Geotechnical Engineering-II

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Lecture 5

Sheer Strength of Soil IV

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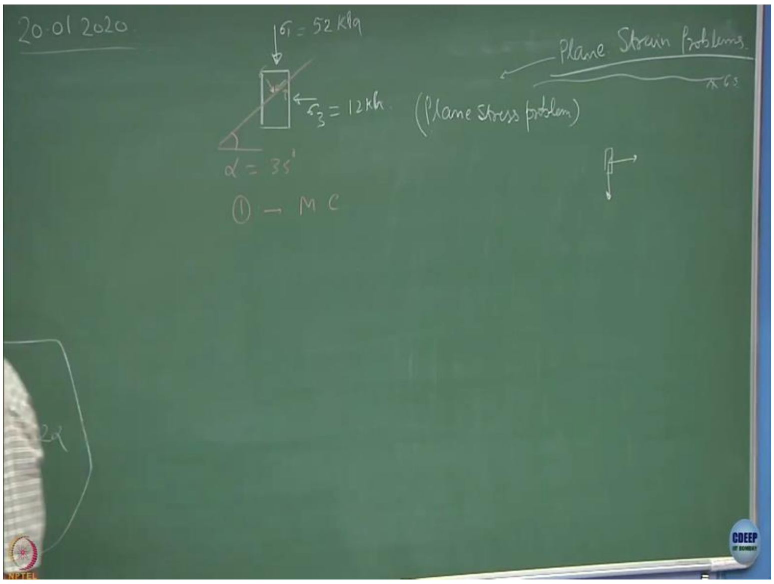

Now, what I am drawing all the time is a triaxial element. Why triaxial? You have the ground surface. You take any point over here and take out the sample. Truly speaking the sample is getting exerted by the stresses in 3 directions. Clear? So, this is what is known as a triaxial state. Three stresses acting on any element in the material, triaxial; very realistic situation but highly complicated to deal with.

Because if you remember, first of all, we are dealing with a very complicated material like soils where we assume them as isotropic-homogeneous. Clear? We assumed isotropic homogeneous. You might be violating these situations. Clear? So, the material is very complicated and then the complication comes in the form of the three-dimensional analysis.

So, what I said is, let us make life comfortable. Let us try to understand 2-D situations. We are dealing with 2-D situations by assuming that the effect of the stress which is perpendicular to the plane is ignored. These types of problems are known as plane strain problems. So, these are the plane strain problems. It is in a plane.

The reverse situation would be a plane stress problem. So right now, we are going to deal with only plane strain problems. So, if I take out this element and if I show here as σ_1 and σ_3, the statement of the problem is: find out the state of stress at this plane which is inclined at an angle α of let us say 35°. σ_1 is 52 kPa. σ_3 is 12 kPa. Find out the state of stress at this point which is nothing but σ and τ.

I hope you can solve this problem. Take this as an assignment. Go back to the hostel. Spend 20 minutes. Master this. If you do not master this, I can assure you will not follow anything next time onwards in the class. So, try to solve this. The first thing you have to do is complete the Mohr circle. You can do it very easily. σ_1 is known, σ_3 is known, radius is known, center is known, everything is known.

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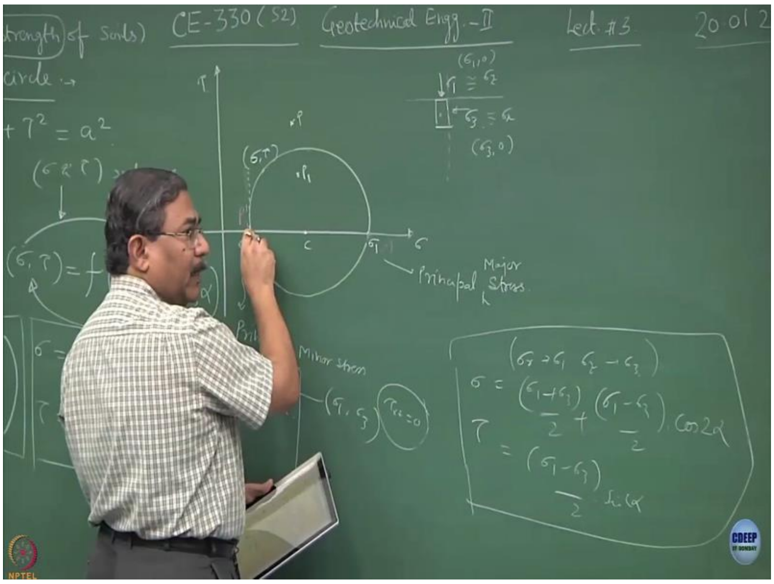

