Geotechnical Engineering - II

Professor D. N. Singh

Department of Civil Engineering

Indian Institute of Technology, Bombay

Lecture No. 47

Slope Instability-II

So, having done this dry sand slope stability analysis, now we will move on to submerged slopes. And when we talk about the submerged slopes there has to be a seepage line. So, we want to understand because of seepage conditions, how the stability of the slope is going to get changed. Say, this is the slope, and this has a seepage line. Now, this seepage line is inclined at an angle of i sorry i we normally take for the inclination of the slope.

So, this is α this is how the seepage line is. Seepage line means, flow line is given and under these circumstances I want to compute the factor of safety of the slope. The direction of the seepage is from top to bottom. There could be a situation where seepage is from bottom to top also reverse direction. Bottom to top we will discuss, but for a quick answer we have done this in seepage analysis, if you remember.

The seepage force acting per unit volume of a control volume or a soil mass. Now, in this case what we have to do is take the slice, this is the slice which I have considered. This surface is parallel to the slope, infinite slope. Now, tell me one thing if I ask you to find out pore-water pressure at this point, can you obtain? What else is required. The flow line is known perpendicular to this is equipotential line.

So, equipotential line is going to be like this. So, what is the relationship between these two? 90°. Equipotential line is cutting the flow line. I want to find out pore-water pressure at this point. So, what I should be doing? Old concepts which we discussed. At this point if I insert a piezometer what is going to happen? This is the height of the cut somewhere. That means actually the way I have drawn is not correct because you have to be careful when you are doing this.

So, wherever the equipotential line would cut the surface now this is the point where if I put the piezometer it will go up to this point. So, this is the piezometric tube and this is the height of the water column. Why it is so? At this point the equipotential line cuts the atmosphere. So, the height of pore water pressure or the height of water which is going to be in the piezometric tube will be in the h_w.

Go back to the basics you will realize that there is a seepage line, and the seepage line is perpendicular to the equipotential line. So, wherever these two cuts and become atmospheric the equipotential line cutting the atmospheric surface that is the value of total piezometric head. So, the main objective is to find out pore-water pressure, h_w. Why? There is a relationship between pore water pressure γ_w into h_w. The rest is simple geometry.

This angle is α what is this angle? 90+i. Is this fine? and what about this angle? This thing is 90° this whole thing this is α-i. So, suppose this is x, can I use the triangle law and triangle ABC, what is the value of now can we compute x? This is known. This is d; d is the depth of slip surface. So, I can say x upon sin(90+i) equal to as AB that is d over sin(90-α).

$$\frac{x}{\sin(90+i)} = \frac{d}{\sin(90-\alpha)}$$

$$x = d.\frac{\cos i}{\cos \alpha}$$

What will be the angle? (90+i+α-i). So, this become 90+α and 90-α. So, what we have obtained is x equal to d sin(90+i), cosi over cosα. Why have we obtained x? Because we want to obtain h_w. it is a good question. Now, what is the relationship between h_w and x? So, use triangle CBD what is the value of h_w? h_w equal to x into sin of this angle.

$$h_w = x.\cos(\alpha - i)$$

This is cos(α-i) because this will become (90-α-i). So, this will be cos(α-i). Please check it. So, x is equal to d cosi over cosα into cos(α-i). This is the relationship which we have got for h_w; h_w depends upon what, incidentally? Analyze this expression height of water or the pore-water pressure at the point B is a function of depth of the point.

$$h_w = d.\frac{\cos i}{\cos \alpha}.\cos(\alpha - i)$$

α is seepage line perpendicular is not possible why? Because if α=90° what is going to happen? So, the analytical solution which we are doing right now is not valid for α=90°. Why? What is the reason? Now forget about this angle. what is the reason? The reason is any guess. If α is 90° equipotential line is not going to cut the atmospheric line. That means point C will not exist.

