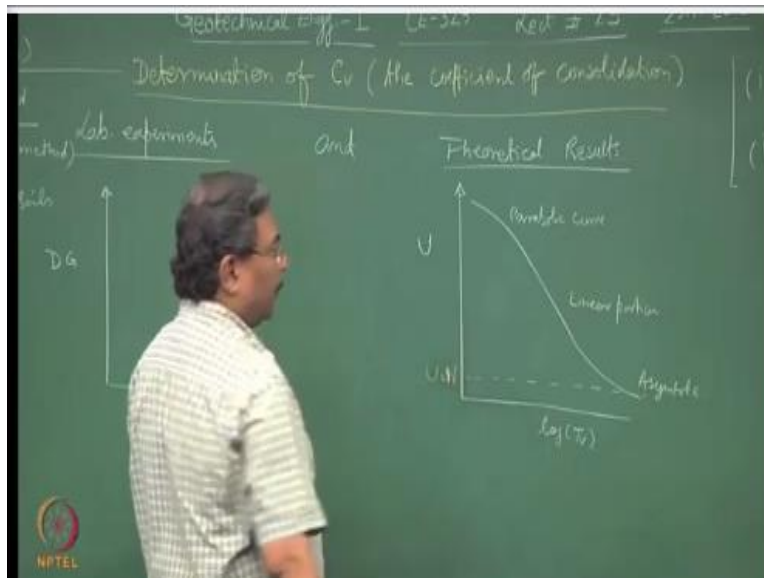


**Geotechnical Engineering I**  
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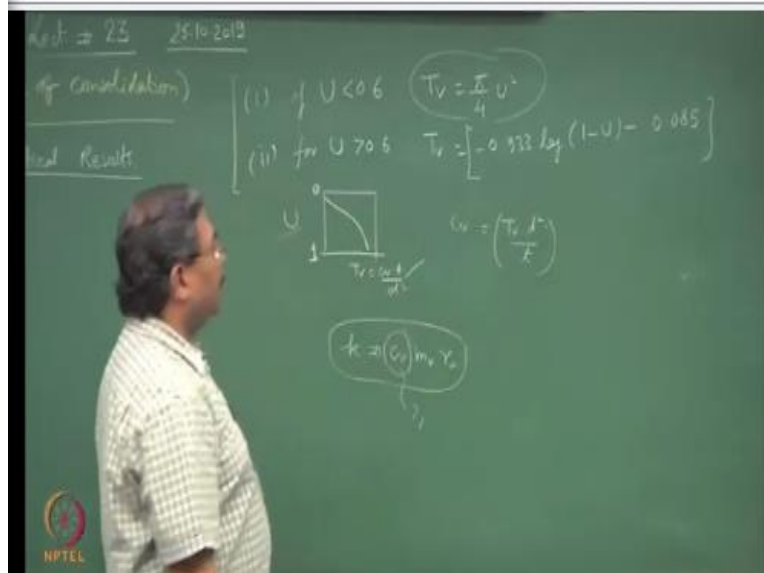
**Lecture-28**  
**Coefficient of Consolidation**

Welcome to lecture number 23, what we have realized is that once you solve the one dimensional consolidation equation proposed by Terzaghi. The coefficient  $C_v$  which we have defined as the coefficient of consolidation remains unknown. And hence we have to conduct experiments to obtain its value. So what is done in real life that we take help of the laboratory experiments and the theoretical results.

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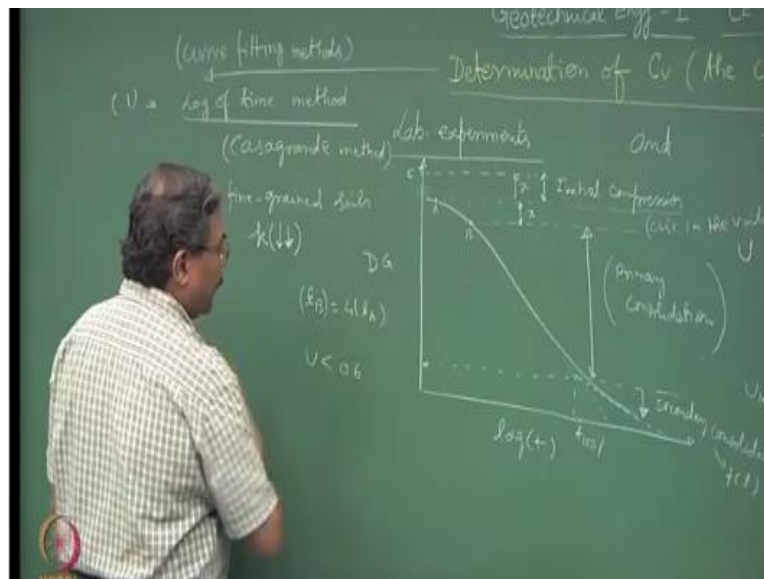


