Geotechnical Engineering - II

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Lecture No. 28

Earth Pressure Analysis (Trial Wedge) I

We had been discussing about analysing the Earth pressures and in the previous lecture, I had talked about the theory, which is proposed by Rankine.

I will continue with this discussion and the discussion topic would be Earth pressure analysis and by employing Rankine's theory, which has already been completed. The second part of the analysis or the second type of analysis, which is normally done to solve most of the issues which occur in day-to-day life is the trial wedge. And this is what I will be discussing today. And the third way or the tool for analysing the earth pressures is the Coulomb's theory which I will be doing slightly later on.

So, as far as the trial wedge is concerned, as the name suggests, suppose if I have a retaining wall which is sitting on the ground, this is a backfill. And in Rankine's theory we have assumed that this is a frictional material and *c* is 0 and we have assumed that the state of stress in the soil mass exists because of the gravity. So, gravity is nothing but the unit weight. And of course, the backfill happens to be dry and another assumption is that this all is at 90° we call this as the Rankine wall provided there is no friction between.

So, if I draw a shear stress component over here. So, this is the shear stress component between the soil and the wall this is equal to 0, smooth condition, no friction between the wall and the backfill material. Smooth wall, vertical wall containing the backfill which is only frictional in nature, no cohesion unit weight prevails dry condition and the backfill is horizontal.

So, inclination angle is equal to 0. So, this is a typical Rankine wall and we have analysed this. Now, in most of the cases what happens is I would like to analyse what is the inclination of the slip surface. So, AB happens to be the slip surface and what we want to do is, we want to optimize value of θ, though we know this value. So, I have written here θ_{critical}. So, I want to find out what is the value of $\theta_{critical}$ for the height of the wall H, there is no friction on the base also, there is no friction between the wall and this, this is a horizontal surface and how to analyse this.

So, this is where people consider trial wedge A, B and C. Now, this is a beautiful application of rigid body mechanics which you have already done, where if I want to understand what are the pressures which are going to come on the block because of the wall. It is an action reaction. So, the soil is pressurizing the wall and wall is trying to contain the pressures coming from the backfill.

So, the best way to deal this type of situation would be if I isolate this block which is in local plastic equilibrium, you remember? we have differentiated between global and local plastic equilibrium. Global is a phenomenon which happens because of the natural activity, tectonic motions. Now, this is a local phenomenon which is mostly prevailing in the manmade structures where the dimensions are finite. So, if I isolate the block which is in either active or passive state of equilibrium plus equilibrium.

So, what we do is we consider this as the block. So, this is the A, B and C and this is θ angle. So, a statement of the problem is what is θ when there is a weight of the block acting? Is this determinate or indeterminate parameter, determinate is it not. So, if I know the value of θ, this angle is going to be θ, this is going to be H cotθ, this is going to be H.

So, weight is equal to half into γ. Intentionally I am writing γ because I am not sure about the condition of the unit weight which is going to prevail in the backfill. I can always alter the situation by creating this is water up to this limit or the whole soil mass is dry or there is a partial submergence and whatever. So, let it be γ multiplied by H into H cotθ multiplied by 1 is this, ok?

So, 1 happens to be the third dimension perpendicular to the board. So, this is the total weight of the system. That means, this is half $\gamma H^2 . \cot\theta$ as simple as this. What this indicates is that the weight of the block which is under active or passive conditions of failure would depend primarily on the height of the confinement or the walls multiplied by the critical θ angle which is slip surface.

Now, if I ask you to find out or to complete a free body diagram, first of all. So, what we will do is, if I show P, is this correct? Basically, what is happening is if this is the wall and this is the backfill under active earth pressure conditions, what is going to happen? Active earth pressure condition is the condition in which the wall is going to be pushed out by the backfill. If I see the interface CA and if I detach the wall with the backfill material, this is how it will look like. So, this is the wall and this is the block, a wedge. Now, wedge is trying to push the wall outside. So, that means the wall is pushing the wedge inside.

So, this is a pressure which is coming from the wall onto the wedge, is this part clear? So, what we have done is we have isolated two forces W and P. Now, body cannot remain in equilibrium because of the two forces there has to be a third force also. So, imagine if this block has a tendency to attain active earth pressure condition, this block is trying to slide down on the surface AB. So, when it is sliding down the direction of the friction would be upwards. This is the shear strength, which is getting mobilized in the system, is this part clear?

So, if I show a resultant force, which is going to act on this slip surface as R. Now, this resultant force is going to have two components, one is the shear force and the normal stress. So, this is the net resultant force. So, I can exhibit the components of the R over here. So, this is your T, the force which is getting mobilized on the plane or the slip surface having the components of the shear strength, and this is the normal stress. Is this part clear? Now, the body is going to be in equilibrium because we have three forces, and these three forces are P, W and R. Now, this is a typical state of equilibrium which we draw for active earth pressure condition. What is going to happen in the case of the passive earth pressures?

