Geotechnical Engineering - II
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Planar Failure Surface

So, I have been discussing about the slope instability and discussed a lot about the infinite slopes, different conditions starting from drained conditions to the slopes having seepage in the downward direction, upward direction, parallel to the slope, under partial submergence, and the complete submergence of mostly dry-sandy materials which was extended to these situations where the water table is present.

And we have done detailed analysis to find out the factor of safety. These were the infinite slopes. Now, in most of the situations which we as geotechnical engineers deal with the slopes are of finite nature.

So, now onwards I will be discussing about the analysis of finite slopes. You remember we were talking about what is the difference between the infinite slopes and the finite slopes and there we had talked about the material properties as a function of depth vary, they do not remain constant, state of stress does not remain constant in finite slopes as compared to the infinite slope where they remain constant.

Peculiar finite slopes could be embankments, which you are making by compacting the soil mass for railway tracks, for highways, roadways, runways and so on, runways are normally not done on the embankments. But the finite slopes could be because of excavations mostly. So, here we talk about 2 situations one is the planar failure surface and the second one would be circular failure surface.

So, let us begin with our discussion on planar failure surface. These types of situations occur due to excavations and excavations in the stratified soil deposit. What are the second characteristics? This is going to be the weakest plane parallel to the strata and this could be a combination of 2-3 planar surfaces also, maybe a combination of 2 to 3 planar surfaces.

One of the examples of this type of situation would be, if I consider a slope, this is a natural soil mass. This is of inclination *i* and the backfill here is of i'. This happens to be the slip surface the planar slip surface of height H. So, height is taken up to the topmost point over here. The inclination of the slip surface is assumed to be as what θ always. Now, my question to you would be draw the free body diagram of the forces which are going to act on this.

The easiest possible way would be drop a perpendicular from A and designate this as h. Now, you can compute W_1 easily, W_2 can also become computed. So, basically, I am writing this as W. So, W is known. What are the other forces which are going to act on this? Given a chance the entire soil mass or the slope would have a tendency to slide down and this is what is being protected by c and ϕ which is getting mobilized in the system.

So, this is the first time I am writing c and ϕ which are getting mobilized. Mobilization means the entire c and ϕ of the material is not going to be used against instability. Certain fraction of this is going to be used. That means, if I say that C mobilized is equal to c over a factor of safety associated with cohesion. And similarly, if I say ϕ_m is the mobilized friction angle which is equal to tan inverse tanϕ over F of ϕ.

$$c_m = \frac{c}{F_c}$$

$$\phi_m = \tan^{-1}\left(\frac{\tan\phi}{F_\phi}\right)$$

So, I am using 2 factors of safeties. One is the factor of safety for cohesion another one is the factor of safety for friction. Ideally, F_C should be equal to F_ϕ. So, we are going to talk about these types of situations subsequently, just to begin with let us show the forces acting over here. So, this is the component of C_m. Now, C_m is going to act on surface BC. So, multiplied by length of the BC the surface on which it is acting. So, this is BC.

What are the other forces acting on the system? There has to be a reaction. So, this is the reaction let us say. And what is the relationship between the action and the perpendicular, if I draw over here? This is going to be ϕ_m. Now, this friction angle mobilized is what is that we have computed from here. So, factor of safety can be obtained, factor of safety can be utilized to find out what fraction of cohesion and friction getting mobilized.

Now, once this is done, we know the Ws, W can be computed. The only thing which we have to keep in mind is I can utilize this H and basically this is a function of h and h can be related to H and what else, i, θ, i' will not come to the picture. So, ultimately what is that we want to prove? We want to prove that this $\theta_{critical}$ value is the one where the failure is going to take place.