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Now what we observe from here is the degree of consolidation depends upon the time factor  $T_v$  and  $T_v$  is linked with the coefficient of consolidation. So we can that  $C_v = T_v$  into  $d^2$  square upon  $t$ .

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Now in real life there are 2 ways of determination of  $C_v$  number 1 is we are known as curve fitting methods. And the first one is what is known as log of time method, this was proposed by Casagrande, Casagrande's method. As per this method, in laboratory experiments the results if they are plotted as a function of dial gauge readings and with log of time, why log of time. Because this method is normally used for fine grain soils for which the  $k$  is extremely small.

And hence it takes lot of time for the soil mass to consolidate, so that is the reason we plot it on log this scale, fine. So the equivalent between this and theoretical result is that if I plot  $U$  because from dial gauge reading ultimately I wanted to know  $U$  which we have already discussed. Here on the x axis log of  $T_v$  is plotted, because  $T_v$  is also a sort of a non dimensional time factor, this has 3 components, the first one is the parabolic curve, this is followed by a linear portion.

And ultimately it becomes asymptote and this becomes asymptote on approximately  $U = 90\%$ . Now what you have to do is the experimental data which you get from the one dimensional consolidation test, we have to do curve fitting in such a manner that these 3 portions get reflected on this. So suppose if I get result like this, the caution here is that please follow a book what I am going to discuss here for the clarity of understanding of the figure though I will try my best to do the best possible.

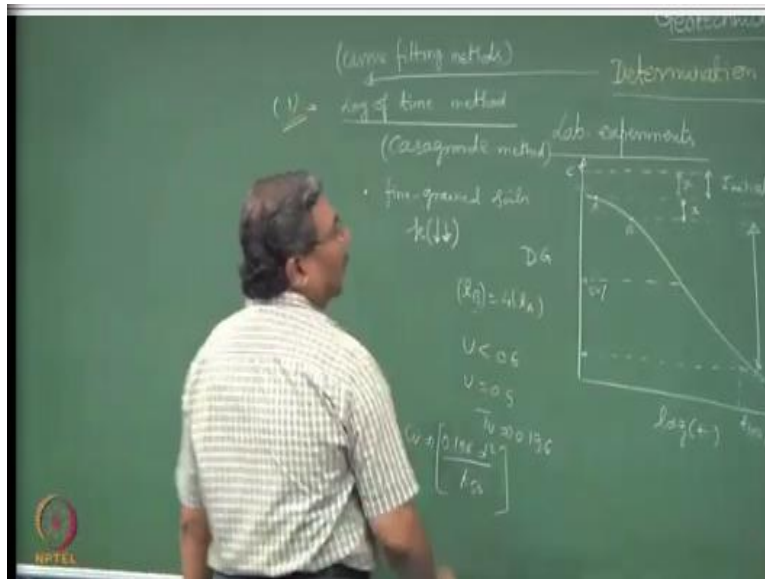
But sometimes not really possible to be 100% precise value when you draw these type of graphs on the board. See what happens is you consider a point in such a manner on this graph if it is A and a point B in such a manner that  $t_B = 4 \text{ times } t_A$  alright. So this is let us say  $x$  what is normally done is this  $x$  is incremented on A to get another point let us say  $t$  or  $c$ . now  $c$  to A becomes initial compression, so this is equal to  $x$ .

On the graph or the experimental data which you have got you consider A and B point in such a manner that  $t_B = 4 \text{ times } t_A$ . In process I get the value of  $x$  increment A by  $x$  you get this point and this becomes the initial compression, the whole thing. The linear portion of the graph if I extend and the tail of the graph which I have got if I draw a tangent over here, this gives 100% of I am sorry this should be 100% here.

