Geotechnical Engineering - II

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Lecture No. 29

Earth Pressure Analysis (Trial Wedge) II

I have been discussing about the Earth pressure analysis by using wedge or trial wedge. So, we already studied the Rankine's earth pressure theory. And the second procedure or the methodology, which is normally utilized to find out the earth pressures on the earth retaining systems is what is known as trial wedge method. And that is what I am discussing since last lecture. And the last one I will be discussing would be the Coulomb's method. And there are some graphical techniques also, which have become outdated in the due course of time with the advent of all powerful computational devices.

So, in continuation with where we left in the last lecture, we were talking about the earth pressure determination by using a trial wedge and I had drawn the free body diagrams for now, this is the trial wedge and in most of the cases, the statement of the problem is as long as this wall is vertical and smooth we have to find out P and this P could be active or passive.

So, a corresponds to active earth pressure and p corresponds to passive earth pressure with the weight of the block known and what we wish to do is we wish to optimize the inclination of the slip surface. So, this is the slip surface about which the failure takes place we drew the free body diagram for this situation. So, this is the shear force, which is getting mobilized, it's a normal on the slip surface. So, this is the N component normal component and the reaction of the two is resultant of the two is the reaction R.

Similarly, I drew the earth pressure for passive case. So, in passive case the direction of the shear force will be changing and then you have the normal which is acting, and this is the resultant. So, once you have done the free body diagram, life is not so difficult. As I mentioned the previous lecture, the principal unknown is P_a or P_p, and this is a function of weight of the block and weight is the function of height of the wall. We bring in θ component over here. And principally this is what the statement of the you know this the objective.

There are three forces which are acting on the block ABC and there are few knowns. This angle is friction angle, hope you know internal friction the τ could be $c + \sigma \tan\phi$. So, this could be equal to $c + \sigma \tan\phi$ and σ is nothing but the normal component and right now, we are dealing with the pure frictional case. So, c tends to 0 we are not going to take into account this. So, once you have this objective function, we can optimize the things. This is where I introduce the concept of friction also.

So, if the wall is not as smooth in that case friction gets mobilized and the only difference here would be if this is the wall, and this is the block which is in contact with the wall. So, one is the friction which is getting mobilized between the wall and the block this part we have analysed and if I draw a perpendicular on the surface for active earth pressure condition this is the direction of P_a, we call this angle as δ the friction angle between the wall and the block.

In case of the passive earth pressure what will happen is this will be the direction of P_p. Now you need not to remember this it is easy to obtain, this will be again δ value because of the friction which is getting mobilized between the wall and the block the direction of P_p and P_a changes. This part we discussed in the previous lecture.

So, if the wall is in the active state this block is moving down the friction is going to be upwards and hence you have a normal component you have a shear component the resultant is P_a in case you are having a passive earth condition the block is moving up inside the wall, inside the backfill the shear stress is going to be acting like this. So, these are simple. See in passive situation the wall is pushing the block inside, movement within the backfill. So, what is going to happen if this is a surface the tendency of the block is to move up slide up. So, that means, the reaction on this is going to be like this, this is how the shear stress going to mobilized. Remember the hump formation is that peculiar characteristic of the passive earth pressure. And in case you have a depression here.

So, this would be let us say the hump formation which is associated with the passive earth pressure and in case you find any depression over here, this is going to be because of the movement of the material outside, away from the backfill this is going to be active earth pressure. This part we have discussed quite in details. Now, what I will do is I will just try to analyse the simple situations and then slowly and slowly we will move on to the complicated cases which are more practical cases. Now, this is a system where three forces are acting and hence you can do the equilibrium analysis.

So, let us try to do one of these cases active earth pressure let us say. So, I hope you understand now, this is ABC weight is acting like this, there is a reaction coming like this, this is the normal component this is the shear component this now with all of you, this is P_a. So, I can use this concept of mechanics equilibrium of the soil mass or the block. So, I can say H, or V. I have two equations, this is how you do the most of the rigid body analysis.

One thing which you have to understand is, if the height of the wall is known, let us say this height of the wall is as H, Rankine's earth pressure theory gives me the point of application of P_a which is H/3. So, this is known. So, first of all you try to identify what are the things which are unknowns.

So, as I said over here, the principal unknowns are the pressures which are coming on the wall because of the backfill number 1. Number 2 unknown is the θ value, because this has to be the critical value at which the slip surface will form, and the failure will occur. I am sure you will realize W is a known factor. Another principal unknown is capital R. I hope you agree with this, because you do not know what the magnitude of R is is, why? several issues.

