Geotechnical Engineering I
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Lecture-27
Consolidation of Soils

We have been discussing the compressibility characteristics of soils in the previous lecture and this is where I had introduce the concept of consolidation also, a specifically I talked about the difference between consolidation and compaction. And I think I highlighted that these are 2 mechanisms or different mechanisms which occur in the different states of the material. After talking about the computation of settlements in the soil mass because of the external loading.

I discussed about the 1 dimensional consolidation theory which is proposed by Terzaghi and this is the spring analogy which I discussed in details followed by the assumptions which are made in the one dimensional consolidation theory proposed by Terzaghi. In today's discussion I will be talking about the first of all derivation of the one dimensional consolidation equation which is used to obtain the pore water pressures as a function of time and z.

And once the pore water pressures are known, I can obtain the settlements undergone by the soil mass because of the external loading.

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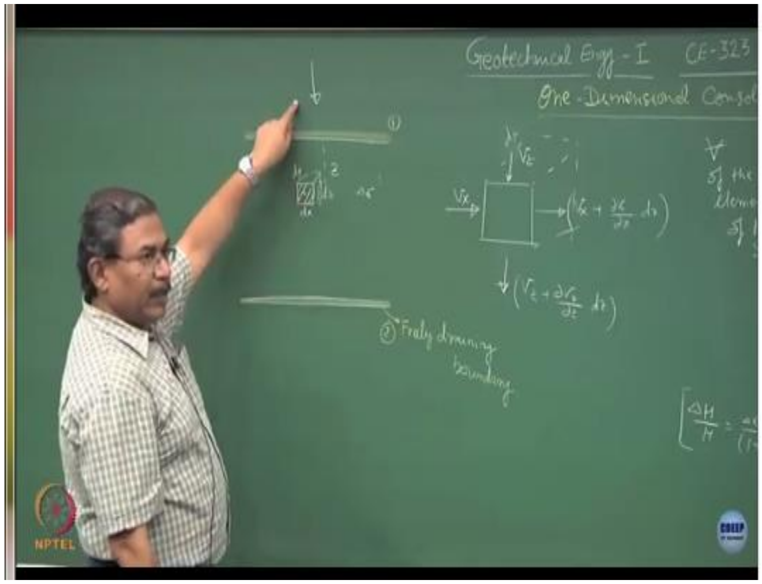

To begin with, let us try to derive the one dimensional consolidation equation. If I have the soil mass and if I take an element of the soil mass of depth, dz located at a depth of z up to the c z. And as you remember, what we did is we exposed this soil mass to external loading, maintaining the 2 boundaries, which are freely training. So this is the mountain number 2 and number 1 and there is an external load which is applied on the system which is causing delta sigma to occur at this point at the middle of this element.

So if this is d x and the third dimension, if I consider to be d y because of the loading, the pore water pressures are going to develop and because of the development of the pore water pressure and due to the vicinity of the training boundary, there will be a tendency of the soil mass to dissipate the pore water pressures. So suppose if I assume that in this element, there is a influx of the seepage are the discharge and this is v x + del v x upon del x into dx.

So this is a continuity of the flow which is entering in the system, in this direction I can say this is del v z upon del z into d z, so I have to reverse the directions of the arrows. So this is v z and this is a discharge is comes out. I can consider this element as a 3 dimensional element the way I have shown earlier and this axis is d y.

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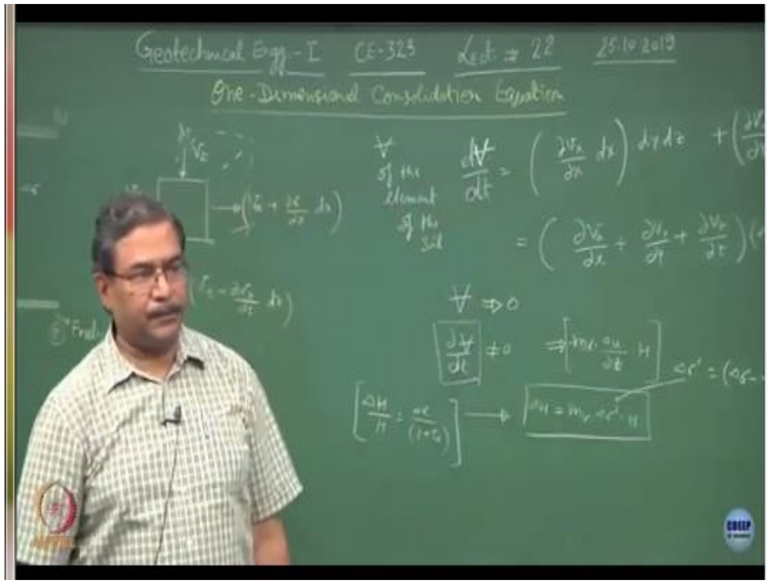

I can write that is d v by d t is the change of volume, so let me put this as capital V. So, this is the capital V which is the volume of the element of the soil. I can prove that this d v by d t = velocity into the area. So that means del v x upon del x into d x multiplied by d y d z + del velocity these are all velocities del v by del y into d y del x del z + del v by del z into d z into d x into d y. I can simplify this as del v x upon del x + del v y over del y + del v z over del z into d x into d y into d z alright.

