Geotechnical Engineering - II

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Lecture 10

Selection of Parameter (Shear Strength, SS) I

In previous lectures I have talked about state of stress and a bit of the Mohr-Coulomb envelope. I have discussed quite in detail the direct shear box test. We talked about the limitations of these tests and the strength of these tests and I was trying to derive a relationship between σ_1, σ_3 by using the concept of Mohr-Coulomb envelope which also I defined as a failure envelope.

So, to pick up from that point onwards if I plot τ versus σ and if this is the Mohr circle where I have defined this as σ_1 and this as σ_3. In the generalized form, so if this is the Mohr circle and if you draw that failure envelope, so this is what we discussed in the previous lecture. This angle we have defined as internal friction angle, this thing we have defined as c, this is the perpendicular from the tangent, this angle is 2θ, this point we have defined as (τ_f, σ_f), f corresponds to the failure, and this is the Mohr-Coulomb envelope.

Now, this line is also defined as K_f line. Now, the state of stress which we have been talking about is where,

$$\sigma_v = \gamma z$$

You remember we have taken out an element which is buried at a depth of z and this is the state of stress σ_v and σ_h and this is the depth z. And corresponding to this, we have defined this as,

$$\sigma_h = K.\sigma_v$$

So, if you remember the expression which I derived σ_1 as a function of σ_3, ϕ and c, where we have defined this term,

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi} = \tan^2\left(45 - \frac{\phi}{2}\right)$$

So, if I transpose it this becomes,

$$K_p = \frac{1 + \sin\phi}{1 - \sin\phi} = \tan^2\left(45 + \frac{\phi}{2}\right)$$

Now, what you are realizing is that this is the angle α. So, if I say that,

$$\alpha = \theta = 45 + \frac{\phi}{2}$$

So, let me replace alpha with the theta term. So, basically this function can be written as now,

$$\sigma_1 = \left(\sigma_3 . \tan^2 45 + \frac{\phi}{2}\right) + 2c . \tan\left(45 + \frac{\phi}{2}\right)$$

Now, there are two ways of interpreting the whole thing. When you are doing direct shear, box test I asked you to plot the initial state of stress. What we do is we apply a normal stress first and when you are applying normal stress there is no shear stress so we are dealing only in the x axis. So, if this value happens to be,

$$\sigma_1 = \sigma_v = \gamma . z$$

$$\sigma_3 = \sigma_h = K_0 . \gamma . z$$

So, in direct shear test, what we have done is this is σ_1 we are maintaining the normal stress and we are shearing the sample.

That means keeping this σ_v constant if I start shearing the sample what is going to happen, somewhere at this point the failure is going to occur. Now, this point has to lie on the K_f line, that means if this is the failure line if I say this is the K_f line keeping σ_1 constant which is equal to σ_v if I shear the sample, so this is the pre-shearing stage and this is the shearing stage, so the moment you sheare the sample over here, it fails over here and this becomes the K_f line, rest of the things remain same.

Now, what I am going to introduce today is the concept of switching over from a two-dimensional to three-dimensional situation, that is from plane strain to the three-dimensional situation. Where it will be very difficult for you to change σ_1 and hence what is done normally is we change σ_3.

So, the moment you change σ_3 this becomes a triaxial test. σ_3 is also termed as the confining stress, sometimes we also write this as σ_c, this is also known as cell pressure meaning thereby in the direct shear box test we could not control the confining stresses, from where the confinement is coming in the form of σ_h because what we have done is we have assumed this to be a state of stress as a state of stress at rest.

What I have discussed now can be depicted in this form. If I start by keeping σ_3 constant what I will have to do? I will have to keep on changing σ_1 to achieve the failure, look at this. Is this correct? The second stage of stress would be something like this, σ_3 is constant.

So, what I am doing is I am confining the sample to the same stresses and I am shearing it. So, truly speaking from one to two to three to four is nothing but depiction of how to achieve failure of the sample by shearing it. Is this part clear? So, the moment this circle touches the K_f line the failure is going to occur. All these points which are at the peak they become the part of the or the components of the stress path, we use the word stress path.

That means if I start shearing a sample from the state of hydrostatics, when the shear stress is 0 that means this is the hydrostatic condition, at this point your σ_1 is equal to or let me put it as

$$\sigma_v = \sigma_h$$

and hence the shear stress is 0. So, this is the typical hydrostatic condition, starting from hydrostatic condition if I shear the sample the shear component is coming up and building up like this.

So, what is going to happen? This is how the failure is being achieved and somewhere here it goes and hits and the failure is achieved. So, what I have depicted is the stress path. Now, I am sure you must be realizing that it is very difficult to draw the Mohr circles all the time, it becomes very complicated.

