aGeotechnical Engineering - II

Professor D. N. Singh

Department of Civil Engineering

Indian Institute of Technology, Bombay

Lecture No. 34

Earth Pressure Analysis I

So, we have been discussing about earth pressure analysis, finding out the active and passive earth pressures which are acting on the walls. And in most of the situations, we have talked about the Rankine earth pressure where the wall was smooth, vertical, backfill was horizontal, and in the previous module, I introduced the concept of sloping backfills and this is where we had computed the K_a value.

So, the situation was like this that you have a retaining wall and rather than being the horizontal backfill, the backfill is inclined at an angle of i and then, I had shown a conceptual diagram that how the element or the triaxial element which normally we consider as this where we have σ_1, σ_3 acting, gets modified to a different form of the element which looks like this in case of the sloping ground or the sloping backfill.

So, this is the ground surface. Alright? Very close to the hills, you may find this type of a situation. One of the practical examples of this situation would be you have upside hillock and then this is where the road is. Alright? So, in the hilly terrains and what we do is we create a retention scheme over here. So, this becomes a retaining system. Now, one thing which is important over here is the free-body diagram of this element which I did not draw in the previous module. I hope you will realize that at this depth z, we have σ_z which is acting. And what we the value of this σ_z?

So, let us complete the free-body diagram first. We have σ_z, we have the earth pressures, P_a or P_p depending upon the direction. Alright? And then once you have lifted this element or the ground surface by angle i, what is going to happen? There will be shear stress which gets introduced. So, this is how the element looks like. Now, from this point onwards, we should be knowing how to compute the value of σ_z.

So, normally the procedure is if I consider this as a backfill and I want to find out the value of stress at a depth z. Can you compute this? Normally the sign convention is along the length of the slope we take the slice width as b. This is one of the slices which is constituting the entire soil mass.

So, this is one of the slices which I have taken. The entire continuum has been divided into small-small slices and this is how one of the slices looks like. So, we take the width of the slice along the length b. This angle is i. So, this becomes b cos i. This is a convention which I will be using henceforth in most of the inclined situations. Now, what is the stress component which is going to come over here? This is perpendicular to the surface. So, σ_z is perpendicular to the surface. So, this is equal to what is the weight of the block unit weight multiplied by b into z and cos of i divided by b. So, this is a stress which is σ_z and this will be equal to gamma into that into z into cos i.

$$\sigma_z = \frac{(\gamma . b . z) \cos i}{b} = \gamma . z . \cos i$$

So, what we did is we took the help of the Mohr circle and we employed the concept of axis rotation. So, if you remember what we did in the previous class is, this is the Mohr circle for the material. This being a pure frictional material, we have assumed this as the friction line. And then we used the concept of rotation of axis. So, this is the plane on which the new σ_z is acting. So, this angle is i and we computed this was O, A and B.

At this point the value of σ_z is equal to γ into z. Now, what you are observing is the σ_z on plane inclined at an angle of i, that is at point B the value of γ_z is γ into z in the cos i. So, what is the shear stress associated with this? This is the shear stress, and this is the normal stress axis. So, if this is γ z, this is γzcos i. The shear stress associated with this would be equal to γ.z.cos i into sin i. Fine?

$$\tau = (\gamma . z . \cos i) \sin i$$

Now, this concept will be utilizing in slope stability analysis also everywhere. So, we know the normal stress which is acting on the plane. We know the shear stress which is acting on the plane. And the best thing is, if you substitute i equal to 0, you can cross-check whether you are going to get the same state of stress or not.

So, if i is equal to 0, what is the shear stress? Is 0. That means, in this case where i is equal to 0, this element assumes the form of principal stresses. Shear stress becomes 0 and you have σ_1, σ_3 acting on the system in which the equilibration is being done. Is this part, okay?

So, please remember this analysis is what we have done for purely ϕ material, frictional material. c component cannot be included over here because it is a Rankine wall, the only thing which we are defining is the inclination of the angle, i. This is one of the cases. Now, you must be realizing that it is not so easy to deal with non-Rankine earth pressure situations. So, under these circumstances, what we need to do is we need to introduce the concept of Coulomb's earth pressure theory.

So, I repeat- in most of the situations what you notice is neither the retaining wall is going to be vertical, in most of the real-life situations. So, this is a retaining wall. You must have noticed when you cross by the highways, you know there is some batter, inclination. This is taken as β, and the backfill is also not horizontal. So, this is also inclined at an angle of i. Now, this face could be vertical depending upon my requirements. So, this becomes a simple gravity retaining structure and what we want to analyse this how much earth pressure is going to get mobilized on this system.