I hope you will realize that I have already subtracted the discharge which is coming out of element with the discharge which is entering into the element, so this is the mass balance alright. Otherwise this will be v x plus this term multiplied by d y dz minus this term, now if I consider that the soil element is incompressible under that condition what is going to happen, this will be 0 alright. But most of the time, soils are compressible as we discuss, so del v by del t is not equal to 0.

Now the question is, if I assume that a one dimensional consolidation occurring in the system, can I obtain a relationship, where I can show, what is the value of rate of change of volume with respect to time. Now this is where we can take help of the relationship which we are developed delta h upon h = delta e upon 1 + e 0 equal to you know this was our basic premise, is this ok. Now analysis of this leads me to a situation where I would say delta h = m v into delta sigma prime into h you are agree.

This we derived in the last lecture, what is delta sigma prime, delta sigma prime is the effective stress which is developing at the c g of this element which we have considered because of the external loading. So this delta sigma z = delta sigma - delta u and this delta u is nothing but delta u w fine. That means, now if I try to write this function del v by del t, can I write this that del v by del t will be equal to.

If I differentiate this function by substituting it over here, this will be m v is practically constant we are assuming multiplied by delta sigma will be constant, what is changing is delta u w. So this becomes minus delta u I will remove the term w now is understood that the pore water pressure, so this is going to be del of t multiplied by h is constant ok. So what I have done is, I have got a relationship between the coefficient of volume compressibility the pore water pressure which is developing in the system and it is rate of change multiplied by thickness of the sample.

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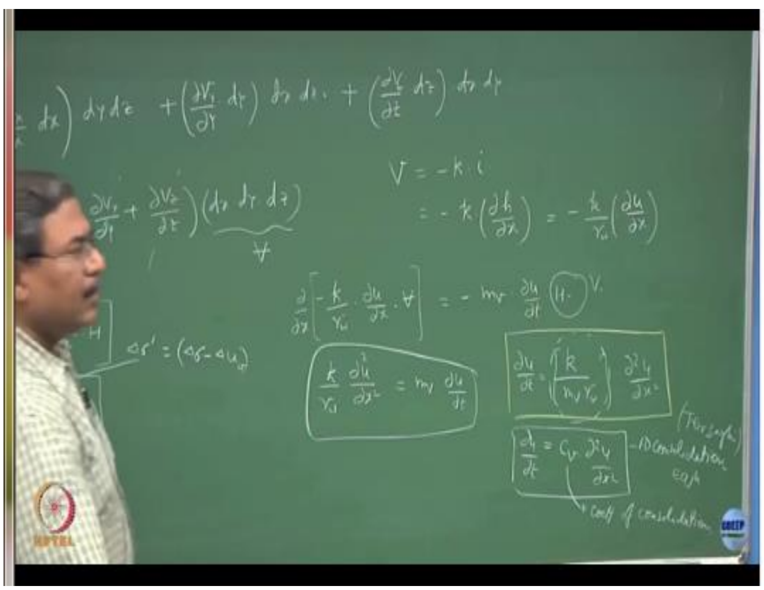

Now is there a way to get rid of the velocity vectors, I hope you will realize that velocity is nothing but - k into i. So I can write this as - k into del h upon del x, where h is the head and this can be written as - k over gamma w into del u by del x, is this part clear. Because h = gamma w into h into gamma w = u pore water pressure, so this is a relationship I am using. So if I substitute it over here, I can and if I assume a one dimensional situation, other terms will disappear.

And this is the volume term of the element, I can write that - k upon gamma w into del u by del x into volume alright. And what we are doing is, we are taking derivative of this function, so this function has to be differentiated with respect to x. So this will be equal to – m v del u by del t into h, is this ok. So this is your rate of change of volume with respect to time we have obtained. Now this is also going to be a function of volume because in one dimensional we have written like h.

So what I can say is that del u by del x square into k upon gamma w, this will be equal to m v into del u by del t. Now this expression is of some use to us, what we can do is, we can write this as, del u by del t = k upon m v into gamma w into del u square by del x square. Now this equation is known as one dimensional consolidation equation which is proposed by Terzaghi, this term which is appearing over here is also defined as C v.

So where C v happens to be the coefficient of consolidation, now this is what is known as 1 dimensional consolidation equation which was proposed by the Terzaghi. Now interesting thing here would be, this is a transpose in the one dimensional case. So this volume in 3 dimensional is volume this has to be multiplied by area of cross section because this equation is valid for delta h upon h, remember area of cross section is constant.

