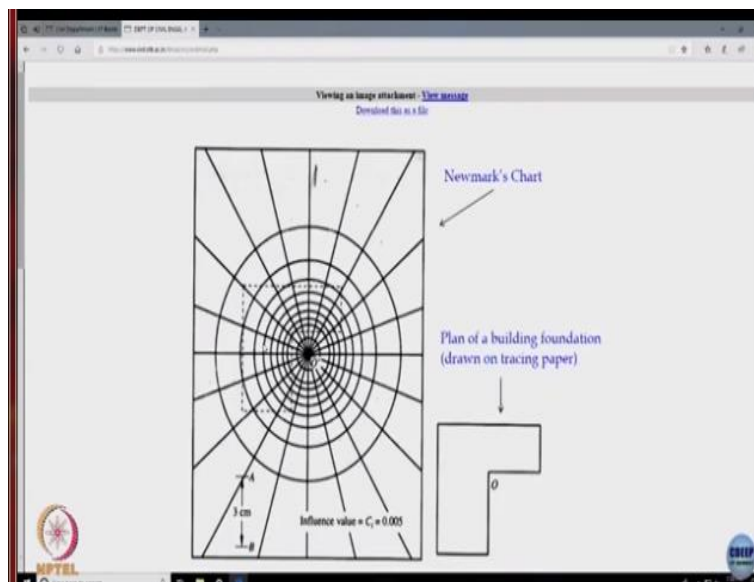


Geotechnical Engineering I
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Lecture-23
Stresses in the Soil mass due to External Loadings-I

I will just start discussion on a new topic which is stresses in the soil mass due to external loadings. Until now, we have discussed about the internal stresses in the soil mass and these internal stresses are let us say pore of water pressures or because of the sulphate.

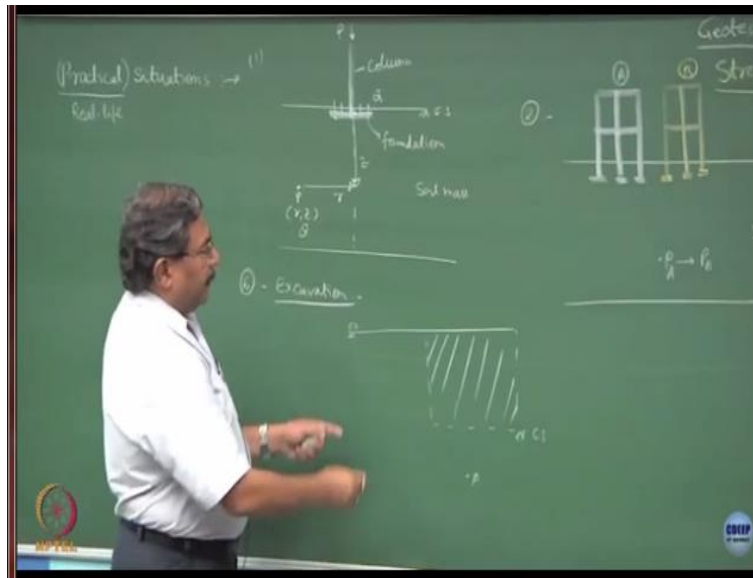
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Now what we are going to do is, we are going to talk about the situation where I would like to know, how the soil is going to respond when external loading is applied to it. Now in engineering applications, this concept has lot of significance and most of the problems which are being tackled by the consulting geotechnical engineers would require these type of concepts to be practiced.

So let me first bring to your attention the types of situations, where the stresses in the soil mass due to external loadings becomes very, very important.

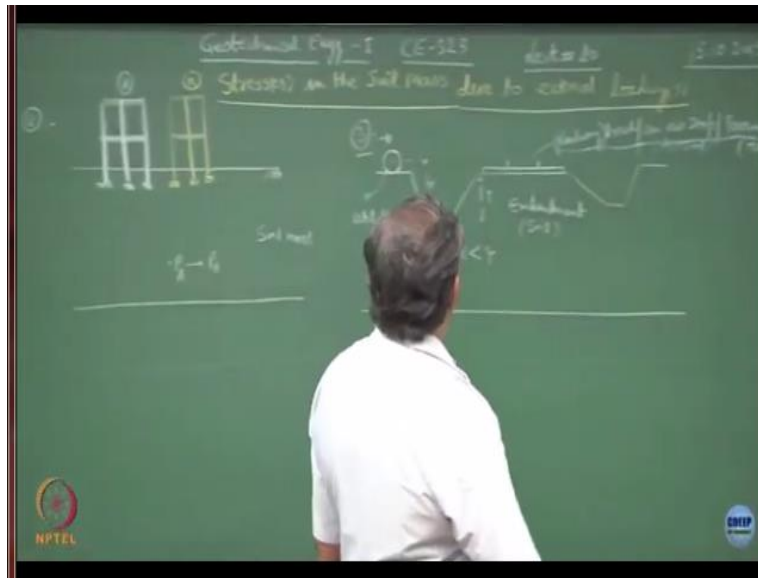
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The first situation could be or we may say this is the real life say I have the ground surface and somewhere here there is the footing which is being placed. So this is the column and this is the foundation and this happens to be the soil mass. It is to understood that this column is going to take load p and hence the footing is going to transmit a pressure Q on the soil mass. I would like to understand, what is the state of stress at this point p .

And this point p can be defined in such a manner that I can say that this is the radial distance r and the depth of this point is z . So basically it has the coordinates r, z I can include θ also here, it could be r, z and θ . So this is situation number 1, when you construct foundations, I would be eager to know what is the state of stress in the soil mass.

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There could be another situation like suppose if I have a multi storey building, so this is a multi storey building and situation like let us say corporate and most of the populated cities in the world, where the cost of land is extremely high. So you cannot afford you know higher spacing between the buildings. So suppose if I ask you a question, what should be the safe distance at which I should be designing or I should be constructing another building, remember space is extremely important, very expensive alright.

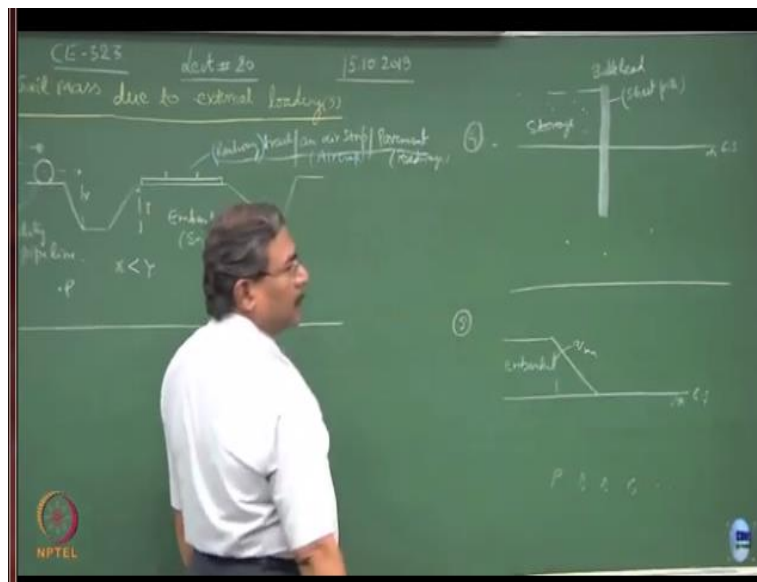
So this is the building which is sitting on the foundation system. Now because of construction of building B if the building A was already existing, what is happened, the state of stress at a given point p which I have taken over there has changed. So p A has changed to p B alright and I would like to understand whether this system is soil is capable of taking this type of loads or not alright, so this is again the soil mass.

