Geotechnical Engineering - II

Professor D. N. Singh

Department of Civil Engineering

Indian Institute of Technology, Bombay

Lecture 9

Direct Shear Interpretation of Test Results II

Now let us complicate the thing. Rate of shearing plus depositional history. The way the formation took place, how the sandy deposit was formed, and this is where I will introduce a term which is known as K parameter and to be specific, I will say that this is K_0. We write this as earth pressure at rest. Truly speaking this K is a function of $\left(\frac{\sigma_h}{\sigma_v}\right)$. That means,

$$\sigma_h = K_0\sigma_v$$

What is the meaning of at rest? I am not shearing the sample at all, at rest no movement, no displacement. So, this is at rest condition. From here we will derive active and passive situations later on K_a and K_p. So, K_p will become passive earth pressure condition, and this becomes active earth pressure. So, to begin with, I will deal with only K_0 condition. What is the interpretation of this?

The interpretation would be the initial state of the stress in the sample is the moment I applied σ and there is no shear the value of σ_v is σ and value of σ_h will be $K_0.\sigma$. Before shearing process at rest condition, the sample is just static. Draw the Mohr circle for this. You can draw it easily no issues.

And normally K_0 is going to be a number which is less than 1, so that means on a τ-σ plane, the initial state of stress of the sample would be, this is let us say σ value and this will become K_0 times σ, initial state of the sample. Now comes the shearing part.

So, when you are shearing the sample, what is going to happen? Your τ is picking up, so starting from this point of σ_3 if I draw a vertical line what is going to happen? I require a higher value of σ_v to fail the sample. Is this, ok? Is this correct? So, that means what I am going to do is, if I have to increase the value of σ, K_0 σ also changes, it is not going to be feasible.

So, what should I do? That is a big question. What will be the answer now anybody could think of this, K cannot be changed because K is the ratio of the two states of the stress. Now, please remember σ_h is going to be a sort of your shear strength or the shear stress, now σ_v is equal to γ.z, of the material unit weight multiplied by the depth of the point, so that gives you σ_v value.

Under all possibilities your σ value is going to be constant. But now I want to shear it what is going to happen? Look at this point, as long as this situation is concerned it is fine. Now, the moment you shear it what is going to happen? This is σ_1, this is σ_3 and just now we have discussed that this is the Mohr-Coulomb envelope, this is the point of tangency, point of tangency indicates that the failure has occurred, peak point in this case it is going to be the residual value.

So, this is the state of the stress where you have σ_f, τ_f. Is this part, okay? The failure. If this is the centre this becomes a radius, this is 90°. Where is the Pole? Now, this state of stress you are forcing to be acting on the horizontal plane remember and that is the biggest limitation of the whole test.

That means this state of stress if it is acting on the horizontal plane what I have to do is I have to draw a horizontal line. What is this going to give me? Starting from the given state of stress you get the Pole. Agreed? What have I shown here? I have shown the rotation of the state of stress on the sample.

Now, if you draw a tangent, so not tangent sorry, if you join this point what this becomes? So, passing from the Pole if I connect σ_3 this becomes the minor principal plane. If I draw σ_1 passing through P what is going to happen? This becomes the major principal plane, this is what we call as rotation of planes.

So, this is where σ_1 is acting and this is where σ_3 is acting. If you have followed these concepts solving numericals is not going to be a big issue. So, that part is left up to you, please solve the problems, I think I have discussed all finer aspects over here and should not be a difficult task.

Any questions here? Hope you are enjoying the strength of the materials, where the materials are soils, lot of ambiguities, a lot of expert eyes are required to understand what type of interpretation we are going to do with the results. Because you have to understand the material. If this is clear, should I move ahead?

If interlocking comes so do one thing you please do like this, this is the interlocking effect and now try to give torque, no, no there should not be slippage interlock them, this is the interlocking and now rotate what is happening? So, it is a sort of apparent cohesion. Interlocking as I said would always give you apparent cohesion.

Friction is because of the rubbing of the particles together. Here you have locked the two particles, they are not going to slip over each other. When slippage occurs then only friction develops. So, we came only up to point number one, point number two and K_0, depositional history, number three the most important thing on which the shear strength of the material would depend upon this the type of the soil.

And the fourth one would be moisture content. Very soon, I will be introducing the concept of OCR. So, depositional history is depicted mathematically as OCR, OCR is over consolidation ratio, over consolidation ratio. Another interesting way to characterize the soils which would include the type of the soil, the history of formation and history of deposition also mathematically, very advanced way of characterizing the materials. So, over consolidation ratio, until now we have talked about only consolidation characteristics.

Now, this is the first time I am using the term how the soils are formed. So, suppose if I take you to the geological journey of the soils. Remember these are the Himalayas and then this is the Gangetic plain and somewhere here you have Deltas, Bay of Bengal. And these are the Himalayas.