So I can say this is area and this is also area, so this becomes del v by del v this is how this volume term gets cancelled with this volume term ok. This is one dimensional case we are talking about only height, area of cross section of the sample remains constant. So what we have done is, we have obtained the one dimensional consolidation equation and now to understand that what we should be doing with this equation number 1, what is the form of this equation.

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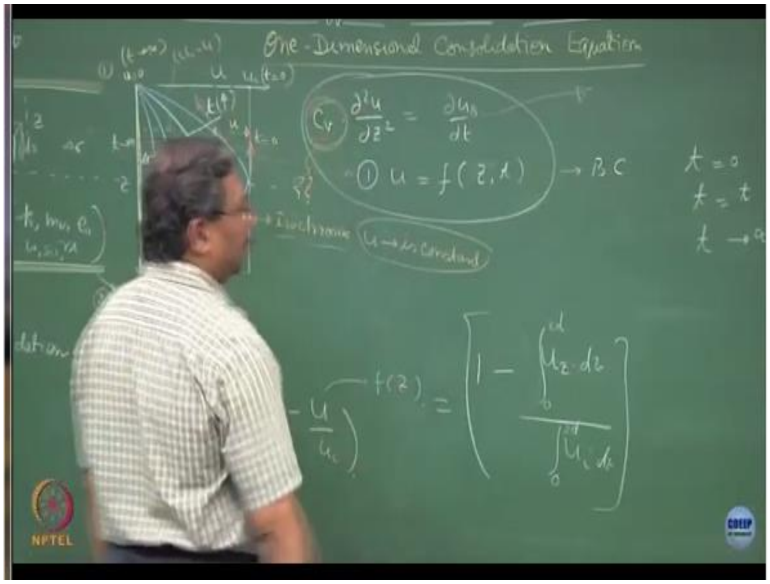

So suppose if I say del square u by del x square multiplied by C v = del u by del t, truly speaking if I solve this expression or this equation, I hope you understand what is second law of fix or the fix second law you must have come across somewhere is not. So this is u the pore water pressure, I can substitute it with a concentration I can substitute the temperature or whatever you want and then this is your second order fix equation.

So truly speaking u is a function of x and t fine, x happens to be incidentally the z. So to make it more apt to our discussion, I will say this is z comma t. In simple words, the one dimensional consultant equation is the value of pore water pressure at a given time and at a given depth in the soil mass. So that means, if I consider a given point at a given time, because of the external loading, whatever pore water pressure is developing in the system is depicted by one dimensional consolidation equation, is this part clear.

So what we have to do is we have to solve this differential equation to get u, so this is the first objective when we solve this u as a function z and t, I have to apply some boundary conditions, what will be the boundary conditions, the moment I apply load because of the hydraulic conductivity and this is saturated soil. So the assumption is the moment you apply load delta sigma gets generated and this delta sigma is at t = 0 initial condition.

So delta sigma is equal to or it is generating delta u, now if my initial level of u is 0 delta u is nothing but u, so this becomes u initial and if I say minus of this. So basically delta u is nothing but the u i term initial pore water pressure is 0 clear, this is the first equation which I am having, Now this is existing at z = 0 at this boundary as well as at this boundary because both the boundaries are draining. So this is t = 0, delta sigma equal to this and delta u = u i in the entire domain.

So this will be less than equal to d, normally we define the thickness of the layer as 2 times d for a specific reason I will tell you why. So if the thickness of the layer is 2d, so the boundary condition is going to be z less than equal to 0 in the entire domain. The physical significance of this is the moment you compress the whole system by applying external loading, the initial pore water pressure develops which is equal to the delta sigma value and at t = 0, the entire system shows a constant pore water pressure, alright, this is the initial condition.

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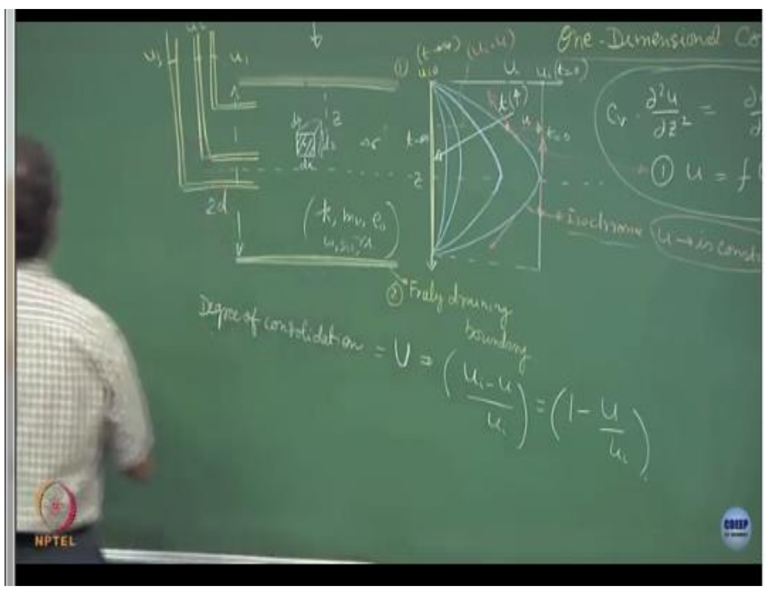