And if point C does not exist, we cannot get the h_w that means I have to do some other method. So, this is the solution which is not valid for α=90°. You got this point? That means h_w tends to infinity. So, we have obtained h_w now what else can be done with h_w? So, in most of this analysis where we are doing slope stability. What we want to do is we want to find out the effective shear strength or total shear strength at the slip surface.

So, one more thing which we should realize here is that this shear force is the stabilizing shear strength. The weight which is acting is destabilizing. So, the W sini term is destabilizing and w cosi is trying to negotiate with the pressure acting at the base, normal stress. Is this fine. So, suppose if I know u and if I know h_w can I compute the value of pore-water pressure? u at base, what will be equal to this equal to this is in the form of the pressure.

We are dealing with the forces. So, this has to be multiplied by the area. So, this will be equal to $\gamma_w.h_w$ multiplied by if this is b what is the length of the inclined length of the base this happens to be b into seci. The convention is that we always take lateral directions horizontal or the width of the slice as horizontal and whatever the depths are we take them in the vertical direction. For the same sake of simplicity.

So, now this thing is getting multiplied by which term? This is going to get multiplied by b seci. So, this is the value of the u_b. Can I write down the factor of safety term now?

$$u_b = (\gamma_w . h_w).(b. \sec i)$$

What will be the factor of safety relationship? The shear strength offered by the material so this is c=0. So, this is going to be N' tanϕ' τ, c=0 for dry sands. Oh, I am sorry this is not dry sands this is submerged condition but sandy slopes. So c equal to 0. We have to compute N' I am just coming to that. Once you have got u_b value you can compute the N' value and what is the stabilizing force?

This is the stabilizing force, shear strength and what is the destabilizing force? The W component which is acting parallel to the slope. So, what is that?

$$F\,S = \frac{N' \tan \phi'}{\gamma . b . d . \sin i}$$

Now, compute the value of N'. So, what is the value of N'? From here we can take the vertical component, vertical to this base. So, this is w cosi w sini. so, w is γ.b.d cosi minus N' is what we are computing. So, pore-water pressure in pore water pressure term what is appearing? What we have written is that u_b equal to the base pore water pressure is γ_w h_w b.seci. h_w is this so this is the big expression what you have to do here. This will be equal to γ_w into d cosi over cosα into cos(α-i) γ_w h_w. So, this is h w portion multiplied by b sec i. This is okay. N' tanϕ' has to come in the numerator. So, this can be written as γ.b.d cosi minus γ_w into d into b cos(α-i) over cosα. I can write this as γ.b.d cosi minus this will become γ_w by γ into cos (α-i) over cosα.

$$N' = \left[\gamma.b.d\cos i - \frac{\gamma_w d\cos i}{\cos\alpha}\cos(\alpha - i)\,.\,b\sec i\right]$$

$$N' = \left[\gamma.b.d\cos i - \gamma_w.b.d.\frac{\cos(\alpha - i)}{\cos\alpha}\right]$$

$$N' = \gamma.b.d\ \left[\cos i - \frac{\gamma_w}{\gamma}.\frac{\cos(\alpha - i)}{\cos\alpha}\right]$$

So, what is the factor of safety term? The factor of safety term would be if I define this as now, I will substitute straightaway. I think we can realize this. So, the factor of safety term will be equal to this term multiply by tanϕ' and what is coming in the denominator? γ.bd into sini.

$$F.S = \gamma.b.d\ \left[\cos i - \frac{\gamma_w}{\gamma}.\frac{\cos(\alpha - i)}{\cos\alpha}\right].\frac{\tan\phi'}{\gamma.b.d.\sin i}$$

So, this becomes the factor of safety term. I can further simplify this as take out cosi term that is intentional. So, when you take out cosi term this will become 1 minus γ_w over γ cosα minus i over cosα into cos i multiplied by tanϕ' over tani. Are you happy with this? What is the similarity between this situation and what we have derived? That means this term is a sort of a penalty or a factor which has been imposed on this curve. Very simple to realize the situation.

$$F.S = \left[1 - \frac{\gamma_w}{\gamma}.\frac{\cos(\alpha - i)}{\cos\alpha\,.\cos i}\right].\left(\frac{\tan\phi'}{\tan i}\right)$$

In case of dry sandy slopes, the factor of safety was tanϕ over tani. Submerged situation what is going to happen this becomes your effective stresses. We have filtered out the effect of water by using this concept effective stresses, normal stresses substituting over here and I get this relationship. What can be done further? Imagine, what is this term? In simplified form go back to your 10 plus 2 trigonometry.