So this point intersection gives the  $t$  100% anything beyond this if I draw a horizontal line over here if I consider this point here. This is the what is known as the consolidation, primary consolidation and anything below this point is termed as secondary consolidation. Now this is a function of time and we ignore this, when we do the consolidation analysis we are more interested in the primary consolidation.

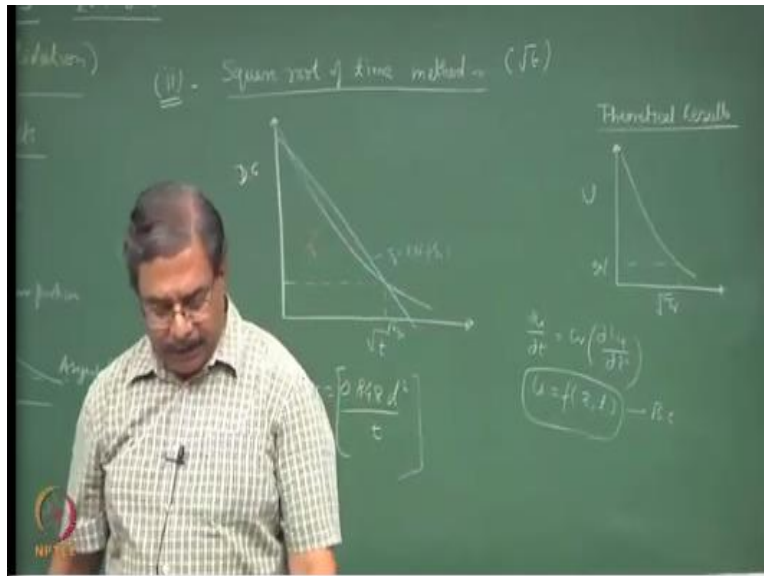
The initial compression may come because of the presence of the air in the voids of the sample alright. Now this is based on the assumption what we are getting from the theoretical results, I can use this relationship as long as  $T_v$  is  $U$  is less than 0.6. So if I want to find out let us say for 50% consolidation I can use this function that for  $U = 0.5$  the  $T_v = 0.196$  and hence  $C_v$  will be equal to  $0.196 d^2$  upon  $t_{50}$  where  $d$  is the drainage path and  $t_{50}$  is the time required for 50% consolidation to occur.

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Please refer a book to understand the different aspects of the geometry which I have done over here, that would give you a better idea about how the construction is done. The second method is the known as the square root method, a square root of time method. So this is the first method of curve fitting to determine the  $C_v$  value.

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And the second method is square root of time, the way this method was used for fine grain soils with very, very low permeability whether consolidation times are going to be extremely high, we plot them on a log scale. This is going to be for relatively less time taking soils for consolidation, that means the permeability are going to be higher. And hence we deal with the under root t method under root of time.

So theoretical results here would be if I plot U or says T v under root, you will get a graph like this, the linear portion followed by a curved linear portion. In experimentation, what we will do is we will plot the dial gauge readings either function of under root time in minutes, if I have a relationship like this which I get as per this method. The linear portion of the graph has to be extended back wherever this cuts the dial gauge axis alright, find out the slope of the line.

If this is slope s, increment the slope by 15%, so this is the line with slope 1.15 times smallest let us say, this is s 2, this is s 1. So from this point you draw a line the slope 50% more than s 1 wherever this cuts the experimental results this gives you t 90. And corresponding to this dial gauge reading we can say the degree of consolidation is going to be 90%. For 90% consolidation the C v comes out to be 0.848 into d square upon t.

So this is the relationship utilized, so these are the 2 methods from where you can get C v value. Once you have obtain C v value you can substitute it in equation, and you can solve this function

U as a function of Z and t by keeping in view the proper boundary conditions. All this is about one dimensional consolidation test, odometer test you get so much information from one dimensional odometer test.