So that means if I want to plot let us say, this is z and suppose if I plot pore water pressures, so in the initial condition, what is going to happen is the value of pore water pressure which is developing air as well as air is going to be a constant, I hope you agree with this. So this is what is equal to u i, the moment you applied the pressure, the pore water pressure = u i at t = 0, now what is going to happen.

Something intermediate is going to happen at t = t let us say, very difficult to estimate or if I want to estimate, what is the value of pore water pressure at if time, at this time, what I will have to do is, I will have to solve this expression, so this is what f z t would be intermediate state. The third state is when I said t is standing to infinity. So the moment you said t tending to infinity what is going to happen. Because these 2 are drainage boundaries, the pore water pressures at this point in the soil mass and at this point in the soil mass is definitely going to be 0.

And the pore water pressure is going to be completely 0 because t is infinity that means the other bound of the plotting would be u = 0 which is t tending to infinity. And this is the case when you have t = 0 initial case, is this part clear. In between what is going to happen, let us try to discuss what is the form of this equation, what type of question is this, it is a parabolic function alright. So that means, in between whatever is going to happen, is a parabolic distribution of u as a function of z and t.

So suppose if I consider the line of symmetry of the sample, this line of symmetry also defines the symmetry of the drainage boundaries. So whatever I am doing right now is valid only for the drainage boundaries 1 and 2 put together. If I change the boundary conditions of drainage, this solution is going to change. Now it so happens that the moment t increases from 0 to some finite value, this is how, so this is how it is going to look like, is this ok.

Now this line is known as an Isochrones, what we have done is, we have just represented the u as a function of depth for a given time. So this boundary please remember, corresponds to t = 0 and this boundary corresponds to t tending to infinity alright, so these are Isochrones. As time progresses, what will happen, the Isochrones will keep on shifting towards the left hand side ok.

So in this direction, the time is increasing and a stage comes when the pore water pressure becomes 0 at equal to infinity fine, so this is that line which you are meeting this is the initial value. So what is the interpretation of this Isochrones, if I consider a point over here on this graph or on this curve, can you tell me what this point is going to correspond to. This point is nothing but this equation, where is the z, this is a z and where is the time.

This Isochrone corresponds to a certain time, ok, so at this point the pore water pressure is this value. There is a symmetry of the Isochrones along the axis of the sample provided the boundary conditions are same both sides draining. Now if I draw a tangent over here, what is this going to give me and if I draw tangent over here, what I am going to get del u by del z at a given t, what is del u by del z velocity vector, got it.

So this is the velocity of the water which is present in the pores at this point and what is the direction it is moving up this is moving down because there is a free draining condition here, there is a free draining condition over here. At this point what is happening if I draw a tangent, this is a velocity vector and is the velocity vector ok. So this is the interpretation of the Isochrones, there is one thing more which we would like to discuss about the Isochrones.

And how they can be utilized in day to day practice of geotechnical engineering, I hope you are realizing one fact that Isochrones are nothing but a form of a solution the graphical form of this equation. And ultimately as t tends to infinity starting from t = 0, Isochrones are also going to tell you how the consolidation process is occurring. That means, if I draw a lateral horizontal line suppose here, now this is the value of out of so much of the pore water pressure, look at this x axis is u i alright.

Suppose certain this is the value of u now what is remaining is u i – u, that means if I want to define a term degree of consolidation ok. So degree of consolidation is defined as normally capital U and this is equal to the total amount of pore water pressure which is available to get dissipated to the one which is remaining. So I will be defining this as u i - u over u i, have you understood this.

Now this can be written as 1 - u upon u i, imagine that I wanted to find out, what is the settlement undergone by a soil mass. Because of the external loading of known properties like k m v, initial void ratios, gamma d and all these things are known moisture contents, saturation is 1 because we have saturate the sample ok all these properties are known. What I have to do is, I have to get the values of pore water pressure as a function of z and t, is there any other way to do this rather than going for all these mathematical analysis experimentally yes, there is a way.

Ultimately what I am measuring is pore water pressures which you are measured until now in several cases. So what I have to do is, I have to just insert several piezometric tubes all along is this ok. And whatever amount of pore water pressure comes over here, I can use this for drying this contours. It is very difficult to conduct this type of experiment because the size of the piezometric tubes is quite big and the size of the sample, if you remember is hardly 25 mm in the laboratory.

