Geotechnical Engineering - II

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Lecture No. 30

Earth Pressure Analysis (Trial Wedge) III

Now, coming back to this situation. So, what we have done is, we have got the $\theta_{critical}$ once $\theta_{critical}$ is known, $W_{critical}$ is known, what is $W_{critical}$? $W_{critical}$ is the weight of the critical block, which is going to be under plastic equilibrium and on the verge of failure.

Now, if you want to compute R it is not difficult, because there is a relationship between P_a, W and R also, is it not? So, suppose if I write an expression, you can do by this also and you can also compute the value of… that,

$$\frac{P_a}{R} = \sin(\theta - \phi)$$

So, that means,

$$R = \frac{P_a}{\sin(\theta - \phi)}$$

That is it.

I know the value of ϕ, I know the value of $\theta_{critical}$ I know the P_a value I have already computed I can get the value R. So, I have solved the problem. Reaction acting on the slip surface is known. It's inclination is known only point of application of R is not known. Use the third equation take moment being a free vector about any point and get the value of R. Application point, you know, application of P_a this, W you can obtain on a graphical paper, the CG of the wedge problem is deterministic. Any questions?

Student: What is critical plane?

Professor D N Singh: We would like to find out because look at this. If you remember, when I was doing this explanation of the model, what I did, this was a box if you remember, I filled up the soil over here. And then I said each one of them is a critical plane, it depends on where the failure is going to take place. This is the condition at criticality.

There will be several failure planes where the critical conditions do not agree, what is the critical condition, do not exist this is the critical condition. That is it? So, what we are trying to do is we are trying to find out the criticality which is all this system can fail from here also. This is a hypothetical plane if you remember.

So, you are walking on a hilly terrain, a small boulders may collapse. Subsequently, what will happen? a big chunk will come out. Again rains come next portion, next portion. And if you see the slip surface, they are all parallel. Why? Because this is the plane on which the entire story is happening. Because you are trying to find out under what circumstances the whole thing is going to be the critical.

Student: So, is this more critical?

Professor D.N. Singh: Yes, because this under active conditions. So, state of stress at every point on this plane is going to be guided by this equilibrium. So, if I have understood your question correctly or maybe another way of looking at your question would be you are trying to play with W, let us say the logic which I have given you a small block W is going to be less. If W is going to be less P_a is going to be less, agreed, there is a relationship between P and W keeping $\theta_{critical}$ constant, why? Because that is a state of active earth pressure. So, there is inter relationship between W and P_a which we wrote like this P_a and W, is it not? I can also write a relationship between P_a and W

So, truly speaking P_a comes out because of W so this concept we are going to use later on finitesimal, you understand what is the word finitesimal, as small as possible. Finitesimal clear? W tends to 0, very thin slice P_a tends to 0. P_a becomes so W becomes finite P_a has to be finite. I think this is fairly simple, is it not? So, all concepts I have given to you. So, you have to start from the basics and just develop these edges and free bod diagrams, force vectors and that is it.

Now, let us move on to the passive earth pressure case. Passive earth pressure case is going to be slightly tricky. Imagination is required. So, what people do normally is they analyse everything for active earth pressure and they convert it to passive earth pressure, which is not correct. So, let me start with the passive earth pressure case.

Now side by side you draw it on your paper and see whether you are getting the same answers or not for the free body diagram, this block remains same P_p is this. Vertical smooth Rankine wall W, point of application is known or we are assuming whatever the most tricky thing is reaction, passive earth pressure case, wall is moving in wall is pushing the block in. The tendency of the block is to go up. The reaction is going to be like this, is this okay? That is the crux of the problem.

Then you have a normal component over here. And then you have the reaction. This is what is tricky, you see the way I have shown reaction, and this is what I showed I was demonstrating in the last lecture that if I have to lift a piece of a block, how would I lift it? You know, your callipers have a horizontal section also. So, you have to lift it like this, and you have to detach it from the parent body. So, the more emphasis is on the R, this body or this block is going to be under equilibrium of P_p, W and R and what you are observing is how they analyse this further R and N, what is included angle?

What indication it gives you when I was talking about this ϕ_m and other things and somebody was asking the question somebody asked a question why to optimize thing which one is going to be easy ϕ_m less than 90° or ϕ_m going to be not 90. I will say becoming much more as compared to the ϕ. Now what you are observing here is ϕ_1 is going to be higher than what you can visualize. So, if you draw a free body diagram or sorry force diagram, this is P_p, this is W, this is R, what next?

Compute the angle between R and W. Can you conceptualize this thing that R and W are going to be $\theta+\phi$ over here. Is this correct? If this correct, how your component of the shear stress and normal stress are going to get mobilized draw it. Are you getting the triviality associated with this system? So, that is what the issue is because the way you have plotted here $\theta+\phi$ cannot be this. But let us prove that what mistake we are doing mistake means is not mistake what triviality we are dealing with, it is not a mistake. So first now you draw T and N components and then you will realize what is happening

What is the angle between R and N ϕ, this is $\theta+\phi$ that means your this is going to be, and is this correct? So, that means this is going to be ϕ by definition the resultant and the normal are incline at ϕ angle where is T? T is going to be somewhere here. I hope now you have understood in an acute angle you are trying to show that there are two parts of that.

So, the way we have drawn it, if you really do it on a graph paper, you will realize that this thing will pop out. Your direction of R will be taken care of somebody was saying clear and what is going to happen now, this is 90° this is ϕ, this is $\theta+\phi$ check with N and W, what is this angle? So, you have to conceive this idea that how R really acts on the system.