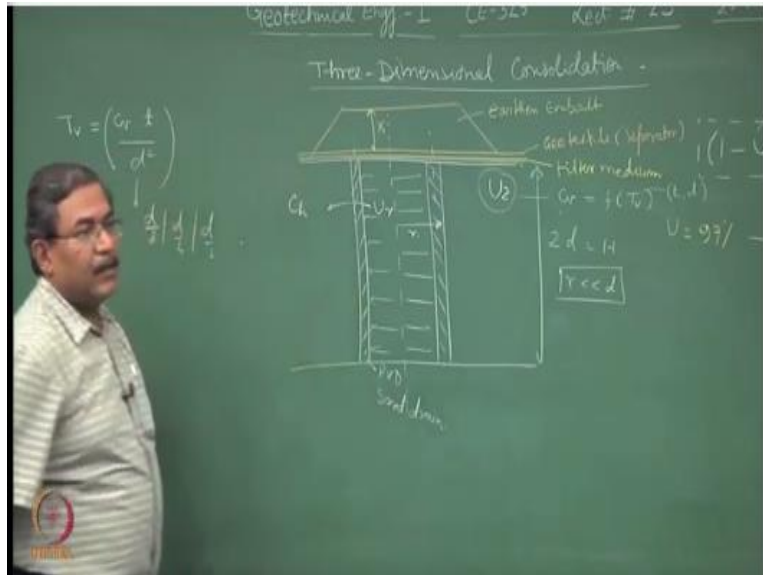
I hope you can realize now, once you have obtain  $C_v$  value you can get the hydraulic conductivity. Now this hydraulic conductivity is going to be much, much precise and you know better in the way because it is a controlled experiment. And hence if I know the  $C_v$ , if I know the  $m_v$   $\gamma_w$  is known, I can obtain the hydraulic conductivity in a better manner.

So this way I will be getting a better and precise value of k, the beauty of 1D odometer test is you can simulate the boundary conditions which are drainage conditions and you can obtain the k value which was quite difficult in the case falling head test or constant head test. So with this I finish my discussion on one dimensional consolidation test and the only thing which I have like to discuss for there is 3 dimensional consolidation which is quite contemporary.

And all of you should be aware of what happening in the country when most of the infrastructure projects are done, very rarely it so happens that one dimensional consolidation is going to be valid in the real life. If you remember the basic premise here was that you have taken a steel ring in which the sample has been contained and then you are applying load from one side. It is not a very practical situation in field because you do not have rigid boundaries you know confining the soil mass.

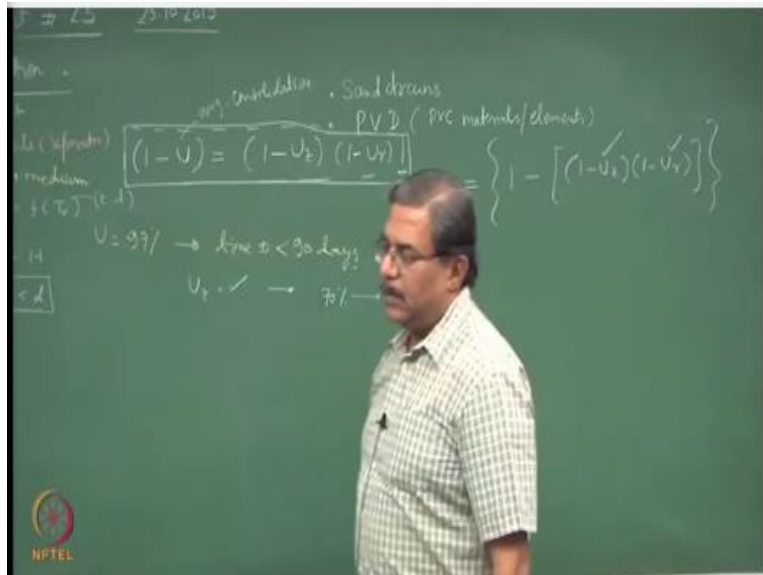
So truly speaking the consolidation is 3 dimensional in nature, now in order to accelerate the consolidation and to create infrastructure facilities, normally what people do is they resort to 3D consolidation procedure.

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This is normally achieved by inserting the sand drains or sometime we call them as PVDs alright, pre fabricated vertical drains.