But suppose if you want to see what is happening in the field, you can insert these type of potentiometers or sort of you know capillary tubes and you can obtain what is the pore water pressure which is developing because of the external loading, so you are free to do whatever you want to do. The most interesting thing which we have got from this simple analysis is, first of all the equation it is nature.

The solution of this equation going to give me the pore water pressure and using this pore water pressure I can find out the degree of consolidation. Now degree of consolidation is going to tell me how much the settlement of the system has occurred, that means degree of consolidation can also be defined as.

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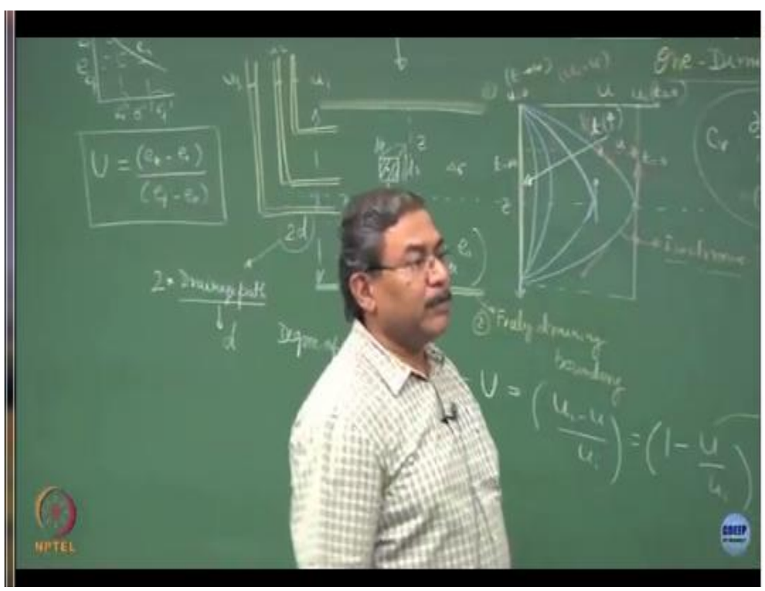

Suppose if I extend this analysis to e sigma prime curve starting from e 0 I attained e f and corresponding to e 0 the sigma is sigma not prime and this is sigma f prime ok. So this is a sort of a situation where I can say e f - e 0 this will be in the denominator and something intermediate which I can define as e t will be coming over here. So this is another equation which I can use for defining the degree of consolidation, is this ok.

That means, the void ratios at a given time - initial void ratios upon e f - e 0 I hope you understand e f is going to be lesser than e 0 it is going to be lesser than e 0. So negative will cancel out and this will become e 0 - e t upon e 0 - e f, so there are several ways of doing this. Now let us complicate this function a bit, because in real life these type of boundary conditions are rare.

So what we have discussed until now is, the laboratory setup, where if you remember we had taken a consolidometer ring we call it as a odometer ring. We kept the specimen there, we put 2 porous stones there, saturate the entire thing, apply the external load clear. So this equation is specifically simulating what you did in the laboratory. If the sample or the specimen in the water bath can be maintained at elevated temperature.

If I do the whole exercise by increasing the temperature of the sample, now we have a coupled effect, you agree, no you are not asking this is what people are doing in the western world, is this ok. THMC of everything that means this whole test has been done under STP standard temperature pressure conditions. I can raise the temperature of the sample to certain the value and I can see what is going to happen to the consolidation or the pore water pressures are going to change.

So those of few who might get a chance to work in thermo active structures will be still analyzing the whole thing. Only thing is that you would become a function of temperature also and that becomes a coupled phenomena which is complicated. So at undergraduate level you are not supposed to learn on this things, a bit more of analysis of this fact which we were discussing. So at t tending to infinity, what is going to happen.

The value of u tends to 0 and this is going to happen only for z = 0 and z = 2d because in between we are not sure about what is happening. So we have to do it either analytically or experimentally. Now suppose if I say, here we have well defined distribution of the pore water pressures. Now if I say that the u is a function of z I do not know what type of variation of the pore water pressure is.

So I can also write this as 1 minus if I say u z and from 0 to 2d it is varying in certain fashion, I can average this out and I can integrate it from 0 to 2d into dz the u i remain same. If I integrate u i also in the same form, what is just going to happen, this 1 upon 2d will get removed and this function will also get changed to 2d into u i into dz. That means, if I say that the initial pore water pressure is also a function of z, so this type of manipulations can be done, the triviality comes.

