Geotechnical Engineering - II

Professor D. N. Singh

Department of Civil Engineering

Indian Institute of Technology, Bombay

Lecture No. 26

Mechanisms of Development of Plastic Equilibrium in Soils

In the previous lecture I started discussion on the plastic equilibrium in soils and I gave you basic concepts regarding what causes the plastic equilibrium in the soils. In continuation with where I stopped yesterday, and I was discussing about the global state of equilibrium in the soil mass which is normally caused by the natural processes under the action of gravity, and we define that as global state of plastic equilibrium in the soils.

Now, our interest is to understand how the manmade structures develop the state of plastic equilibrium and this is what is known as local plastic equilibrium in soils. So, what we are going to discuss today is a subset of what we discussed yesterday that was for the natural phenomena. And now, I am going to focus on the mechanisms which cause plastic equilibrium to develop in the soils under local conditions.

So, the best way to understand the mechanisms is if I consider a box which is resting on a horizontal surface, let us say this is made up of perspex, perspex is a plastic sheet which will not offer any friction, and this happens to be the top of the box and this is where I am filling up the soil mass. Now, the way sample is created, this is by sedimentation process or by very slow packing, controlled packing which represents the elastic state and we have quantified the state as K_0 at rest.

So, the first mechanism in which the plastic equilibrium state might occur in the soil mass would be if this wall which I define as let us say AB slides out, this is what is known as sliding failure, very common type of a failure where the wall AB would get slid to a finite distance and under this situation what is going to happen? This much of strain is caused in the system which I define as Δl. So, this is the first mechanism.

Now, what has happened here is? the wall has moved out of the backfill. So, as we discussed in the previous lecture, this is going to be a situation which is termed as active earth pressure. Now, at this case or in this situation what is going to happen is because the soil mass is also moving out, there will be a slip surface which is getting generated, and this slip surface gets generated in such a manner that if this is the P_a we define this as active earth pressure that is P_a. This angle is going to be 45+ϕ/2 we have proven this.

Now, if you take an element somewhere inside the soil mass of the soil and if I denote this as σ_v σ_H what has caused this type of a situation is that σ_v happens to be greater than σ_H.

So, this theory was given by a guy known as Rankine and we call it as Rankine earth pressure theory this was in 1847, I think long, long back where what he assumed is that the shear force acting between the element and the hypothetical wall is zero that means for a smooth surface. So, there is no shear force which is acting between the contact. There are few assumptions which this guy has made, he says that there is a relationship between σ_H and σ_v, (ii) this is a homogeneous isotropic soil mass, (iii) this theory is valid for the frictional material that is pure frictional material that means c tending to zero, which is dry. We call this as a non-cohesive material.

Another condition which he has put over here is that the wall is smooth. There is no friction getting mobilized on the wall itself because of the backfill and the backfill is horizontal. There could be a situation where the backfill would be inclined at an angle of let us say β. So, this is what we will be studying later on.

This is the wall angle at 90° so, when this type of state of stress develops where $\sigma_v > \sigma_H$ and $\Delta\sigma_v$ remains 0. I hope you can realize that the equation which we derived was σ_1 equal to σ_3 1 plus sinϕ over 1 minus sinϕ plus 2C cosϕ over 1 minus sinϕ, is this okay?

$$\sigma_1 = \sigma_3 \frac{1 + \sin\phi}{1 - \sin\phi} + 2c\frac{\cos\phi}{1 - \sin\phi}$$

So, in this case, σ_1 happens to be σ_v and σ_3 happens to be σ_H and σ_v is equal to γ into z.

$$\sigma_1 = \sigma_v = \gamma \text{h};\ \sigma_3 = \sigma_H$$

So, that means the equation which I am going to use for active earth pressure would be equal to γ into z you have to do a bit of manipulation.

So, σ_3 goes on this side which is equal to P_a all right? This becomes K_a minus 2 C root of K_a. So, this you can derive as easily.

$$P_a = \gamma . z . K_a - 2c\sqrt{K_a}$$

Now, in this case, what is going to happen, because of the movement of the wall away from the backfill, what you will observe is this whole block is going to be under a state of plastic equilibrium and we have yesterday talked about by using the concepts of Mohr circle we have shown P_1 and P_2 and how these slip surfaces are going to be inclined.

