Geotechnical Engineering - II

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Lecture No. 25

Plastic Equilibrium in Soils

The first major part of this course, I have already completed that was the shear strength of soils. And I spent about 13 lectures discussing the importance of shear strength, how to determine this, what are the tests that are to be conducted and how to interpret the results and at many places I have given you the importance of parameter selection, so, that they can be utilized in real life projects, which I hope would be very useful and helpful for all of you who are into the field of consulting and advanced research in the realm of geomechanics.

So, today onwards I am going to start the discussion on the second major topic of the course that is the plastic equilibrium in soils. So, needless to say it is understood that this is the state of the stress which is causing the plastic equilibrium in the soils and as a geotechnical engineer, we would like to understand what causes the state of stress to develop in the system and what are the applications of this type of concept which is quite prominent in the practice.

So, when we talk about the plastic equilibrium in soils, the first question is where are we going to apply this concept? So, the applications are basically this is the application of how to use the parameters. So, designing of retaining walls particularly. I normally call this as retention schemes and when we talk about the retaining walls and the retention schemes, the most prominent one which we are going to discuss would be the sheet piles.

We will be talking about the bracings and before we talk about the bracings, we will be discussing about the cuts in the soils. These are also known as excavations. We will be talking about different types of sheet piles or the retention systems. This would be anchored sheet piles and you may say non-anchored also.

So, this is the first subtopic of the plastic equilibrium in soils. Having introduced the concepts of plastic equilibrium in soils today, I will move on to the analysis of retaining walls from the next or next to next lecture onwards.

The second major subtopic or the application would be analysis of slopes and this is where I will be talking about two types of slopes, the finite slopes, and the infinite slopes. The basic objective is to analyse the slope so that the failure does not occur. So, rather than talking about the stability analysis, normally this thing is also called as stability analysis of slopes, alright? so it is always better to talk about the instability analysis.

So, instability is included. So, more focus is to study how the instability occurs. So, that as an engineer and as a designer, I can stop the instability to occur. Then we will go for different types of analysis methods, and I would like to spend enough time on slope stability analysis. It is a major topic which one should be studying during the undergraduate.

The third major application would be the bearing capacity of the soils, or you may say geo materials because later on some of you might explore the possibility of studying the rock mechanics and then soils becomes a misnomer and this can be applied to the rocks also.

However, as I said some time back, this is beyond the scope of this course. And there is a specialized course which you might be doing in the fourth year that is the foundation engineering. However, one thing is common in all these problems.

And what is common is that the objectives are common, and these objectives are, I want to find out what is the cause of failure and how this failure gets stopped, clear? So, that means, what we call them as stabilizing forces and the second is destabilizing forces. So, this is the principal objective. In all these problems or the class of problems, what we try to do is we just try to analyze the system for two types of forces.

One is the forces which are stabilizing a situation or the failure and another one is the destabilizing forces. So, most of the time it so happens that destabilizing forces are gravity and so, destabilizing forces will be the gravity number 1, it could be rain so, during the monsoons in the newspaper, you keep hearing that in the Western Ghats landslides have occurred, buildings have collapsed, dams are failing whatever during rains.

So, one of the destabilizing forces is rains, it could be snow also. We are lucky that we are in the temperate climate, and we are we do not normally come across snow forces much except for the northern reaches and the northeastern part of the country. Otherwise, this is a major issue in most of the entire world and of course, the earthquake. Manmade so this could be what we call it as a human intervention.

So, one of the good examples would be earthquake comes systems fails. Nice because they give a little force, human intervention as you said, vehicular movement yes, on the hilly terrains, you are chopping up the hills and then you are moving the vehicles vibrations get induced. And because of that, destabilizing forces generate.

Now, you must be realizing that the stabilizing forces are quite in isolation, they do not have a big company. So, truly speaking, the stabilizing forces are what, any guess, what could be the stabilizing force, which would resist the failure? Very good, who has answered this, excellent, nice. So, you have understood the materials very well. So, this is the shear strength. So, it was worth spending 14, 13 lectures on understanding the material why? because this is the lone fighter, understand.

So, the entire stabilizing forces in the system get mobilized because of its shear strength and we have done a lot of justice with shear strength, we have tried to understand what is shear strength and how to determine it, how to interpret it and all those things. So, now, we are well equipped to move ahead and solve all these real-life complicated problems. Alright? or what else could be the stabilizing forces?