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So these are basically PVC materials, elements which are inserted into the ground or install into the ground to accelerate the consolidation process. So if this is the ground and if I wait for one dimensional consolidation to occur this will take me several hundreds of years or tens of years. Rather what is done is if you realize that this equation indicates that if I want to achieve quick consolidation what I should be doing.

For auctioning a certain value of consolidation I require a certain amount of time, now if I reverse the problem that within a certain amount of time if I want to achieve the same degree of consolidation what I should be doing, I should be playing with the  $d$  parameter. So if I decrease the value of  $d$  to let us say  $d$  by 8 or  $d$  by 4 or  $d$  by 2 whatever. So what is going to happen this will become  $d$  upon  $x$  square ok.

So that means I get an advantage in terms of the time  $x$  square times, so where do you use this concept. If I install a vertical drain over here, so this becomes a PVD or a sand drain, earlier without installation of this the drainage path was  $2D$  which was the height of the clay column. Now by inserting this what I have achieved is I hope you will realize that if this is the axis of these PVDs or the drains, I have reduce the drainage path.

So drainage path is now going to be somewhere in the middle of the system, if I rotate this whole thing by 90 degree it becomes the situation which I have already discussed. So these are the drainage layer alright the pore water pressures are going to be 0, we solve this problem in one dimensional Oedometer test. The same situation is exiting in the vertical direction, what is the drainage path, the drainage path is only  $r$  and this  $r$  is going to be much less than  $d$  is this ok.

So by reducing the drainage path I am getting the advantage of accelerating the drainage process and hence the consolidation gets accelerated, this is the concept. So I am sure you must have seen most of the projects which are being done the country right now particularly in the coastal regions you must be seeing big, big machines inserting something into the ground. So with the help of big machines we can insert the PVDs, this could be up to 20 meters, 22 meters in the ground and they act as a drain, so they take out the water.

So the water is now going to get collected in this and half of this is like this and hence the consolidation process becomes faster. But what we have done in the mean time is earlier we were talking about the  $U_z$  that these consolidation which is taking place in the vertical direction. Now this another component of the consolidation which is taking place in the radial direction, so this is  $U_r$ .



So by doing this type of a manipulation I say that  $1 - U$ , if  $U$  is the average consolidation undergone by the soil mass this would be equal to  $1 - U_z$  into  $1 - U_r$  ok. So this is a relationship which is utilized, the procedure is having inserted these drains in the ground. We provide a filter layer and on the top of the filter layer we provide a geo textile. So geo textile acts as a separator and separator between the soil mass which is going to be consolidate and the preloading which is done.

So having install this and laying the layer of the filter media, normally granular materials, sands or dust of the stones crush dust and this is the earthen embankment. So you create an embankment of an certain height  $X$  and design the entire system. In such a manner that I achieve about 95% to 97% consolidation in the desired time. Normally people do not want to wait for more than 3 months. So here what you are seeing is in the process of constructing this embankment on the top of the system which is having the pre fabricated drains.

I have the component of  $U_z$  which is coming because of the embankment and which can be analyzed by the using the concept of  $C_v$  is this ok. So this  $U_z$  is linked to  $C_v$ , I will say now coefficient of consolidation in the vertical direction. And this  $C_v$  is related to  $T_v$  and this  $T_v$  takes into account the time and the drainage path, is this ok. Now what we have done extra is we have induce  $U_r$ , so that means this philosophy is valid for  $U_r$  also provided.

