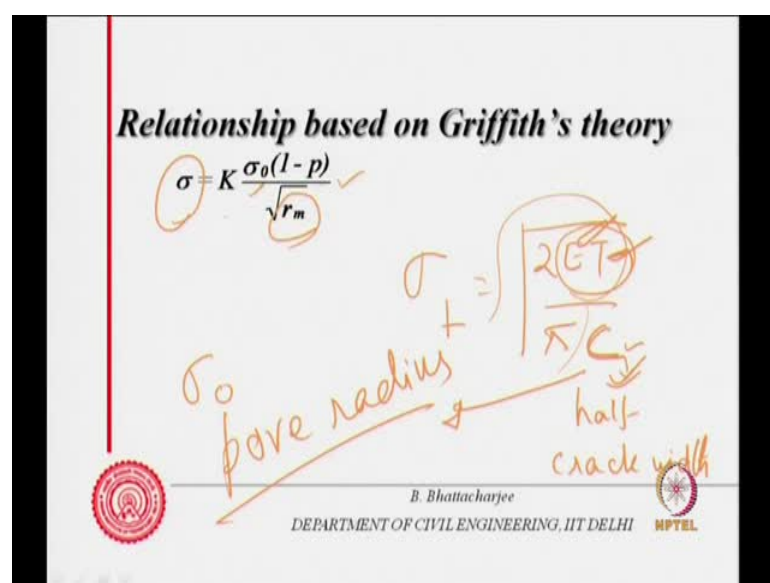
So therefore, there is phase which will call as interfacial transition which is important also we looked in to what could be the failure mechanism. We have seen that in such material theoretically estimated strength is much higher than actually that is realized. And this must be because of some kind of flaws so stress concentration around flaws of the holes existing. That one can look into and fracture process can be thought of as generating new surfaces, creating new surfaces which require phase energy.

So rate of strain energy will release. If it is more than that require to create new surface energy require to create new fracture will propagates spontaneously under tension. Now concrete under compression also fails in tension in the transverse direction. And that is what we have to load it to so, just we will follow it up further that is let us look into more of it. So, crack goes in compression due to transverse tension that is what is most important to understand.

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So therefore, if we look at the previous 1 you know it was sigma T was something like under the root 2 ET divided by pi into C so, C is the half crack width. And if we assume that we have do not have really shrinkage cracks in the situation, shrinkage cracks are not occurred. Which actually do not necessarily not large shrinkage cracks cannot occur in concrete. Then this C would correspond to something like, a pore radius, radius of the flow that is existing. So it, will something like pore by ds therefore, some sort of a measure of the pore radius. If I have C will be related to pore radius T off-course, is surface energy. Now E is modules of elasticity of pore flee solid pore flee solid and

surface energy is also is that of pore free solid. Therefore, if pore free solid I considered and I find out  $\sigma_0$  for the pore free solid. For a porous solid it can be simply related to this, portions can be actually related to this constant etcetera. Can be related to  $\sigma_0$  into  $1 - p$ .

Because more the porosity this effective  $\sigma$  would get reduced, effective  $T$  also get reduced. So, this is simple model actually get could be much more complex model that I can fit. In empirical kind of relationship for compressive strength with some  $\sigma_0$  minus  $P$  into another constant.

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**Relationship based on Griffith's theory**

$$\sigma = K \frac{\sigma_0(1-p)}{\sqrt{r_m}}$$

$$\ln r_m = \frac{\sum_{i=1}^n V_i \ln r_i}{\sum_{i=1}^n V_i}$$

$$\sigma_t = \sqrt{\frac{2ET}{\pi c_l}}$$

$$\sigma_t = \sqrt{\frac{2E_0 T_0 (1-p)}{\pi \sqrt{r_m}}}$$

$$\sigma_c = K_1 \frac{(1-p)^b}{\sqrt{r_m}}$$

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Now,  $\sigma_0$  is also is a strength at 0 porosity which is, almost difficult to determine.  $r_m$  is a measure of average or some kind of a mean pore size or some sort of a mean pore size. You know this tell us that it would be universally proportional to the forces, so simple relationships of this kind were tried out. And this is called you know average pore size mean distribution radius is called mean distribution radius. Where  $V_i$  is the volume of pores corresponding to  $r_i$  radius. So this is the sum total of all volume of pores and this actually logarithmically. So, this is logarithmically  $\sum V_i \ln r_i$  so mean logarithmic pore radius. Because radius pore radius varies in logarithmic scale, you know it will vary from as usual.