There could be a situation number 3 where I am sure when you travel all along the highways or the railways, what you observe is, there the pipelines which are carrying water. So suppose this is the embankment of the railways, so this is the let us say embankment on which the train is moving. It is an embankment made up of soil and this is a railway track, it could be a an airstrip also or this could be a highway, a pavement alright.

And somewhere you must have noticed what we do is we carry, let us say some utilities. So this is the utility pipeline, and I want to see whether do not go by the elevation this could be, you know x this could be y . Most of the time x is going to be less than y and hence I will try to understand because of the movement of the traffic what I have done is I have simulated 3 type of traffics. One is the railways, another one is the aircraft, the third one is roadways.

So I would like to understand whether this pipeline is safe or not. That means because of the movement of the vehicles here, what is going to happen to the pipelines alright, this could be buried pipelines also. So I would like to find out what is the state of stress somewhere here again, the point p shifts. Now point p could be in anywhere in that domain and what I would like to do the analysis, there could be a third situation we have been talking about fourth situation

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This is a ground surface and as we discussed in the last lecture the application of sheet piles I would like to create a bulkhead over here. So this becomes a bulkhead, this is the soil mass of water this could be storage and I would like to know, what is the state of stress at a point let us say somewhere here. Now suppose, if I remove the piles this is the sheet pile and if I say that in a simple situation, where I have the ground surface and I would like to retain an embankment the same embankment which you have created over here.

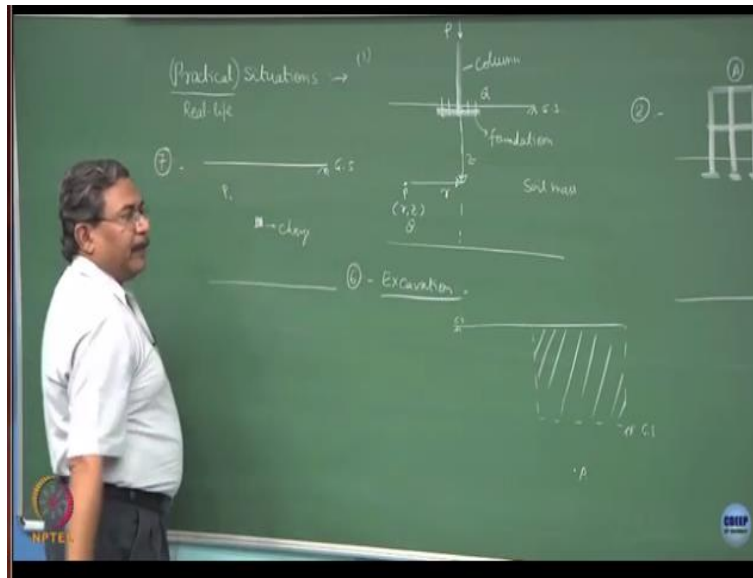
So this is the embankment and this embankment can be utilized for creating let us say oil foundation, tank foundations, storage whatever. So suppose if I want to find out the moment I have created this embankment, what is the state of stress at different points in the soil mass. So p might be varying here, you know p , p_1 , p_2 , p_3 and so on, what has changed over here is the type of loading which we have, has become a triangular loading.

At this point the loading intensity 0, as you go this side the loading increases to q_{\max} and then it remains constant. So this becomes a typical loading condition which you know imposing on the soil mass, there could be another situation where we might be doing excavations. So this is let us say, the loading of the soil mass and this could be let us say excavation.

I would like to excavate the soil and I would like to create a facility like this. So this is the initial ground surface and this becomes the final ground surface. If I remove this much of the material, this much material has been removed and I will not to find out after removal of this material what is the state of stress at this point. Because ultimately this system has to stay stable it is unsupported, here this is supported with the help of a sheet pile alright.

So these are different examples of the situations which we might come across in today's you know civilization. What I want to know is, what is the state of stress at point p , define the state of stress at point p and why we want to do this. Because if I know the state of stress at point p , I can link it with the yield strength of the soil, you may think of any other situation which you think is practical, I am a create a situation like this also.

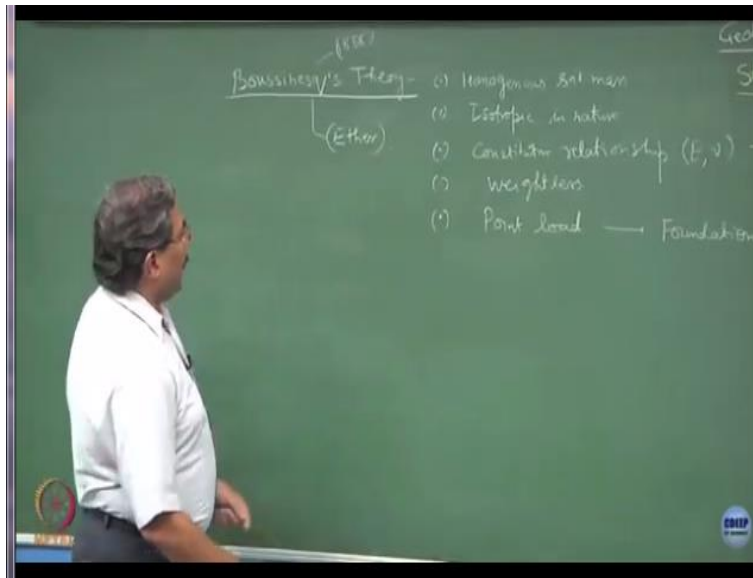
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Suppose if I do a let us say, the detonation of an explosive alright. So what I will do is, I will bury the charge over here and I can charge it. So what is going to happen, this is going to create shear waves, compression waves and because of this the state of stress is going to change at point p. So these are very critical situations which you may come across alright. So what we will do is, the general principle remains same except for the situation, where I have talked about the blast loading which is a different scenario altogether.

Rest 1 to 6 we can easily solve by considering the concepts which I am going to discuss in the class today alright. And as a undergraduate student, as a beginner in the subject, you should be aware of situation 1 to 6, how to analyze them fine. The best way to define the state of stress in the soil mass because of the external loading is by considering the Boussinesq's equation or Boussinesq's philosophy. So as I said, we are talking about the stresses in the soil mass due to external loadings.

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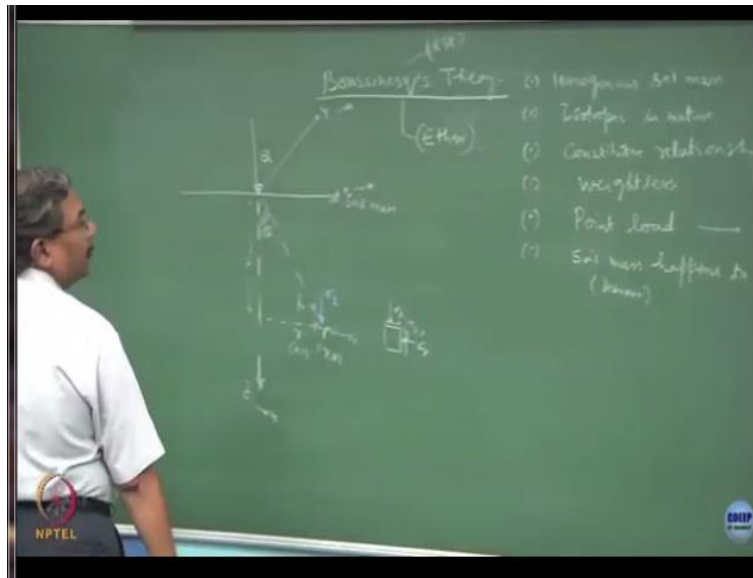


This is where the Boussinesq's theory is used in 1858, this was you know proposed by Boussinesq what this theory states is. That for homogeneous soil mass which is isotropic in nature which follows the constitutive relationship. That means elastic modulus is defined, Poisson's ratio is defined and hence we are assuming the soil to be linearly elastic material ok. One interesting thing is that this soil mass has been assumed to be weightless, did you notice this, that the soils mass has been assumed to be weightless, why.