So, if you compute this W will be equal to half into γ into H into l_{BC}. This is the one equation which I can obtain. And the second equation could be is there relationship between h and H, think. So, h is also related to H in what way? h can be written as AB into sine of i minus θ, is this correct, check it out. This angle is i minus θ, so this h is equal to AB into sin(i-θ).

$$W = \frac{1}{2}.\gamma.h.l_{BC}$$

$$h = AB.\sin(i-\theta)$$

$$H = AB.\sin i$$

And what is capital H? AB into sini. So, these 2 expressions are going to help us in finding out the weight. Is this correct? So, what we have done? Now, we can relate h to H as h upon H equal to sin(i-θ) over sini and this can be expressed as h equal to this. BC is known, I can draw this section on a graph paper, I can find out the length otherwise also I can compute the weight of the block by plotting it on a piece of graph paper. That can also be done by graphically or by analytically you can obtain it like this.

$$h = \left[\frac{\sin(i-\theta)}{\sin i}\right].H$$

Now, further what we have to do? Let us complete the force diagram. So, how the force diagram will look like? Yes, try to complete this.

We have a component of the weight and then we have a reaction and this C_m into l_{BC} is the cohesion force which is getting mobilized on the plane and this then becomes the r value. Is this angle known? What is the value of this angle? We have discussed several times, active condition this minus ϕ_m, prove it or check it out whether it is correct or not. And what about the other included angle between the C_m value and the weight, C_m value and the weight what is the included angle, this is θ, this is 90-θ.

What about the third angle? Correct? I can use this relationship to obtain, what, relationship between W this is of ABC which we have obtained. So, W_{ABC} divided by sin(90+ ϕ_m) equal to, what is the principle unknown, how much C_m is getting mobilized, I do not know. So, I have to obtain c value, ϕ value. I do not know even factor of safety sometimes, so I have to assume and then go ahead.

So, this will be equal to C_m into l_{BC} divided by $\sin(\theta-\phi_m)$. What is that I am going to get from here? I am going to get the value of C_m from here. So, once you have got the value of C_m, how are you going to use this? any idea? This C_m, if I correlate it with the factor of safety or stability number both ways, I will prefer to use the stability number over here. So, what will be the value of stability number?

$$\frac{W_{ABC}}{\sin(90+\phi_m)} = \frac{c_m . l_{BC}}{\sin(\theta-\phi_m)}$$

C_m over γH. Is this okay? Just quickly compute what is the factor which are going to get, this will be equal to 1 by 2 because of half γ H^2 which is coming over there. So, this will be equal to, yes, $\sin(i-\theta)$ into $\sin(\theta-\phi_m)$, this whole thing divided by sini into $\cos\phi_m$. So, this is the expression which we have obtained. Now, what is that you want to do by obtaining this function, you want maximum stability, or you want minimum stability, both ways I can try.

$$Stability\ Number = \frac{c_m}{\gamma . H} = \frac{1}{2} . \left[\frac{\sin(i-\theta) . \sin(\theta-\phi_m)}{\sin i . \cos\phi_m}\right]$$

So, suppose if I want to maximize this function. So, what I will have to do? I will have to differentiate this with respect to what, what is the principal unknown here, the principal unknown is the θ value. So, that means, this has to be maximized with respect to θ. So, if you differentiate what is that you are going to get, θ will be equal to i plus ϕ_m divided by 2. Prove this. Is this, okay?

$$\theta = \frac{i+\phi_m}{2}$$

And this θ is normally written as theta failure that mean this is where the failure is going to take place. And at failure what we can do is, we can also assume that the value of ϕ is equal to ϕ_m, the most critical condition that means the factor of safety becomes 1. So, θ_f is the slope of the critical failure surface or critical slip line. It is basically a geometrical manipulation which has been used to obtain the stability number here. I might be having different types of combination of the planar surface.

So, here this is 1, I can create a situation where you have a slope like this, which again goes further and so on. This could be the typical critical surface. So, what I am going to add is one

ABC followed by another ABC. If it is a multi-layered system what is going to change, the value of c_m and ϕ_m.

So, there are 2 ways of doing this analysis either you average out the c and ϕ for different layers by using their thickness or if you want to be very precise, then the slip surface should be passing through each section. And then ultimately what you have to do. You have to do piecewise computation of c_m into l_{BC} which is acting on the base followed by the weights, choice is yours.