See from nano size to fraction of millimeter size. So, over a large range so it is better to express them in logs in logarithmic scale. And this is the log mean radius is weighted the



average with respect to volume something of this claim is this is called mean distribution radius. So sigma T if you recall something like this and therefore, 1 can possibly write Eo To. Where Eo To stands for pore free modules of elasticity of pore free. Material To is a surface energy for pore free material. When it is some porosity already existing in the material.

So, both this can be written as Eo into 1 minus P and this can be multiplied into To into 1 minus P. So, if you take it about this will be the constant. And 1 minus p will be come out and critical stress will be something like this. So therefore, from this it can be understood that the strength is a function of porosity higher. This relationship is the too simple but, still it is good to understand qualitatively that higher the porosity strength will be lower. And more the larger the pore sizes strength will be again lower so critical stress because critical stress is related to this.

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**Relationship based on Griffith's theory**

$$\sigma = K \frac{\sigma_0(1-p)}{\sqrt{r_m}} \quad \ln r_m = \frac{\sum_{i=1}^n V_i \ln r_i}{\sum_{i=1}^n V_i}$$

$$\sigma_t = \sqrt{\frac{2ET}{\pi c_l}}$$

$$\sigma_t = \sqrt{\frac{2E_0T_0(1-p)}{\pi \sqrt{r_m}}} \quad \sigma_c = K_1 \frac{(1-p)}{\sqrt{r_m}}$$

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Now, when does you know one can possibly write compressive strength is functional base critical stress tensile critical tensile stress as well. Because compressive failure is also failure due to tension in transverse direction. So, with all this you know one can empirical relates sigma C to 1 minus Pp divided by under root rm. This is a simple relationship qualitative just to understand. That compressive strength cube compressive strength or cylinder compressive strength will be a function of the porosity of the

concrete. And also a function of root over mean radius some sort of mean radius which we have calling as mean distribution radius.

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**Relationship based on Griffith's theory**

$$\sigma = K_2 C^{\frac{(1-p)}{\sqrt{r_m}}}$$

$$\sigma = K f_{ca} f_e f_a f_T C^{\frac{(1-p)}{\sqrt{r_m}}}$$

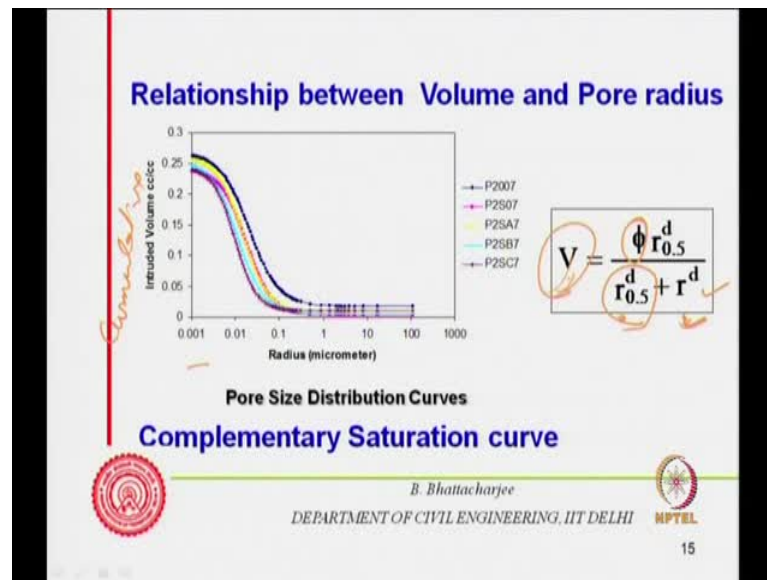
**-Strength of concrete increases as porosity and pore sizes decreases**

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You know mean distribution radius and whereas been relationship develop along this direction while putting this as cement content or various other factors etcetera. So one thing is understood most important what I like to point out here, is the strength of concrete increases porous it increases and porous size decreases. Increases strength of concrete increases porosity increases and porous size you know strength of concrete increases porosity decreases strength of concrete increases. As porosity decrease porosity and pore size decreases. I think that, what it is strength of concrete increases as a porosity and pore size decreases both. So, porosity decreases strength will increase pore size decrease strength will increase. This is what you have must understand that is the fundamental.

