Geotechnical Engineering - II

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Finite Slopes Friction Circle Method

So, I have discussed about slope stability analysis, and I was talking about 2 situations, one is the infinite slopes where we did lot of analysis for different types of infinite slope conditions, then the second situation was finite slopes. And I talked a bit about in my previous module, how to analyse the slopes which are finite in nature. And this is where I introduced the concept of factor of safety, you must have noticed, and that factor of safety we had defined as factor of safety associated with the friction angle and the factor of safety associated with the cohesion. And we have denoted this as F_ϕ and F_c.

And the problem which I solved in the class, I showed you that the best possible optimal solution would be when $F_c=F_\phi$, and that becomes the condition where you have the equal weightage for the cohesion and friction to get mobilised for the soil mass. Another interesting example of this concept of mobilisation of cohesion and friction is what is known as friction circle method.

So, I will introduce today the concept of friction circle. And basically, friction circle method is applicable to the cohesive soils, purely cohesive soils. We call it again as a total stress analysis. It is understood that the $\phi_u=0$, and undrained cohesion is equal to 0, but then we will extend this concept to a general c-ϕ soil. We assume that the failure surface is a part of circle. We define this as arc of the circle. I introduced this concept of arc and the chord in the previous lecture also. You remember we have shown the mobilisation of cohesion along the chord of the slope. So, I will explain it again today.

So, in short, suppose, you have a slope here. So, this is the ground surface, this is also the ground surface. And we have been talking about different modes of failures of this slope. So, there could be a situation where the slope might fail like this, typical face failure, the failure is of this block, there could be a situation where the failure might occur like this, the slip surface is passing through the base of the slope. And there could be even deep-seated failures also.

Now, I am sure you will realize that these types of situations are going to be valid for higher c values. So, that means there could be a transition from 1, 2 to 3, as c increases.

There is another interesting situation when we talk about the slope stability of finite slopes could be, many a times people ask you a question suppose if I rest a foundation over here, there is some facility which I want to construct. First question as a designer, which comes to the mind is, whether this is going to be a slope instability problem, or whether this is going to be a bearing capacity problem related to the foundation of the system. So, a good designer would be checking for both the situations depending upon what? What is the critical criteria, which would differentiate between a shear failure, which could be for the bearing resistance, or which could be for the slope instability.

So, suppose if B is the foundation width, and if this is located from the edge of the slope at a distance of x. So, what is going to happen? If you keep on decreasing the value of x, the foundation comes and sits at the edge of the slope, one situation. And what is going to happen then? We assume semi-infinite soil mass, but then you are defying that. So, that means, if I say that this is a hypothetical surface, and this is a continuum, if this foundation is going to sit over here, what is going to happen? I have removed this much portion. So, only this portion is contributing, it is not going to be a bearing capacity problem. The slope is going to be unstable because of this type of situation.

So, that means x becomes a very critical parameter. However, if x keeps on increasing, it becomes 3 to 5 to 10 times to 20 times the value of B, what is going to happen? This keeps on shifting on the right-hand side, and this becomes a typical bearing capacity problem. Now, extending this surface, I want to analyse let us say the stability of the slope because foundation designs and analysis you will be doing in another course, not in this course, this is an introductory course, for soil mechanics II, geotechnical engineering II, where we talk about the properties of the soil mass, and how these properties particularly the shear strength parameters are utilised for designing the systems like this.

So, I am going to restrict my discussion only on the stability of the slopes which happens to be the topic for discussion. So, we are assuming that this is the typical slip surface, circular. It might be passing through any of the points either 1, 2, or 3. In general, what we do is we normally use graphical methods, and we define a point or the axis of rotation. So, point of rotation means, this is the point about which the slip is going to take place, look at the motion of my hands. Now, this is the point about which the material is going to slip off. Have you come across this problem somewhere? In 10 plus 2 physics, yes your JEE. Yes.

Have you come across this situation? There is a bowl, and in this bowl, there is a small ball, simple harmonic motion. Yes, that is right. So, suppose if I drop a ball like this, and this ball would be rolling down and then following a simple harmonic motion, and then what happens? Stops over here. Similar problem, see physics of the material remains same, the material changes, this material and the ball material is different than what we are discussing over here the mechanism remain same. So, when this is the axis of rotation, I can complete the figure like this. So, this becomes the included angle. So, this angle is normally taken as β angle, rest of the things are same, I might be having a slope of height H, this angle is some you may say some θ.

Normally, we take β. You are right. But in this case, it does not matter. What are the forces are going to act? The weight of the block, and what is this block A, B and C, this B is the width of the footing. Yes, you are right. So, this is the width of the footing. So, this is the point C, through which the slip surface initiates passes through the A point. Same thing I can do for any other situations also 1, 2 and 3. So, in practice, the chances are the failure might take place through any of these surfaces you never know. So, most of the slope stability analysis problems become iterative in nature. That means, you select a slip surface follow the procedure which I am going to discuss over here and get the factor of safety terms.

