Geotechnical Engineering – II
Professor D.N. Singh
Department of Civil Engineering
Indian Institute of Technology, Bombay
Lecture No. 03
Shear strength of soils II

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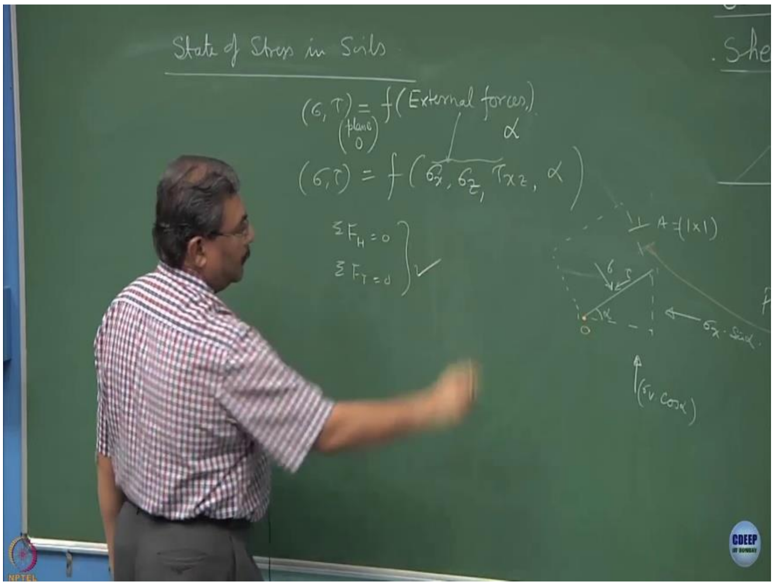

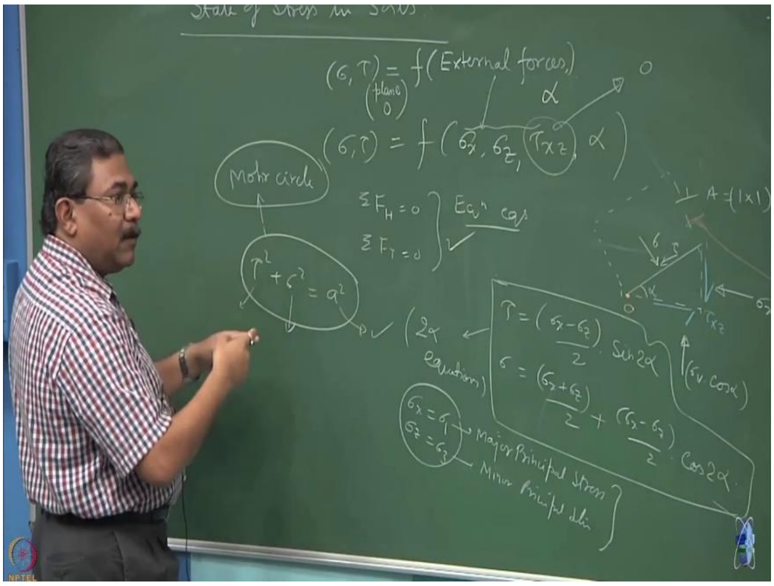

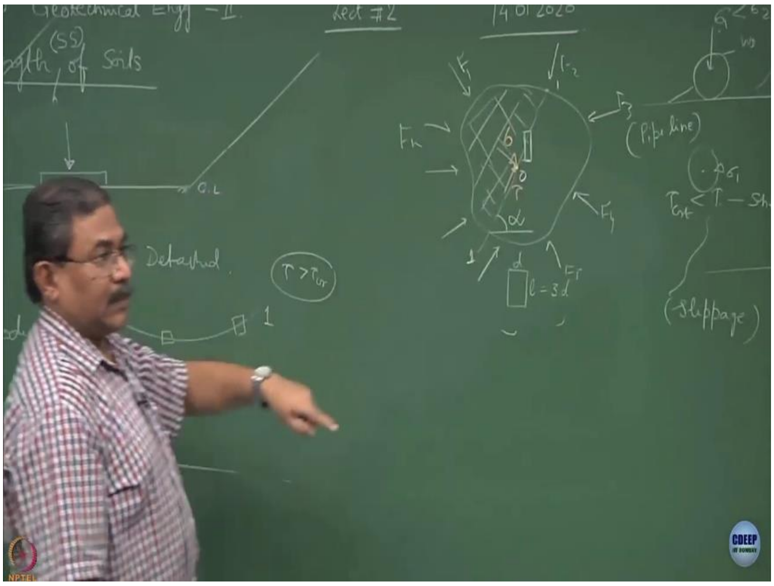

So, now we will be talking about the state of stress in soils, at this point what is happening… the end effect of all these forces at this point is there is going to be a normal stress I write normal stress as σ and there is going to be a shear stress I write it as τ. So, the question number 1 is, when we say the state of stress in soils can we find out sigma and tau for as a function of external forces, is this okay?

And this σ and τ is always on a plane, why? Because I am interested in understanding the detachment of a block from the parent body and this plane gets created because of the criticality which we are talking about in the soil mass and the condition is each and every point within this soil mass is going to be critical simultaneously.

So, sigma tau acting at alpha so, plane indicates alpha value and then there has to be a point. So, point O external forces and alpha is the statement of the problem clear? I can modify this as σ_x, σ_z. Now, you might say why not three dimensional? Again, my answer would be let us do first two-dimensional cases, which are more prevalent in engineering because most of your structures are two dimensional in nature, though they appear to be three dimensional. Why does so, embankments soil mass they are all extending up to infinity in the third direction is it correct.

So, we are taking a slice of this and hence, the three-dimensional problem is being converted into a two-dimensional problem. And once you become an expert in solving two dimensional problems, you can extend them to three-dimensional problems easily. That is not a very difficult thing. So, σ_x, σ_z, what else is going to come? τ_{xz} very nice. What else? α?