Take a point somewhere here. What is the state of stress? Something let us say, σ_{v} and σ_{h}. What is happening here? Weathering process, soils get defragmented, water very high speed of the water. So, truly speaking most of the erosion takes place over here, when erosion occurs the state of stress at this point which is today. So, σ, state of stress I am writing today is always going to be less or more, less as compared to what it was in the past.

The ratio of the two is OCR. So, this is fine?

Correction: Read it as:

(*OCR is the ratio of the maximum stress the soil subjected to the state of stress acting on the sample today.)** These materials are going to be under consolidated, oh sorry, over consolidated. By the time soil particle moves and comes over and gets deposited over here now what is happening is a reverse process.

Now, today's stress is going to be more than the stress in the past. Why? Because rivers are bringing continuous loads of sediments and these sediments keep on getting deposited one over the other, consolidation process might not be over. So, in this case the OCR value is going to be 1.

So, what we have done is**, (Correction: *** yeah this is less than 1, today’s is more than past, so OCR is less than 1, OCR is greater than 1 over here because today is less than past).** So, I have now used the state of stress to characterize the soil deposits and I have a strategy now in my mind rather than doing all these sorts of tests which I have been talking about by using only the state of stress in the past and present I know how to characterize them and how to deal with them.

So, this itself is going to guide to the shear strength of the soils. Now, one of the ways to depict all the results would be if I plot τ versus σ and the way I have done over here I will be getting let us say for **(Correction: ***pure sands, when I say pure sands friction angle I am assuming to be greater than 0),** this is the coefficient of internal friction, rubbing of particle with each other causes friction.

Take a water bottle, fill it up with water, keep on adding drop some particles of sands, close the bottle and shake it, sound comes. Why? Because of friction, that is the easiest way to explain to you what friction is, keep the particles of the soil on your hand and rub it.

Now, if you if you look at this there is enough sound coming out of it, granular material this is the frictional mobilization, friction mobilization of the soil, it may cut your palm also. Why? Because all sudden done particles of the sands are quartz and the crushing strength of quartz is 20, 30 MPa, so you have to be absolutely careful when you are rubbing this thing in your palm. A good Geotechnical Engineer would go to the site, take a sample put beneath the palm, rub it and he can tell you what is the c, ϕ parameter, but never do this.

So, I hope now you understand that this is what is going to be for the peak strength, if this angle is ϕ_{peak}, what I am doing is I am consuming this value τ_{peak}, σ is fixed, so $\left(\frac{\tau_{\text{peak}}}{\sigma}\right)$ is a very rough estimation, this is what is going to give me the peak shear strength profile or envelope. However, this is for the, for dense sands and this is for the loose sands.

If I remove this effect of peak I will say ϕ and then what I can do is I can subscribe this to RD relative density which is a function of void ratio. Now, let us analyze a bit more on the friction angle, there is something known as angle of repose, α. What is angle of repose? If I take a funnel and if I pour sands into it and if I open the other end and let the sand fall out approximately let us say 30-centimetre distance, the distance should be small.

Whatever heat gets formed you must have seen in most of the production units this could be grains, it could be sugar, it could be rice, it could be metal slags, it could be soil or whatever. This is what is known as the hopper. The slope which is formed naturally when you drop sands from a certain distance. Why distance is so important? So, that there should not be any impact of falling grains on the grains which are already sitting in the heap, you are not imparting any energy. This slope is defined as α, this is known as angle of repose.

Angle of repose is approximately equal to internal friction angle of the soil, coefficient of internal friction of the soils. So, this concept is very useful in making different type of silos for storing grains or the granular material. And then we can link all this information with the model which we are going to develop now on a τ-σ plane. If this is the Mohr circle, stop writing and please concentrate for one minute on the board, your concepts will be forever.

If this is the Mohr-Coulomb envelope, this is the point of tangency which corresponds to state of material at failure. If I connect it with the point σ_3 which happens to be the Pole of the Mohr circle, this is the failure plane, this angle is theta f, if I connect this point, sorry, if I draw a perpendicular to this Mohr-Coulomb envelope starting from the point of tangency, it cuts the x axis at this point which is the centre of the circle, Mohr circle, this is $(\frac{\sigma_1-\sigma_3}{2})$ if you remember, this is $(\frac{\sigma_1+\sigma_3}{2})$, this we have just now defined as *c* the apparent cohesion and this angle is ϕ.

Now, if this is the theta f failure plane inclination, this angle is going to be what? 2 times θ_{f}. Let us do some simple construction now. And before I move on to that there is a peculiarity about this material. If I draw a perpendicular from this centre and extend it in such a manner that it cuts the Mohr coulomb envelope what you are observing?