I did this analysis in the previous discussion, you remember when we were talking about the Taylor's chart, Taylor stability chart. That was also ϕ_u=0, total stress analysis. So, I might be having several situations like this, thousands, hundreds of slip surfaces, very closely spaced, and what is going to change? What is going to change is only the centre of rotation. So, if you plot this factor of safety associated with this point of rotation for which there is a unique slip surface, what am I going to get? I am going to get a plane in which, you will be having different values of factors of safety associated with different slip surfaces.

So, your generation is very lucky that you need not to do these things, or this analysis manually. But when we were students, we used to take some 10 slip surfaces, we used to analyse them by following the method which I am going to discuss in today's lecture. Now, there is everything is software based, they are very good, very potential software which are available in the market. I will be talking about that also. But before I do that, just let me give some concepts and try to follow the concepts so that you can use the software which are available commercially in a better manner.

So, one of these slip surfaces I have selected over here as AC. So, in simple words, or if I want to show this this is how it will look like. So, this is A, B, C and what we did is? Last time we considered this as the straight line, I am sorry for. AC happens to be the chord, and the curved AC, we differentiate it like this, happens to be the arc. This is the axis of rotation, so, what is going to change? If I take another slip surface just now I have shown the axis of rotation in 2 dimensional plane is given by x,z, or x,y. So, its location will change, and β will change. Yes so, you are right. So basically, the slip surface is a function of the type of material. Yes, and what else? Axis of rotation, and its location basically. So, this is the location of axis of rotation r, this is r.

What else? Geometry of the slope θ, H, and of course, the included angle β. So, these are the parameters which would influence the location of the slip surface. Now, what is going to happen on the slip surface? So, suppose if I take an element over here, of let us say very small length c, and what I have done, I have discretised the entire arc in small, small segments, which are linear in nature. Linear means, these are lines. Yes, so, circle can be considered to be constituted of several infinitesimal sections of linear sections. What are the forces that are acting on this? Try to draw the free body diagram. So, at this surface, which is let us say A and B, what are the forces which are acting on the surface? Yes, so, the one is going to be C_m into l, correct.

What about the next force? Normal stress, and when there is a normal stress what else is going to come? There will be a shear stress. Now, what should I write here ϕ or something else? This cannot be the total friction angle which is getting mobilised, this is going to be ϕ_m, that is right. So, this angle is going to be ϕ_m. So, what is we have? What is that we are assuming? We are assuming that only certain fraction of cohesion and friction is getting mobilised in the system. So, you remember the factor of safety term, which we have talked about.

So, suppose if I say $\tau_{failure}$ upon factor of safety is what? This will be equal to $\tau_{mobilised}$. So, this will be equal to C_u undrained shear strength, total stress analysis plus σ tanϕ_u yes, upon factor of safety F. So, this can be written as C_u upon F plus σtanϕ over F, this F is normally denoted as F_c, this F is normally denoted as F_ϕ. And what did we assume last time? Fc is equal to F_ϕ equal to factor of safety for optimal solution is this fine? So, this becomes C_m plus σtanϕ_m. So, what is the relationship between tanϕ_m, and tanϕ_u? ϕ_m tanϕ_m equal to tanϕ_u over Fϕ. So, this becomes tanϕ_m. Now, this C_m is getting mobilised in the form of C_m at a very infinitesimal length of the arc, and this friction angle ϕ_m is the included angle between σ and the shear stress which is getting mobilised.

$$\tau_m = \frac{\tau_f}{FS} = \left(\frac{C_u + \sigma \tan \phi_u}{F}\right)$$

$$\tau_m = \frac{C_u}{F_c} + \frac{\sigma \tan \phi_u}{F_\phi} = C_m + \sigma \tan \phi_m$$

C_m is coming because of the component of the cohesion which is getting mobilised on that length particularly. So, this is the basics of the whole thing. Now, what we do is, we assume that the total cohesion is getting mobilised somewhere at a distance of, this is the friction circle method. Let us say r_1. R will be reaction, which I will be using subsequently. Now, this is the force component and what we are saying is or assuming is that c is acting parallel to AC. So, c force is parallel to AC. Is this part okay? And what will be the value of C? This will be equal to C_m into yes L_c, capital L_c. Yes, so, L_c is equal to what AB, and this is a chord so, that is how we are depicting it. So, what we assume is that the total cohesion is getting mobilised and its direction is known, and the magnitude is going to be equal to this.