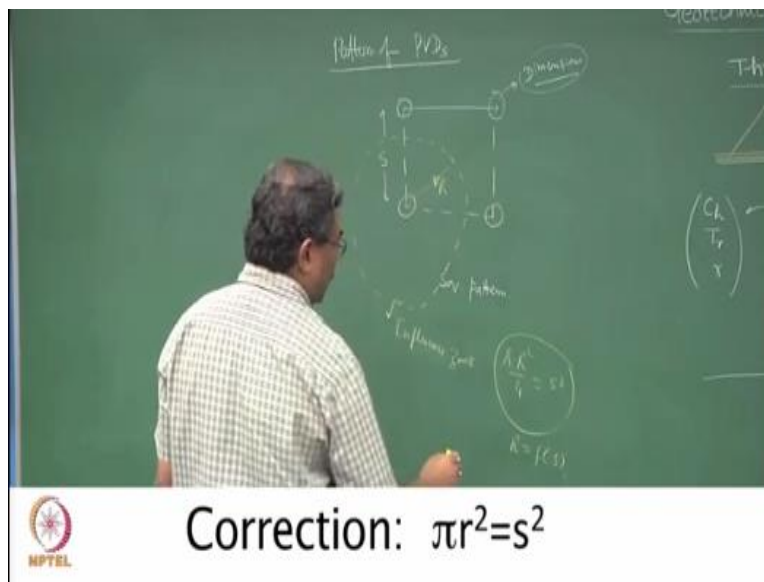
We define another term which is  $C_h$ , so  $C_h$  becomes the coefficient of consolidation in the horizontal direction,  $T_v$  will get replaced by  $T_r$  this is the time factor in the radial direction. And we have a term  $r$  which is the drainage path in the radial direction fine and ultimately what governs the entire consolidation process is this equation. So that means the moment I compute  $U_z$ , I will compute  $U_z$  and suppose if I get some amount of settlement let us say 70% settlement I am getting, 30% which is remaining can be obtain by inserting these drains.

Or I can revert the problem also, this 70% I will attaining after let us say 2 years, so a good design problem can be created out of this by saying fix the time. And within this duration let me see how much vertical settlement I could attain and rest of the settlement I will attain with the

help of 3 dimensional consolidation, is this ok, this is how the design is done. So the remaining portion of the settlement is going to be now taken care of by  $U_r$ .

So I can say that this expression if I know the design  $U$  value, design  $U$  value will be given by the designer that I can tolerate only let us say 3% settlements after 2 years railway tracks, you cannot really take a chance. So I can write this as  $1 - 1 - U_z = 1 - U_r$ ,  $U_z$  can be obtained easily by using the vertical consolidation concept  $U_r$  I will tell you how to obtain, is this part ok. Normally these drains are fitted or fixed or inserted into the ground in 2 manners.

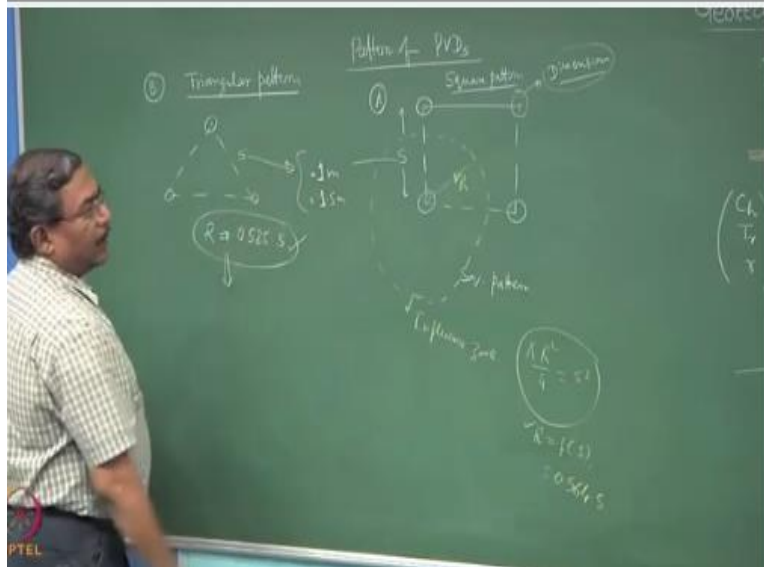
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So either they are inserted into the ground in a square pattern, so this is the square pattern and if I consider the spacing as  $S$  and if I know the dimensions of these drains say manufactured by the industry. So they give you all the dimensions and the properties like hydraulic conductivity of the element of the drain which you are making. What I can do is, I can obtain an effective  $r$  which is known as the influence radius of each drain, so this becomes my capital  $R$ .

So  $R$  is the radius of the influence zone and then I can go for equivalence, I may say  $\pi R^2$  by  $4 = S^2$  and I can get a relationship between  $S$  and  $R$ . So if you solve this function you will be getting 0.564 times  $S$ , So  $R = 0.564$  times  $S$ .

