Geotechnical Engineering - II

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Lecture No. 50

Circular Failure Surface

Let us start discussion on the circular failure surface. So, historically, the circular failure surfaces are basically the rotational failure of the slopes. So, this is the planar. Now, we are going to talk about a rotational failure. So, this is the slope, and the failure is going to be along this surface which is a part of circle, or it could be a circle depending upon the material properties. So, the genesis of this analysis is 1916, its so old.

Mostly this is subscribed to the people who are working in the Sweden and Swedish group that is we call this method also the Swedish circle method by Pettersen. So, this is attributed to the Swedish geotechnical society, this method. These are the societies of different countries like in India we have Indian geotechnical society which monitors and controls the activities of geotechnical engineering professionals in the country and the profession also.

The basic philosophy here is now, if I consider a failure surface like this ABC. What are the governing forces on this block ABC? One is the weight, another one is the cohesion which is acting all along the surface. So, that means, the cohesion which is acting on the surface BC. We show this direction as C in the form of force. So, I will use a capital C here as the force. The direction of C is always taken as parallel to the chord BC.

This is the arc, and this is the chord. So, the direction of C is assumed to be parallel to BC. So, one unknown is less, the direction is known magnitude is not known. Now, suppose if I ask you to complete the triangle force triangle, we have c' and we have W. What would be the third force? The reaction which is acting at the base. Now, there is something very interesting. Suppose W decreases or increases what is going to happen?

So, basically W directly is proportional to the c' or the c' is, what is c'? The cohesion which is getting mobilized on the surface. So, that means, I can always say that W controls the magnitude of c, the direction is known, lower the value of W lower c is required. So, you remember what we did when we were discussing about the tension cracks which are occurring in pure cohesive materials. This is the tension crack let us say.

From the lateral side there is no force which is coming on the block. That means W is going to be balanced by the cohesion which is getting mobilized on the surface. So, if W tends to 0 c also tends to 0. Correct? If W tends to 0 c also tends to 0. That means a smaller the weight is smaller amount of cohesion is required or gets mobilized to stabilize the whole thing. So, what we can say is c upon W is a term which remains a constant.

Any idea why we are using the term c upon W? Look at the fact stability number? So, stability number is defined as c over γH, γH is associated with the weight of the destabilized block. So, can I write this expression as c_m into length? And what is length? Length itself is a function of H height of the slope. And when you say c_m what will be the c_m value, c_m will become C_u over factor of safety associated with cohesion because this becomes C_m.

So, C_m is getting mobilized along the slip surface, which is a function of H. H is the height of the total slope. And what is W? The w is a function of γH^2. So, that means what we are going to get is a function I said. So, that means I am going to get stability number as, what is the stability number? C_u over factor of safety against cohesion multiplied by the γH, that is it.

Incidentally, this C_m is also same as what we have written over there. The interpretation was different. The C mobilized is going to be the total cohesion under undrained conditions divided by the factor of safety. Why undrained conditions? Because the failures are going to be very quick, instantaneous failures, undrained conditions. When these are undrained conditions, we have to follow total stress analysis.

Now, I am sure you are realizing how to interpret C-ϕ parameters which you have got from different types of triaxial tests and then how to include them in totality to depict a situation or failure. Normally, these types of problems are defined as the short-term stability problems. Is this part clear? So, what we are trying to say is C_u upon F_C into γH is a constant which is known as instability number.

$$Stability\ Number = \frac{c_u}{F_c.\gamma.H}$$

For undrained situations, where you have pure cohesive failures. Now, one more thing which you may would like to understand is that this function will be equal to C_m into length of the chord along which it is that thing. So, length of the BC as a chord. So, this is the chord. This hypothesis is attributed to Taylor, name of the person who has proposed this and we call them as Taylor's Stability Chart.

First of all, they will be applicable for total stress condition. And truly speaking, these charts have been derived based on the friction circle method which I will be talking about later on. You have done this friction bearings in your Engineering Mechanics course, there is another application of this concept. So, by definition, the way the Taylor's chart has been defined, we use the term N_s, this stability number is defined as N_s.

$$N_S = \frac{c_u}{F_c.\gamma.H}$$

And this is commonly written as equal to C_u over F into γH. It is understood that F is basically associated with the factor of safety of cohesion. And we take this F as the minimum value so that the factor of safety term or the stability number gets maximized. As per Taylor this Ns is a function of the slope angle β and the ϕ_u value. So, the stability charts which we will be talking about, they look like a relationship between, this is the factor of safety or the stability number, this is the β value, and these are the values of ϕ_u.

Which direction ϕ_u will change? Suppose friction angle increases from top to bottom or bottom to top which one is correct? that you have to think of. So, suppose friction angle is 0 is it the topmost line or the bottom most line. For the same value of β, which slope is more stable the stability number? Yes, that means, even the steeper slopes can stabilize, can stand alone without any support, if the friction angle is more, if the friction angle is less you require less steeper slopes.

So, for ϕ_u equal to 0 there is a special condition we will be discussing about this. And normally the ϕ_u increases in this direction. And for ϕ_u case, it would depend upon the something known as depth factor. So, suppose if this is the slope and this is the hard strata, height of the slope is H, inclination of the slope is β. One of the failure mechanisms which is going to be most critical would be the failure like this.