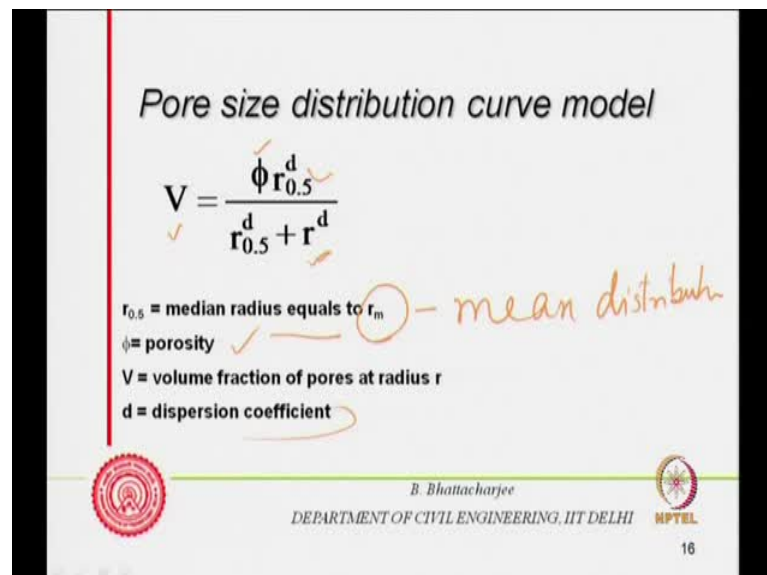
Now, volume of pore and pore radius one can actually you know obtain from, What is known as pore size distribution? I will not going to basis of this. But, what I would like to tell you that it you know the volume of pores, can be related to this mean radius. Some dispersion coefficient and overall porosities, so volume of porosity corresponding cumulative volume of porosity can be related to mean distribution radius. You know b can be expressed, b is the cumulative porosity at any points included volume for marker introduction porosity volume you get.

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Let me say this is cumulative porosity can be related cumulative porosities from small pore size to other pores porosity. Can be related to porosity cumulative porosity, can be related to pore size and mean through in terms of mean distribution radius and over all porosity.

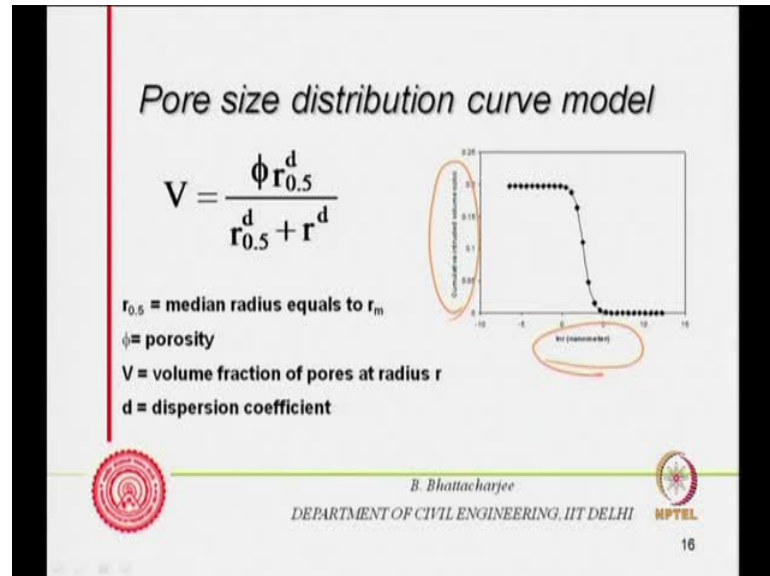
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And therefore, I can express you know this is called mean radius which was same as  $r_m$  this  $r$  point 5 medium can radius equals to  $r_m$  or it is called mean distribution radius.  $V$  is the volume fraction of pores at radius  $r$  and this is some dispersion coefficient this is

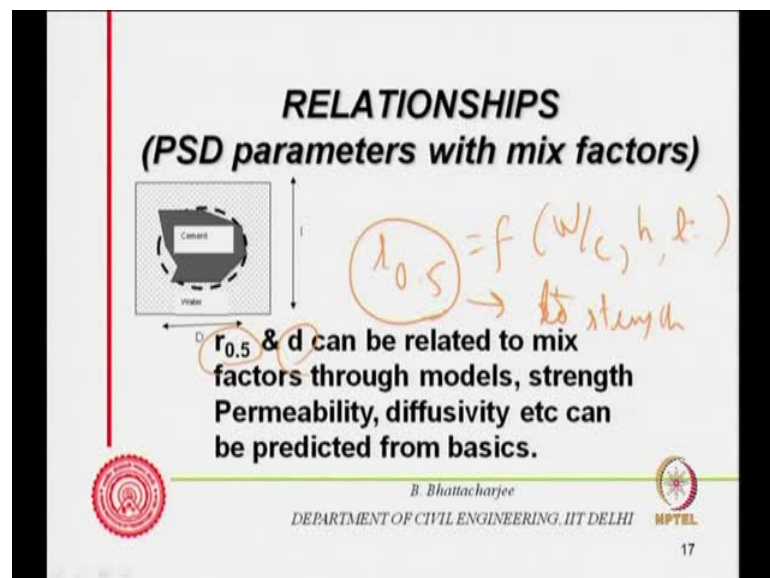
overall porosity. All I am trying to point out that porosity this you know there can be relationship existing between cumulative porosity and  $r$  and from this what happens.