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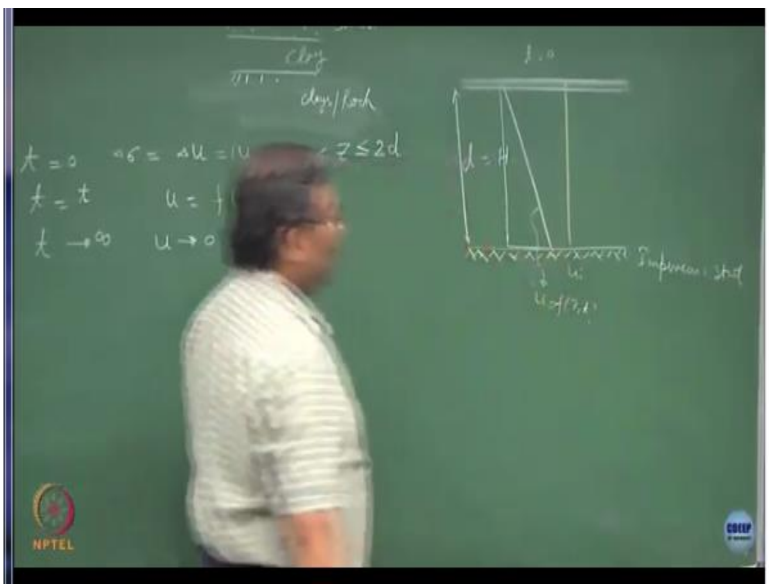

Suppose if I give you a situation where the clay sample is draining only from one side, this is the impervious strata alright and this side is draining. In nature how it is going to happen and when it is going to happen suppose there is a clay layer clay seam, which is sandwiched in sands, so this is sands and this is sands. Both sides draining condition what we have discussed over here, the porous stone simulates both side draining.

I change the context of the problem, the clay layer is underlined by, let us say another silty layer no not silty. Let us say this is another clay layer or a rock and hence this is going to be impervious. Now if this is a situation, this becomes one side draining, so when I said intentionally I am using this 2d term this 2d is defined as drainage path. So drainage path is going to be half of the 2d value, why.

Because there is a line of symmetry and we have already prove that all these points are they going to drain not, this point remains confused whether it should get drained here or whether it should get drained over here. So in other words, how many of you have baked a cake ever in your home. So if you really want to learn consolidation process go and make a cake and I hope you will realize that despite good amount of heating, which you have done in the microwave oven, what happens was center portion of the cake it remains uncooked why, this is the answer.

So look at the water molecule it does not know whether it should go on this side or it should come on this side. So that means the central layers are always going to remain wet there is no draining taking place. So it is very difficult to consolidate a sample fully alright, so the drainage path is defined as half of the 2d let us say d. So this is 2 times the drainage path and the d becomes the drainage path through with the drainage is taking place.

In this case, the drainage path is going to be equal to H because this is one side draining only. Now suppose if I ask you to plot the variation of u with respect to time what is going to happen, start from the basics at t = 0. The entire load comes on the system, the pore water pressure develops, this is the value of u i which is equal to delta sigma, external load. As time passes by what is going to happen, the top layer will quickly dissipate the pore water pressure.

But this being a impervious surface this is not going to dissipate anything and hence what you have done is, you have created a sort of a profile of pore water pressure like this, is this ok. So, I will say that, this is the profile u as a function of z and t, this case is known as a partial drainage case or half drainage case. And as I said this is possible when the clays are sitting over impervious system or a clay system which is much more impervious than the clay in which you are trying to find out the settlements.

The one more thing which is still bothering us is C v is unknown you agree, this is a bigger culprit. So we do not know how to obtain C v because this is some coefficient, now you can always say that if I conduct one dimensional consultant test and if I use this expression which I had written for C v.

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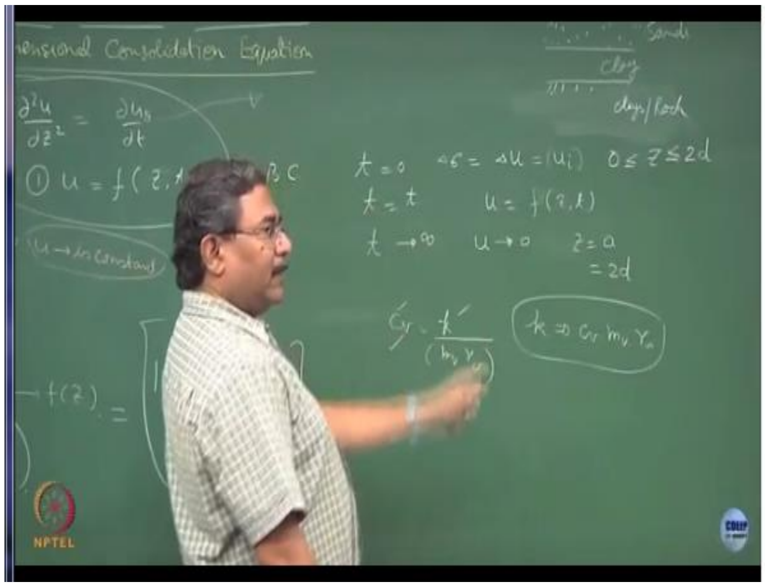

So C v = k over m v into gamma w alright, if I know the hydraulic conductivity of the soil, if I know it is m v value which I can obtain by conducting the one dimension consolidation test gamma w is known, C v can be obtained simple method. Normally this is not done we never obtain C v like this, what we do is, we obtain C v we knows m v, we know gamma w, we compute k. So k = C v into m v into gamma w hydraulic conductivity can be obtained by knowing m v value, C v value and gamma w.