This line cuts over at this point which is point number A, the Mohr-Coulomb envelope says that this itself defines the state of stress of the material at failure. So, theoretically a state of stress at failure at point A occurs, but practically before point A is reached sample has already failed.

Why? Look at this, before you achieve point number A, the failure is already occurred at F point. Now, this is your σ_f, sorry τ_f and this is σ_f. So, this is a fictitious point. What it indicates is you have enough strength for the material even corresponding to this confining stress. Because the shear strength available theoretically is this much.

Suppose if I say O, so OA happens to be a theoretical strength which is available for the material, but it so happens that the material fails before theoretical shear strength is achieved and hence this becomes the point of failure. I can define the term factor of safety now. First time I am introducing this concept.

So, what factor safety says? The factor of safety says that you have so much of a strength available, but the failure is occurring at this strength only. So, if I say this is tau f and this is the total strength which is available theoretically, the factor of safety is going to be the ratio of the two. And this is the term which people utilize for designing most of the foundations and Civil engineering structures. Is this part, okay?

There is another interesting way of defining the factor of safety. There is something known as brittleness index of the soil. This is normally depicted as I_B, if I consider this strength as τ_{peak} and this strength as $\tau_{residual}$, the brittleness index is defined as;

$$I_\mathrm{B} = \frac{\tau_p - \tau_r}{\tau_p}$$

And this becomes another way of classifying the soils when you use them as engineering material.

What it shows is the state of the material where the brittleness is too high, dense systems will always show higher brittleness as compared to loose systems. So, I have defined now factor safety's two forms, this itself is a factor of safety term. How much safe I can be without failing the systems. And another factor of safety now I have defined in this way.

So, for all practical purposes this is your $\frac{\sigma_1-\sigma_3}{2}$. Now, if I say that this is point B and this is point P, can I use simple relationship geometrical relationship $\frac{\sigma_1-\sigma_3}{2}$ equal to if this is the friction angle this is going to be this is c tanϕ. So,

$$\left(\frac{\sigma_1 - \sigma_3}{2}\right) = \left(\left(\frac{\sigma_1 + \sigma_3}{2}\right) + c.cot\phi\right).sin\phi$$

Is this okay? So, solve this expression and what you will be getting is,

$$\sigma_1 - \sigma_3 = (\sigma_1 + \sigma_3)sin\phi + 2c.cos\phi$$

I can reschedule the terms, so this becomes,

$$\sigma_1(1 - sin\phi) = \sigma_3(1 + sin\phi) + 2c.cos\phi$$

This answers your question Shashanka what you were asking last time.

We have started from a state of stress where σ_v is equal to σ_1, σ_v happens to be a major stress, so this equation is valid for a condition when σ_v is σ_1 and σ_h is σ_3. The moment we reverse this situation the Mohr circle will change; we will discuss about this later on.

So, now you do this analysis and what I can do is for pure frictional materials I can put c equal to 0. So, ϕ only prevails pure frictional material so this becomes,

$$\sigma_1 = \sigma_3\left(\frac{1 + sin\phi}{1 - sin\phi}\right)$$

This term is K term, K parameters. The way we have used K parameter now is we have to relate σ_3 by σ_1. So, you have to transpose it and this will be written as,

$$\sigma_3 = \sigma_1\left(\frac{1 - sin\phi}{1 + sin\phi}\right)$$

So, this becomes K_a, coefficient of earth pressure at active stage. Is this part clear? Two more things before we disperse today. If I am assuming a pure frictional material this line the Mohr coulomb envelope is going to pass through the centre and if I draw this line over here like this the point where the envelope touches the Mohr circle becomes the point of maximum obliquity. Is this, okay?

So, this becomes the maximum possible friction angle a system can mobilize, in reality what is going to happen is some fraction of ϕ will be getting mobilized and the material is going to fail. Few limitations and strengths of direct shear box test, so I am sure you must be realizing that a lot of things can be obtained from a simple direct shear test.

The limitations are as we discussed, drained conditions cannot be simulated, not valid for fine grained materials, area of cross section changes and hence you cannot shear it much, the pressure keeps on changing at the interface, it is not constant, because the area of cross section keeps on reducing and so on.

The strengths are simple test and whenever you are dealing with the field situations and suppose there is a stratified deposit, what I can do is I can take out a sample from here capturing the stratification, bring the sample, put it in the box, shear it and obtain the shear strength along this plane, that is the beauty. It could be inclined plane also, does not matter, if their strata are like this I will still try to take out a sample from here, these are the exploration techniques.

I will bring the sample in the lab, fit it in the direct shear box and I will get the shear strength at this plane. So, in short there are lot of beauties of doing this test, though the disadvantage is many. And what we will do is to overcome these disadvantages we will now switch over to triaxial testing, that will be from next lecture onwards.