Geotechnical Engineering - II

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Lecture No. 23

Stress Paths

I have discussed quite in details, triaxial testing and interpretation of test results. I have tried to expose to you the types of problems which we normally encounter in geotechnical engineering while designing and analysing the systems which are made up of soil or which are located in the soil or which are coming on the soil mass. And that was a very comprehensive discussion regarding the testing procedures, what care should be taken to select the parameters and so on.

Now, in today's discussion, I will be covering the concept of stress path which you will find very useful as a designer or a consultant.

So, there are three types of stress paths which normally we talk about, and I have introduced this concept a bit may be when we were talking about the Mohr-Coulomb envelope and this is where I had demonstrated what is the application of stress paths and how analysis becomes simple when you are following the stress paths.

So, the three types of stress paths are the total stress path we call it as TSP. The second one is the effective stress path that is the ESP, and the third one this is what is known as the TSSP and the full form of this is total stress path minus static pore pressure. So, I will introduce the concept of these stress paths to you and then try to analyse those situations where these stress paths become very important.

One thing to remember is when we plot TSP, you remember what we did is, we transformed our results of the shear test from a τ-σ plane to a p-q plane. So, this is a transformation which we did and the added advantage was that rather than plotting the more circles which appear over here, which is a cumbersome thing to do. Particularly when you have several number of samples which you have tested, what we did is very conveniently we picked up the (p,q) points where q was defined as σ_1 minus σ_3 by 2 and p was defined as σ_1 plus σ_3 by 2 and σ_1' plus σ_3' by 2 and of course, the pore water pressure.

$$q = \frac{\sigma_1 - \sigma_3}{2}\ ;\ p = \frac{\sigma_1 + \sigma_3}{2}$$

$$q' = \frac{\sigma_1{'} - \sigma_3{'}}{2} ;\ p' = \frac{\sigma_1 + \sigma_2}{2}$$

So, this will be minus u component, this will be p and p' value. So, what we have written is that p' is equal to p minus u.

$$p' = p - u$$

So, when you transform the τ-σ plane to p-q plane we get points and each point corresponds to a circle, Mohr circle and then what we did is we plotted this line and we term this as a K line, sometimes it is also known as K_f line, the failure line, which is equivalent to the Mohr-Coulomb envelope.

On this plane, we have this Mohr-Coulomb envelope, failure envelope where we have defined c and ϕ in general, I hope you understand why I am using the word general and here we have transformed this to q and p, if you remember we have done something like *a* and β. Now, what I will do further is I will get rid of this type of plotting of the results and I will deal with only p-q plane now.

Now on p-q plane, the K_f line has some slope and this slope is β. So, one of the ways of defining β would be if I say Δq by Δp and this is nothing but the function of tanβ.

$$\tan\beta = \frac{\Delta q}{\Delta p}$$

Because of your slope of the line in general form and this can be written as,

$$\tan\beta = \frac{\frac{\Delta\sigma_1 - \Delta\sigma_3}{2}}{\frac{\Delta\sigma_1 + \Delta\sigma_3}{2}}$$

$\Delta\sigma_1$ minus $\Delta\sigma_3$ divided by of course 2 and this will be $\Delta\sigma_1$ plus $\Delta\sigma_3$ by 2. If initial values are zero, I can write this as σ_1 minus σ_3 divided by σ_1 plus σ_3.

$$\tan\beta = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}$$

So, that means, that tanβ has been related with the state of stress existing in the sample if I introduce this concept of K parameter, which is the earth pressure coefficient I can write,

$$K = \frac{\sigma_3}{\sigma_1}$$

tanβ equal to 1 minus K over 1 plus K and this will also give me K equal to 1 minus tanβ over 1 plus tanβ.

$$\tan\beta = \frac{1-K}{1+K}$$

$$K = \frac{1-\tan\beta}{1+\tan\beta}$$

A little bit of geometry which we are discussing over here.

So, it depends either way you can work if you know the value of K, I know the value of β, I can use the transform function and if you remember we derived this was, sinϕ equal to tanβ and a equal to c cosϕ I think this is what sineϕ. So, we have all these types of mathematical transformations associated with this.

$$\sin\phi = \tan\beta$$

$$a = c.\cos\phi$$

Now the total stress path is defined as particularly a $\overline{CU}$ test if you conduct, this is q plane, sorry q-p plane. The characteristics of total stress path is that always incline at 45° and we will prove it subsequently alright. So, the basic characteristic of PSP is that it is always inclined at 45° clear. So, suppose if this is the K_f line, the failure line, this is the starting point of the test, hydrostatic condition σ_1 equal to σ_3, shear stress is zero. What I have written is TSSP and this is equal to total stress path minus static pore pressure. Static pore pressure is normally defined as u_0.

Now when you are doing a consolidation test, $\overline{CU}$ test during consolidation u_0 becomes zero no issues. There could be a situation where you are applying back pressure alright. So, during application of back pressure, the value of u_0 will be equal to the back pressure which you are applying, there could be a situation I am taking the sample from the ground and there is a water table.

So, because of standing water table, there will be a pore water pressure which we have computed. So, that will be the value of u_0, the static pore pressure. So, in other words, this TSP would get shifted to the TSSP and the difference between these two would be u_0 value. Now, if I start shearing the sample under undrained any situation, this is how the ESP would be nonlinear curve. Because soils are non-linear in nature, this becomes ESP effective stress path what it indicates is from this point onwards, if I start shearing the sample, one of the ways to fail the sample under effective stress condition would be like this.