Can I convert it into tan components tanα, tani try doing that. Yes, very nice so this becomes tanα plus tani term. One plus so that means you are right, so this is equal to 1 minus γ_w over γ 1 plus tanα into tani and of course the very well known factor that is tanϕ' over tani. So, simple analysis or the logic is that whatever is happening in terms of submergence and seepage is to reduce the factor of safety.

$$F.S = \left[1 - \frac{\gamma_w}{\gamma}.(1 + \tan\alpha . \tan i)\right].\left(\frac{\tan\phi'}{\tan i}\right)$$

It is understood. The submerged slopes are going to be less stable as compared to the dry slopes. So, the factor of safety is going to reduce. i=0 and α=0 what is going to happen horizontal ground and seepage line is parallel to the surface. We are not interested in such type of a situation. Because this is a very critical situation which we are trying to solve. Usually, γ_w upon γ is taken as 1 by 2.

Another interesting situation could be when α=i so what is going to happen? i is the inclination of the slope also. So, alpha is in such a manner that this is just at an angle which is same as the angle of the slope parallel to the slope. So, in that case what is going to happen is this will become what alpha vanishes then, this becomes $\tan^2 i$.

So, here I can write this as factor of safety will be equal to 1-γ_w over γ and what is 1+$\tan^2 i$ into tanϕ' over tani. What is the value of this bracket term? This is normally written as 1-γ_w over γ into 1 upon $\cos^2$ i tanϕ' over tani.

$$F.S = \left[1 - \frac{\gamma_w}{\gamma}.(1 + \tan^2 i)\right].\left(\frac{\tan\phi'}{\tan i}\right)$$

$$F.S = \left[1 - \frac{\gamma_w}{\gamma}.\frac{1}{\cos^2 i}\right].\left(\frac{\tan\phi'}{\tan i}\right)$$

In this case if your α term is 0 what is going to happen parallel flow condition. So, α=0=i then what is going to happen?

If you put α=0 this becomes a situation where the direction of the flow is parallel to the slope. So, what is going to happen in this case this will become i is not becoming 0 like that, but because α=i so this term disappear this will become 1 minus γ_w over γ multiplied by tanϕ' over tani and usually we take γ_w over γ as half, so this is equal to 1 by 2 tanϕ' over tan i.

$$F.S = \left[1 - \frac{\gamma_w}{\gamma}\right].\frac{\tan\phi'}{\tan i} \approx \frac{1}{2}.\frac{\tan\phi'}{\tan i}$$

What happens to the factor of safety? It has reduced by half under submergence. For the critical condition can I write that i=ϕ'/2. Approximately, this will remind of 10 plus 2 geometry and the trigonometry which we used to do to solve these problems. The concepts are simple. Just to wrap up the things what we have done is we have taken a slice, we have computed the weight, weight is known find out the normal stress which is acting on the surface.

Where it is acting? This is N' and then we wanted to find out what is the pore-water pressure use the concept get the h_w converted into the force acting on the base N is known N minus pore water pressure is going to give you N'. This is shear strength of the material which is stabilizing force divided by destabilizing forces or the stresses and then this is simple analysis.

One of the expressions could be, here I can get rid of $\tan\alpha$ and I can make it $1+\tan^2 i$ also. Which is more advantageous for us because i is the inclination of the slope with respect to horizontal. So, factor of safety is 1 minus γ_w over γ 1 plus $\tan^2 i$ multiplied by $\tan\phi'$ over $\tan i$.