We are making the soil mass devoid of it is weight because weight causes the internal stresses in the forms of either self fix or the pore water pressures. So it has been assume that, as if we are computing the state of stress in ether, sometimes these type of soils are also known as ponderable soils, check it on net, what is the meaning of word ponderable. These soils are talking to themselves they are pondering they do not know the way they have been modern their properties are different, but somebody is modeling them in a different manner.

But if these relationships are valid homogeneous, isotropic, constitute relationship is valid, weightless material and the point load. The external load which is acting on the soil mass is a point load, where do you come across point loads, as I said foundations of columns. So the moment I say raft this is not a point load. So foundations of the columns are point loads alright, if I converted into a raft this becomes a uniformly distributed load area wise.

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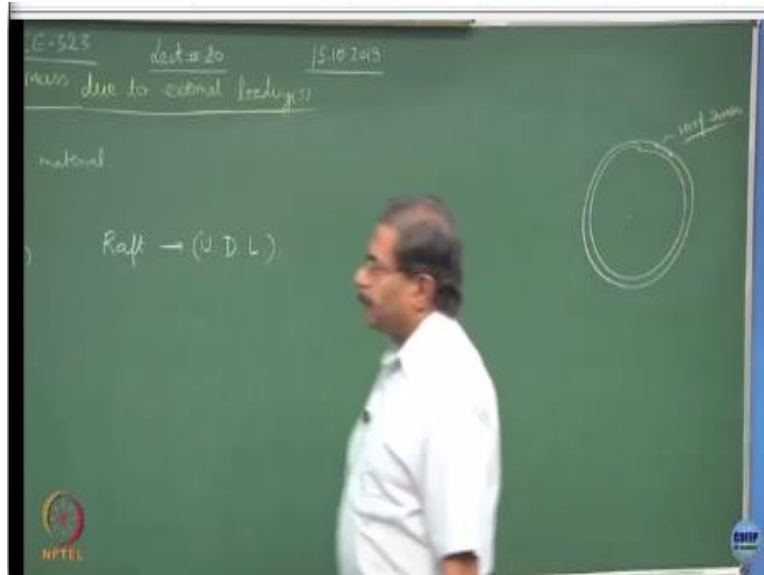


So as per this, if you have the soil mass which is exposed to an external loading of Q and if I want to find out what is the state of stress at a point p in the soil mass, the radial distance is r , this distance is capital R . And another assumption here is that the soil mass happens to be semi infinite soil mass. That means the extent of the soil mass is semi infinite, we call it thus as a semi infinite soil mass alright, soil mass happens to be semi infinite.

Semi infinite means, if this is the direction of z , this is the direction of x and if this is the direction y we are considering the domain. So soil mass is nothing but the domain in which we are doing this stress analysis, this happens to be alright semi infinite. So z is tending to infinity, x is tending to infinity, y is tending to infinity. So this is a domain in which we are doing this stress analysis, what is the state of stress at point p .

The state of stress in point P would be normally let me change the nomenclature we take this as z and this as y , this is as z . So basically this is σ_z , the normal stress which is acting at point p σ_r . I am sure you are doing a course in RCC also, this semester.

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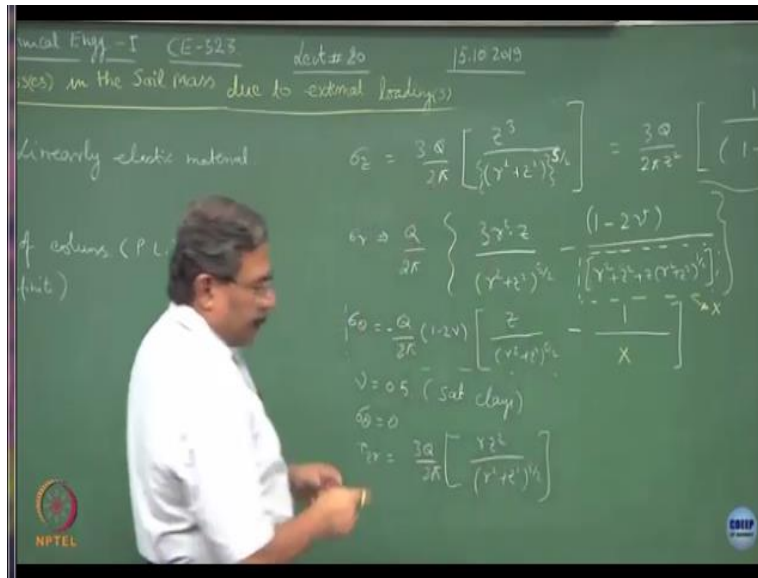


So suppose if I give you a ring of finite thickness alright and suppose this is a pipe which is carrying some fluid and if I ask you to show the annular stresses we call them as hoop stresses. So hoop stresses will be acting in the periphery, circumference alright, so these are the hoop stresses. So we have σ_z , we have σ_r there could be a stress in the direction also but because I have assume this to be isotropic in situation and hence $\sigma_z = \sigma_y$ clear.

If you draw the state of stress at this point you have done a engineering mechanics and you have done mechanics of solids. This is σ_z , σ_r , we will be having the shear forces τ_{zr} ok. And if I want to come out of this coordinate system I can always use the coordinate system if I define this theta and if I say that this is r and theta, is this ok. So this I am assuming as z this distance and if you look in the 3 dimensional at point p , there is a component of r which is a function of x and y , is this ok.

If you look at in 3 dimension, this is a state of stress has been defined as this will be x and y correct, so this will be in $x-y$ plane because z we have already taken alright. So r will be under $\sqrt{x^2 + y^2}$, so this is the plane in which this point is sitting.

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So as per the Boussinesq's the equation is for the point load $\sigma_z = \frac{3Q}{2\pi} \frac{z^3}{(r^2 + z^2)^{5/2}}$ upon $r^2 + z^2$ to the power $\pi/2$. This can be simplified to $\frac{3Q}{2\pi} \frac{z^3}{z^5}$ and this becomes $\frac{3Q}{2\pi} \frac{1}{z^2}$. I think this one is easy to remember alright, Q is the load, z^2 is a sort of area. So this becomes a pressure term, $\frac{3Q}{2\pi}$ is the solution.

Normally what we do is we replace this term as I_B and this is what is defined as the influence factor alright, influence factor I_B . I am sure you must be realizing that state of stress in the soil mass is totally devoid of its properties, what are the properties. Normally we define the properties of the soils or the soil mass as density, void ratio, porosity, saturation, unit weight or pore water pressure ok, what else Poisson's ratio engineering properties.