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You can actually obtain you know  $\ln r$  versus cumulative porosity this sort of graph can be obtained.

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But, what is implication in our case, we can actually find out porosity by assuming simple model. For example, this is the, cement this is the water, wall outside and we can  $r$  point 5 and  $d$  we can express to mix factors. So through some models and then if I

know  $r_{0.5}$  as a function of. Let us say  $w$  by  $C$ , degree of hydration, etcetera age  $t$  etcetera. Then I can get the full pore size distribution and we know that  $r_{0.5}$  is related to strength this is related to strength.

(Refer Slide Time: 40:28)

**RELATIONSHIPS**  
(PSD parameters with mix factors)

The slide features a diagram of a cement particle with a dashed line representing the water-cement interface. Handwritten notes in orange ink include:

- $r_{0.5} = f(w/c, h, t)$
- $r_{0.5} \rightarrow \text{to strength}$
- $f(\text{strength}) = f(r_{0.5}) \Rightarrow \phi$
- $\text{Strength} \Rightarrow f(w/c, h, t)$

At the bottom, it says: B. Bhattacharjee, DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI, NPTEL, 17.

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**RELATIONSHIPS**  
(PSD parameters with mix factors)

The slide features a diagram of a cement particle with a dashed line representing the water-cement interface. Printed text below the diagram states:

$r_{0.5}$  &  $d$  can be related to mix factors through models, strength Permeability, diffusivity etc can be predicted from basics.

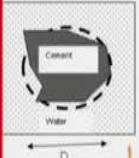
At the bottom, it says: B. Bhattacharjee, DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI, NPTEL, 17.

So therefore, I can relate strength to I can actually relate strength to strength can be related to strength can be expressed as a function of  $w$  by  $C$  age and  $t$ . Because strength is a function of  $f$   $r_{0.5}$   $r_{0.5}$  and  $5$ . Now,  $5$  can be estimated from powers module,  $r_{0.5}$  if I can relate to water cement show I debriefed. This in time then I can relate

strength to all this. So it is possible to use simple model. To arrive at you know  $r$  point 5 and  $d$  as the function of water cement ratio.

(Refer Slide Time: 41:13)

**RELATIONSHIPS**




$$I = \left[ k_1 D^3 + k_1 D^3 \frac{\rho_c w}{\rho_w c} \right]^{\frac{1}{3}} \approx k_1^{\frac{1}{3}} D \left[ 1 + \frac{1}{3} \frac{\rho_c w}{\rho_w c} \right]$$

$$\alpha = 1 - k_2 (D/I)^3$$

*degree of hydration*


$$\alpha = 0.217 \ln\left(\frac{w}{c}\right) + 0.061 \ln(t) + 0.618$$

$$r_{cap} = D \left[ -0.215 + 0.423 \left(\frac{w}{c}\right) - 0.041 \ln\left(\frac{w}{c}\right) - 0.011 \ln(t) \right]$$



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And this just as a qualitative term I do not think I have going to the expression of algebra. But, what I mention is that,  $r$  can be related to this is the degree of hydration  $\alpha$  degree of hydration,  $\alpha$ . This can be related to age and water cement ratio and  $r$  capillary been capillary porosity can be related to water cement ratio and age.


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**SEGMENTATION AGE**

W/C	0.3	0.4	0.5	0.6
Age (days)	0.03	0.5	9	199
Lit		3	14	180


$$r_{0.5(gel)} = a_1 \frac{(w/c)^1}{t} + a_2 (w/c)^2 + a_3 (w/c) + a_4$$

$$r_{0.5} = [r_{0.5(gel)}]^{p_{gel}} \times [r_{0.5(cap)}]^{p_{cap}} \quad \text{or,} \quad \ln(r_{0.5}) = p_{gel} \ln[r_{0.5(gel)}] + p_{cap} \ln[r_{0.5(cap)}]$$