So we differ the problem, the problem is not solved yet, we still do not know C v clear. One interesting thing you must have notice, if I ask you what is the dimension of C v. So this is del square u by del z square del u by del t, u is truly is being a pore water pressure parameter. So the units of C v will be L square by T and hence this is known as the diffusion coefficient. So those of few who might get a chance to do higher research in the THMC, THMCB and all those series of contemporary geo mechanics.

You can replace this term by different types of coefficients. If I replace u by c concentration gradient in the direction of z concentration gradient as with respect to time change of concentration, diffusion coefficient D clear. If I am interested in doing how heat migrates in the geo materials, del square theta is the temperature, rate of change of temperature with respect to distance, rate of change of temperature with respect to time and this coefficient is going to be thermal diffusivity clear that becomes alpha.

So this is how you can interpret the equation now let us come back to the consolidation theory and relook at what we need to do further, is this part ok. If I solve this equation, in analytical form the solution of one dimensional equation can be written as.

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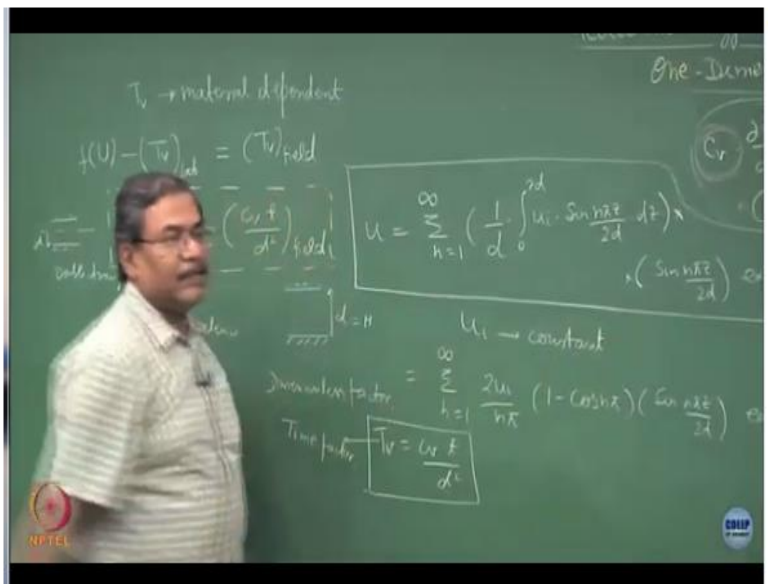

The pore water pressure = n = 1 to infinity, 1 upon d the half of the drainage path 0 to 2d u i sine n pi z upon 2d into dz. And this term is multiplied by sine n pi z upon 2d exponential - n square pi square C v into t upon 4d square. So this is the solution which is known as a Fourier series which you obtain when you solve one dimensional consolidation equation, if I simplify it is further.

If I assume that u i is constant all throughout this can be written as n = 1 to infinity 2 times u i upon n pi 1 - cos of n pi sine n pi z upon 2d exponential this term. If I assume that T v = C v into t upon d square this is what is known as a time factor fine, it is a dimensionless factor. Now what is the interpretation of this parameter T, normally we have used v term here and this defines the drainage which is taking place in the vertical plane because of the vertical loading.

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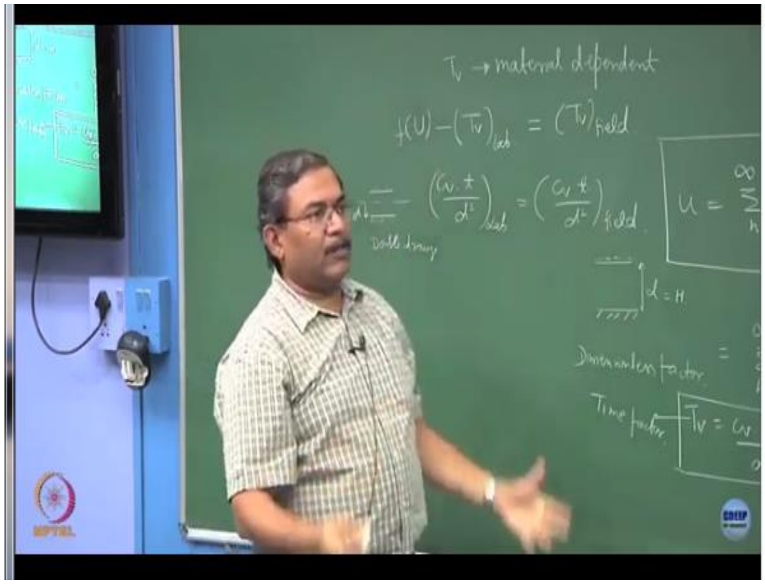