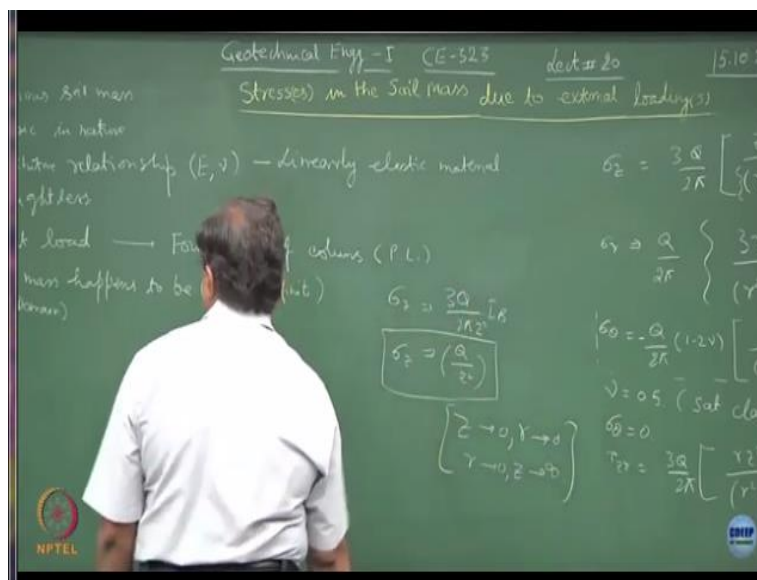
These are the fundamental properties bulk properties, engineering properties are elastic modulus and Poisson's ratio. So these are the soil property none of them appear in this expression because the way the Boussinesq's theory has been derived this is for weightless material alright. Now $\sigma_r = \frac{Q}{2\pi} \left\{ \frac{3r^2 z}{(r^2 + z^2)^{5/2}} - \frac{(1-2\nu)}{[r^2 + z^2 + (r^2 + z^2)^{3/2}]} \right\}$ but yes if you can remember that is ok, $\frac{1-2\nu}{r^2 + z^2 + (r^2 + z^2)^{3/2}}$ alright.

σ_θ is defined as $\frac{Q}{2\pi} \left\{ \frac{1-2\nu}{[r^2 + z^2 + (r^2 + z^2)^{3/2}]} - \frac{z}{(r^2 + z^2)^{3/2}} \right\}$. If I use this term as, let us say capital X , so this becomes

capital X. Now there is interesting thing, if Poisson's ratio is equal to 0.5 which is for saturated soils, saturate clays particularly sigma theta becomes 0. The fourth stress which we have not still considered is tau z r the shear stress.

The shear stress would be $3Q \text{ upon } 2\pi r z^2 \text{ over } r^2 + z^2 \text{ to the power } 5/2$, if you solve this equation, you know the Boussinesq's equation which has been used you will get the sigma theta as the negative term the state of compression and you must be realizing that all these stresses are compressive. So let us say if you take a plate and then compress it from both the sides, there could be a bulging.

So the bulging indicates the negative pressure, so sigma theta is going to be tensile in nature. Now what we can do is, so you can replace all these coefficients with influence factor alright I B. **(Refer Slide Time: 27:54)**

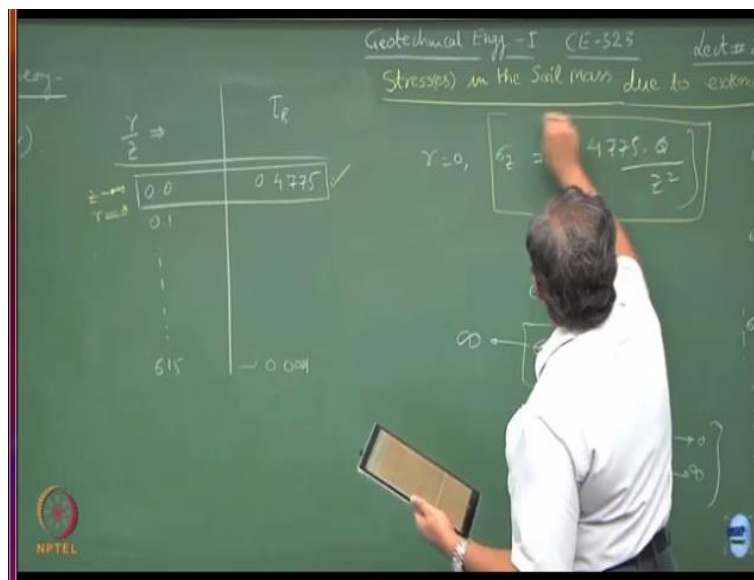


Hence I can write sigma z = $3q \text{ upon } 2\pi \text{ into } I B \text{ and } z^2 \text{ also } 2 \text{ upon } 3 \text{ upon } 2\pi$ also I can take into I B and I can say that this is a function of Q upon z square. Now do you find any abnormality in this expression, see we started with a point load because the Boussinesq's equation is for the point loads. Now suppose if I say that z tends to 0 or for that matter if let us say I say r tends to 0 and z tends to infinity, where this point would be r tends to 0 means on the z axis and z tends to infinity.

This is one of the situations in which I will be interested the second thing which I will be interested is z it tending to 0 r tending to 0. So z tending to 0 r tending to 0 is this point clear, so what is this stress at this point σ_z tends to infinity ok clear. So this is the issue because what you are doing is when r and z tends to 0 this becomes a point load this stress at this point is going to be infinity.

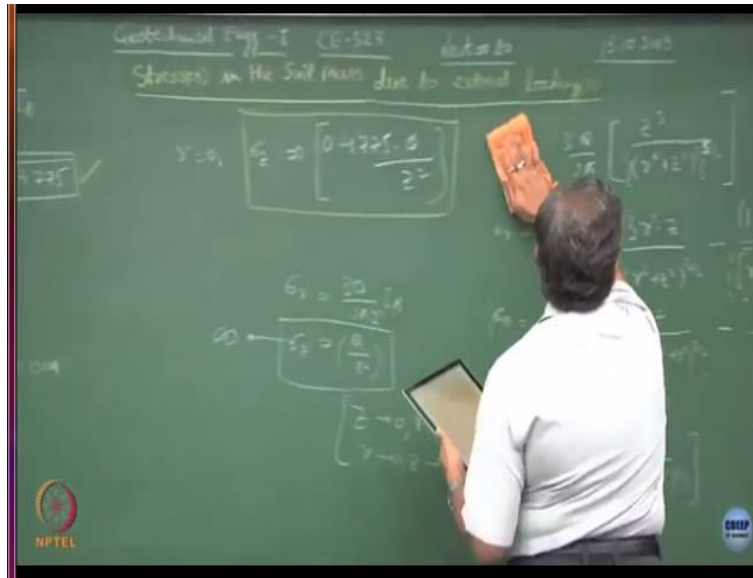
One of them abnormalities of the solutions any way no problem, we as an engineer, you will still go ahead with these solutions and we will try to see how these things can be sorted out, so if you look at this influence chart.