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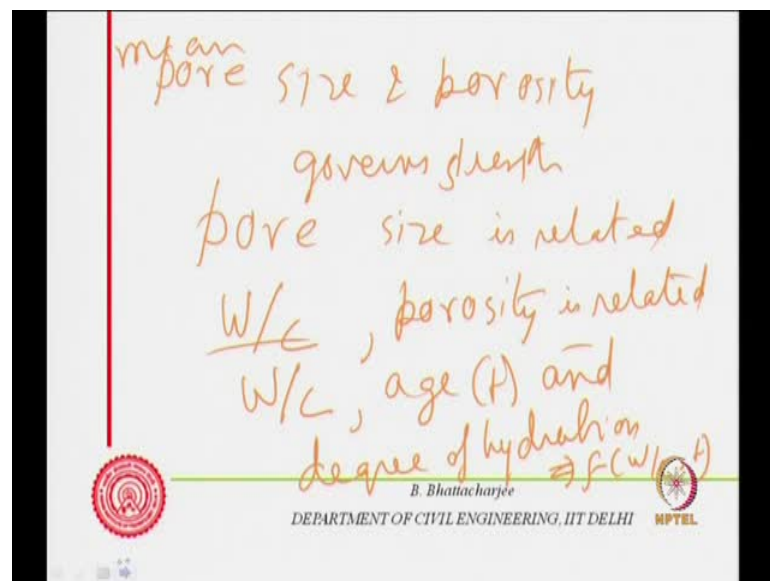


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And overall  $r$  point 5 also can be related to overall  $r$  point 5 for the gel force it can be related happened 5 can be related to water cement ratio. Because this is related to water cement ratio and this is also related to water cement ratio and time age. So, using this kind of a model one can calculate the segmentation age. If you remember we talked in terms of curing and theoretical and estimated segmentation age seems to match somewhat. Any way that is not my concern but, I am trying to point out that  $r$  point 5 can be related to  $r$  point 5 can be related to water cement ratio.

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That means the pore size is related to  $w$  by  $c$  porosity is related to  $W$  by  $C$  age and  $t$ . That is you know age is  $t$  and degree of hydration age is  $t$  and degree of hydration. Now, degree of hydration is itself is a function of  $W$  by  $C$  and  $t$ . So degree of hydration is function of therefore, porosity and pore size is little water cement ratio  $n$  I. And we have seen that pore sizes mean pore size  $m$  porosity strength. Hence, strength is governed by water cement ratio and age.

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### SEGMENTATION AGE

W/C	0.3	0.4	0.5	0.6
Age (days)	0.03	0.5	9	199
Lit		3	14	180

$$r_{0.5(gel)} = a_1 \frac{(w/c)^4}{t} + a_2 (w/c)^2 + a_3 (w/c) + a_4$$

$$r_{0.5} = [r_{0.5(gel)}]^{p_{gel}} \times [r_{0.5(cap)}]^{p_{cap}} \quad \text{or, } \ln(r_{0.5}) = p_{gel} \ln[r_{0.5(gel)}] + p_{cap} \ln[r_{0.5(cap)}]$$

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So, just let me put it in words now, let me put it in now words, you see what we have seen the  $r_{0.5}$  is a function of  $r_{0.5(gel)}$  to the some gel porosity. And it is a function of water to cement ratio and age. Similarly,  $r_{0.5(cap)}$  is also a function of water cement ratio and age. And we know the porosity itself is function of water cement ratio. So in other words what we are trying to say qualitatively quantitative we leave out.

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### Factors effecting Paste strength

- Strength of paste is depended on bond strength and number of bond & in addition on pores i.e. porosity and pore sizes.
- ❑ Capillary & total porosity is dependent on w/c, so, Strength is governed by w/c given by Abraham's law.

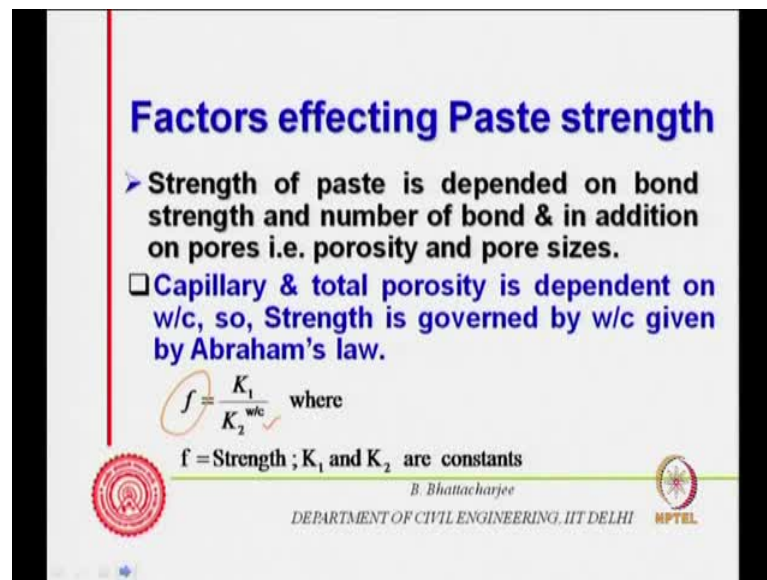
Porosity increases with w/c  
Pore size increases with w/c  
hence strength will decrease