The difference between these two points is known as pore pressure. I will write u only, u at failure, now this is a typical response for a NC material pore pressure is always positive, clear. So because u_f is positive, this corresponds to normally consolidated soil. On the other hand, if I am dealing with let us say OC material, what would happen is the pore water pressures are going to be negative if this is the failure line, this will be the total stress path. This would be the TSSP and then when you plot the effective stress path you remember the relationship which we plotted for u as a function of percentage strain in triaxial testing in $\overline{CU}$ testing and what else we did is we plotted A versus axial strain and this is how the graphs work, NC material, OC material and this was also for NC material and OC material.

So, what is going to happen is when you plot ESP for this is going to be something like this because of the negative pore water pressures. So, wherever this cuts the K_f line this pore water pressure is going to be u and depending upon the stress level this is either going to be positive or it is going to be negative. So, sometimes you will find u_f coming out to be negative as well. Now, all this is done normally, when we do monitoring of the pore water pressures.

Now, what we are going to do in further discussion is we are going to use these concepts to see how the stability of the system gets governed.

So, if I start from this equation, the Skempton's equation of pore water pressures where we defined u equal to $B\Delta\sigma_3$ plus A $\Delta\sigma_1$ minus $\Delta\sigma_3$.

$$u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

When you are doing triaxial test what is going to happen? $\Delta\sigma_3$ is 0 clear? Normally, we do not change the σ_3 value. So, what it indicates is that u will be equal to the this term will be 0 and this will be equal to Δu upon $\Delta\sigma_1$ and remember this term we have defined as u_2 during shearing.

So, one of the applications of stress paths is when you do triaxial testing, you can draw the variation of A as well or you can obtain A value from $\overline{CU}$ test.

For an example, suppose if I do a little bit of geometrical construction over here now, this value is going to be equal to let us say x, y, and z.

So, Δu_2 term will be equal to u parameter or I can write as x y z and what about $\Delta\sigma$ term, $\Delta\sigma_1$ is on q plane which truly is half of the value so, this is going to be 2 times the value of x y, correct? What it indicates is, if I plot the variation of A line this is how A equal to 1 upon 2 will look like this is the stress path for the triaxial test where the A is varying in this way on the right hand side of this you will be having u as negative, on the left hand side you will be having 1 and this would be greater than 1.

So, this is one of the applications of stress paths in obtaining A parameter and I hope you understand what the importance of A parameter is. These are new concepts they will take some time but anyway I mean you can just follow the notes and follow some book and then I think things will become quite simple.

This was the application 1 and of course, now from this point onwards let us move on to another concept.

I hope you can realize that if I am doing isotropic compression and if I ask you what the TSP and ESP will look like interesting question to ask. So, under isotropic condition, what is going to happen there is no shearing.
That means, one of the situations would be that your stress paths would get translated like this. So, this will become the TSP and if I know the pore water pressure this is the value of u, this will become ESP. So, this is a peculiar situation of isotropic compression.

We have been talking about construction activities. And we have been computing the types of stress or the state of stress exists in the soil mass this we are doing since long.

Suppose if I give you a situation at this point P which is at a depth of z in the soil mass. Ground surface or sometimes they call it GL, we have σ_V and σ_H acting fine, all of you know σ_V we have computed as γ into z and σ_H we have computed as K times σ_V which is equal to K times γ into z.

$$\sigma_V = \gamma . z$$

$$\sigma_H = K\gamma . z$$

Now, suppose if I start constructing something on the top of this what will happen σ_V is going to change, correct? So, that means, if this is the activity of the construction you know I will say loading so, what loading results. First of all, loading results in enhanced σ_V value clear. So, this indicates that $\Delta\sigma_V$ is going to be positive, is this part okay? keeping σ_H constant, perfectly alright?

That means $\Delta\sigma_H$ is equal to zero, is this part clear? When you are loading the soil mass from the top confining stress remains same, what is going to change? Only the vertical stresses I would like to depict this whole process of loading and unloading in the stress path so that I may not compute all the time I just want to know if I start loading it what is the limit of loading on this soil deposit before the system fails under shear, clear?

So, two things are inbuilt. I have set system fails that means, the failure envelope is known, you are dealing with a soil mass that means, properties are known, γ, z, void ratio, R_d, c, ϕ, c', ϕ', C_u, C_{CU} everything is known. So, material is known, now, I want to utilize it the way I wanted to utilize it. So, the question is some politician will say why not a train 100 bogies should run on the railway track, so somebody has to resist no?
As such it is not possible you can only have a train of 56 bogies or 75 bogies why? There must be a reason, clear?

So, loading on the tracks, traction on the tracks, all these things are interrelated. Somebody will say he wants to make a 200 storey building in Khambala Hills what is going to happen here fine. Yes, it can be done either I will change the ground conditions, ground modification and I will satisfy you if you are not spending money there, I can't help you. And then you have to live with whatever is possible.

So, one of the situations which I have created is if I load this soil mass $\Delta\sigma$ is always positive $\Delta\sigma_H$ remains zero.

Reverse process of this, suppose if I say excavate it. So, if you are excavating this is a situation, so this becomes an excavation, let me put it up to this point only because our point of interest is only up to this point. So, again, you have excavated z, what is the state of stress at the point P? what has happened to $\Delta\sigma_V$, excavation, unloading, clear.

So, this becomes a case of unloading. Is this part, okay? Removal of material causes stresses to decrease, external loading causes vertical stresses to increase, keeping σ_H constant.