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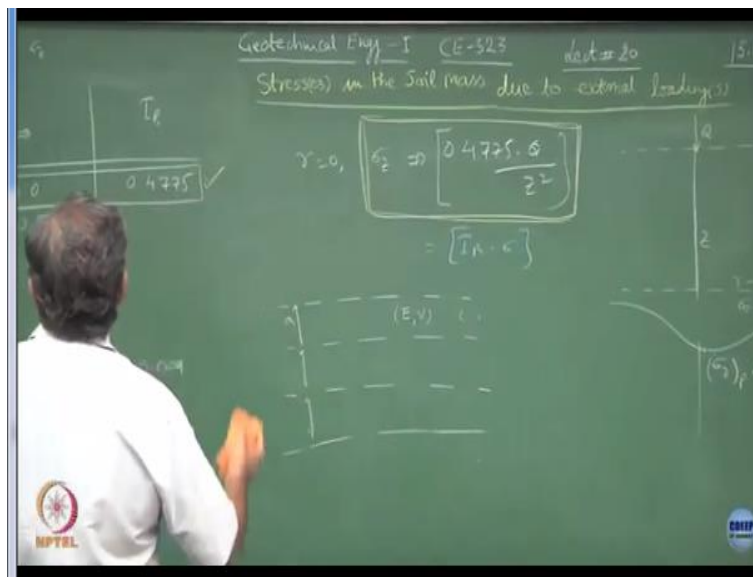
If I substitute the values of r and z and if I get the values of I_B , in that expression for r by $z = 0$, the I_B is 0.4775. You can compute this for different values up to 6.15 and this will be tending to 0.0001. So, for all practical purposes, I can write that at $r = 0$, now this is a very interesting solution which you have obtained. There are 2 possibilities of getting this either z tends to infinity then only we can get r by $z = 0$ or a better possibility will be $r = 0$ itself.

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So this is a solution when $r = 0$, we say that $\sigma_z = 0.4775 Q$ upon z square. Now the point is this is the solution which normally is used to obtain the σ_z now what I want to do is I want to use the Boussinesq's theorem or Boussinesq's relationship for developing 3 things.

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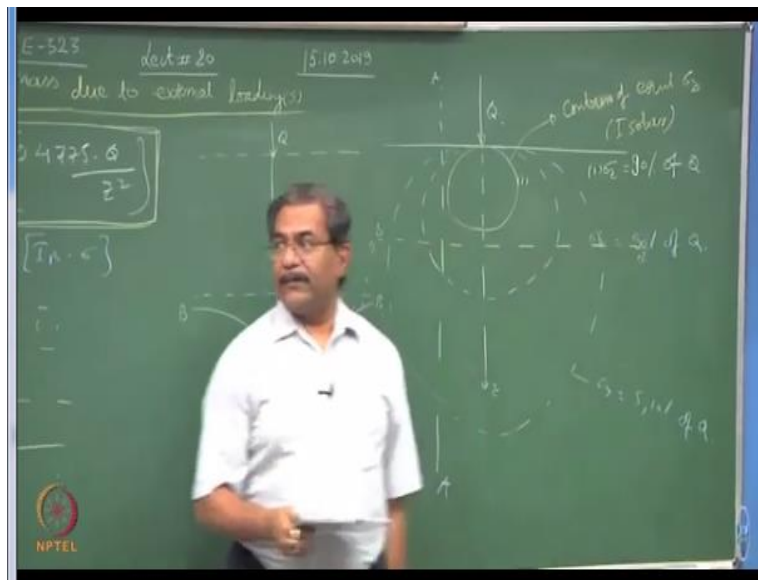


The first thing which I want to develop is the isobars for the point load we call it as a isobar diagram. The second thing is we want to do the pressure distribution, so when I say pressure distribution this is your σ_z , σ_r , σ_θ , τ_{rz} this is a state of stress acting at a point p because of the external loading Q . So pressure distribution on a horizontal plane and the third thing would be to get the pressure distribution on a vertical plane, fine.

So these 3 things I want to do, now let me introduce the concept of the isobars, isobars are the contours of equal vertical stresses. That means, if σ_z remains constant and if I plot the pressure lines or the pressure contours they become isobars. So these are the contours of equal σ_z vertical pressure, how do plot them. Now, I am sure you can realize from here if I what is a form of this equation between σ_z and z , Q is a constant, is it not.

So what is the form of this equation, if I write like this z^2 into σ_z is constant. Suppose this is y^2 into x is equal to constant, what is this graph hyperbola clear. So that means all the contours of the stresses are going to be hyperbole functions.

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That means, if I take let us say Q and this is the point and this is my z direction. If I plot let us say 10% of Q as σ_z 90% let us say, where this would be sitting, it would be sitting like this. Please mind the discontinuity at this point which we discussed just now. So at this point the function is discontinuous, it would look like this, it is not a circle, it is a hyperbolic function. So this is corresponding to 1 as the intensity of the pressure decreases, what will happen.

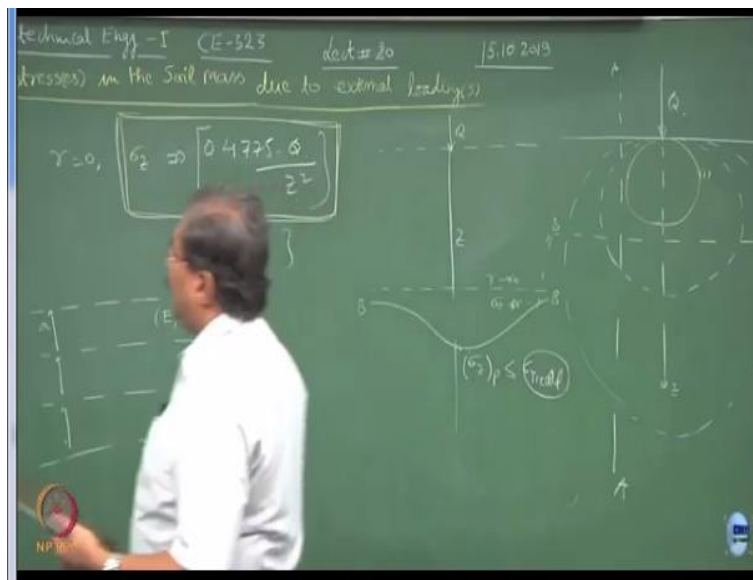
Somewhere here, you will have this would be let us say $\sigma_z =$ some 50% of Q and this will go up to certain value, where I must may that σ_z has become let us say 5 or 10% of the Q value because this influence factor is doing all the tricks you are right. But see, if you look at this

point again this is nothing but the point p 1, p 2 p 3 and so on. So r is changing z is changing, which is getting absorbed in this term, I B is a constant term is represents r r by z.

So that means, a better way of writing this would be I can say I B into Q upon z square the stress term, so this I can write as stress term sigma. So I B is a function of r z, so the basically the contours of this pressure bulbs we call them sometimes we call them as isobars are inclusive of r and z and they depict the state of stress alright. Now this is one thing which you wanted to do, the second thing which you want to do is, we want to do the pressure distribution.

So if this point is Q and if I ask you to draw the vertical pressure distribution and the horizontal distribution of the pressures.

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This is the ground surface at a depth of z, if I create a plane this is a plane at which I want to see the variation of sigma z as if you are cutting all these pressure bulbs along this axis. So if there is a plane which is passing through this, is the same plane as what I am showing over here. So the pressure distribution would be like, why the wetting takes place of the pavements, this is the answer.