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What we are trying to say is strength of phase depend on off-course, bond and number of strength. But, it is maze mostly depend on porosity and pore sizes. This is will be main contributive factor in normal strength concrete. And capillary porosity and total porosity is dependent on water cement ratio that is what we have seen. And pore size is also got bind by water cement ratio. That is what just I have seen. So pore sizes both pore sizes so porosity increases with W by C. Pore size increases with W by C. And it porosity increases pore size increases strength decreases. Hence, strength will decreases and strength will be decreased. So that is what is a Abrahams Law you know total porosity demands. So strength is demand by water cement ratio that is what is given by Abrahams Law.

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**Factors effecting Paste strength**

- Strength of paste is depended on bond strength and number of bond & in addition on pores i.e. porosity and pore sizes.
- ☐ Capillary & total porosity is dependent on w/c, so, Strength is governed by w/c given by Abraham's law.

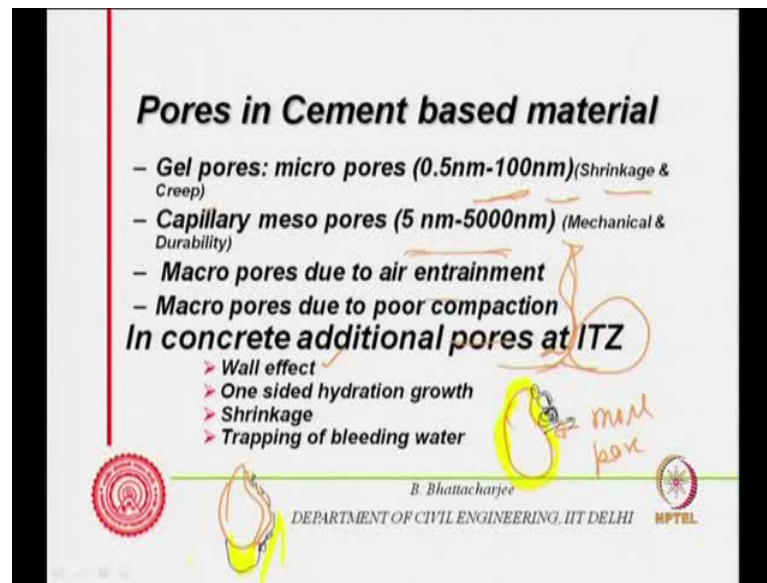
$$f = \frac{K_1}{K_2^{w/c}} \text{ where}$$

$f$  = Strength ;  $K_1$  and  $K_2$  are constants

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And Abrahams Law is something like this, strength is a function of inverse. You know kind of reduces with water cement ratio these are 2 constants. So, Why does it reduces water cement ratio? Because higher than the water cement ratio my porosity will increase. And my mean distribution radius will also increase and we have seen that strength is a function of porosity higher the porosity, strength is lower 1 minus P higher the mean distribution radius strength is lower. Because it is related to under root 1 over under root mean distribution radius. So, that is why strength is a function of water cement ratio.

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That is what was discovered by d Abrahams in 19 30 20s and 30s you know very early in 20s actually. So gel force the 1 of the force in cement based material I think I have talked about, this earlier. Gel force is 1 of them capillary force in the cement paste. Then you can see the orders and micro pores due, to air entrainment and pore compaction. So, all these will affect the strength. All this will this off-course, will affect the strength less because size is very small. So, because largest size pores affects the strength therefore, there affect of strength is less. But, they actually has a bigger role in shrinkage and creep.

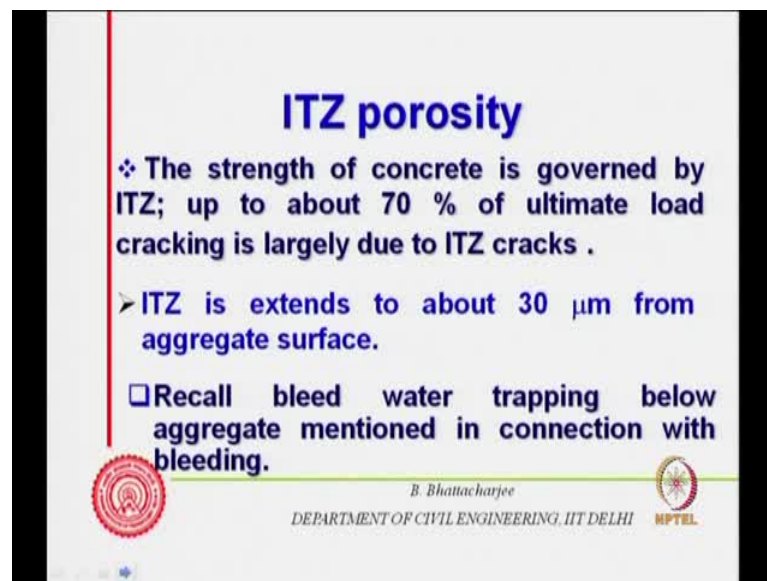
And off-course, in concrete you will have ITZ porosity and this ITZ porosity counts because of all affects. Which we have already talked in relation to you know when we are talking of aggregate packing. So, at the boundary of the aggregate you will not have good packing of the cement particle. Then 1 sided hydration growth hydration growth hydration product will go from the bulk piece side, not from the aggregate side. And if, shrinkage occurs page shrinkage aggregate do not bleed water. Water has the tendency to come up because, it is specific is low and if, it get step below aggregate, There will be bleed water taping.