And it is acting at a perpendicular distance of r_1 can I obtain now a relationship between r_1 and r and AC and AC arc? Suppose if I take moment about this axis, what is going to happen? C into r_1, capital C into r_1 will be equal to, try. All these C_m, L_cs are going to get summed up into length of correct, AC. Is this fine? So, that means, C value I can substitute as C_m into length of cord. So, length of cord I am writing is as AC into r_1 this is equal to $\Sigma C_m L_c$ into AC arc. So, C_m gets cancelled out, because C_m is constant. So, what is the value of r_1 you are getting? All these summations of small, small chords is going to be what is the value? Yes, compute it.

So, no sorry this is going to be C_m into I have not taken the moment. So, you have to take the moment about this also multiplied by r value. So, this is r. So, this becomes what? AC arc divided by AC into r. So, that means, by using this concept we have obtained the point of application of C also, hope this part is clear? Is this, okay? So, this r_1 is nothing but the point of application C, AC chord r can be obtained, AC chord is known, r is known, I can obtain r_1.

Normally, this type of analysis is done on a piece of graph paper. So, when you are plotting a section for a given slip surface, what you know is the weight, draw the force triangle. Yes, so, force triangle would be W, what are the attributes of W? Direction is known. Magnitude is also known. Very good. So, direction is known, W direction is known, and magnitude is also known, what else is required to complete the force diagram?

The second component is C, you are right. So, C is direction is known, magnitude is unknown, why? I do not know how much cohesion is getting mobilised that is the reason, what is the third force which is going to balance the 2? the reaction, yes. Is the magnitude of R is known? Direction is known. that we have to find out so, what is going to happen? W direction is known, magnitude is known, C direction is known, but magnitude is unknown, R we have to obtain its direction should be known and its magnitude should be known. So, as if what you are doing is you are plotting let us say W, the direction is known magnitude is not known. So, if I superimpose on this, the resultant force with the known direction and magnitude, the triangle can be completed.

How will you obtain now R, that is a big question. Now, this is what actually friction circle method talks about. So, friction circle method talks about this in such a sense that R, application of capital R first of all it is going to be tangent to a circle which is known as friction circle, so suppose if I draw a circle of radius r_m this component of R is going to be tangent and passing through this in such a manner that if you draw a perpendicular from on any point it is going to pass to the centre. So, if this becomes my normal stress, if this becomes the normal stress, what is the angle of inclination between R and N? ϕ_m. Now, what we assume is that r_m is equal to some multiplier, multiplied by r $\sin\phi_m$ so, this multiplier actually comes in the form of a constant.

Now, what is this constant? There are charts which are available to obtain the value of k, normally k is dependent upon β, and this relationship is something like this, exponentially increasing starting from 20° onwards, you can refer to a book and you can obtain the value of k, and k is normally placed between 1 to 1.2, and β could be 120° also. So, we will obtain k value from here, multiply it over here, R is known for a geometry ϕ_m what we will be doing with ϕ_m? Remember ϕ is always assumed, we assumed in the previous analysis is also what we did in the previous module or the lecture. So, we assume F_ϕ, and F_ϕ you know how to obtain it? And then you get F C value, and see whether both of them are same or not?

This method is known as friction circle method. I know there is a lot of clutter on the board, but then if you follow these steps, you can still understand this. So, just to repeat, what we have done is we have made a force triangle, W direction magnitude are known, C we have assumed to be parallel to the chord AC, its magnitude is going to be C_m into L_c, L_c is known, but C_m is not known for C_m we have to obtain C_m, that means, knowing the direction of C we have to know the magnitude, R can be obtained. And once the R is obtained, then how will be obtaining us, yes a good question. So, we have to do like this F_ϕ, the moment ϕ_m is known, what we will be doing you can draw R value, which is passing through this, drop a perpendicular from this point complete this friction circle, get the value of r_m, r_m will be equal to k into r $\sin\phi_m$, k we will obtain from here, r_m is known.

And then once the r_m is known, can you not obtain the value of R, capital R? We can obtain capital R also. So, this is how we normally do this analysis. Please follow a book to read the complete methodology, I hope you can solve this. So, when you are doing these types of problems stick to the basics, take a graph paper draw the geometry of the slope, compute the area of cross section of the slope and along with the slip surface, get the point O, get the W, and rest is all known to you. Sometimes this angle β is also defined as the central angle.

So, what is the relationship between central angle and the length of arc? That you know, is this not so, $(2\pi r/360)$ multiplied by the central angle will give you the arc length, that is it. So, you are just maybe analysing this type of situation. So, in other words, weight is known, what we can obtain is length of the arc is known, length of the chord is known. So, this point is known r_1 point of application of r_1 is known. So, this is where the C is acting, compute the value of c_m, and the moment you have c_m, F_C will be equal to yes, F_c will be equal to total c over c_m, and this C is nothing but you are undrained cohesion.