So, excessive loading the tire pressure from the vehicles creates the maximum intensity of the pressure at a certain distance z just below the point of application, and beyond which what

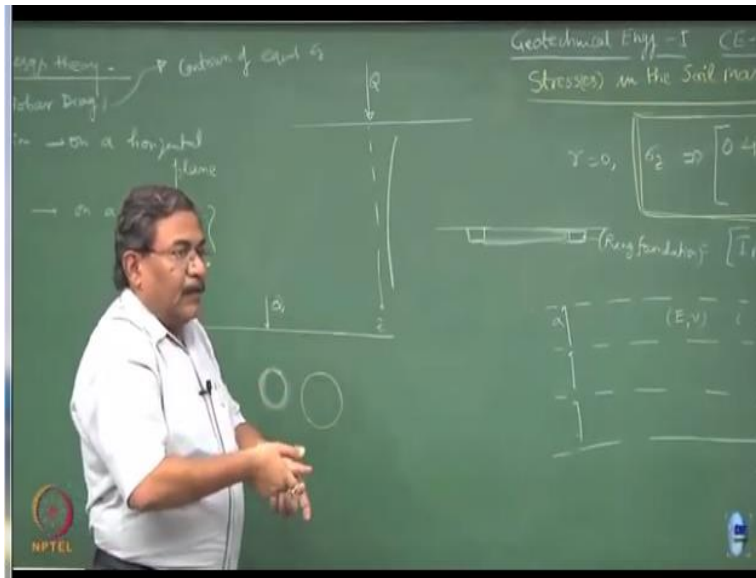
happens it fades out. So the moment r tends to infinity what happens to your σ_z value, this tends to some constant value at this plane, otherwise this is going to be the maximum value. So, this is the pressure distribution in the horizontal plane.

As a transportation engineer particularly if I am working on the materials I will be very much eager to know. If I am designing a runway, let us say I always give this example when I travel particularly with pilots, I do not know whether you are aware or not, but most of the runways are designed based on the layered theory, multiple layers of the material sitting over each other.

This is a layer number 1, number 2, number 3, number 4 we play with the stiffness of the material. So stiffness of the material is defined as Poisson's ratio and elastic modulus and the thickness ok. So if I want to minimize the influence of the σ_z on a horizontal plane, what I should be doing is, I should be playing with this cross section, so that I can nullify the effect of σ_z .

And I can make it a point, that at this point, σ_z is going to be much less than σ_{yield} , this is what you will be studying in your transportation in course, this is how the pavements are designed fine. Now another interesting thing which I would like to do is I would like to look the variation of the stresses on the vertical plane. So as if I am cutting this whole thing across a vertical plane, so this is let us say A A, this was B B, so this is B B.

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And the A A would be if this is Q, these are the contours of this is what I have written there, these are the pressure bulbs contours of equal σ_z 's known as isobars. So, this whole yellow color bulb is going to be representing $\sigma_z = 90, 95\%$ of the stress which are getting transmitted to the soil mass not the inside part these are the contours of the points, you know he was discussing each and every point on this surface is going to be of 90% Q value.

So, that is why we call them as contours of equal σ_z 's fine. So as you come deeper, the pressure and density is decreasing, this becomes only 5 to 10% of the Q value and you can use this expression which I had written over here to compute what is the state of stress and you can plot it over here. So this would be a function like this it cannot increase, so I have to curtail it somewhere here ok.

So, starting from a very high stress value the σ_z that is decreasing as z increases. So this becomes the variation on a horizontal plane, this is the variation on a vertical plane. Now depends upon what type of problem you are dealing with. Suppose I am designing a pipeline buried pipeline, there could be a situation that this is my buried pipeline. This project I did recently for Santa Cruz airport. I hope you are aware that they are produced to be shut down for several hours during March and February.

So it is a active run way at run way cannot be stopped permanently. But I am sure next time when you are travelling, you should see there are been 2 pipelines which of the oil which have been taken across the active runway. That means the landing and taking of operations are occurring and you would like to take out some utility across the runway, simple losses like this can help you. I can define what is the zone of influence of the pressures.

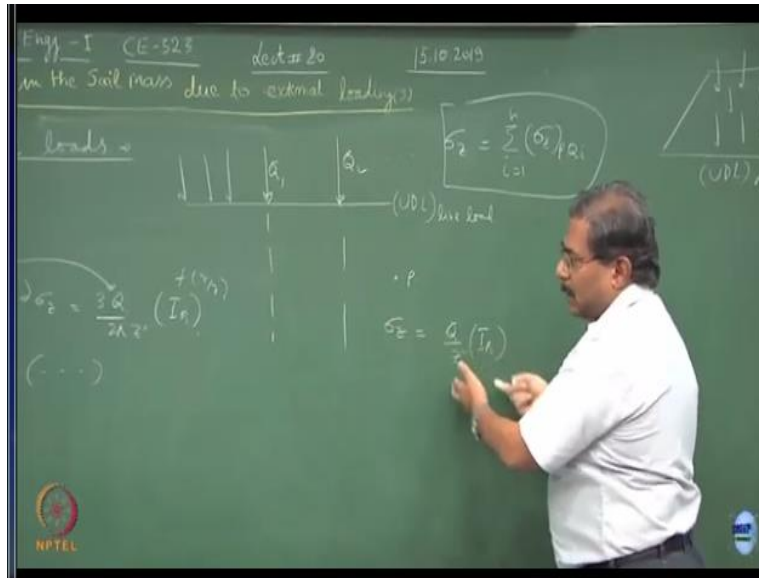
And I can negotiate by locating the pipelines there, every time I cannot be so lucky. So then you have to do engineering based on the metal properties and whatever. Anyway, so give me an example, suppose this is how the load comes alright and I want to see whether this pipeline is going to get influence because of external loading or not. If I know the value of Q , I can find out what is the state of stress at the crown of the pipe, pipe is made up of some element either structural element would be steel or mild steel or concrete or whatever.

And then I can compute what is the yield strength of the pipe and then we can design it accordingly fine is another good example of where the application of these type of computations would be in real life. So is always the vertical stresses which are going to be much more detrimental as compared to in plane or perpendicular to the plane stresses. But suppose, if I am designing a tank foundation which you must be seen big tank forms in your structure.

So this becomes a ring foundation, normally oil the tanks are seated on the ring foundation, I need not to spend so much money because rafts are going to be expensive, what I will do is I will cut them into ring, is this ok. So as if a ring is wearing the entire load of the tank, in this case your σ_θ is going to be very important r_θ . Because this is going to be the hoop stress clear, so this design will be based on σ_r .

However coming back to your question, why we are only interested in σ_z because σ_z happens with the most critical stress which is going to govern the stability of the structure as well as the soil mass. In case yield occurs at any of these points, your structure cannot remain stable, we will discuss these things in a second course fine.

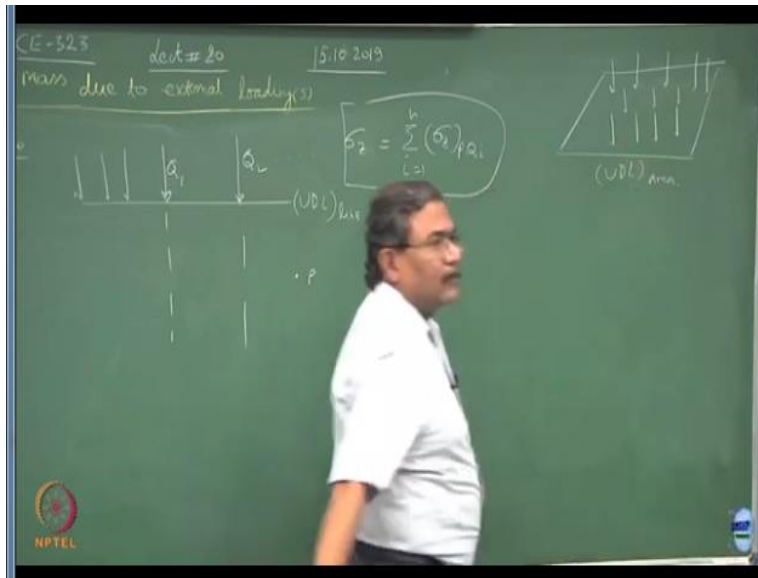
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Now let us discuss the concept of superimposition of the loadings, how many types of superimposition principles you have studied until now. So there you superimpose is it not, suppose there is a cantilever and I can superimpose the load in such a manner to get a certain value of the deflection, what is the name of this method, in structure analysis. Suppose, I started with the first situation there is Q_1 acting here and this is Q_2 acting here and find out what is the state of stress at any point let say somewhere here.

