Geotechnical Engineering - II

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Lecture No. 48

Slope Instability-III

So, I mean this is a good example of how you would start from the basics and analyse the factor of safety for the infinite slopes. Now, suppose if I reverse the direction of the seepage, if it happens to be upwards direction. And that is what I cited sometimes back.

We have discussed this type of a situation there is an element of soil mass and then there is a seepage force which is acting at the base. So, the way we define seepage force is, if I know the hydraulic gradient multiplied by, yes, γ_w per unit volume of the control volume or the soil mass, per unit volume of the soil mass the total seepage force is i into γ_w. So, if I know the volume of the slice, if I multiply it by $i.\gamma_w$, I know what the pore pressure in terms of the simple force.

$$seepage\ force = \frac{i.\gamma_w}{volume\ of\ the\ specimen}$$

So, can I compute here, if the direction of the seepage is upwards N' will be equal to what, yes compute it quickly. Yes? Yes, compute it. So, this will be γ_b cos i minus, of course b term is missing, yes you are right. γ_b was buoyant. What is the value of the pore pressure here? If critical gradient i_c, if I depicted i_c, i_c into γ_w into volume that is b into d. So, this is i_c into γ_w into b into d, that is it, rest is same.

$$N' = [\gamma_b.b.d.\cos i - i_c\gamma_w.b.d]$$

So, what is the factor of safety in this case? So, this is the case of upward seepage. N', tanϕ' and divided by this thing, this whole thing is this divided by what, Wsin*i*. So, γ_b into b.d sin*i*. Solve this expression, what is that you are going to get? 1 minus cosi also I will take out and this comes down. So, this becomes tanϕ' over tan*i* multiplied by i_c γ_w over, what is the term which you are going to get, γ_b cos i.

$$F.S = \frac{[\gamma_b.b.d.\cos i - i_c\gamma_w.b.d]}{(\gamma_b.b.d.\sin i)} = \left[1 - \frac{i_c\gamma_w}{\gamma_b.\cos i}\right]\left(\frac{\tan\phi'}{\tan i}\right)$$

What is the significance of this? γ_w upon γ is taken as half. So, what is going to happen to this term, 1 minus i_c over cos*i* multiplied by here we are assuming that γ_w over γ is equal to half. So, what is the significance of this if i_c is the seepage gradient, yes, seepage force. So, the more the seepage force acting on the system the factor of safety is going to decrease. That is right.

$$F.S = \left[1 - \frac{i_c}{2.\cos i}\right]\left(\frac{\tan \phi'}{\tan i}\right)$$

So, simple application of what we studied in the seepage theory, how to find out the force acting per unit volume of the seepage and seepage pressure we have defined and seepage force and then we can find out the factor of safety.

The last situation which I would like to discuss in case of infinite slopes would be, let us extend this model to c-ϕ material, infinite slopes. And then see what happens. So, principally what is going to happen? Wherever we have assumed c=0 this component will come nothing more than that. So, if you are considering a c-ϕ material when c≠0 your τ shear strength which is available would be c component plus σ tanϕ component. Include c over here and that is it.

This becomes a c-ϕ soil. Let us draw the free body diagram for the different types of pressure which are acting on the slice of the infinite slope. The only tricky thing here would be computing the pore water pressure acting at the base of the slice. You can use the same concept still and mobilize c.

So, for a c-ϕ soil mass and infinite slope. Let us draw the free body diagram of the slice or the element which we have taken and show different forces which are going to come on that. This is the ground surface, sometimes people call this as ground level also, does not matter. There is a standing water table, that means there is a seepage which is parallel to the slope and then I want to find out what is the stability of the system.

So, this is a water table, this becomes a standing water table, this is the base of the slope, infinite slope, which is the critical one where the failures will be going to take place. You have shear stress, normal stress and that normal stresses because of the water table, effective stress. Let us take the element. How will you compute the pore-water pressure at the base? Put the piezometer and let it cut the phreatic surface, this is a phreatic surface.

What is the intuitive feeling? If you place the piezometer over here, what is the height up to which it will go? up to here? Never. up to, this line is cutting, this is the phreatic surface, draw a, nothing doing, it is not going to happen like that. So, this is your flow line, phreatic surface flow line. Draw the equipotential perpendicular to the flow line passing through the point which is sitting at the base of the slice. Is this correct?