Once this is known 52, 0 is acting here. 12, 0 is acting over here. All is known. Clear? Second part of the problem. What it says is, find out the state of stress at 35° angle. I have two ways to solve this problem. State of stresses known: σ_1, σ_3. This point is 35°.

What should I be doing now? Where this plane is going to lie? Can I project this plane on the Mohr circle? Do that and obtain the σ and τ value. So, the first stage is going to be Mohr Circle. The second is going to be finding out σ and τ. There are two ways of doing this now. I want you to do all these things and hence I am skipping steps.

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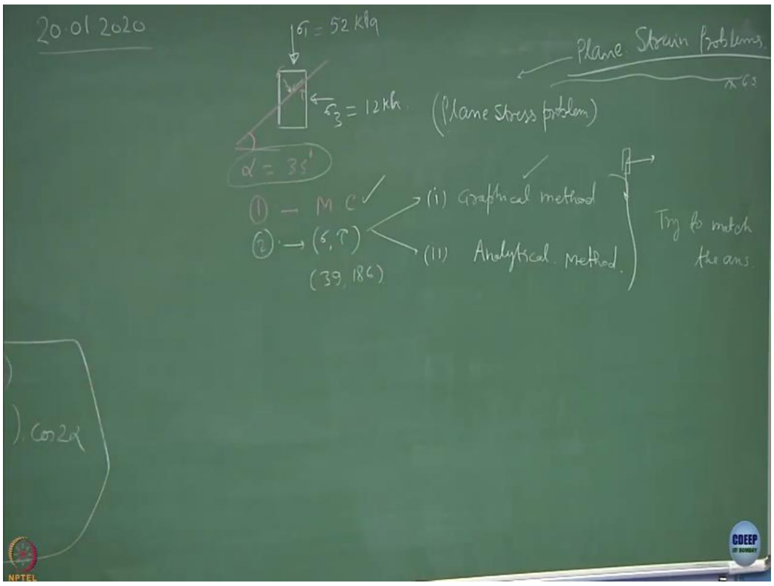

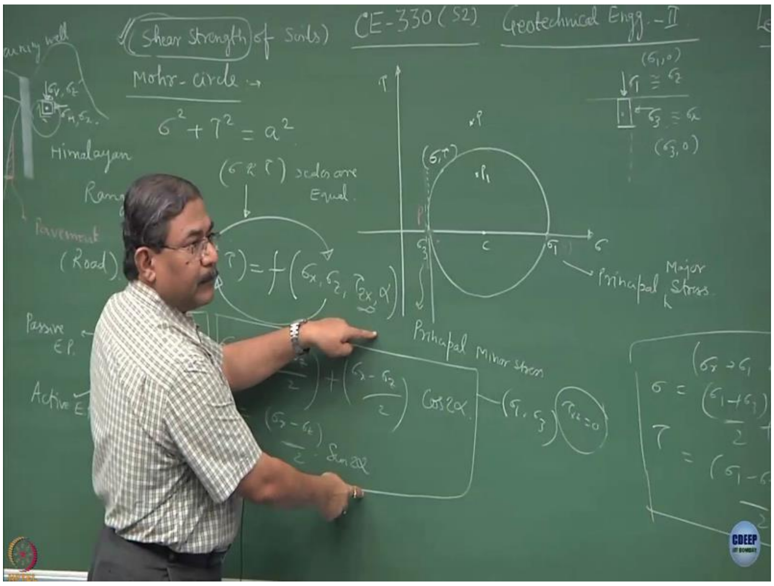

I can obtain σ and τ by two methods. One is graphical. This is what we have been doing taking the help of the Mohr circle. Second is analytical. So, this is analytical solution. σ_1 is known. σ_3 is known. α is known. σ,τ is known. Clear? So, if you do the analytical method which is nothing but your equations. Try to match the answers and see whether you get the same answer or not. The answer you should be getting is state of stress would be 39 kPa and 18.6 kPa.