One is shear strength; shear strength could be very weak. So, remember you go to the offshore environment or the mud flats where most of the construction is happening in Bombay city right now, you remember my statement that most of the best possible land has already been utilized by us or our grandfathers.

Now, what is left for you guys and up generations to come is all marshy land, which cannot be rehabilitated so easily. That means it is understood that the shear strength is almost either negligible or extremely poor. And hence, they fall in the category of challenging problematic type of soil deposits. So truly speaking, the fun is to deal with a situation where I can create stabilizing forces in the system.

Despite the fact that shear strength is almost tending to zero. A beautiful example of this is marine deposits where you cannot even stand, forget about walking. Because if you try to stand over there you sink. So, this is the objective, and this is what the art of practice of geomechanics is. What I have used the word as art of practicing the geomechanics clear. In the worst possible situation also normally, the criminal lawyers do not raise their hands. And they accept the client fully knowing the fact that this guy is a criminal. And he is going to be hanged. So, this is a very similar situation which we deal with clear.

So, shear strength almost tending to 0, marine deposits, but look at this the economy of the country and the world depends upon whatever structures you have created in the offshore environment, not onshore that is where the money is. Fine? So, these are general concepts, which I have talked about the applications in so many areas and objectives. So, for that matter, one of the objectives of stabilizing forces enhancement could be let us say ground stabilization or ground modification or soil improvement.

Starting from shear strength 0, they will bring it up to a value where C_u becomes greater than let us say finite and this finite value is obtained because of ground modification, because of soil improvement. So, if this algorithm is clear to you, now, what we will do is, we will start describing the plastic equilibrium in soils.

So, until now, whatever you have studied was all elastic state of the material. Though I have introduced this concept of K_a and K_p and in the previous lecture, if you remember, we have tried to relate σ_h and σ_v with K parameter, clear? So, what we define as is if I define,

$$K = \frac{\sigma_h}{\sigma_v}$$

I hope you understand the connotation. This is the ground level I have taken a point somewhere here at a depth of z, this is point P. The state of stress at this point is σ_v and σ_z. What I have done is I have defined this also, σ_1 equal to σ_3 $\tan^2(45+\phi/2)$ if you remember plus 2C $\tan(45+\phi/2)$.

$$\sigma_1 = \sigma_3 \tan^2\left(45 + \frac{\phi}{2}\right) + 2c\tan\left(45 + \frac{\phi}{2}\right)$$

This we derived some time back using the Mohr circle. We drew the Mohr circle and then we took σ_1, σ_3 and then we did some geometry and from there we describe this, clear? and this is where I introduced the concept of role reversal of σ_1 and σ_3. So, in today's lecture, I am going to talk about this much more in detail. So, please be focused and try to understand this concept once for all. Once you have understood this concept, nothing can beat you.

Now, this parameter is known as N_ϕ and this is also equal to K_p which is also equal to 1 plus sinϕ over 1 minus sinϕ,

$$K_p = \frac{1 - \sin\phi}{1 - \sin\phi} = \tan^2\left(45 + \frac{\phi}{2}\right)$$

The way I am using this ϕ terms over here is this is generic term, depending upon the situation this could become ϕ' this could become ϕ, C_u, this could become $\phi_{undrained}$, $\phi_{drained}$ and all clear, just implicit. The reverse of K_p is known as K_a and this is defined as $\tan^2(45\text{-}\phi/2)$ and this can also be written as 1 minus sinϕ over 1 plus sinϕ.

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi} = \tan^2\left(45 - \frac{\phi}{2}\right)$$

So, truly speaking, this equation itself is defining the state of plastic equilibrium in the soil. So, only thing what I have do is if I write an equation let us say K is K_p is K_a, is this understood. So, this term K_p is defined as the passive earth pressure coefficient and K_a is defined as active earth pressure coefficient. Sometimes they also call it as coefficient of active earth pressure coefficient of passive earth pressure.