Then only you can find out the pore water pressure. So, meaning thereby, the equipotential line has to be touching this point, crossing this point, and perpendicular to the flow line. So, this is the point where I want to find out the pore water pressure. If you put the piezometer here, this is cutting, this is the equipotential line which is cutting the top flow line. So, this point has been obtained. So, this was misleading, why, because you are to draw the equipotential line and the flow line and the intersection of the two.

So, what is the value of hw here? Perpendicular from here to this, that is it, this is a concept. So, this remains b. Yes, you are right. This is d, the depth of the slip surface. This is the height of the water table normally we define as. This is the slip surface. There is a factor which is defined as n which is equal to z by d. So, can you compute now, what is the value of pore-water pressure at this point? This is z, yes.

So, this thing will be z cos i and again projection of this on this plane. So, this is the value of h_z. So, that means u_w will be equal to what, z $\cos^2$i and because we are finding out u_w multiplied by γ_w. So, u_w is as you know γ_w into h_w, that is it. The moment u_w is known what we have to do. Put the factor of safety term. So, factor of safety here will be equal to the shear strength available divided by the force, which is acting, destabilizing force, what will be that value, w sini.

Whatever steps I have followed over here, you have to follow the same steps for computing everything the only thing is that this shear component will be equal to c multiplied by the length. What is the length? The base at which it is acting. So, what is the base length? This will be b sec*i*. So, c' multiplied by sec*i* and then effective stress upon w sin*i*.

So, if you solve this expression, what you will be getting is you will be getting, I am just skipping the steps and writing the final step, which will be equal to… the factor of safety will be equal to c prime over gamma d sine i into cos i, this term remains almost fixed, sine i cos i into gamma d, c prime by gamma d itself is a non-dimensional number. Cohesion divided by gamma d. Now, what is going to happen?

$$F.S = \frac{c'}{\gamma . d . \sin i \cos i} + \left(1 - \frac{n\gamma_w}{\gamma}\right)\left(\frac{\tan \phi'}{\tan i}\right)$$

Here we had this term 1 minus, this term is becoming some parameter like n. So, this will become 1 minus, what, n times γ_w over γ multiplied by tanϕ' over tan*i*. So, if you solve this expression by putting factor of safety equal to 1 because this is the most critical condition. So, for F.S equal to 1 what is going to happen, c' by γ into d can be written as $\cos^2$i tan*i* minus 1 minus n γ_w over γ into tanϕ'.

$$\frac{c'}{\gamma d} = \cos^2 i \left[\tan i - \left(1 - \frac{n.\gamma_w}{\gamma}\right)\tan\phi'\right]$$

This is an expression which we will be getting. This term is defined as r_u the pore pressure parameter. So, in the simplest possible form this expression can be written as c' by γ into d will be equal to $\cos^2$i tani minus r_u, 1 minus r_u into tanϕ'. So, such a complicated situation we have brought down to a simple relationship and here the way you will read this as this is the critical depth.

$$\frac{c'}{\gamma d} = \cos^2 i\,[\tan i - (1 - r_u)\tan\phi']$$

So, d may tend to become H_c, the critical depth of the surface. So, this d may attain criticality. This is also a sort of a stability number c' by γ_d. If you remember when we are talking about the unsupported height of the vertical cuts, I use this term c by γ_d is a sort of a stability number. So, this can be written as c' by γ at H_c equal to $\cos^2$i tani, yes, c' by gamma H_c will be equal to $\cos^2 i$ multiply by tan*i* minus, yes, c, this r_u term is n γ_w over γ.

$$\frac{c'}{\gamma H_c} = \cos^2 i \left[\tan i - \frac{\gamma_b}{\gamma}\tan\phi'\right]$$

And suppose if I say n=1 fully submerged case, z becomes equal to d water surfaces on the ground surface. What will happen then? So, this will become 1 minus γ_w over γ, γ minus γ_w is γ_b yes, over γ multiplied by tanϕ', very good. So, this is a situation for a totally submerged slope. What is the value of γ_b by γ? This is equal to 1.

So, what is the significance of this, c' over γ H_c equal to $\cos^2$i multiply by tan*i* minus tanϕ'. How many parameters contributing to the stability of the slope? This is what actually you have to find out. You should realize this by the fact that this term remains same, c' by this term actually gets added up. So, what cohesion is doing, mobilization of cohesion is giving more factor of safety to the slope and that is right.

$$\frac{c'}{\gamma H_c} = \cos^2 i\,[\tan i - \tan\phi']$$

So, when both cohesion and friction are getting mobilized both c and ϕ are coming into the picture factor of safety get enhanced. What cohesion does? It induces pore-water pressures and the draining condition of the boundary conditions. So, this water table because of the presence of partial submergence of the slope. The factor of safety has been computed. And what we have done is we have extended this analysis to a situation where the entire slope has been assumed to be submerged.