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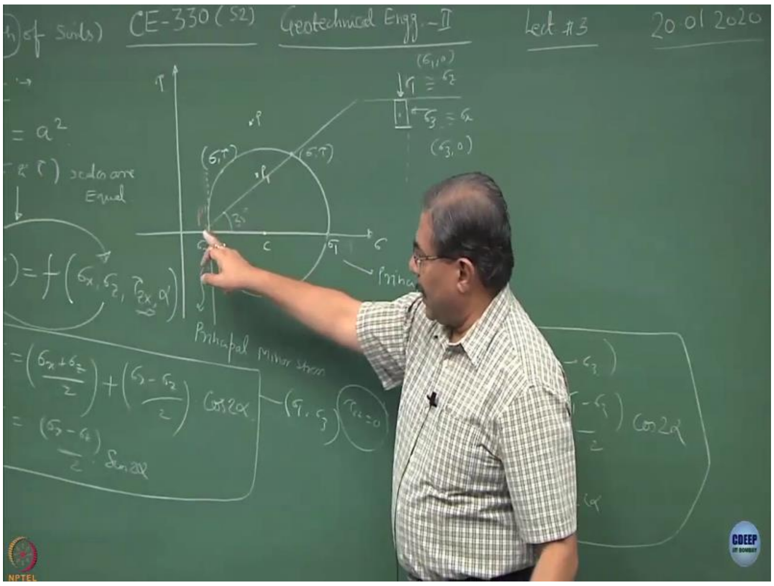

Where is your guess? Where the pole is, pole we have already identified. 35° line if I draw, this is the place where σ and τ is acting. So, by definition, the state of stress has given me pole and pole is giving me a plane which is cutting the Mohr circle where the state of stress is to be identified. Is this part clear?

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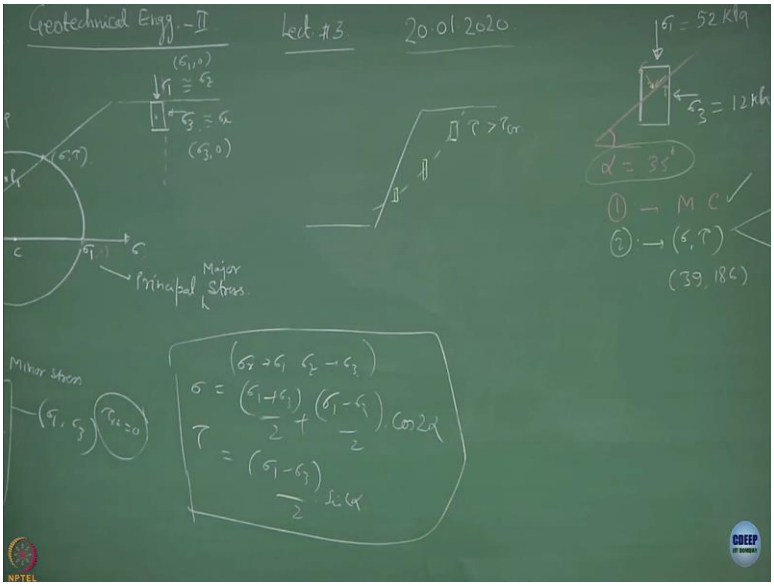