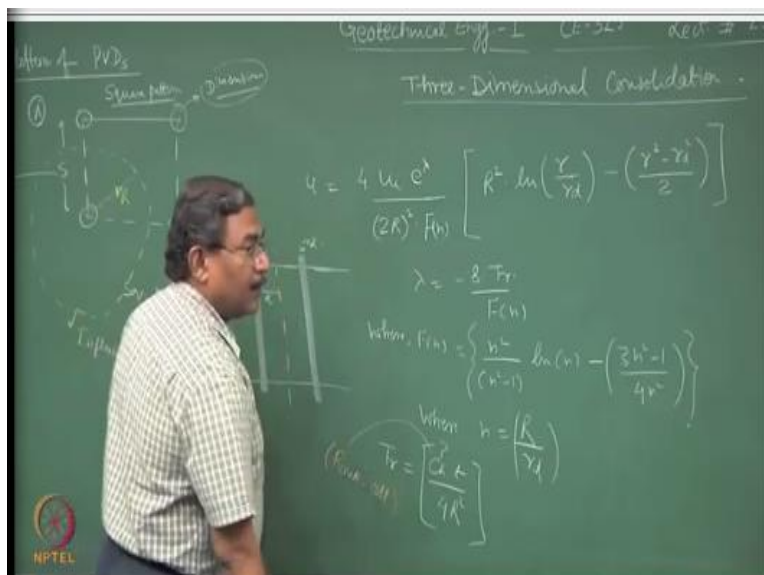
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Normally what we do is we fix S center to center spacing is fixed as 1 meter, 1.5 meter, so S is known, R is known. Once R is known we will proceed further, the second pattern which is normally used to install the PVDs is triangular pattern. So this is a square pattern and this is the triangular pattern, again the spacing between the 2 PVD is less and get the value of R this turns out to be 0.525S, yes.

So once you have got this S there also we normally fix S as 1 meter and this I can compute R. Now we have to use some design charts and you need not to mug up this equations these equations are available.

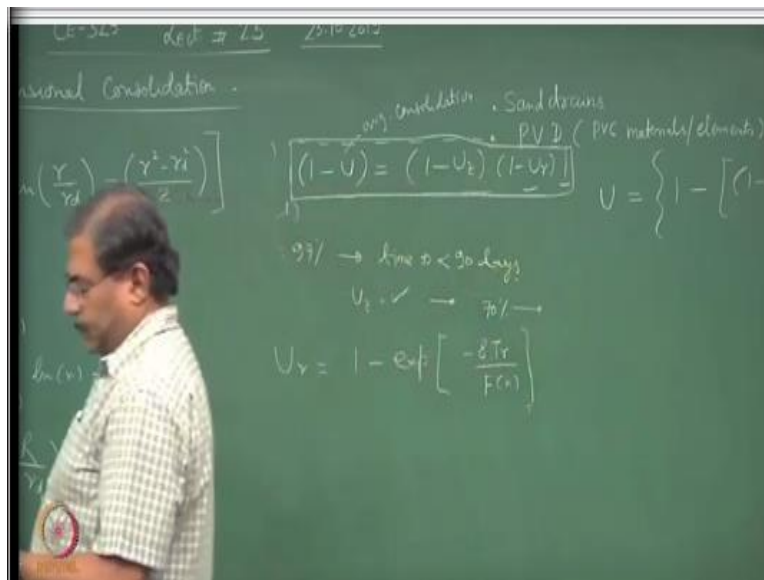
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So the pore water pressure is defined as  $4u_i$  i.e. to the power  $\lambda$  upon  $2R^2$  into  $F_n$ ,  $F_n$  is the function,  $R^2$  into  $\log$  of  $r_d - r^2 - r_d^2$  upon  $2$ . So this is the function which gives you the value of the pore water pressure where  $\lambda$  is equal to  $-8T_r$ ,  $T_r$  is nothing but the coefficient of radial consolidation  $T_v$  we have used for vertical consolidation upon  $F_n$ .

