Geotechnical Engineering - II

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Lecture No. 33

Sloping Backfill

See, another situation which normally we come across while designing the earth retention schemes is when the backfills are sloping. So, far you must have realized that we have not talked about the backfills with inclination, because we did not want to defy the Rankine's theory. So, a Rankine wall is a smooth vertical wall and the backfill is always horizontal and which is retaining the frictional material as a backfill material. So, one of the deviations we did from the Rankine's earth pressure theory is that we have introduced the concept of a c-ϕ soil keeping a vertical wall and horizontal backfill.

Now, we are going to deviate further from this situation, where I would say find out the earth pressures for sloping backfills. A very complicated situation would be, you have a sloping retaining wall and it is retaining a backfill at an inclination of i and this angle we can define this as β. The slip surface is going to be now like this, a complicated case.

So, if you do the analytical solutions, it will be difficult for you to obtain the earth pressure coefficients K_a and K_p for this situation and hence the P_a and P_p. Better would be either we go for Coulomb's method which I will be talking about later then we can create a Coulomb's block and mechanistically we can analyse this. This we will do subsequently.

Another way would be go for the graphical schemes which I said in one of the lectures that they have become outdated nobody uses them due to the advent of very precise numerical codes, finite element methods and boundary element methods which people are using for solving these type of problems. There are several softwares which can give you these analysis very quickly.

So, let us slightly make this problem easy by assuming that still this is a Rankine's wall and in this Rankine wall which is vertical, the backfill happens to be inclined. So, this is the initial case when the backfill was horizontal. If I would have taken an element of the soil mass very close to the wall, this is how it would have looked like triaxial element and we would have been having σ_v σ_h due to the condition when the backfill is horizontal. But now what has happened?

The backfill has got inclined with an angle of i, so geometrical compatibility is lost, and the new geometrical compatible system would look like this, element would look like this, is this, okay? Here σ_v is acting perpendicular to the plane not in the vertical direction, this says σ_h which could be P_a or P_P. We like to analyse this situation and by using simple concept of axis of rotation of the principal stresses which you had studied long back. You remember? We did several cases for finding out the Mohr circle?

If this is the element of the soil, σ_v is known, σ_h is known, this is the inclination plane θ and then you are supposed to find out the state of stress here at a point O which is passing through a plane inclined at an angle of θ. So, you are supposed to find out σ-τ as a function of σ_v, σ_h, θ and we did a reverse problem also. What we did is, we created a situation where the element itself was inclined at an angle of θ.

So, that means if this is the element of the soil which is sitting at an angle of β and then being acted upon by σ_v, σ_h and then you are supposed to find out, it could be anything it could be with respect to horizontal or I can also define the state of stress which is parallel to the surface inclination β. So, it could be anything. Now what we can do is we can use the concept of Mohr circle very easily to solve this type of problems. Any guess how?

So, draw the Mohr circle, there is a Mohr circle for frictional material. So, this becomes the failure envelope ϕ angle, what is the state of σ_1, σ_3? This is σ_3, this is σ_1 that means this is σ_v and this is σ_h and σ_v is if this is the depth of a point from the horizontal backfill at a depth of z. So, this was equal to γz.

Now, what has happened? Because of rotation with i angle, the state of stress which you have shown as σ_h and σ_v has got rotated. So, this is the new state of stress which is going to act. The σ_v has gone from here to here and σ_h has gone from here to here. That is the only difference, and this inclination is i.

So, what I need to find out? I need to find out let us say K_a, coefficient of earth pressure under active earth pressure condition. Can I use the geometry to solve this problem? Perpendicular from the point of tangency at the centre, draw a perpendicular from this centre to this chord. This is perpendicular, we can connect the chord by the radius, is this, okay?

We normally define this as O, C and we can take A and B as the two points here. So, this is A, and this is B. We can have this as D and E. The most interesting thing here to follow is when i was 0, what was the value of K_a? K_a value was σ_h upon σ_v, correct? So, that means σ_3 divided by σ_1 was the K_a value, is this alright? Now, this is equal to OA divided by OB in the new system, is this fine or not? Have you understood this thing? Rest is all geometry.

So, the concept is axis rotation can still be depicted on the Mohr circle as a rotation of the planes, the value of K_a will be OA divided by OB. If this point is clear rest is all simple to follow. What is OA equal to? OA is equal to now you can write in terms of OD, OD minus DA divided by OB OD plus DA and DA is equal to DB. So, this can also be written as DOD minus DB and DB, correct?

$$K_a = \frac{\sigma_3}{\sigma_1} = \frac{OA}{OB} = \frac{OD - DA}{OD + DA} = \frac{OD - DB}{OD + DB}$$

From this triangle OEC what can be obtained? This is ϕ, OC is common? So,

$$EC = OC \, sin\phi$$

This is fine? From triangle ODC, we can obtain DC equal to OC sini and remember here we have used the equality,

$$AD = DB$$

EC is nothing but the radius of the circle. So, now can you substitute for OD, what is OD value?

$OD = OC \, cosi$

Because this angle is i.

$OD = OC \, cosi$. And

$$DB = AD$$

So, how will you find out DB? We can use another property of triangle DBC which is,

$$BC^2 = DC^2 + DB^2$$

So,

$$DB = \sqrt{BC^2 - DC^2}$$

So, if you solve this expression what is that you are going to get and if you substitute it over here, you will be getting K_a will be equal to, can you try this? Cos i minus under root of $\cos^2 i$ minus $\cos^2\phi$ divided by positive summations of these terms, is it not? So, here this will be plus and rest of them will be same. So, this is the value of K which we have obtained.

$$K_a = \left[\frac{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i + \sqrt{\cos^2 i - \cos^2 \phi}}\right]$$

Now, let us check it quickly, if i is 0 what is going to happen? This will be,

$$K_a = \left[\frac{1 - \sqrt{1 - \cos^2 \phi}}{1 + \sqrt{1 - \cos^2 \phi}}\right]$$

This becomes cos of 0 is 1, $(1-\sin\phi)/(1+\sin\phi)$, correct? So, that means whatever we have derived is alright. This is a good example of how rotation of axis can be utilized to compute the earth pressures which are acting on the walls. So, we have discussed a lot of cases for finding out the earth pressures which are acting on the retaining systems and now we are going to use these concepts to solve the problems which are happening in real life. Any questions?