So, what I have done? All these forces, external forces, I have converted them into stresses in geomechanics, engineering sciences and engineering technology, we do not talk about the force ever, we always talk about the stresses. Though the forces are acting on a system, we convert them into stresses, and then we put them over here like this.

So, what I am interested in is, I am interested in finding out σ and τ as a function of σ_x, σ_z that τ_{xz} , α. This is the state of stress which is existing at a given point and this point lies on the plane at which the failure is going to take place. Let us start this process. I will isolate this whole system now, and go for the free body diagram of this, it is a point and the point is lying on a plane correct?

So, suppose if I say that this is the point O, and this is the plane, 1 - 1 and on this plane, the normal stress and shear stress are acting, clear? And then we have combination of σ_x and σ_v. So, what I have done all f1, f2, fn, and I have combined in σ_x and σ_v, so, truly speaking σ_x is your horizontal stresses divided by area of cross section, and this is also equal to vertical stress divided by area of cross section.

So, henceforth we will not talk about this, v and h as I said, we normally do not talk about the forces, we talk about the stresses. Now, if I take the projection of this plane and if I assume that this plane extends up to unity in the third direction clear, and this is of unit length and this is also unity. So, area of cross section is 1. So, area of cross section of this 1 is the square 1, if this is the angle of α, this is going to be the horizontal force is going to be now σ_x multiplied by sin α and this is σ_v multiplied by cos α.

Can you do simple equilibriums and try to prove that τ will be equal to

$$\left[\frac{\sigma_x - \sigma_z}{2}\right] \times \sin(2\alpha)$$

and σ will be equal to

$$\left[\frac{\sigma_x + \sigma_z}{2}\right] + \left[\frac{\sigma_x - \sigma_z}{2}\right] \times \cos(2\alpha)$$

This is what is known as the equilibrium equations and these are known as 2 α equations, I hope you can realize that if τ is this and σ is this truly speaking, this is the form of an equation $\tau^2+\sigma^2=a^2$

and this is nothing but a circle that means, these equations are the generalized form of the circle which is known as the Mohr circle. What we have done in last 3 minutes and Mohr circle onwards you are aware of everything in the mechanics you have done.

So, now soil does not appear to be a foreign material. Now, we can handle it easily because strength of material is already done. And you started with Mohr circles. So, I brought down everything to the Mohr circles and I will be using this as a tool to define the state of stress which is getting induced in the material causing failure because of the external forces the stresses and what is the assumption it is a rigid body.

So, we are not talking about any type of deformation, settlements, compression now, it is the 100 percent shear failure, this is a simplified solution which we have obtained for this type of a free body diagram here I have taken this as the radius so, you can interpret it the way you want and then this is τ and σ.

So, what he is suggesting is I am not taking into account this thing. I am just coming to that, good that you have hinted on this. So, this equation is valid provided your σ_x is σ_1 you know what is σ_x equal to σ_1 and σ_z is equal to σ_3 so, if this is the condition which I am assuming over here were this happens to be the major principal plane principal stress and this happens to be the minor principle stress. Now, I think your question is answered. And by virtue of being the σ_1, σ_3, σ_x , σ_z , τ_{zx} stands to 0.

So, what I have done? A complicated problem I have further resolved into a situation σ_x σ_z happens to be the minor and major principal stresses that means there is no shear stress which is going to act on over here. So, your point is correct, if I want to generalize this thing, I will have to show a shear stress component here. And I have to show a shear stress component over here. And this is what is going to be τ_{zx}.

We need not to enter into these complications at all, I will tell you why, because what I have done is I have created a subset out of this and if I can define this circle where σ_1 σ_3 correspond to τ and σ this is going to be a solution to a problem which I can generalize further on.

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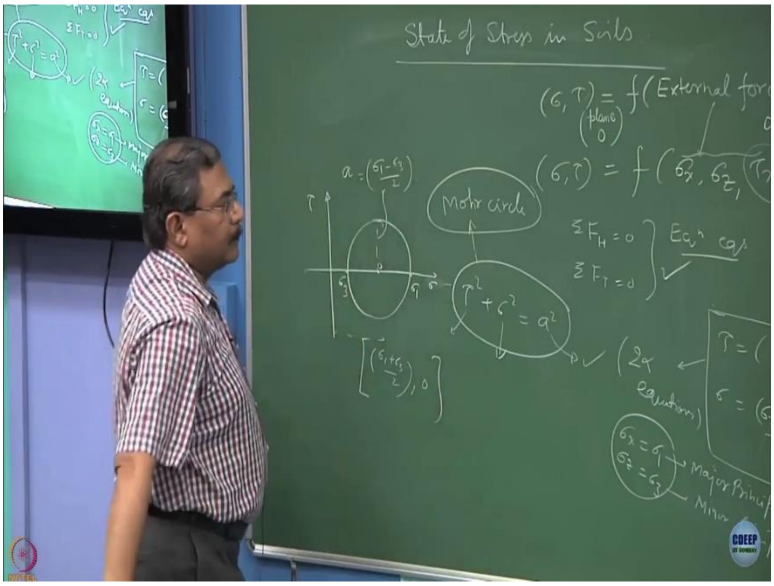

So, this is the situation where you have this is as σ_1 and this is as σ_3. So, what we do is we plot it as a function of τ and σ on the τ, σ axis, this is a centre O, so by virtue of this, it becomes $\left[\frac{\sigma_1 + \sigma_3}{2}\right]$ the co-ordinate of the centre and the radius is $\left[\frac{\sigma_1 - \sigma_3}{2}\right]$. So, this is the coordinate of the centre and this is the radius.