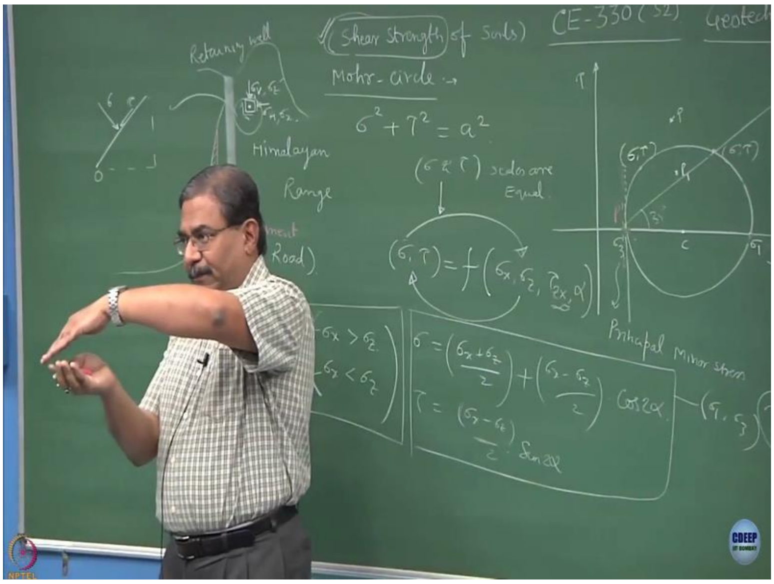

Now, most of the time in geomechanics, the problems are not going to be so simple. What is going to happen? Like in this case, if I consider this as the embankment, railway embankment and this happens to be the critical plane. You remember the definition of the critical plane? At this plane, τ is getting or becoming higher than τ critical. So, the failure has to happen. At this point, the element looks like this.

At this time, this point the element looks like this. At this point, the element looks like this. Truly speaking what is happening is the axis is getting rotated. So, we call this as the rotation of the axis and what is going to happen if I am analysing this type of situation from point 1 to 2 to 3 to n, how results are going to change.

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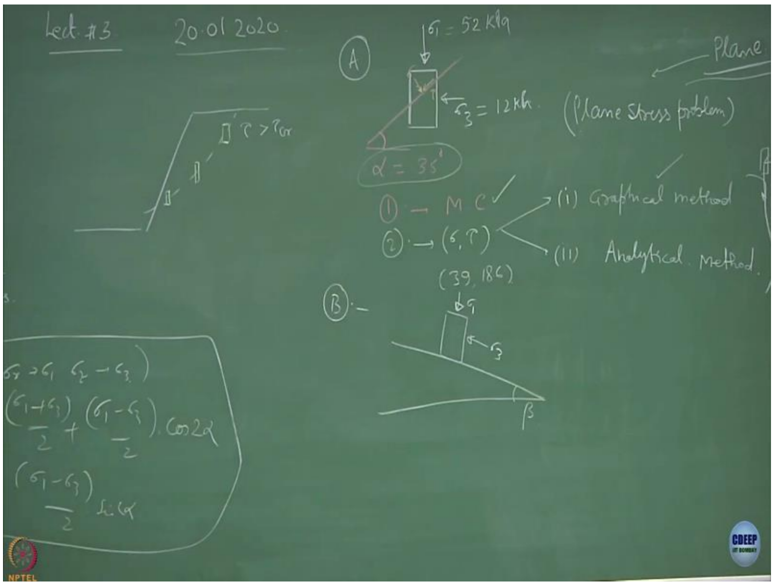

So, the second class of the problem which we normally talk about is; this is the first category of the problems. The second class of the problem would be if there is a complete rotation of the plane. That means this element rather than sitting horizontal, it might be sitting on a plane which is β. The rest of the things remain the same.

I might be having σ_1, σ_3 and again the statement of the problem is: find out the state of stress at a plane which is let us say α equal to 35°. This is what is known as axis rotation. This plane 11 has been rotated by an angle β and the question is whether τ, σ or σ, τ is going to be the same or changed when I do the same analysis for 35° angle. Do you think something substantial is going to change or everything remain the same? I can still start with σ_1, 0, sigma σ_3, 0. Where are these types are going to act? on the planes which are inclined to the horizontal plane now. Try to solve this problem.

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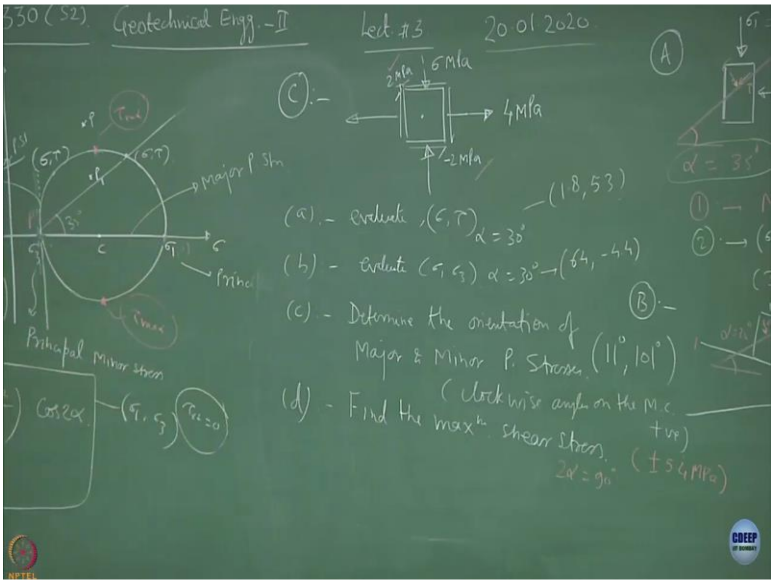