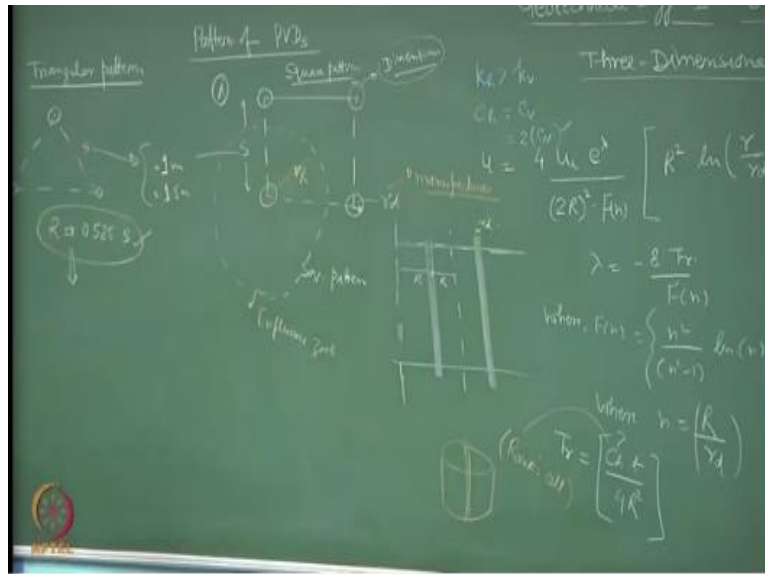
Where  $F_n$  is  $n^2$  upon  $n^2 - 1 \log$  of  $n - 3n^2 - 1$  upon  $4n^2$ , where  $n = R/r_d$ , what is  $r_d$ , the radius of the drain. So this is the  $r_d$ , now  $T_r$  is defined as  $C_h$  into  $t$  upon  $4R^2$ .

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And hence  $U_r$  can be defined as  $1 - \exp(-8T_r \text{ upon } F_n)$ , now this becomes a very interesting design problem. Just try to understand the steps which are involved in this and then you can be a perfect designer I mean this is the most art people are using it in the field every time in every project in the country. The big question is how to obtain the  $C_h$ , how would you do it there is something known as row's cell.

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So now row's cell is use to obtain the  $C_h$  value, what it does is there is a sample which is maintained in a ring or a container which is cylindrical and rather than applying the vertical load there is a mechanism by which I can squeeze the entire sample radially in. And the drainage is applied at the center path, expensive device but this is the only way. Or the second way would be, we know the relationship between  $k_h$  and  $k_v$  which one was more, yes,  $k_h$  is greater than  $k_v$ .

And  $k_h$  controls  $C_v$  directly because what we have seen is  $C_v = K \text{ into } M_v \text{ gamma } w$ , so for a given material  $M_v \text{ gamma } w$  is constant. So truly speaking  $k_h$  and  $k_v$  relationship can be extended to  $C_h$  and  $C_v$  also. For some type of soils  $C_h$  is equal to  $C_v$ , another type of soils normally we take  $C_h = 2 \text{ times } C_v$  radial consolidation is 2 times the vertical consolidation, assumption, please go alright.

So this  $C_v$  I can obtain from one dimensional consolidation test and once I know the  $C_h$   $C_v$  value I can get the  $C_h$  value. Once I know  $C_h$  value  $R$  can be obtained because  $R_d$  is known this is supplied by the manufacturer, it is a property of the PVD. So if  $R_d$  is known,  $R$  is known,  $n$  is known I can substitute in this  $F_n$  is known,  $F_n$  is known,  $T_r$  is known,  $\lambda$  is known I can come to this, I can use this expression  $T_r$  is known,  $F_n$  is known,  $U_r$  is known, this is how it is done, fine.

Truly speaking this is a process of optimization because everything will depend upon  $n$  value and  $n$  will depend upon what type of drain I should be using, what should be the radius of the drain and its properties. So if I know  $r_d$ , if I know the spacing because  $R$  is also a function of  $S$  you must have noticed. So what we have to do is we have to fix  $S$  get the value of  $R$ , fix the dimensions of the drain which we are going to use.

So that  $n$  gets fixed, once  $n$  gets fixed everything is a simple procedure, this is what is being done in most of the projects which are happening all along the coastal regions where you have marine clays which will take a lot of time to stabilize. Well, I will close this course now at this juncture, we have discussed a lot of things and all the best for your examination, be prepared with because end sem is really going to be a real test of your metal. And I hope you must have realized that this course was a different course.