And mathematically what we have done is we have put n=1. So, this water table goes and sits over here. So, as the water table rises in the slopes the factor of safety keeps on decreasing. Suppose if I ask you to draw the dependence of like this is a function, I can always plot c' by γH_c as a function of i. Is this correct? So, more the inclination angle what is going to happen to c' by γH, it is going to reduce.

As i increases, what is going to happen to this function? it is going to decrease. As ϕ' increases, what is going to happen? So, more of the value of ϕ' what is going to happen? The factor of safety is again going to decrease. And we use this term n over here, pore-water pressure parameter. So, I can plot this function with respect to pore pressure also.

What is the contribution of the pore pressure? So, if n is lower, this term is going to be higher. So, what is going to happen to the factor of safety? Optimize it, did you realize this situation. So, we have discussed situations where infinite slope is made up of dry sands and then there is a seepage pressure, there is upward seepage pressure which is acting on the system. And we have also talked about the submerged situation.

And we have extended this situation to a complete submergence. And what we are observing is that how the factor of safety can be obtained. This is a complete submergence because this water table have reached up to this point. What is critical here is just to obtain the pore-water pressure function and once you have obtained this, rest of the things are simple.

So, if I solve this equation further c' over γH_c, this is equal to $\cos^2 i \tan i$ minus $\tan\phi'$. One of the ways to interpret this would be that if I plot H_c, the critical height of the slope and as a function of i, normally the solutions are valid for i>10. So, at 10°, i if you plot the variation of H_c with respect to i, this is how you will be getting it.

So, what it indicates is H_c and i are inversely proportional. And otherwise also you remember this is the slope and this is somewhere we have taken as d which is equal to H_c. So, more the inclination of this slope, infinite slope, which is having water table. What is going to happen? The H_c value is going to drop. So, otherwise we can also plot it as c' by γ H_c as a function of i.

Another way of plotting this could be you have c' and c' is plotted with respect to i. The way we read this graph is, if I have to design a slope or if I have to do some retrofitting for a given angle of the infinite slope, what should the value of c' or vice versa. If c' is known, what is the value of i? So, this can be extended again as 10° and then we have a straight line, and this straight line is c' equal to γ into H_c multiplied by this factor X.

One more parameter which can be plotted with respect to i would be c'. And the way we read this is for a given c' value what will be the critical value of i which is going to be stable. So, here again we will have this 10° as a threshold and beyond which we will have a non-linear line. And what is the function? This function is equal to c' into γ H_c multiplied by X, this is the value of X. So, these types of charts can be utilized quite well in the engineering practice.

Whatever we have discussed so far there could be an alternate situation where the groundwater table, if I say that this is the ground surface, this is the slope and at this point we have ϕ_d, C_d getting mobilized on the slip surface. The water table is somewhere here. And if I say that this unit weight is γ_1 and this section is of H_1 height, the total height of the slope is H, it is the reverse situation. Because what we did in the previous analysis is we took the height of the water column above the critical surface as n times d that was z value. And suppose, if I ask you to find out the stability number. So, you try to prove this yourself by using the same principles what we have discussed so far.

What we will be getting is as C_d over γ into H_c, where H_c is equal to the critical height or the depth of the critical surface and this will be equal to $\cos^2 i$. Now, this term will get modified a bit, 1 minus H_1 over H, γ minus γ_1 over γ and this whole thing multiplied by tan i minus γ_b over γ plus H_1 over H γ_1 minus γ_b over γ tanϕ_d and this is the stability number which we have got.

$$\frac{C_d}{\gamma . H_c} = \cos^2 i \left[\left\{ 1 - \frac{H_1}{H} \left(\frac{\gamma - \gamma_1}{\gamma} \right) \right\} \tan i - \left\{ \frac{\gamma_b}{\gamma} + \frac{H_1}{H} \left(\frac{\gamma_1 - \gamma_b}{\gamma} \right) \right\} \tan \phi_d \right]$$

You should attempt proving this equation. The concepts remain same, the only thing is sign convention has got changed. In the previous problem what we did is we considered this as z. And the unit weight which was beneath the water table or below the water table was considered as $\gamma_{submerged}$. Thank you.