So the best way of doing this would be, you assume them independently find out the state of stress corresponding to Q_1 first at this point Q_2 at this point and sum them up. So σ_z will be equal to summation of if I have series of loads over here, this will become σ_z at point p because of Q_1 , so this becomes $i = 1$ to n , have you understood this. So what I have done, a point load has got converted into a line load, practically there is no difference between the situation you know, where several Q 's are acting all along the line ok.

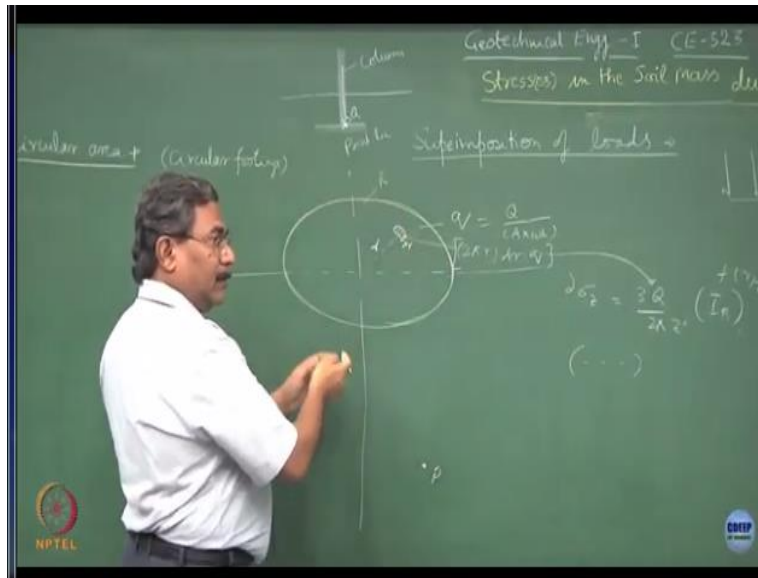
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It could be UDL also, why not, suppose if you take a plane on which there are several Q 's acting several Q 's. Now if I integrate on the surface, this is the integration over the line, now if I integrate over the surface, this is going to give me the net effect of a loaded area of certain area of cross section. So very simply, I have converted the context of the situation, from point load where we were defining the stresses at $r = 0, z = 0$.

I have created a uniformly distributed load in the form of the line load and uniformly distributed load on the area is ok rest will all integration, is this ok. So if I integrate over these things I will get the same effect. In other words as long as the situation is very simple, I can keep on clubbing for $Q_1, Q_2, Q_3, Q_4, Q_5, Q_6$ and I will say that this is a total sigma that acting at this point.

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So suppose if I ask you to find out the second situation from point load, we will move on to the uniformly distributed circular area. Until now we have talked about a foundation system you know, where this is a column. So this is the point load the best possible approximation would be that the Q is acting at a given point though the foundation also is of finite but this is a approximation as compared to the domain of semi infinite soil mass clear.

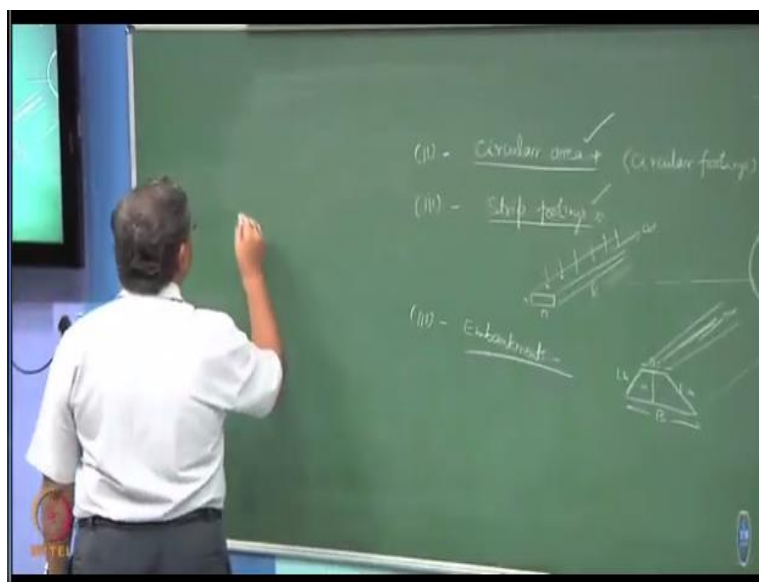
Now when I come to circular area circle footings is a good example, I might be having a situation like this, this is the area which is carrying a uniformly distributed load of q , where q is equal to capital Q upon area of cross section. And suppose if I ask you to find out, what is the state of stress at a given point p , so this whole thing is resting on the ground. And because of this, you want to find out what is the state of stress at point p , you have done lot of integration in your 10 + 2 or whatever (()) (51:12) 10 + 2.

So suppose if I consider an element added up as a radial distance of r small r this is the radius of r the entire area. And if I consider a small element over here as dr ok r into dr clear, area of section of this can be known that will be $2\pi r$ into dr . And if I know the loading intensity q , it gets multiplied by small q this is the load nothing, so great. What I have done is the entire circular area which is uniformly distributed loading, I have converted again it to a point load.

And I am assuming here that the circular loading is also an assembly of several point loads acting on the soil mass fine, is this ok. So your function $\sigma_z = \frac{3Q}{2\pi z^3} I_B$ can still be used, I will replace this by $\frac{\partial \sigma_z}{\partial r}$ and now I_B is the tricky term. So this is a function of r by B r by z clear, so this Q is equal to this one. So I can substitute this value over here and I can integrate.

So I can get the expressions for the σ_z due to the circular loading is this part clear. Fortunately, I hope you will realize the way we assumed I_B over here in case of point loading. If you remember what we did, we simply said $\sigma_z = \frac{Q}{z^2}$ into I_B ok. So this term you can form a table and that table you can always refer to for different r by z and you get the values of σ_z . The same thing you can do over here also, conceptually things are same fine.

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Another interesting thing sometimes what we do is, rather than having a circular footings we might be having, let us say strip footings. Whenever you see your fences in the hospital or in the buildings you know fences are maybe boundary walls or compound walls, they are all sitting on a foundation which goes in the 1 dimension, why. This is the foundation part, which will be extending up to infinity, when we say infinity, when the width of the foundation v .

