

**Geotechnical Engineering - II**  
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**Lecture No. 27**

**Mechanisms of Development of Plastic Equilibrium in Soils II**

You must have noticed, when you go to the offshore environment particularly let us say Marine Drive in Bombay, you must have gone inside on the parapet wall. And what you see on the seaside is tetrapods, any idea why tetrapods are designed over there? One is of course, they act at the breakwater. Number 2 of course, they avoid erosion of the soil. Number 3 they will not allow the entire force of the wave to come on the wall. Number 4, which is more mechanistic, and the answer is here, and that is the beauty of you know, practicing of geo-mechanics.

So, a careful understanding of this diagram will exhibit if I can nullify this by putting overburden, are you getting this point? So, why tetrapods are kept? You want to negotiate with this  $-2c\sqrt{K}$  which is the culprit. Because this creates an easy path for environmental activities to happen inside the backfill.

In cold countries, what will happen? The water will enter it will become ice, and subsequently we will study that this block is going to be acted by the water pressure on the surface. So, the shear stress is going to be 0 because of the cracking, the hydrostatic pressure is going to act on this block of the soil which was retained by the soil mass.

So, one of the ways to get rid of this would be you go for nullification of this by applying external loading. This could be a sort of a tetrapod. I hope now you can design the tetrapods easily. So, the external pressure which you are going to apply on this soil mass will be equal to  $-2c\sqrt{K_a}$ . So, this whole term vanishes and you have compressive forces acting in the soil mass, life becomes simple, is this fine? Now, I will take simple cases to extend this theory given by Rankine and try to analyse some situations which are very very prevalent in geo-mechanics. Hope this part is clear.

So, what you should be careful about is the transformation of  $\sigma_1$  and  $\sigma_3$ , how do they interchange this we have been emphasizing since long. What you have to do is you have to simply overload the soil mass. So, if this is your marine drive wall and this is the parapet on which you go and sit.

So, what you will observe is they put tetrapod over here, why? Because of this reason, so, this  $-2c\sqrt{K}$  component I can nullify by applying this loading. Tension cracks will not develop. Imagine in this wall, if there is a tension crack which develops on the side the water ingress and all sorts of problems will start.

So, let us try to attempt to solve now, the application of Rankine earth pressure theory. The wall which I had drawn is known as Rankine's wall, vertical wall having no friction, backfill sloping 0 and this becomes a typical Rankine wall. So, if this is the wall, this is the pressure distribution because of the total wall and, what we depict it as is  $P_a$  and  $P_p$ .

If the whole wall is  $H$ , this will be acting at  $H/3$  and under this condition this excess of the pressure will be  $\gamma \cdot H \cdot K$ . And because you are dealing with active earth pressure this becomes  $P_a$  and when we are dealing with passive earth pressure this becomes  $P_p$ . Now, rest all things you can do easily.

Until now, you have been dealing with the fluids, water, you have done lot of problems in mechanics, where the water or the gas is contained in a container and then you are trying to find out the pressure coming on the vertical plane or horizontal plane depends upon the problem. So, in case of water what is going to happen to  $K$  parameter? This will be equal to 1.

So,  $K$  parameter will be equal to 1 and hence, this becomes your simple hydrostatic case and then you can solve this problem. Now, the complications will start. Let us say now, this point onwards do not write and just look at the board and try to understand once for all, how the real life situations are dealt with.

Now, suppose if I say that there is a UDL, uniformly distributed load, what is going to happen? So, you have the soil mass in which this type of pressure is developing active or passive depending upon the condition. Now, this UDL acts as a surcharge and suppose if I define this

as  $q$  kN/m or  $m^2$  depends upon the type of loading. Because this is the line loading, I will take it as  $a$ , it could be aerial loading also. So net effect gets added to this in the form of  $q$  multiplied by  $K_a$  or  $K_p$  that is the only difference.