A slip surface which is circular, particularly in pure cohesive soils. And if this is H, we define this as D into H the depth factor, so D is the depth factor. I will show you these graphs so that you can use them for doing the analysis. So, these stability charts were proposed in 1937 by Taylor. And how do they look like? I am going to project it over here.

Yes, so, this is the first stability chart. There is an embankment or there is a slope. And this is the critical circle which is passing through the toe of the slope. There could be different cases. The slip surface may not pass through the toe, which is case one. There could be an outcrop. So, what you are observing here is the slip surface passes in such a manner through the foundation of the embankment or base of the embankment or the slope and then there is some outcrop.

So, this outcrop is depicted as nx into H where H is the height of the slope, β is the face value or face angle. So, there are 3 cases, the first case is passing through the toe, second is touching the hard strata intersecting the face of the slope, this is what is known as face failure, this is what is known as toe failure. And the case 2 is depicted as the base failure for the same height H. Now, stability numbers are defined based on Zone A and Zone B, sort of analysis.

So, if you look at this line, which is the dotted line which is passing like this starting from about 25°, the right-hand side zone is Zone A, left-hand side is Zone B, Zone A is the critical circle passing through the toe. And Zone B has 3 cases, the case one which we discussed critical circle through toe full line, case 2 critical circle below the toe as the dotted line, and the third is case 3, where we have a very strong stratum. What I have depicted there as D into H which again is the dashed line.

X axis is slope angle, Y axis is stability number N_s, the friction angle under undrained condition increases from top to bottom. And for the first case you will realize, it depends upon the depth factor. So, as the depth factor increases, what is going to happen? If the depth factor increases in this graph for $\phi=0$, you will have different lines in this direction, the depth factor will increase. So, let me define this as depth factor.

There are some more cases of stability number which have been discussed over here $\phi'=0$ corresponds to undrained or the total stress analysis, depth factor is here treated as n_d which I have taken as DF into H will be the total D of the or what we call it as the depth of the deepest point of the slip surface. You have the stability number, we have n_d value and then for n_x we can use the dotted lines for a given slope angle and then we can compute the factor of safety.

So, you can use these graphs for analyzing the stability number for different types of slope conditions. Now, we will solve one problem to showcase to you how the analysis is done.

So, one example problem would be suppose if I take a 60° sloping surface, embankment of height 6.5 meter, the soil properties are 18 kN/m^3, 28° and c is equal to 20 kPa. Find the factor of safety with respect to shear strength and use Taylor's chart. Suppose for the sake of simplicity. Subsequently, you will have to understand which charts to be used.

So, depending upon the material property, you might have to apply your intelligence to select the right chart and go ahead with the analysis. The general principle of analysis is like this that we always assume a value of F_ϕ say 1.5, 1.6. This is the start starting point. So, once you assume the value of F_ϕ you can compute ϕ_m, yes, tan ϕ', tan inverse tan(28°) divided by 1.6. Now, this comes out to be approximately 18.4°.

So, what we have obtained is we have obtained the value of friction angle which is getting mobilized. Out of 28° friction angle which is available, the mobilized value is only 18.4. Now, what should be done, we can use Taylor's chart. Is β known? Yes. What else is required? ϕ_m we have obtained. Can you interpolate between the 2 lines? β is known here on this graph.

So, β is known for 60°, go up 18.4, you have to come somewhere here. Can you obtain the stability number? N_s is equal to 0.1007. I mean you have to be very careful while you are using the numbers and usually you should go up to the four-decimal place. So, it is always better to use the empirical relationships, analytical solutions substitute the values and obtain it, rather than seeing the graphs.

But nowadays you need not to bother much because most of the software which are commercially available include these parameters which are inbuilt. So, if I know the value of N which is equal to C_u over F_C into γH. So, what I can obtain from here? I can obtain the value of C_u which is nothing but C_m, The mobilized value. So, this will be equal to 0.1007 into γ, γ is 18 into H, H is 6.5.

So, this turns out to be 11.78 kPa. What is the factor of safety for F_m? The total value of c is known as, is this correct? So, this F_m is basically F_C, factor of safety for cohesion and this is mobilized. So, this will be equal to 20 over 11.78. And this comes out to be 1.698. What this indicates? This indicates that the starting point of F_ϕ as 1.6 is not equal to F_C. But it was not a bad assumption.

We started with F_ϕ as 1.6 and we landed up with F_C value as 1.698. What I should be doing then? Of course, this depends upon the value of F_ϕ which you assume to start with. Now, suppose if you assume F_ϕ as 1.5 you might have to go for 2, 3 iterations to satisfy this condition that F_ϕ is equal to F_C. So, until this condition is obtained, we keep on doing iterations.

One of the ways to plot this is, one of the ways to optimize this would be if I plot F_ϕ versus F_C ideally these 2 values have to be same. So, you keep on assuming the value of F_ϕ and compute F_C, you get a point over here. The second trial what we will have to do is 1.698 and 1.6. So, I will do this whole analysis by assuming F_{phi} equal to 1.65 let us say. So, from 1.65 what is the value of F_C we are going to compute.

So, you keep on computing this and wherever this 45° line is cutting this curve this is where F_C is equal to F_ϕ. So, this was the first trial which we did. This whole exercise is to be repeated until or unless your F_ϕ becomes equal to F_C. This is what in the simplest form the application of Taylor's method is. Now, if you solve this problem, you will be getting the right answer as 1.671. Please try this yourself.