So, these types of situations can be taken care of very easily by adopting to Coulomb's earth pressure theory. So, Coulomb's earth pressure theory was given in 1776 quite much before the Rankine's earth pressure theory and what it assumes is again the similar assumptions homogeneous soil mass, isotropic, granular material. Only thing is that the wall is inclined and the backfill is inclined. So, there are a few assumptions which are made in Coulomb's theory. What we assume here is that the wall is rough. So, the first assumption is that we have a rough wall and the backfill is sloping. What is the significance of this?

The roughness of the wall is defined by an angle of δ and this δ is approximately equal to ϕ/2 to 2ϕ/3. Its limiting value could be 10° or 15°. All right? And I hope you can realize that normally $i < \phi$. This condition is imposed everywhere. Even here also when I say I, i is going to be less than ϕ angle. It is very important to understand how the wall friction is imposed on the retaining wall. Like we did earlier, there are two cases. One is active, and passive situation. Active earth pressure and passive earth pressure.

So, activate earth pressure when we talk about is, draw the perpendicular on the wall surface and then under active pressure what is going to happen? This is going to be the slip surface and this angle we assume as θ. Now, under active earth pressure, the block is pushing the wall out. So, that means the direction of the pressure would be like this. So, the reaction which is going to come on the block, or the wedge would be opposite to this.

This angle as we defined earlier is δ which denotes the wall roughness, and this is what is taken as the positive batter. So, this is how the earth pressure gets mobilized. What we assume is that the direction of the P_p and P_a is known, and the point of application of P_p and P_a is also known, and which is assumed to be H/3. So, if the height of the wall is H, we assume that the pressure is acting at H/3 which is reasonable. Now, if you do not assume the direction of the pressure, alright, then the problem would become redundant.

Another thing what we assume here is that this slip surface passes through the heel of the wall. This is the heel of the wall. So, the slip surface passes through the heel of the wall. This is the third assumption. The first and the fourth assumptions would be it is an isotropic material, cohesionless, homogeneous, and uniform material.

So isotropic, homogeneous material and which is cohesionless. The basic theory was on cohesionless material. Anything else which is very important and which we have missed out? One more assumption here is that the wedge which is sliding either down or up so when I say passive pressure, in this case, what is going to happen? In this case if this is the wall, this is the perpendicular to the wall at H/3. H is the height of the wall. The tendency of the wall is to push the block up, inside the backfill. Correct?

So, how that is going to happen? That is going to happen when the point of application of P_p is like this. So, this is the angle δ. This is the sign convention which is normally used. So, under these circumstances, what is going to happen? Remember we had talked about when we are discussing the active and passive earth pressure theory, this is a case where the scooping effect is coming in the picture. That means, when this type of pressure gets mobilized in the real life, the wall is moving inside the backfill, you will be having a sort of a hump formation on the surface.

In this case, what is going to happen? In this case, there will be a sort of a depression because the material gets adjusted. So, this volume of the material gets adjusted in the space which is getting created because of the movement of the wall on the left-hand side and on the right-hand side. Suppose if I asked you to draw the slip surfaces, actual slip surfaces because of the mobilization of friction of the wall. So, imagine when you are having a passive earth pressure acting like this, the slip surface is bound to get become slightly deeper. Difficult to draw.

And this is the point where you have, so what is the angle associated with the passive earth pressure 45-ϕ/2. So, this angle is going to be 45-ϕ/2. This angle is going to be 45-ϕ/2 provided the backfill was horizontal. Clear? So, in this case we have to compute these values. So, this is how the log spiral curve surface gets generated.

What about the case in the case of active earth pressure? So active earth pressure, the material has a tendency to slip down. It has to be a bit of a concave surface and then it would join somewhere here. So, this becomes the surface. If I define this as convex, this becomes a concave surface. So, this is the influence of the earth's pressure on the wall.

So, by all means, we are assuming that these blocks are rigid bodies. When you are assuming this to be a rigid body, we can use the concept of limit equilibrium here to solve the problem, and hence the concepts of limit equilibrium can be employed. So, now I think you understand why I had emphasized upon the fact that we assume the point of application of the earth pressure at H/3. Because if you do not assume that, that becomes another unknown.

So, the magnitude of P_a is principal unknown. θ is unknown which can be obtained analytically. β and δ will be known. The best way to obtain δ would be, you conduct a direct shear test. Make half of the block of the sample by using the same material and half of the block of the sample would be of the soil and shear it along that surface to get the value of δ. These are real-life problems where, you know, people will approach you to help them.