So, what we do is we transform from τ-σ plane to something which is known as a p-q plane,

$$p = \frac{\sigma_1 + \sigma_3}{2}, \text{ for 2-dimensional case and,}$$

$$p = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}, \text{ for 3-dimensional case}$$

Sometimes we call this also as the average stress.

And if I put a condition that,

$$\sigma_2 = \sigma_3$$

A typical triaxial condition, so what happens? In that case this will become,

$$p = \frac{\sigma_1 + 2\sigma_3}{3}$$

What happens to the q axis? The way q is defined is this is the deviation of σ_1 with respect to σ_3, so we call this as deviator stress. Have you understood these two things?

So, this is the deviation from the σ_3 of σ_1 which is causing failure of the sample, so this is also written as σ_d.

$$q = \sigma_\mathrm{d} = \frac{\sigma_1 - \sigma_3}{2}$$

Now, what is the beauty of projecting the Mohr circles on a p-q plane? If I point out a point over here like this each point is corresponding to what? a Mohr circle.

So, what I have done? From a complicated graphical situation, I have now converted the entire thing into a simple point wise depiction of each sample and if I connect this line, this line becomes the K_f line because at this point the failure is taking place for each sample. Remember here starting from the hydrostatic condition we have sheared the sample to achieve this point, so this point is going to be unique, each of this point is going to represent one sample.

So, I have done identical four, five samples testing and then I got these results. Now, if I define this as let us say α and this intercept on the y axis as a. Can you prove that,

$$\tan\alpha = \sin\phi$$

Simple geometry and,

$$\mathrm{a} = \mathrm{c}.\cos\phi$$

Now, this is a transformation which we have done. Now, one hint for you would be I can write this as,

$$\frac{\sigma_1 - \sigma_3}{2} = \left(\frac{\sigma_1 + \sigma_3}{2}\right).\tan\alpha + \mathrm{a}$$

This is first equation.

$$y = mx + c$$

The second equation comes when I have this relationship over here. So, I can also say that the way we have derived this function, so we defined this as

$$\left(\frac{\sigma_1 - \sigma_3}{2}\right) = \left[c.\cot\phi + \frac{\sigma_1 + \sigma_3}{2}\right]\sin\phi$$

Please check my expression it might be incorrect also. Hope it is alright. If you solve this expression what you realize is we are basically transforming a τ-σ relationship to a p-q plane.

Now, let me introduce the concept of the effective stresses. Sometime back we said that in direct shear box test the pore water pressures cannot be measured or even if you measure it is very difficult to introduce the concepts, or introduce the equipment. Now, suppose if I say I am interested in finding out the effective stresses, this is the first time I am using this term in this class.

This happens to be a sort of a dry situation number one, pore water pressure does not come in the picture or this could be a drained condition, where I have allowed all the pore water pressures to dissipate. But suppose if I ask a condition this material happens to be not so permeable which is not valid for direct shear test case, the boundary conditions are so that there is no dissipation of pore or pressures taking place and pore or pressures develop. So, that means if I say,

$$\sigma_1' = \sigma_1 - u$$

Remember effective stress concept?

$$\sigma_3' = \sigma_3 - u$$

$$i.e., q = q'$$

What happens to p? When we say p',

$$p' = p - u$$

So, today onwards now what we will do is, we will talk about the effective σ' also and when you are dealing with effective σ', the best way is to transform everything on a p, p' plane and q happens to be q'. So, this becomes simple. So, I have introduced three things today in the class.

One is the concept of K_a and K_p, this is what is known as coefficient of earth pressure under active conditions or active earth pressure and this is what is known as coefficient of earth pressure under passive conditions or passive earth pressure. Later on, we will discuss that K_0 will sit somewhere in between.

So, starting from rest I can achieve failure both under active and passive failure conditions. Only thing is this is the question which you were asking in the previous lecture I hope you are getting this point. So, this is the interplay between the K parameter which is going to define σ_3 and σ_1, the way you denote them σ_1 is always the major stress, σ_3 is always the minor stress.

But what is going to happen is, there is a switch over between σ_v and σ_h. So, remember when σ_h is greater than σ_v, K_p conditions prevail, passive earth pressure. When σ_h is less than σ_v, the K_a condition prevails. So, all these analysis which we are doing now is valid only for the active earth pressure case, when σ_v happens to be greater than σ_h, when I start discussing about the state of stress in the soils and the plastic equilibrium I will introduce the concept of how this Mohr circle will get transformed to a passive state where σ_v becomes lesser than σ_h, the horizontal state of stress increases and vertical remains constant.