A better way to understand this would be this whole situation can we transform like I can always go for equivalent height of this  $q$  provided if it is not kN/m<sup>2</sup>. What I can do? I can replace this whole thing by a situation where this is the wall of height  $H$  and on the top of this there is another layer of the soil which is compacted and let us say this is the height  $z$ . If the unit weight of this material is known, what I have done? I have done an equivalency between  $q$  and  $z$ .

So, I can compute the height of the wall which is got extended because of the  $q$  loading as  $q/\gamma$  this type of transformations normally we do whenever the situation calls for it. So, no difficulty, this small portion is going to create this pressure. What is axis here? This axis will be equal to if  $\gamma$  is known, this will be  $\gamma \cdot z \cdot K$  depending upon active or passive situation.

If the unit weight of the material and  $C-\phi$  parameters remain constant. Right now, let us say I am dealing with the pure frictional material to make things easy. If this is  $\phi$ , this is also  $\phi$  there is no contrast. So, what is going to happen? This overloading will remain constant and if you remember in the previous case what we have done. We have taken this as an overloading because of cohesion components.

Now, this is the overloading in terms of the first layer plus the second layer. So, this is how it looks like. So, this remains as  $\gamma \cdot H \cdot K$ . And this component gets added up to this. So, this is  $\gamma \cdot Z \cdot K$  and  $\gamma \cdot Z$  is nothing but the surcharge which is acting on the system. So, I can say this is  $q \cdot K$  in total this will be  $q + \gamma \cdot H \cdot K$ . Which is equivalent to this function. Now, rest of the things are simple mechanics you can find out the pressure, you can find out the force and you can apply the equilibrium conditions that will come much later.

So, if you venture into the real-life situations, I will create few situations and I would like to analyse them. What we call it as an effect of water table in the backfill. If there is a wall here of height  $H$  and this is a backfill and suppose there is the water table over here. Now, what is

going to happen? It is a situation when it has rained overnight and water got logged in the backfill.

So, is this situation good or bad? That is what actually we are trying to analyse, there is a complete waterlogging sometime back I said I write B in a C- $\phi$  material with a predominance of c, what would have happened? Now, whole system might become saturated. A saturated material under self-weight is going to consolidate very difficult to compact cohesive materials, cracking which we discussed. The moment crack develops water seeps through and exerts pressure on the wall. So, those are the complications. Now, when we analyse these types of situations particularly for pure frictional materials, because Rankine theory is valid mostly for pure frictional materials, what we did is we extended it to C- $\phi$  soils, in general.

So, rule of thumb is always filter out the effect of water. Remove the effect of water first. So, it is a case of submergence. So, the pressure diagram would be I used the word submergence, this will be equal to  $\gamma'$  submerge multiplied by H multiplied by K. And where the effect of water will go, that can be added up to this. So, this gets added up in the form of the pressure from the water. So, this will be equal to  $\gamma_w$  into H. So, the total effects would be the submerged or the effective pressures which are going on the system and the water. What we call it as a perched water table

Let us, talk about a partial submergence, water table drops down to this level. So, this is a partial submergence. The moment partial submergence occurs, the backfill behaves as if there are two layers of the system that is it. Rest of the principle remains same fine. So, what is going to happen? This much soil mass is going to be a function of type of the soil and type of the soil is going to govern the capillarity. Yes, or no? Both. Agreed? So, the more and more I compact the material, what I am going to do? I am going to create a capillary zone and this part we have discussed partially saturated system starting from this point, this point below everything is submerged.

So, drawdown of a water table creates a multi layered system in the soil mass. What type multi-layered system this is creating? Even if the material properties c- $\phi$  remain constant, what is going to change is  $\gamma$ . So,  $\gamma$  could be a function of z, depending upon the type of the material, is this part clear? So, the simplest possible situation would be when you are dealing with sands, coarse sand, not fine sands even, fine sands will have some capillarity.

So, coarse sands what is happening, that is a special case when you have coarse sands as a backfill material and the water table drops in no time this whole soil mass will become dry and this remains submerged, that is it. So, I have created a multi-layered system wherein you have the wall and this wall is having a layered backfill. So, let us say this is  $\phi_1$  and this also remains  $\phi_1$  no issues. This becomes  $\gamma_{dry}$  and this becomes  $\gamma_{submerged}$ , because of the water table.