So, R is a big question mark. Because what we have assumed is that this is angle friction, which might get mobilized or not-number 1. And then later on we will talk about the factor of safety associated with c and ϕ parameters. So, the chances are that the entire friction may get mobilized or not. In c-ϕ soils, what is going to happen is the shear strength might get mobilized because of mobilization of cohesion and friction 50-50, 70-30, 30-70 whatever combination. I do not know. So, overall, R is unknown, and its point of application is unknown. Got it. So, it is a beautiful situation where R is unknown as a vector and its point of application is unknown.

So, most of the time, these types of analysis are done to obtain these questions answer to these questions by using the concepts of the mechanics, where is the third equation which I can use, I can use the free moment concept and that also I can equilibrate to 0. Now, this method is known as limit equilibrium method. That means you are just talking about the limiting equilibrium of the block. And where is the limit coming from? It is not the failure has occurred its at the verge of failure and mechanics you use the term incipient motion, is it not? At the verge of failure, at the verge of movement. What is going to happen to the system?

So, same thing is happening here also, this block under all circumstances remains in equilibrium, but it is on the verge of failure because of the plastic state of equilibrium getting formed in the backfill, is this part, okay? So, I can use these concepts of mechanics quite easily to solve the problem. I can take moment about any point I have two equations, which are going to help me quite a lot and then the third is the moment equation to find out what is the application of R provided I know P_a provided I know τ provided I know N and R in fact and of course your θ term.

So, the simply simplified model is where I am dealing with let us say a fictional material, no c is coming in the picture right now, that part we will deal later on because the moment c comes in the picture in the backfill, what happens? Tension cracks. Because of the less permeability, consolidation will take place because of compactability is not possible differential settlements and so on pore-water pressures development. So, we are not going to talk about the cohesion right now, we are simply assuming that at the time of failure, the shear strength gets mobilizes because of the friction and 100% mobilization.

So, subsequently, I will be using a term that ϕ_m, ϕ_m is nothing but the ϕ mobilized. Imagine there is attack on the country you need not to send all the troops on that particular place, you reserve some of the troops and then you spend maybe 10%, 20%, 30% depending upon the situation.

Now, this is what the material does material will not mobilize the entire shear strength, it will only mobilize certain amount of shear strength depending upon several situation which we have already studied. So, this m corresponds to mobilization or mobilized, mobilized means utilized. Remember this concept that when we drew the Mohr circle, this is what we were discussing, the failure takes place at this point.

So, this is the τ_f of σ_f. But, if you draw a perpendicular from here, you will realize that there is more strength which is available at this point. Now, I think it looks better. So, at this point at this normal stress, you still have this much the shear stress, which is available, but the material has a tendency to fail much before that, is this part clear? We discuss the ratio of the two is going to be a factor of safety.

So, that means, ϕ mobilized will be equal to ϕ divided by some factor of safety. And this is my choice as a designer what factor safety I am going to use. So, today for the first time, I have introduced the concept of factor safety in applications until now, we just talked about the material properties.

So, the sample fails, or the material fails at this point. But at this normal stress, you have so much of shear strength still available, which is higher than the failure strength. So, what we will do is, I know the W value this is half into H into H into cotθ, is this okay? So, if this is H this is going to be H cotθ CB. So, I know the W value multiplied by unit weight. Now unit weight is a characteristic you know how to compute it, there could be a total submergence, partial submergence depending upon the material variable submergence and so on. So, you know the value of γ you can compute it very easily. We have done this in the past.

Now, what is another equation which I will be getting. So, suppose if I say σ_{FH} equals 0, I will be getting P_a will be equal to something you know you can resolve this and you can compute this similarly, you will be getting R equal to something you can do it very complicated method, is it not? So, what we will do is, we will go for a simplified way draw the force diagram and the force diagram says just see how I am interpreting the force diagram. P_a is acting over here the first weightage is given to active earth pressure passive earth pressure. It does not matter whether it is P_a or P_p is acting in the horizontal direction. This is the W, is this part clear.

Where is the third force which is balancing these two R, what is R? The reaction which is getting mobilized on this slip surface balancing W and P_a. So, this is your R, is this part clear? Simple. Now, what we were discussing the last lecture, suppose if I overload the system and if I say this is the surcharge q_s one is used the analysis that we did last time.