What I have done is I have switched over from a plane strain to a three-dimensional situation. So, I told two things, you remember in a direct shear box test what we did is we applied normal stress first, agreed, and then we sheared it two stages are there. So, when you applied σ, this is σ_v which happens to be σ_1 what is going to happen to the horizontal stress, this is σ_h which will be equal to K_0 times σ_1, this is a state of stress at rest when you are not sharing the sample at all.

So, this circle corresponds to this state, failure has not occurred because you have not sheared the sample. Now, slowly from this stage onwards if I apply the shear stress what is going to happen, this point is going to jump over here and this point is going to jump over here and until it meets the failure line. So, this is the path which is traversed by the state of stress to cause the failure in the sample.

Now, two things I introduce here as long as I am doing direct shear box test, I keep σ_1 constant maintaining σ value I am shearing the sample, you are traversing from here to here directly.

The second option is kept σ_3 constant which can be done in the triaxial testing, we will discuss this and then shear the sample. So, what is going to happen in that case? σ_3 remains constant which is the cell pressure, triaxial testing and σ_1 keeps on changing and ultimately you meet the failure, so these are the two ways of failing the sample, but the conditions are different.

The first one happens to be a 2d-plane strain, third one, second one happens to be a three dimensional. Suppose this is a small strip, this is a small system, let us say this is the sample. I can fail it by tearing it, very difficult to do soils by tearing them apart, the reason is soils do not have tension. So, this is ruled out.

Second possibility is compress it and see when the failure occurs this is normally we adopt. Compress the sample and see when the failure takes place along this plane this is what we have been discussing. So, unfortunately there is no other way to fail the sample. Is this correct statement? So, either you have to play with the σ_1 or you have to play with σ_3.

This is the Mohr circle corresponding to what state please understand first thing, this is no shear case and now you are slowly and slowly sharing it, your assignment number 2 is on this concept only. So, what I have done is wait, so what I have done is I am shearing the sample δ_h is picking up, δ_v is picking up and I am trying to make you understand how the failure is being achieved.

So, please remember another good way of depicting this was starting from this state when there is no shearing and now you are slowly shearing it, but sample is not failing. So, what is happening? You are picking up slowly, keep on picking up, picking up, picking up, picking up and ultimately the failure occurs at this point. And what happens beyond this post failure, dense sands.

Other case, you keep on shearing the sample there is no defined peak but the sample achieves the residual strength, failure. So, all these states of circles which I have plotted over here might be depicting each incremental loading in the form of τ because $\delta\tau$ is nothing but delta of this term. So, there are two ways of achieving this either keep σ_3 constant σ_1 vary, or keep σ_1 constant and vary σ_3, this is a very interesting concept which we are discussing try to understand this.

The first situation which I talked about is when σ_3 can be changed is only in the triaxial sample, we will discuss this so you will follow it. So, you take a big chamber, keep the sample inside and pressurize it. So, I am changing σ_3 and then I am seeing when the failure is going to occur because σ_1 is accordingly getting changed because the tau value is getting changed.

When you do direct shear test this is in your hands, this is not in your hands. So, there are two ways of failing the sample. Oh, in direct shear test is going to be difficult that is what I told you because what you do normally is, what is the limitation of direct shear text you must have realized, you know your σ_1-σ_3 planes keep on changing when you are shearing the sample.

I think I discussed in the last class, starting from this state, this is your initial condition of the sample, this is σ_1 and this is $K_0.\sigma_1$ and this is the point of the failure, it is okay, what did we do, the state of stress is acting on horizontal plane, so this becomes the Pole, this is our σ_1-σ_3 plane, unfortunately the plane themselves are getting rotated. So, we do not have much control on the state of stress which you wanted to have on the sample.

And that is the reason we are coming out of this and switching over to the triaxial state. In short, intermediate Mohr circles cannot be drawn for direct shear box test, this flexibility you get in triaxial testing and where you can observe how stresses are getting, what, how sample is responding to the state of stress which you are imposing on them. And hence, triaxial testing is supposed to be the best way to test soil examples.

That is right, so that means failure is not occurred. So, what is going to happen? This is the state of stress, is it not? This is also a state of stress, this is also state of stress but this happens with the critical one which we have defined as (τ_f, σ_f) this is some σ_1 σ_n τ_n, σ_2 τ_n, σ_1 τ_n, but not the failure.

So, they exist, the state of stress exists but it is not critical, it is not causing the failure. So, these are the two ways of representing one way of representing failure is this, another way of representing failure is this and that is where the K_f line becomes handy. In short starting from this state I am achieving the failure by following this path, path can be changed, I can have another path like this, I can have another path like this, this we will discuss later on.

I might be having another path where I can touch this failure line by pulling it out, look at this, the easiest way to fail the sample would be tear it apart, this becomes my stress path, this becomes my stress path, this becomes my stress path and so on. That is the magic we have to do with the material.