I hope you can realize this K can be termed as K_0 this becomes coefficient of earth pressure at rest, what is at rest this is the elastic state. So, if this is the elastic state and we are saying at rest now, if I use this relationship, the lateral strain equal to 1 upon E σ_h minus μ σ_h plus σ_v

$$Lateral\ strain = \frac{1}{E}\left(\sigma_h - \mu(\sigma_h + \sigma_v)\right)$$

I hope you have understood how this equation has been evolved in a two-dimensional space. So, this corresponding to ε_h, radial strengths. Now, when this tends to zero at rest condition, no movement, natural deposition of the sediments, rivers are bringing sediments they are getting deposited in the offshore environment just under Stoke's law conditions.

No transmission of energy from the next load to the previous loads of the sediments, clear? elastic condition, no shaking, no moment nothing, elastic situation, what it indicates is during the elastic state, there is no strain in the horizontal direction, confinement at that time what happens σ_h will be equal to μ times σ_h plus σ_v that means, I can write this as σ_h 1 minus μ equal to μ into σ_v.

$$\sigma_h = \mu(\sigma_h + \sigma_v)$$

$$\sigma_h(1 - \mu) = \mu\sigma_v$$

In other words, I can write the σ_h equal to μ over one minus μ into σ_v.

$$\sigma_h = \frac{\mu}{1-\mu}\sigma_v$$

So, this thing I have derived and what it gives me is I am trying to now relate the state of stress existing in the material at its elastic condition, elastic equilibrium, Poisson's ratio is involved over here. And then what I can do σ_h upon σ_v I can write this as equal to K.

$$K = \frac{\sigma_h}{\sigma_v}$$

So, very intelligently what we have done we have derived a relationship with the K parameter using the Poisson's ratio, this relationship is normally utilized to obtain the K parameters. There was a further simplification which came from a person known as Jacky and this simplification states that K_0 equal to 1 minus sinϕ.

$$K_0 = 1 - \sin\phi$$

So, all these terms and the equation which we are using right now, though they are derived from the elastic equilibrium they are all valid for the plastic state of the material. Now, let me define what is the plastic state of the material, is this part okay? And of course, if you remember in the previous lecture, we discussed this K_0 is a function of you know what R_D, OCR, type of soil, water content and so on, is this okay?

Now, the question is how to depict this state, remember, we derived this equation A by this construction so, this is the τ-σ, σ plane, the Mohr-Coulomb envelope at this point you have σ_v and σ_z, depth is constant that means σ_v equal to γ into z hydrostatic condition. On this plane if I draw a Mohr circle, we have already analysed the situation the pole is at this point, this is equal to σ_1 which is equal to γ into z equal to σ_v, perfectly all right.

This point is σ_3 okay and then what we did is, so this is the Mohr-Coulomb envelope, fine? The material remain same only the state of stress is going to change. At this point if I draw a perpendicular this becomes the centre what is the inclination of the failure plain, enclose, very good, those of you who have not understood please understand it, go back to your hostels and try to follow all these things to avoid disasters.

So, this is the failure plane at this point, the failure is occurring what is this angle? very good 90+ϕ, excellent here what is this angle? a plane passing through the pole cutting the Mohr circle is the failure plane at which the state of stress corresponding to the failure is occurring perfectly all right. So, this is a failure plane now what it indicates is as long as your σ_v is greater than σ_h this is going to be an active earth pressure condition.

Why because K is going to be less than 1, look at this, fine? So, that means in this case your K_a is going to prevail. So, what we have done is we have proved the state of the stress acting in the soil mass in the plastic equilibrium though I have not defined yet what is plastic equilibrium I will define it and what I have said is this is equal to σ_h upon σ_v. σ_v by virtue of the point located at a depth remains constant that means, σ_h equal to K_a into σ_v.

$$\sigma_h = K_a \sigma_v$$

Now, this discussion we had some time back about the role reversal of σ_1, σ_3 clear or let us say σ_h and σ_v. Suppose, if I am interested in knowing what is going to happen when this condition gets violated, and K_a tends to become K_p starting from K_0 value. Yesterday we discussed this so, if you realize what I did is this was the K_f line, K_f line and somewhere here if I have K_0 line.

If this is the state of the equilibrium which I want to achieve from K_0 I can go to K_a or I can go to K_p, stress paths. Now, keeping this σ_1 constant, because this is γz if K happens to be more than K_0, what is going to happen? your σ_h term is going to be more than σ_v where how are you going to plot this now, this is what is going to get plotted like, good I could manage.

