Geotechnical Engineering - II

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Lecture No. 24

Stress Paths II

Now the question is plot it on this p-q plane and see what happens. So, depending upon whether I am doing drained analysis or undrained analysis, where is the K_f line if I am working with let us say NC material or sands, what is going to happen? The friction angle is, this is going to be the K-line, is this part, okay? Line just below this is going to be K_a line and this K_a corresponds to active earth pressure.

Now, somewhere here, now, tell me one thing where would be K=1 line. If this term becomes K=1 and β becomes zero what happens to the β-line, β value zero, correct. So, what is the meaning of this, this becomes K=1 line and what is K=1, isotropic compression, fine?

The question is where would be K>1? So, this is the K_f line the same material can fail under active state of stress what you are realizing is under K=1, the material is never going to fail because of your compression. Somewhere here this will be K>1 and this is what is known as passive earth pressure line. So, you have transformed the state of stress from first quadrant to the fourth quadrant.

Now, we will analyse these situations, clear? Is this part clear? Each one of them is a failure line. Except for this. Because this happens to be the isotropic compression where the failure is not going to take place, is this concept clear? This failure line corresponds to the situation that this is the material, clear? this is a situation where you have σ_V is greater than σ_H, this is a situation where you have $\sigma_H > \sigma_V$.

Now, when I am dealing with earth pressure theories, it is always beneficial to assume that K_f condition is corresponding to K_0 line, and this is the elastic equilibrium. We call this as earth pressure at rest. So, in nature what is happening? The rivers are bringing these loads of sediments, they are getting deposited in the ocean bed, natural synergy between the particles.

So, that means, this is the ocean bed particle, first layer of the particle gets deposited, second layer and there is no impact of next coming mass of the particles on the previous layers because this is all sedimentation going on.

So, you have studied critical velocity, Stokes law, no energy is getting imparted from the next batch of the particles onto the previous deposited particles, K_0 condition, elastic condition, at rest condition. System is at rest. Now, if you come and shake it, earthquake, the at rest condition gets disturbed.

And then the chances are that there will be a transformation from K_0 line to Ka line starting from here I can achieve the failure like this I can achieve the failure like this also and that is all governed by the combination of σ_H and σ_V. So, with this preface and with this information in mind let us do some mathematics.

So, suppose if I ask you plot hydrostatic condition. Where is the hydrostatic condition now? All of you know that hydrostatic condition is going to be. At this point. So, this is the point A which corresponds to σ_H equal to σ_V, is this okay? From this point if I start shearing it the sample there are several ways. Where is my failure line, this is the K_f line, and the mirror image of this line would be somewhere here. Because the Mohr circle is a circle, clear, what we did is for the sake of convenience we took only half of it.

So, we took only the positive portion of the Mohr-Coulomb envelope. Truly speaking there would be a mirror image both sides you have an envelope, fine? Suppose if I put a condition that stresses are changing in such a manner that $\Delta\sigma_V=\Delta\sigma_H$, try to understand what is the significance of this?

Starting from the hydrostatic condition if I start increasing σ_H and σ_V in such a manner that both σ_H and σ_V are same all the time, the increment has to be depicted in the form of a stress path which is going to show whether the sample is going to fail or not. So, can you draw it? Draw it now, I have written something over here. So, you can get the increments of q and p and plot in terms of β.

You take this box okay triaxial sample and I am keeping all σ_H σ_V increment same, isotropic compression, hope you will realize that this is how you will move, this is the stress path. Are you going to achieve the failure? This line is not going to cut failure line anywhere. So, what should I do to fail a sample? Now, suppose if I change the situation now, if I say stress path number 2 is ($-\Delta\sigma_V = -\Delta\sigma_H$) Please, for God's sake do not cancel the negative signs because negative sign indicates something it is not mathematics pulling a part, system from both the sides.

So, how do you break something, by tearing it apart? Is it not? So, suppose if I pull it from both the sides and from these two sides also the chances are system are going to fail. Why? Look at this. It is okay. And this is definitely going to cut the failure envelope at somewhere. And what is going to give me? is going to give me the tensile strength of the material. Fine?

So, I have shown you know two states of material the way you wanted to utilize them by following the two paths. Let us make slightly more complicated situations. It is no fun having these types of situations in engineering practice.

Now, suppose if I say this point A was for the case when $\sigma_V = \sigma_H$. Now suppose if I say $\sigma_H \neq \sigma_V$. What is going to happen? You are violating the hydrostatic condition. So, point A is going to jump to point B somewhere over here. I can create a situation where I would say these are the linear relationship material is nonlinear clear.

I will put a condition σ_H is a function of let us say $\Delta\sigma_v$ and this function could be non-linear. So, all these lines, which I am drawing a straight line, they will become nonlinear curves. So, what it indicates is if I put a condition $\sigma_H \neq \sigma_V$, this becomes my starting point. And then can I superimpose this condition or not? Yes, you can. So, starting from this point is going to be easier for me to fail the sample as compared to this point, how?

Different ways, this is one of the ways, this is one of the ways, this is one of the ways, this is one of the ways, this is one of the ways, this is one of the ways, this is one of the ways, so on. So, what I have done? I have created different types of stress paths, if I follow the state of stress along them, I can fail the sample.

In practice, what do we want to do, we do not want to fail the sample. So, it is a reverse process, if I understand under what type of loading combination the material is going to fail, I can be conservative while designing it. Longer the path, the longer efforts are required to fail it and the probability of the failure is going to be lesser as compared to a situation where the path is going to be shorter.