So therefore, bleed water trapping and all this makes this zone as we talked about, aggregate, mortar interface, this zone honorable actually there the porosity is more. So, the porosity is more here more porosity more pore. Because of all affect hydration growth will take place for cement particle. You know if I have a cement particle here, it

will grow from cement particle here. That packing will not be very good and hydration will take place more on this direction, very little along this direction. Shrinkage occurs shrinkage will occur there. So even, before you are anything any load has been applied, Even before you have applied any load you might have some sort of fisher existing here. You know you might have some sort of fisher existing here, because you will have materials there. So therefore, you will have fisher of plate like pores existing there.

If, the bleed water coming and getting trapped here bleed water coming and getting trapped here you know bleed water coming and getting trapped here. Because some water trying to go up it gets trapped here. Then, you will have again some sort of fisher existing there. So, they are the 1 which starts the actually from where, my fracture start pore getting. Because there the largest size pore and from there fractures are propagating join some of the pores in the bulk paste. And may all of them have joint actually over all fraction will occur.



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**ITZ porosity**

- ❖ The strength of concrete is governed by ITZ; up to about 70 % of ultimate load cracking is largely due to ITZ cracks .
- ITZ extends to about 30  $\mu\text{m}$  from aggregate surface.
- ❑ Recall bleed water trapping below aggregate mentioned in connection with bleeding.

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So let us progress further we have understand that therefore, ITZ porosity the strength of the concrete is governed by ITZ. Because they will have the largest porosity up to about 70 percent of ultimate load, packing is lightly did to ITZ cracks. Because ITZ would have larger fishers. And from there cracks are propagating as we said it extend around thirty micron from the aggregate surface. And bleed water trapping will aggregate mention in connections of bleeding. If we have we have not talked much about bleeding

but, I have just explained you now. Because when bleeding occurs you see the concrete is made up of materials having different specific gravity. Cement has got a specific gravity of 3.15 aggregate will have something in between. If I am not using very heavy aggregate, like hematite in heavy concrete. So, ordinary concrete aggregates will have around 2.6 2.7 or at best 3, as a specific gravity. But, water is 1 so it will have a tendency to go come up all the time. And if it gets in case where heavy bleeding is occurring high water cement ratio system you know cement is not able to hold the water level by tendency to come up. And it might get trap below the aggregate. So if it gets trap below the aggregate that will be efficient. You know all this we have just mentioned a few minutes before.

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**Porosity in concrete**

- Bulk paste porosity and pore sizes are governed by w/c.
- ITZ porosity is more and pore sizes are larger; & are governed again by w/c
- Hence concrete again follows Abraham's Law.