So, under active condition, you know that this is going to be $45+\phi/2$ and this is the equation which I have derived. Now, one of the features of this system would be that there will be always a depression over here because of the movement of the soil mass. So, once the wall moves out this soil mass being granular would have a tendency to flow out there will be a depression and what happens is this material would like to get accumulated over here in this triangle.

Meaning thereby, active earth pressures are always associated with depressions on the top and bulging on the lateral side. So, this is the first mechanism of the failure. Now, what I am assuming here is that there is no friction getting mobilized on surface AB and the base is also free of friction is smooth surface. So, there is no friction coming into the picture, the principal stress condition remains maintained, because the moment friction gets associated with this, what you will observe in this case is, if the backfill had been inclined at an angle of β, this element would have become like this.

And then we would have a σ_v over here, σ_H over here, and shear stress developing over here. So, this is a basic difference between the two situations, this situation we will be handling later on.

Now, there could be a second situation where I will save my efforts in drawing this picture again. So, there the movement of the wall is inside because of the state of stress, and it may so, happen that the wall moves in to let us say, A" B". As we saw yesterday, once the ball moves into the backfill, this becomes the P_p passive earth pressure.

Now, this is a situation where your σ_v is going to be, so, this angle is going to be $45-\phi/2$, the slip surface, this whole soil mass goes into the state of plastic equilibrium rest of the body remains in the elastic state K_0 condition what happens over here is σ_h is greater than σ_v.

If this is a situation, I can write this term as P_p the passive earth pressure. So, σ_1 remains equal to passive earth pressure, σ_3 becomes σ_v over here you are discussing this thing long back how the reversal is going to take place, and this will become σ_z into K_p plus 2c root of K_p.

$$P_p = \gamma . z . K_p + 2c\sqrt{K_p}$$

So, what we have done is by using the simple model, we have derived how the earth pressures develop in the soil mass.

This was the discussion which was done by Rankine. And there could be other methods of the mechanisms of the development of earth pressure also, what I may assume is that this point B acts as a hinge. What is the characteristic of the hinge, moment about this point zero, correct? If the moment about this point is zero, what could be the mode of failure of the system?

So, the mode of the failure of the system would be you have AB, and this is the backfill if B is the hinge getting formed, the chances are that there will be a rotation about this point and the wall is going to deflect like this under active earth pressure. So, this is going to be Δl.

Normally, the wedge which is getting formed here is defined as AC equal to l and hence the percentage strains which get accumulated in the sample or the material would be $\Delta l/l$.

Now, in this case, what is going to happen? Again, the failure plane develops like this at an angle of $45+\phi/2$. The soil mass has a tendency to flow out so there will be a depression over here. This much volume of the soil mass gets moved out and gets accumulated somewhere here. Typical active earth pressure condition. And I can also assume rather than B as the pin formation or you know a pin joint or the point of rotation, I can also assume the mechanism number 3.

So, this is mechanism number 1 of development of earth pressures, number 2, and number 3 would be you must have studied Sluice gates in hydraulics or something of that sort. If this is the wall, so point A acts as a hinge, these are assumptions. So, if point A acts as a hinge under active earth pressure condition what is going to happen, this is how the failure is going to take place.

Wall is moving away from the backfill, there will be a depression like this of the material. So, this volume gets accumulated over here, this is the third mechanism of the failure where the hinge formation is taking place at point A. So, considering all these types of models, these types of equations have been derived. There is another theory which is proposed by Coulomb and what we call this as the Coulomb's rigid block, wedge theory.

So, basically, this study deals with the free body diagram of the wedge which is getting formed in the plastic equilibrium state and then by using the concepts of equilibrium, you can solve it. This part we will be taking up later on.