So, this is the circle, which is bigger in size number 1, number 2, I have maintained the σ_v value wherever this cuts the x axis this becomes my σ_h though σ_h again become σ_1 now, fine. So, the way I read this is this is σ_h this is σ_v, σ_h is equal to σ_v into K term which is multiplied by K_p. Now, the question is where is the failure plane? And what is the inclination of the failure plane? Determine it, find out the pole first.

So, what is going to happen is the more and more complications in the problems come henceforth, we have to identify the poles that is it. And that is what I told you at that time. Once you know where the poles are, life is simple, you just sit down in your design office know the material properties you have obtained c, ϕ and other things get the Mohr Coulomb envelope plot it over here obtain where the failures are going to take place.

Go back to the basics where σ_h is acting, on horizontal plane or vertical plane, σ horizontal is acting on vertical plane, sigma, this horizontal was acting on vertical plane the failure took place over here the failure is going to be over here. Now, can I prove that the pole is going to be this point, prove this. So, this is your P_{active} and this is going to be $P_{passive}$ this I did it when I was teaching the derivation of poles.

Now what you are realizing is where is the failure plane? any point any plane passing through the pole intersecting the Mohr circle. Is this correct? so this becomes your failure plane under passive earth pressure condition, what is the inclination of the failure plane? Do not mug it up, I think it is easy to understand is it not? So again, draw a tangent from here let it cut over here clear. So, this angle comes out to be 45-ϕ/2, very good.

We will realize that there is some contradiction which we have been talking about. Anyway, as far undergraduate things are concerned you go ahead with this. So, what we have done something 45+ϕ/2, something 45-ϕ/2 what is this? Plane passing through the pole, inclination of the failures is going to be different under two conditions active and passive condition that is what is known as state of plastic equilibrium in the soils. And these two planes are going to be conjugate to each other.

That means the plane 1 1 and the plane 2 2 are going to be conjugate to each other with the intersection between both of them as what 90°. So, if this plane is inclined at an angle of 45-ϕ/2, and if this plane is inclined at an angle of 45+ϕ/2, I think geometrically things are compatible, what happens is when the state of plastic equilibrium develops in the soil mass, the failure planes get developed at either 45-ϕ/2 with respect to horizontal where the principal stresses acting, or 45-ϕ/2 from the plane where the principal stress is acting. This is what is known as the state of plastic equilibrium in the soil.

Yes, oh, do not include the plasticity part, it is a plastic equilibrium. Yes, just hold on, five minutes. I am coming to that. So, first of all, what I have done, I have defined the state of stress, okay good. I will answer your question, truly speaking there is no plasticity coming in picture, but I will explain it to you.

Sometimes in books, you will find that this is also written as flow factor, somebody may ask you during your interviews the material has a tendency to flow, that means what we do is very conveniently we put c=0, a pure frictional material c is tending to 0 and the material is ready to flow, frictional materials flow. Go to any silos where the grains are being stored. So, sugar, rice, wheat, urea, particularly fertilizers, they are all dropped from a hopper.

So, normally what they do is they convey and everything to a hopper belt, which is a conveyor system and then they drop it from the top and this is how the heaps get formed, you know the value of ϕ the repose angle, and if you see the silos, they are normally designed in this manner, keeping in view the friction angle of the material, this is where the application is and this is how the term comes the flow factor, how much metal can flow in a dry state.

Now, I will extend this concept further and this will answer your question. So, by definition, the state of plastic equilibrium in the soils is when each and every point in the soil mass is at the verge of failure following this state of stress, due to gravity only understand, none of these forces are going to come over here these are always super imposed.

So, gravity only plays the important role. Remember, what we have done is the entire derivation is based on the gravitational stresses. The stresses which are getting induced because of the gravity, rest of the forces might play spoilsport, or they may help you in stabilizing the system that is the engineering. But the basics are this so, that means, state of stress in the in the material remains either in active state or passive state.

So, next time onward when you go to these monuments, just come out of the monument, and see look at the walls, the way they were constructed. And I am sure if you are matching the line of wall along with yourself, you will observe that the walls have moved out or they have moved in, clear, or those of you who love hiking and trekking, you must have realized that when you go on the top of the hills, you will find depressions there at the top of the hill and at the base there will be a sort of a bulging.

So, all these things are because of the state of stress which is acting in the system because the gravity only and we are trying to decode all these types of mechanism which prevail in the materials which are made out of, which are in the structures which are made up of soils. So, what we have done until now is we have just defined the state of plastic equilibrium starting from this state of stress.