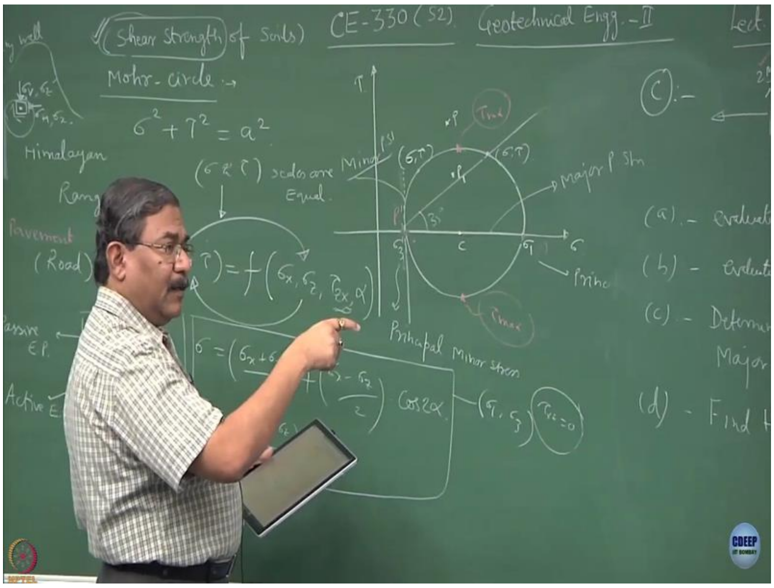

Now, let us take a situation where we call it as a generalized state of stress. So, this is the C type of problem. The state of stress is defined as: this is the element of the soil. You have a compressive stress acting over here. There is a tensile stress acting over here. This is a compressive stress. this is a tensile stress. So, the tensile stresses are always negative; compressive stresses are always positive.

The state of stresses: you have shear stress acting at 2 MPa. The nomenclature is if I consider a point on this stress vector and if the direction of shear stress is clockwise. Alright? If this is clockwise, we consider this as positive. If I consider a point over here and if I see the direction of the shear stress is anti-clockwise this becomes negative.

Solve the same problem and find out the state of stress. Evaluate σ and τ that is a state of stress at α equal to 30°. B, evaluate σ_1, σ_3 when α is equal to 30°. So, this will come out to be, take a graph paper and analyse this. This will be 6.4 - 4.4. Determine the orientation of major and minor principal stresses.

In this case, the orientation of the major and minor principal stresses was 0 because this happens to be the major principal stress and this happens to be the minor principle stress. The sign convention is the angles are always assumed to be positive when they are clockwise. Clockwise angles on the Mohr circle are positive. So, you will be getting this as 11° and 101°. I hope you will realize that the two planes are always inclined at 90° mutually. So, 11° plus 90° would be 101°. They are perpendicular planes. Clear? σ_1 σ_3.

The D problem is: find the maximum shear stress. Just to give you an idea, the maximum shear stress acts at the point where you have this point and this point here. So, these are the points of τ maximum. One is positive, another one is negative. The way we have shown over here. This is positive this is negative. Suppose if I asked you a question on which plane the maximum sheer stress is acting. Not difficult. Once you have drawn the Mohr circle, you get the σ_1, σ_3. Identify the pole.

You know the point at which the maximum shear is acting, this point when connected to the pole is going to give you a plane on which this state of stress is acting which is the maximum stress and its orientation would be this divided by 2 because we have been talking about 2 α. Fine? So, try to solve this. And this comes out to be 2 α equal to 90°. So, α is 45° and the state of stress would be $\pm$5.4 MPa.