We will talk about this later, let us still go on. So, we are not violating anything, we have depicted something with some angles and we are going ahead with this. So, I think now, you understood how to solve this you can also get the only thing is that when we do P_p, P_p will be equal to W and $\tan(\theta+\phi)$. So, now, today onwards you should always remember in passive earth pressure condition the inclination of the slip surface and the friction angle they get added up. And in active earth pressure case subtracted, now what it says is θ truly speaking has to be more than friction angle. Normally, for sands friction angle could be 30° to 35°. So, I do not know whether you are done triaxial test or not.

Somebody was asking this question long back after the testing of triaxial sample, if you take it out and see the failure plane can you show me where the $45+\phi/2$, will be, Mohr circler is there for your help. So, inter so there is interlinking of these concepts, the failure is going to take place from σ_1 plane at an angle of $45+\phi/2$ in your triaxial testing, this is σ_1 this is σ_3, what is this angle? We have proven it just now, got it. For your quick review, inclination of slip surface with respect to the plane on the σ_1 is acting which happens to be a horizontal plane.

Let me read it out over here, instead of stress σ_1 acting on a horizontal plane, the moment you draw this plane cutting the circle at σ_3. Becomes a pole any line passing through the pole at failure is the plane at which the failure is taking place θ_f. θ_f is equal to this θ_f is θ_f is your $90+\theta_f$. So, $45+\phi/2$, this question somebody had asked long long back when I was discussing triaxial testing and I had asked you to wait for some time.

So, now, we have proven, what is the meaning of this? Beneath the foundation in the soil mass state of stress exists which is active in state active earth pressure. Do not goof it up σ_1 is acting on horizontal plane and this inclination is with respect to horizontal for active earth pressure condition.

In case of the foundations what is going to happen is when we will be studying next year foundation engineering you will remember me for whatever reasons. Now, if this is the soil mass, what is going to happen is this is how the failure is going to develop. And the question is what is this angle? So, the question to you would be whether this is active edge or passive edge? This blog is resisting the movement of the foundation. This is the active earth pressure this is the passive earth pressure. Again, we are checking things. All these concepts are interlinked. So, what I am trying to demonstrate to you simple concepts which are studied in Geotechnical Engineering I until now, can be utilized for designing the systems there is nothing great.

Now, just for your quick understanding, let me take one more case before we disperse today. Let us take the effect of wall friction on the diagrams, this case so wall friction case let us say Draw the direction of P_a in your book, you know the direction of P_a, incline at an angle of δ, we call this as a negative batter, this is known as negative batter. This is the word which normally we use in civil engineering practice, inclination which is in the negative direction battered negatively, it defines the slope of that particular line. So, this is the normal we have a negative batter δ. P_a is done, what about W?

Even if I am having q_{sH} and everything that will come over here it gets added up to this. So, you may add q_{sH}, whatever all these things, so this effective load, this is a reaction, can you solve this problem now? Now, tell me quickly what the angle between R and W is? active earth pressure R and W θ-ϕ let us check this. So, as long as your δ is finite, this angle is going to be 90 minus of that, is this correct? For horizontal case what will happen δ is 0. So, P_a will be horizontal this will be 90° perfectly all right. What about this angle?

And the second question would be, what about the included angle? So, if this is I hope you understand this is θ-ϕ, what about this angle then? So, this will be $[180-(90-\delta)-(\theta-\phi)]$, is this okay? So, this will be $[90+\delta-\theta+\phi]$, is this okay? A quick question, what ball friction is going to the earth pressures? Quick quick what happened because of friction.

Now, this realisation you should have. So, more the friction angle, was going to happen? Can you make something from here? Between these two, friction angle does not come to the picture, are you getting this point? Because these are the gravity dependent issues, this is the material the way I will read this this is the material, this is the material R, this is gravity, this is human efforts.

So, do a simple exercise let us go for the sin rule. So, this is

$$\frac{P_a}{\sin(\theta-\phi)}=\frac{W}{\sin[90+\delta-\theta+\phi]}$$

I can straightaway get rid of this and then truly speaking this I can transform over here I can get, clear?

$$P_a=\left(\frac{W\sin(\theta-\phi)}{\cos(\delta-\theta+\phi)}\right)$$

Quick check if δ is 0, what going to happen? Sorry, I have done some mistake over here check it? So, there is something I have done wrong over here.$180\pm\delta\pm\phi$,

Sorry. Same, that is okay, very good. So, if wall pressure is not there, this is going to be ϕ-θ. Cos of minus. So, this is going to be the same thing. So, you just get this value and again your function is going to be same.

So, P_a will be equal to half because let us put it like this W multiplied by K_a and this K_a is going to be inclusive of δ. So, the question is still remains, what wall friction does? And from where the wall friction comes? In natural process or in in real life, whenever you are making a wall there will be some friction which is coming between the wall which is made up of RCC or wood or steel or whatever composites and the backfill material.

So, this is a more realistic picture. But what we have done is we have used the simple concepts of mechanics to solve a complicated situation. So, what is the answer? If the wall friction comes in the picture, what going to happen to the earth pressures? Look at this, I mean, like we drew the free body diagram of the wall and the and the backfill.

So, under active earth pressure, the friction is resisting the movement, is it not? Lifting it up, the tendency of the block has come down and this guy is lifting it up. So, truly speaking, if you draw the free body diagrams properly, this is the component of wall friction, and the tendency of this force is to scoop it up. This force is trying to scoop it up. If I take moments about this point, how it is going to be? This is going to get nullified, but this component is going to be more, or this component is going to be more? This component is going to be more because this is going to be a $\cos\delta$ term. Got it. That is the point you have to remember.