So one of the characteristics of T v is that this is material dependent alright, that means if I conducted laboratory test and if I take out the sample of the soil from the field and the same sample has been tested in the laboratory, I can say that this will be equal to T v in the field fine. There are several possibilities, the possibilities are this T v C v into t upon d square in the lab and this will be equal to C v into t upon d square in the field.

As I said as long as the material is constant same material C v cancels out, so you have a relationship between t and d in the laboratory and the field. So basically you are doing a sort of a modeling of models which you normally do, only thing you have to keep in account is that this should be the drainage path in the field condition. I might be doing a laboratory test by maintaining the you know double drainage condition.

So this is a double drainage condition both sides the drainage is allowed. So in this case, as we discussed just now the drainage path is going to be this however there could be a situation in the field, where the d will become, let us say the total thickness of the deposit, as we discuss some time back, this is equal to H the total thickness of the deposit and this happens to be a draining boundary, the sand layer, so, that is the only difference clear.

So another interpretation could be this is regarding the drainage paths, the second interpretation could be about the time physical time because this T is the physical time you remember, the time at which a pore water pressure is being determined at a given depth. Now what it this might tell you is to achieve a certain value of T v, which is linked with u and u we have defined as the degree of consolidation.

In other words what we are going to get is I am going to get the time required under the laboratory condition and in the field condition which is going to tell me how much amount of degree of consolidation has occurred. In other words, I can simulate by doing a simple laboratory one dimensional consolidation test the real life situation. So this is a equivalence which we use ok, if you solve this equation further, you will be getting some more interesting results.

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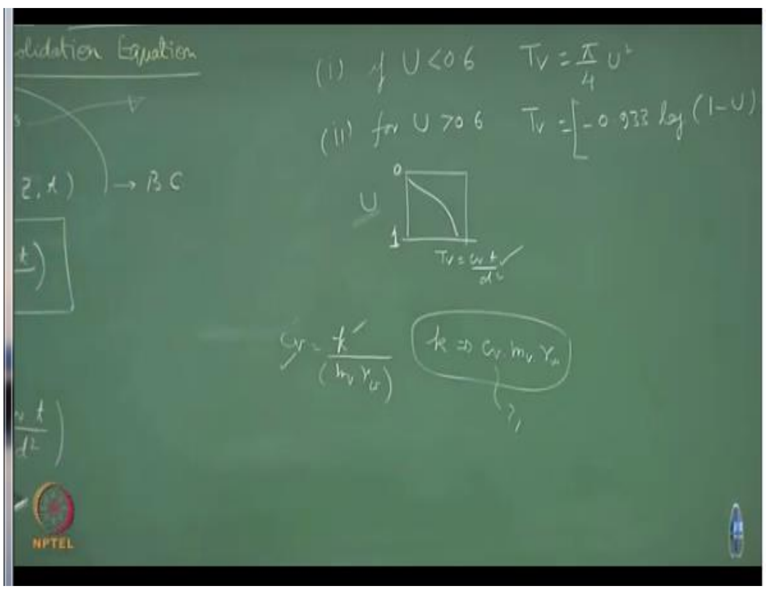

The first one is if the degree of consolidation is less than 60% we get T v = pi by 4 into u square and for u greater than 0.6 the T v value is defined as 0.933 log 1 - u – 0.085. In literature you will get or in the books you will be getting relationships between u 0 to 1 100% I can put it in percentage also or it is up to you. And suppose if I put the T v value here which is equal to C v t upon d square x axis this is how the graph looks like.

The first thing this graph tells you is as physical time increases, the T v value increases means degree of consolidation increases clear and this is what is getting depicted over here. So please be careful as long as the observing the y axis is concerned, this is from 0 to 1 and this is the T v value. I may have different boundary conditions which you can substitute. But the interesting thing here is the relationship between u and T this again is a parabolic equation provided u is less than 60%.

The moment is becomes more than 60% there is a linear nature coming in the picture constant and 1 - u of log this is going to be an exponential term or logarithmic curve. So this culprit is still unknown, is it not, so we have to do something to get C v value. Because what we have done is very conveniently we have used this term over here also to define the non dimensional time factor, which is an integral part of the one dimensional consolidation equation.