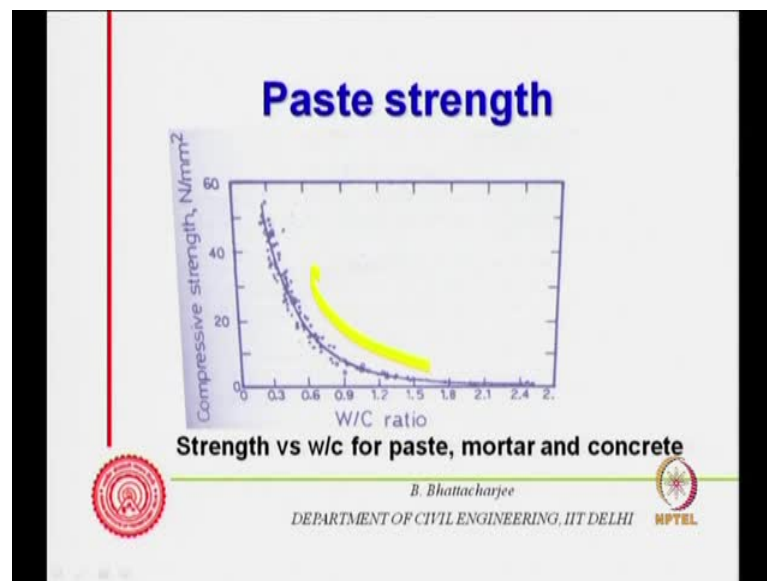
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So therefore, bulk paste porosity and pore sizes they are governed by water cement ratio that is what we have seen. And since water cement ratio governs ITZ porosity is also more and pore sizes are larger. And this also governed by water cement ratio. Larger water to cement ratio bleed water to trap chance could be higher. The shrinkage affect and even you know the porosity in the I T Z is always more than the bulk paste porosity. High water cement ratio means bulk paste porosity is more ITZ porosity will be still more. So it is that that is what governs the strength in the normal concrete. Hence concrete again therefore, concrete follows a, Abrahams Law. So cement paste follows Abrahams Law, mortar will follow Abrahams Law and concrete will also follows Abrahams Law. And that is because higher the water cement ratio I will have more pores



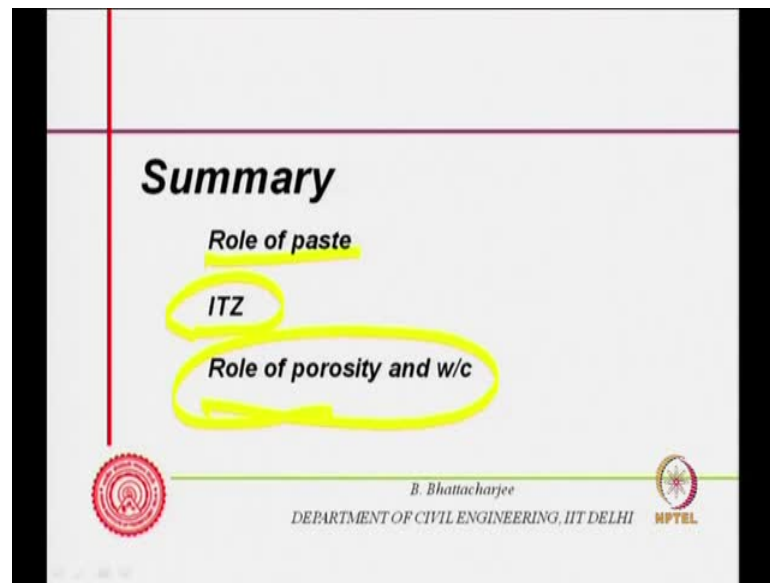
in the system. Because we have seen in you know in the earlier modules that capillary porosity is function of water cement ratio. More the water cement ratio more will be the capillary force. When we looked at the hydration process, we can recall it you know we are talking about expanding classer model. More water cement ratio, more force more capillary force. And size of the pore will also increase with water cement that is what we have seen today. Therefore, higher the water cement ratio, strength will be lower both for cement concrete and mortar.

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As well and this is what possibly the next diagram show this diagram will show paste strength or mortar strength. Or anything of that kind you know water cement ratio higher the water cement ratio. Composite strength reduces composite strength reduces higher the water cement ratio. So therefore, this is the fundamentals so, water cement ratio is the fundamental one which governs the strength of concrete. And that is what we have understood today and why does it affect. Because higher the water cement ratio I will have more capillary porosity. And my pore sizes will be larger and size of the pore governs the strength larger the size, critical stress required to initiate failure from that force will be smaller. Which means the crack will start generating first and propagate again faster. And also we have seen that compared to strength is actually composite failure is because of tensile failure again co tensile failure again. So therefore, this is what we have understood will see the role of aggregate and another things in the subsequent classes.

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So, we have essentially seen the role of paste, role of ITZ mainly role of porosity and water cement ratio. So I think the first lecture gives us this idea why strength is function of water cement ratio is very important to understand. That strength in this all this kind of material decreases with increase in water cement. So therefore, quick in getting high strength concrete is using reduced water cement ratio, reduce it as much as possible. Keeping in mind that you have to also take care of the work ability we said earlier that below the consistency standard consistency.

The water may not be sufficient you know to make in the flow available flow paste flow not at all it will remain dry in consistence paste. So this issues are there but, the technique of getting high cement concrete revolves around this issue. Understanding that, lower the water cement ratio lower will be the porosity and pore sizes. And that is the way you know from that only we proceed to our high concrete any way high cement concrete is not the issue at all at the moment. We have looking at normal strength concrete. So today we have understood role of paste essentially, ITZ and mainly we have looked into this.