So, this is  $z_1$ , this is  $z_2$ , does not matter even if I have to superimpose the effect of cohesion I can take  $c_1$  and  $c_2$ . So, for the sake of simplicity, we will not consider cohesion in the picture right now. Just to give you one concept and once that concept is clear, you can solve any problem

Now, suppose if I overload the system by  $q$ , what is going to happen? Another layer gets added up over here. So, this becomes equivalent surcharge. A layer of thickness  $z$  made up of that material or different material acting as a surcharge. So, this concept is normally utilized when we do three-dimensional consolidation design sand drains or PVDs because after inserting the PVDs you have to preload it by creating this pad.

So, in continuation with this now, let me introduce the last concept and after that you will be quite good enough to apply this anywhere in the life. So, I will consider a simple case of a layer deposit. Suppose this is  $\phi_1, \gamma_1, z_1$  and this is  $z_2, \phi_2, \gamma_2$ . So, if you have followed all this discussion, which we discussed until now should not be very difficult for you to catch up with what I am going to discuss now.

If I ask you to draw the pressure diagram what is going to happen? Starting from this layer the first is this axis which will be equal to  $\gamma_1 \cdot z_1 \cdot K_1$ . It will be  $K_{a1}$  for active earth pressure,  $K_{p1}$  for passive earth pressure. So, I am not writing all those things. Now suppose if I put a condition that  $\phi_1 > \phi_2$ , what is going to happen?

So, the moment  $\phi_1 > \phi_2$ , what is going to change this  $K$  parameter. So,  $K$  is going to get changed. So, that means  $K_{a1}$  is going to be more than  $K_{a2}$ . Is this okay? If fraction is first one is layer is one more so this is going to be like this. Is this okay,  $\frac{1 - \sin\phi}{1 + \sin\phi}$ . So, what it indicates is when you

have a system of soil mass, which is having a higher value of lesser value of friction angle, how this pressure is going to get redistributed in the second layer.

So,  $\gamma$  is  $z_1$  acts as a  $q$ , if this condition is valid  $K_{a1}$  is going to be lesser than  $K_{a2}$  that means, you will be having a step function like this and this will be equal to  $\gamma_1 z_1$  into  $K_{a2}$ . Because  $K$  active has increased. Reverse situation, if I can create a material where  $\phi_1 < \phi_2$ , what is going to happen this  $K_1$  is going to be greater than  $K_2$  and this kink which has gone out will be inside and this becomes the second situation. This will be the first situation.

So, we have got, we have taken care of the first layer and what is going to happen because the first layer. Equivalent model of this would be if I can shift it over here, this becomes a wall and this whole thing I can superimposed as  $q_1$ . But then the material is lost there, because  $c-\phi$  cannot be taken into account, that is the only static pressure. And then followed by this layer. So, this is going to be equal to  $\gamma_2 z_2$  into  $K_2$ . If I want, I want, I can make it more complicated now. What I will do is the whole thing is submerged. No issues you can solve this problem. Filter out the water.

So, the way we did it over here everything becomes effective, submerged. And superimposed the effect of water on this, that is it. So, the moment this happens, you will deal with effective stresses effective weights and superimposed  $\gamma_w$  over there, partial submerges no issues, depends upon the type of material you have capillary fringe, saturation, find out the  $\gamma_z$  as a function of depth of the wall superimpose it over here.

So, this way you can now deal with multi-layered systems. The question is where I am going to use these concepts? So, from the next lecture onwards, what I will do is, I will start analysing more complicated real-life situations which are known as sheet piles. So, suppose if I say that this hypothetical wall becomes a sheet which is embedded in the ground, this is what we will analyse. So, I have created a three-layered system 1, 2, 3 and this side, yes, we have created another type of system. So, it becomes a multi-layered system but the basic conceptual remain same.