Now, this is of some special interest, if I asked you to draw the pressure variation and suppose if this is the wall of height H and if I consider the z value from top and if the height of the wall is H so, what you are observing is there are two components in the earth pressure. So, when z is equal to 0, what is the value of the earth pressure minus 2c root K_a. So, that means, this remains constant all throughout. So, this becomes the surcharge because of the cohesion of the material and this plus the first component which is a triangular variation you must have done in hydrostatics.

So, this is equal to γ into z and z will become now H multiplied by K_a. In short, you can analyze this problem very easily if you have the pressure diagrams, you know what the CG of the application of these forces is. So, this will be H/2 and this is what is going to be at H/3 but normally we do not do it like this.

So, what we are deciphering from this equation is that the total active earth pressure acting on the wall will have two components, if I club them together this is how they will look like. Have you come across this type of pressure distribution somewhere in civil engineering where? Beams. So what is happening here in case of soils? Because of the cohesion, which is getting mobilized in the soil mass, there is a tendency for a tension crack to get developed.

So, this is the z_0 value, which is defined as the depth of tension crack. Any cohesive material, if it is compacted and left on its own in a due course of time the active earth pressure develops and during that what is going to happen? This much portion of the wall is going to get exposed to the atmosphere because of the development of the tensile stresses.

Now, if I integrate this term to get the capital P_a so, this is equal to force per unit length of the cross section. So, this will be equal to 0 to H this term.

So, that mean I am considering a point somewhere here of finite thickness dH and its length is per unit length let us say 1 meter. So, if you do this exercise what you will be getting is, half $\gamma z H^2$ minus $2cH\sqrt{K_a}$.

$$P_a = \frac{1}{2}\gamma K_a H^2 - 2cH\sqrt{K_a}$$

So, there is something known as an unsupported depth of the cut. So, this will be equal to $2z_0$ and I can prove this now. So, if this is the total force which is acting on the system I will try to find out when this becomes 0.

So, this will be,

$$\gamma . z . \sqrt{K_a} = 4\ c$$

Now, this H is the total height of the wall and the z which I am considering is equal to the unsupported depth of the cut. So, basically z will be equal to 4 times c upon γ into $\sqrt{K_a}$ and hope you can realize that this value if I substitute over here as z_0 and z_0 is the point where the point pressure is 0.

$$z = \frac{4c}{\gamma\sqrt{K_a}}$$

So, from this expression I can obtain,

γ into z_0 into K_a equal to 2c $\sqrt{K_a}$ and hence z_0 will be equal to $2c\sqrt{K_a}$ upon γ into K_a. This becomes 2c upon γ into $\sqrt{K_a}$.

$$\gamma . z_0 . K_a = 2c\sqrt{K_a}$$

$$z_0 = \frac{2c}{\gamma\sqrt{K_a}}$$

This z actually will be what so, this this will be Δz sort of a thing. So, this is all right? Now it is fine yeah so, now what will happen is this will be H and γ.

Now it is okay, now this z will not come. So, this z will be basically equal to 2 times z_0, good that you pointed it out. So, now this term will be, it is okay now, $\sqrt{K}$ will be it cannot be because root K_a I have cancelled it. This is actually I wanted to write here K_a term, that gets cancelled.

So, basically what we are proven from here is that the total depth of the unsupported cut is equal to 2 times the tension crack. So, this concept of tension crack I will be utilizing quite a lot. So, AO is defined as the tension crack. What happens at this time? when the tension crack develops in the soil mass this much portion of the soil will not be in touch with the wall, and this is how the tension crack develops. So, this becomes a crack, and this crack gets thrown to accumulation of rainwater. So, what is the message which we have learned from this simple analysis, two messages, three messages.

First thing is if you are dealing with a c-ϕ material as a backfill material and particularly if cohesion component happens to be very high, clay material, it is prone to develop tension cracks, which is not good for the health of the structure. Why?

(i) you cannot compact the cohesive material or the backfill material, (ii) it will consolidate, (iii) there could be water accumulation during the rains, (iv) the entire system will settle because of self-weight. What we call it as a self-weight consolidation. It is not a freely draining system. So, that is a reason normally we avoid using a cohesive material as a backfill material.