Second would be take this component of the loading which is going to come on the system in W itself. So, this will become W plus let us say q_s into H into cotθ, is this part clear? The same thing I can do if I apply a horizontal force coming over here, let us say F_a or whatever. So, what will happen this P_a will get added up with F_a. So, imagine a situation where you are doing a construction and either due to vibrations or due to earthquakes or due to loadings, it so happens that the soil mass gets loaded externally, that is the situation here.

Imagine if you are having a system which you want to construct where you are excavating over here and all the men machinery and your camps are over here you are loading the whole surface and many times it happens during the construction itself the failure occurs, why? You are sitting on the branch which you are cutting, that means you have mobilized mods excessive weight for the less shear strength which is going to cause the failure.

Having done this, let us come back to the simple situations can you compute the angle between W and R in fact sorry, this post diagram is still not complete. Let us complete this first how many other forces are missing from this diagram? Very good. So, normal stress and shear stress. So, τ and N are the components of R if I draw it like this, this is your tangential force and if I join it like this, this becomes your normal component, this is 90°. Let us compute the included angles compute the angle between R and N, quick R and N R and N ϕ. It is okay. What is the angle between W and N?

Quick, normal stress and the weight. So, the weight is acting like this, and you have a normal stress component over here 90-θ and what will be the value of W and R? That will be θ. So, what is the value between R and W theta minus very good. Now, henceforth, you did not do compute anything of this sort. Just remember one thing that when we are talking about the active earth pressure the angles are θ-ϕ, you can prove this.

So, I can write a relationship P_a will be equal to W into tan (θ-ϕ), is this correct? W I know I have got an expression for P_a this was a simple case I got it very easily. What is the earth pressure under active earth pressure conditions half $K_a.\gamma.H^2$, is it not? Can you conclude something from here? What is this K_a value? That is, it. So, simple thing is all these terms are nothing but K_a term. And what is K_a? (1-sinϕ/1+sinϕ).

So, once we have got this objective function, what I would like to do? This is where the designer's concepts are the subjects that you know exposed. Now, my question to you would be whether you would like to have a situation where P_a is maximum or minimum? Is this correct. So, what is P_a active earth pressure the backfill is pushing the wall out just because of gravity, easy thing to happen natural process, is that not? Wall moving inside the system is going to be a difficult situation.

So, truly speaking the component which is low has to be maximized and the passive earth pressure which requires lot of efforts have to be minimized. As a designer you will require these concepts when you deal with the problems. Mathematically it does not matter because what I will do is I can optimize this function now; I am using the word optimization. So, if I say this tends to 0, what I have done?

By solving this I can get θ equal to $\theta_{critical}$, is this part, okay? What should be $\theta_{critical}$? Go back to the Mohr circle this is σ_1 smooth wall vertical wall, no shear stress, agreed? σ_1 σ_3, where is the pole? At σ_3 if I join these two this becomes my failure plain θ_f. And what was the θ value which we computed here? Shashanka you told last time in the class, is this is okay?

So, what you have to prove is that this theta critical under passive earth pressure condition is equal to $45+\phi/2$ which is nothing but mathematics. No complications very nice. I mean, under gravity conditions prevailing or like when the only the weight of the blocks are coming in the picture, the chances are that active earth pressures can be achieved without any efforts.

Most of the hills are falling. Why? Because the gravity is allowing them to slit down the surface, is it not? There is a gravity effect, you know. If I keep a block on a horizontal plane, and if I pick this point, pivot this point, and if I lift it, what is going to happen? Still in equilibrium, the moment I live to $\theta_{critical}$, what is going to happen this will attain an impending motion, that is what is happening.

So, under natural gravity condition, the chances are the system is going to fail under active earth pressure. But all this is being done by the nature. It is not in your hands, the type of stress condition which develop $\Delta\sigma_H > 0$ or < 0. Do not get detached from there that is causing all active and passive earth pressure to come in picture.

For a constant σ_V value, you remember $\Delta\sigma_{V0}$, $\Delta\sigma_H$ could be greater than 0, what are the meaning of this? Wall is being pushed out. Active earth pressure condition. And suppose I reverse the situation, what is going to happen? $\Delta\sigma_H$ is more than σ_V or its increment. Wall is going to come in. So, this is the answer technical answer to your question. And that is why we are maximizing or minimizing mathematically it is remain same.