So, in case of passive earth pressure nothing much is going to change this is the wedge, this remains θ and you know the values of θ are going to be different. I will write this has now P_a and this has P_p, is this correct? The weight is acting the way I have shown, what about the reaction which is acting on the slip surface imagine close your eyes and imagine, how it is going to happen. Now, that visualization is very important.

So, first of all, let us go in steps. If I have to draw this type of a diagram over here, by definition the passive earth pressure is a situation where the wall has a tendency to get moved into the backfill. When it is happening, this block with respect to the parent block remember this is the parent block or the parent backfill which is in elastic condition.

Now, when this block is moving up the friction is going to get mobilized in the downward direction, is this correct? The shear strength. Somewhere here you have the normal stress. So, this is that I was showing let us say R and this is what is going to be the normal stress N, is this clear conceptually? So, we consider the pressure which is going to come on the backfill or on the wedge. So, in that case, what you are observing is this is the direction of mobilization of the shear stress this is going to be the normal component and the way the R is going to act is slightly trivial.

Have you come across this problem anywhere in mechanics? You have, suppose I lift a block it is kept like this. Weight is acting downwards. Now if I have to scoop it out, this is a scooping out action you know what wall is doing wall is trying to push the system in. So, the hole block is going to get scooped out.

So, if I hold it like this, you must have done these problems in mechanics where you are analysing the weight of the block which can be held by callipers or the forceps like this. So, what I am doing is I am just trying to detach this block from the parent body. So, look at the motion, the motion of the block is inside and upwards and hence the friction is getting mobilized downwards. So, as per the thumb rules, when the soil mass gets moved inside, there will be a hump formation. And what is going to happen over here? Because the material is moving out there is going to be a depression.

So, wherever you find depressions and the humps getting formed in the natural systems, you can make out whether these systems are under active or passive state of equilibrium. Now, this part is simple. Now, what we have ignored very conveniently in Rankine’s theory is that this wall is smooth in most situations not going to happen.

Now, we call this as the effect of wall friction on the stability of the block. That means there is a friction component which is acting between the interface of the wall and the back and the backfill block or the wedge. So, now can you draw the free body diagram of the block when the friction gets mobilized. Suppose if I assume that this was the case of let us say, τ equal to 0 at the interface.

But suppose if I say τ is not equal to 0 and this is the rough wall. So, now, what we are doing is you must have noticed that we have started manipulating the theories. It is a deviation from the Rankine's theory. So, the moment you introduce the wall friction over here, what is going to happen? The tendency of the block is to slide down that means the wall is trying to lift it up.

So, there is a component of friction, which is going to come over here the P_a that thing over will not act as it is the way it has been shown and what will happen is, if this is a normal from the surface, now, this will be the direction of the P_a and what we define this as a value δ and this δ is known as roughness of the wall.

So, the two components of the P_a will be one is the normal stress, another one is going to be the shear component which is acting at the interface of the block and the wall. And I think now, you can do the free body diagram very easily if the block is trying to slip down this wall as it tends to push it up this is how the friction gets mobilized the rest of the things remain same. Have you understood this part? So, this is the effect of the roughness of the wall.

But the question is what roughness is going to do? Whether it is going to enhance the active earth pressure which is acting on the wall or it is going to decrease the active earth pressure on the wall. So, in most of these situations, ultimately the objective is to optimize θ obtain $\theta_{critical}$ and once you know the $\theta_{critical}$ value, I can substitute, and I can get the value of P_a. So, the principal objective is P_a which is a function of theta and W and H and γ. Similarly, what we will try to do is, we will try to find out P_p as a function of all this and because I have talked about the roughness in the wall also δ term.

So, δ term will also get included normally this δ is taken as two thirds of ϕ or the limiting value could be about 10°. So, these are the values of δ. So, these were the concepts which are involved and ultimately what we will do from the next class onwards is we will try to utilize these simple mechanistic models and first of all optimize θ value and get the values of P_a and P_p which is most critical.

And incidentally this $\theta_{critical}$ is going to be the slip surface along which the failure is going to take place and we will try to prove that this is theta equal to 45+ϕ/2 or 45-ϕ/2, for what cases?

So, this is going to be for active earth pressure condition, and this is going to be for passive earth condition.

Now I can include as many as complications as possible I will include a surcharge over here. No issues. I may include a lateral pressure also which is coming because of let us say earthquake or which is coming because of some other activity which is going on the site. So, what I can do a certain fraction of W I can include in this direction also which is because of let us say earthquake acceleration. So, once you have done the free body diagram, later on, nothing much is required.