Let us say if this is the soil mass, ground level, remember when we were talking about the granular material long, long back, we considered two rigid boundaries in the soil mass hypothetical. So, I am assuming that these are the two hypothetical planes, which are existing in the soil mass, which is semi-infinite on both sides. What we studied during the compressibility of the material was if it is a granular material, and if the walls happen to be rigid, and if I compact it, what is going to happen, no deflection in the sides only compression is going to take place.

But if the walls are flexible, and if I apply the load, the chances of the material will flow out. I had done this action also. If you apply the loading over here, then the material will flow out like this, provided the walls are flexible. So now I am assuming the first case we analysed compressibility and consolidation in the Geotechnical Engineering-I by keeping a rigid ball assuming that ε_h is equal to 0 fine and not allowing any lateral strains developing in the system.

Two conditions exist, stop writing. So, one is AA another one is BB, look at the motion of the planes. Now suppose, as long as σ_v is constant, there are two possibilities, σ_h will either decrease or it will increase. If σ_h increases, what is going to happen? These two systems will get, they will come closer to each other when σ_h increases, that means what we call this motion as if $\Delta\sigma_h>0$, these two will have a tendency to move closer to each other. This is what is known as passive earth pressure.

So, somebody was asking in the last lecture, how this happens, gravity does this, at global scale geology does this, tectonic motions, they do this. Human beings cannot do it. So, this state of stress exists in the system because of the geology, because of the gravity and tectonic motions. Now, this is what is known as a global plastic equilibrium in the soil. And at this time, what is going to happen your σ_h will be equal to K_p multiplied by σ_v.

Hypothetical plane in the soil mass, boundaries are flexible, the state of stress develops in such a manner that the two planes come closer to each other, what we call this as is the movement of the hypothetical boundary within the soil mass. This is the soil mass, and the boundary is moving within the soil mass. Opposite might happen. So, it so happens that σ_h might become negative, we had talked it about all these situations.

Stretching of the material so, when you are stretching this, what is going to happen? Now, this is what is known as $\Delta\sigma_h$ might be negative or less, let us not put less than 0, sorry, is another state of stress let us say. I mean, it is increasing let us say and this is decreasing, this is better way. Is this correct? Or negative positive I cannot use.

So, in this case, what is going to happen is σ_h will be equal to K_a into σ_v. And this model is valid over here. So, this is the plane 1-1 and this is the plan 2-2. And this is how the state of stress is developing over here. That means, if I draw these lines in the entire soil mass, they will correspond to the failure 1-1.

How would I interpret? All these are the slip surfaces along which the material will have a tendency to slide up. One of the examples would be, if I consider let us say this plane, slip plane, the moment this condition occurs, this material has a tendency this becomes the parent soil mass and this becomes the block or the soil mass which is under passive earth pressure, has a tendency to slide up and you have done mechanics to solve this.

The only thing is that this angle is going to be 45-ϕ/2. The moment I changed the angle what is going to happen, when I was stretching this block from the parent body will have a tendency to get detached and slide down, angle is going to change because of what we have discussed over there. And that becomes your active earth pressure.

When this type of state of stress develops in the soil mass, this is what is known as a state of plastic equilibrium in the soil, it is a misnomer. So, plasticity does not come in the picture at all.

The failure under active condition is going to occur when the slip surfaces are inclined at an angle of 45+ϕ/2 with respect to horizontal, clear, and the failure under passive earth pressure condition is going to occur when the slip surface is inclined at an angle of 45-ϕ/2 with respect to horizontal and what is horizontal, σ_h plane here, here this was σ_v which was cutting over here, σ_v remains constant, the first condition was two hypothetical surfaces are coming close to each other because of formation of this type of stress.

The slip surface which I have defined as this point along with the movement is going to take place as if this block gets detached from the parent body because of gravity only just because of gravity only, clear. So, because of gravity the state of stress acts in the system which moves it up. That means the system is coming close to each other. And why, gravity, gravity plays the trick. Look at these are the tectonic motions. That is what we had been discussing.

This is a situation number one, the second situation is if you stretch it out relaxation of the σ_h values, this block has a tendency to move down, the slip surface will change in compatibility with that this will become $45+\phi/2$. This is what is defined as the state of plastic equilibrium in soils.