So, look at this situation, it is very easy for me to fail the sample like this. Mathematically you can obtain from here.

$$\frac{\Delta q}{\Delta p} = \tan \beta$$

So, the slope of these lines is nothing but the β angle. And then you can compute the β if the K is known number 1, if σ_1, σ_3 is known again, you can compute the beta value by substituting $\Delta\sigma_1$ and $\Delta\sigma_3$, and you can simply compute this. This is fine? Good, that is what I want to hear.

Now, the last part of the triaxial testing would be suppose if I give you the p-p' plane and q plane and this is the K_f line and this is the K_f line yes, so, I will now elaborate upon this concept, this concept and the combination of these two, right. So, starting from the hydrostatic condition, if I draw a line like this, tell me what this will correspond to? It is understood that the βs are different so that you can compute, do not bother about that.

Stress path itself indicates the slope of the combination of $\Delta\sigma_H$, $\Delta\sigma_V$. So, suppose if I plot a line like this what is the interpretation, this is what is known as compression. How will the elements look like? If initially, it was like this, when you are doing pure compression, here, what is going to happen sample is going to get deform. So, this is how it will look like.

Volume might remain constant, as long as you are doing a CU testing, undrained testing. The moment you do drained, it is not possible. I hope you understood this part, then your A_1 V_1 equal to A_2 V_2 plus delta of volume of water which comes out of the sample the equation which I wrote, is this fine?

Now this has been done under σ_V, σ_H is constant what we call this as a one-dimensional compression or loading, so, this is a typical loading curve, loading stress path sorry. What will be the reverse of this? Suppose if I project it on this side, what is going to be or the reverse of loading, unloading under what circumstances, $\Delta\sigma_H = 0. \Delta\sigma_V$. Sorry, I mean you can't put both the things?

So, you have to say that under the constant loading or whatever. So, I will assume this as $\Delta\sigma_v$ is equal to zero and $\Delta\sigma_H$ we will keep as it is. So, here $\Delta\sigma_V$ will be less than this and let $\Delta\sigma_H$ vary. Now, this becomes the unloading or excavation. Now, if K>1 what we have done is we have created two more situations. So, if I draw a line like this and the reverse of this would be, this is what is defined as active earth pressure, and this becomes the passive earth pressure.

If I look at the sample at this point if this is an initial sample because $\Delta\sigma_H$ is more than $\Delta\sigma_V$ what is going to happen? the sample is going to get squeezed in the lateral direction and it will expand in the vertical direction. Good. So, this visualization is important. Sometimes is also known as axial expansion. These concepts we are going to use from next chapter onwards or a better word is dilation if you remember.

We talked about dilation also. The state of stresses like this that the $\Delta\sigma_H$ is higher $\Delta\sigma_V$ is zero under constant vertical stress because the location of the point remains same. The horizontal stresses are much more than the vertical stresses. So, what is going to happen? the system is defying the axial confinement, and this is how the dilation process would be.

In this case $\Delta\sigma_V = 0$ and what about $\Delta\sigma_H$? Here $\Delta\sigma_H > 0$, this will be less than 0. So, these are the four mechanisms which we are going to use quite frequently in the whole geomechanics henceforth. We will get rid of now the material also; I do not want to talk about c, ϕ and all those things why because I am using q,p.

So, material is being depicted now p, q, state of stress is being depicted now, by the stress vector and I know starting from this point, how much amount of stresses are required to achieve the failure under this type of loading, how much amount of stresses are required under this type of loading and the combination of the two. This and this and of course, this also that is always valid because this is the relationship between σ_H and σ_V what you have to understand is what is the state of K.

So, in other words, if K>1 or K<1 or K=1 what really happens? Exactly, so that is what is getting balanced by this ratio. So, I need not to bother about all σ_1, σ_3 now, so, look at this, this becomes my K_p and this becomes my K_a that is it. I have further simplified everything and not worry about $\Delta\sigma_H$ and $\Delta\sigma_V$, σ_1, σ_3 nothing because K takes care of the relationship between σ_H and σ_V. The way it is getting balanced, this is what the important thing is because this shows the formation, this shows the OCR, this shows the shear rate of shearing this shows the type of material, this shows the type of drainage conditions, this shows the type of whatever.

Keeping σ_H constant, you are sitting in a fluid. Only vertical stress is being changed not the horizontal stresses. It's a hydrostatic pressure. This is all what I wanted to discuss about the triaxial testing and the shear strength theory of the soils. Henceforth what we will be doing is, we will be utilizing these concepts of shear strength theory to deal with the stability of different types of walls, what we call them as retaining walls, sheet piles, bracings, struts and so on. The basic concepts I have cleared.

Now, rest is all application of these concepts. The second part of the course which I will be discussing quite in details would be the stability of slopes. So there again we will be utilizing the shear strength parameters, the concept of shear strength theory to prove whether the slopes are stable or not.

So, in short Geotechnical Engineering II deals three major components of the discussion, one discussion was on determination of shear strength. And second is application of shear strength parameters. And when we talk about application of shear strength parameters, we will be talking about the stability of retention schemes and slopes.

Next module of this goes into the Foundation Engineering. So, foundation engineering is nothing but application of shear strength theory to deal with the stability of the foundations. So, this is how the whole scheme of discussion is, fine?