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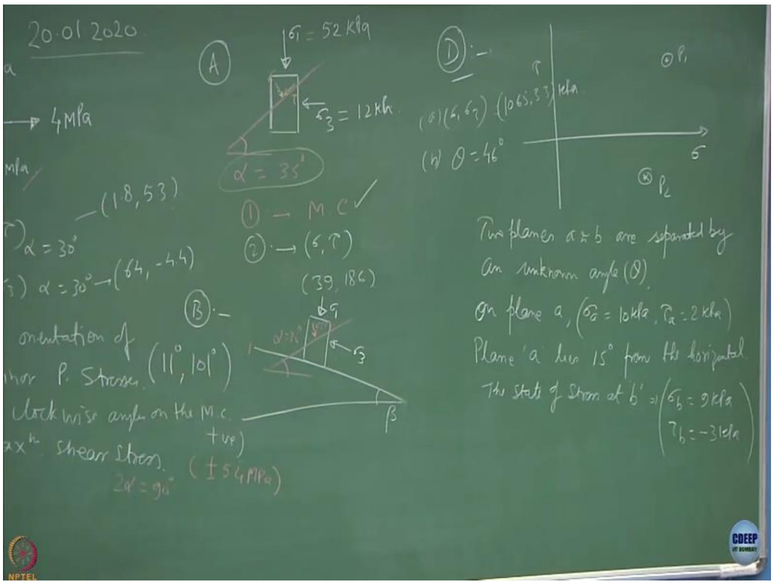

There is another class of problem which is more interesting, more practical but slightly difficult to assimilate, slightly trivial situation. So, the triviality comes from the point that suppose on a τ-σ plane, if I plot two points which are sitting very close like this. Let us say P1 and P2 points. Not very difficult but slightly trivial because you have to do something special to create a Mohr circle here. What property you will be utilizing? Any guess? Use the principle of chords and perpendicular to the chord passing through the center.

So, try to solve this type of problem and the problem statement is: two planes A and B are separated by an unknown angle. Let us say this angle is Θ. On plane A, the state of stress is sigma A equal to 10 kPa and this is 2 kPa. kPa is kilo Pascal. Fine? 100 kPa is 1 kg per cm^2. Plane A lies 15° from the horizontal. The state of stress at point B, σ_B is equal to 9 kPa and τ_B

equal to -3 kPa. The answers for this problem would be σ_1, σ_3 comes out to be 10.65 and 3.3 kPa, and the value of Θ comes out to be 46°. Try to solve this problem.

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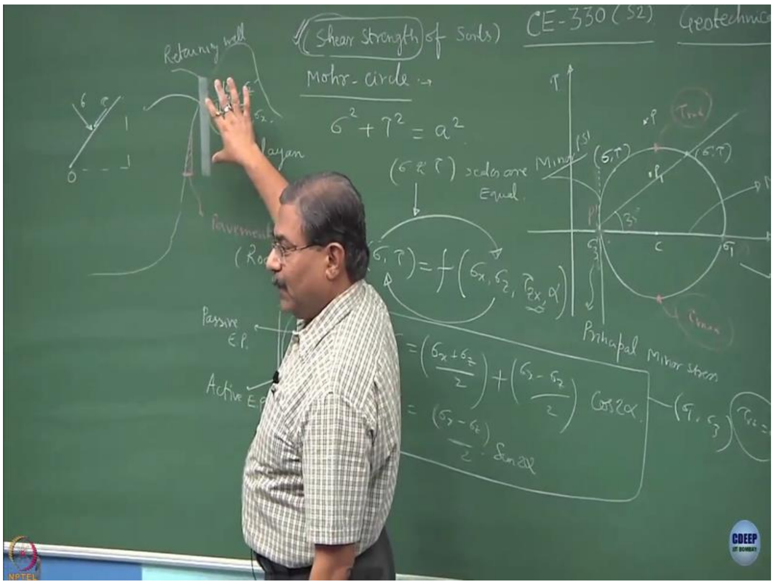

So, what we have done in today's lecture is I have created a situation like this and tried to explain to you what is the significance of the state of stress in the material and then we have tried to answer this question by applying the concepts of Mohr circle because as an engineer as a technologist most of the time you come across the question, if this is a practical problem, and if I have to construct something or provide some utility, how I should be going ahead. Fine?