And thickness of the foundation is much smaller than the length of the foundation clear, this becomes a strip footing 1 dimensional footings. Because their width and thicknesses are very

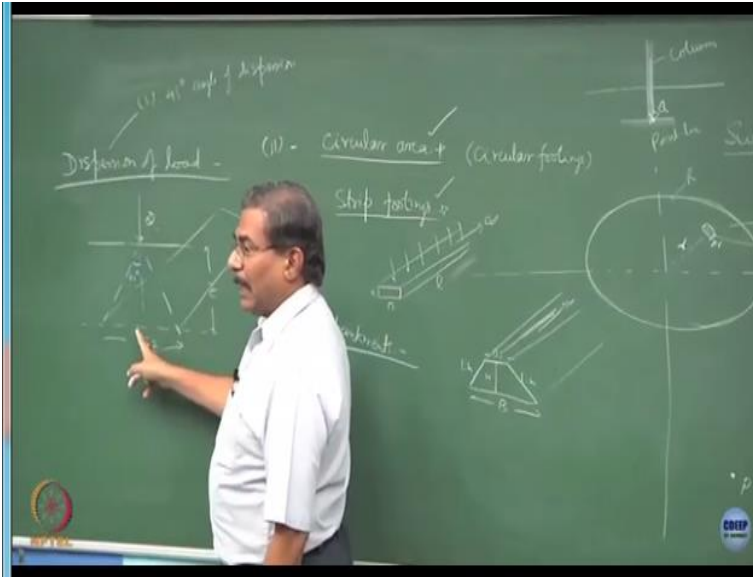
small as compared to the length is you need not to worry. Now this again has become a line load situation clear. So you can integrate over all intermediate q values and you can find out the stresses acting at a point, what is going to change is, I hope you will realize that this dr term which is going to come over here, when you integrate it, this is going to be dr .

So here r has to be from 0 to r clear and what about this z term, this will be from the given point to the point, where you are interest in finding out the pressure, so that means 0 to z . Same thing we can do over here, so this becomes a case of a strip footings circular footings. Now whenever you make embankments as far as the foundation part is concerned, this is nothing but the strip foundations embankments.

Embankments are nothing but the dams which we were discussing in the previous lecture as far as the seepage losses was concerned. So you have certain specific width at the base slopes, $1 : n$, $1 : m$ if you remember, we have to define the height, we have to define the top width of the embankment and this embankment also turns into infinite length, railway embankment. So railways are passing through kilometers of lengths of this type of embankments agreed.

So suppose if I ask you to find out what is the state of stress at the bottom point, no problems you can still model it. The entire loading can be now there is a trick, the load is going to act over here. The railways are going to run over here, the roadways are here ok, aircraft loading is here, we have to delegate the stress at the base of the embankment clear. I use the word delegation. So far a better word for this is the dispersion of the loading.

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So suppose there is a Q load which is acting and I want to find out what is the state of stress in the gross terms as a designer at a depth of let us say z and I do not have time to do these analysis. I mean, I am just being first DPR to tell you, what is the cost of the project. So I do not want to go into the intricacies I do not know how much time it is going to take me to analyze all these things, what I can do simply there are 2, 3 ways of this dispersion of the loads.

The first one is we normally use the concept of 45 degree dispersion angle, that means if this is a Q , I will disperse it on this plane at 45 degree these are thumb rules. So I am sure you can realize now if this is z 45 degree, this is also going to be z clear and this also going to be z . So basically Q has been delegated to $2z$ and then the third dimension is perpendicular to the board. So this becomes your loaded area, what I have done is.

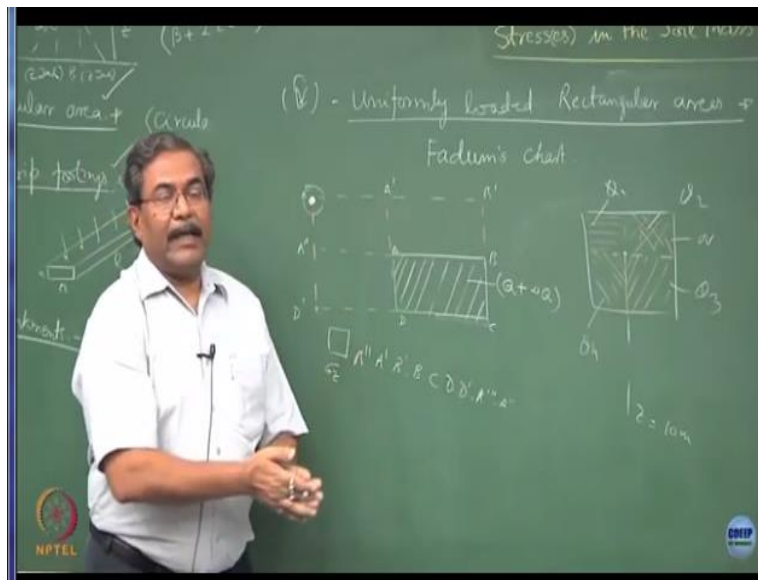
I have delegated the stresses from this point to this, so when you do design foundations normally we give you a foundation pad. So what this is going to do, I get 2, 3 advantages, I have created a pack for the foundation system and I have reduce the stresses, this is the engineering. Another method of dispersion would be, sometimes they have a tendency to disperse it at 30 degree angle. I mean do not ask me from where it has come this is just based on people's whims and fancies.

So this 45 degree becomes a 30 degree angle like alright this is also valid for let us say, there is a finite foundation of width d thickness and if I want to disperse this load on this plane which is at

a depth of z . So if I use this angular dispersion as α this is going to be what, $z \tan \alpha$ and this is also $z \tan \alpha + B$. So this basically, this whole stress is getting transmitted now on a plane which is of area of cross section 2 times $z \tan \alpha$.

You must be wondering, how these constructions are done on the marshy lands, this is a trick, hope you are getting the point mechanistic point, is this clear. If I expose the marshy lands to direct loading nothing can be done, you cannot even stand there. But if I create a good foundation pad, I can disperse the load depending upon the z this area is going to be extremely higher and hence the bearing can be obtained.

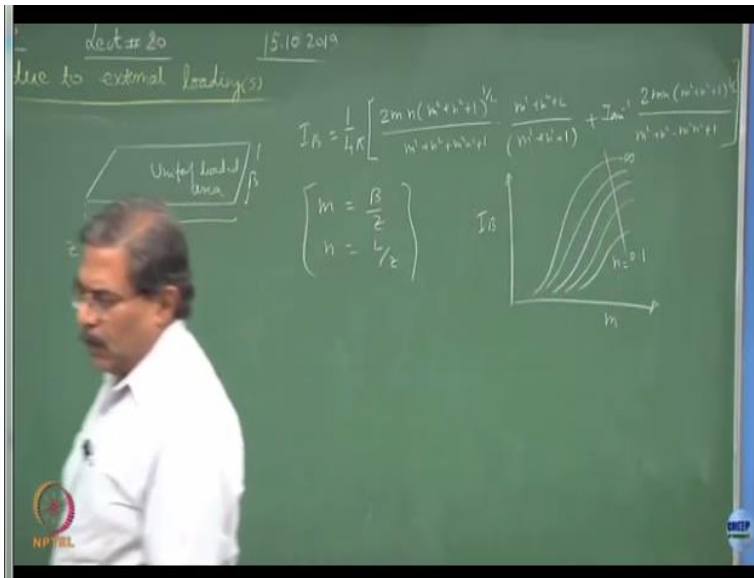
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Now something of interest would be uniformly loaded rectangular areas, to make our life easy there was a person known as Fadum, this person has already given charts. So we call them as Fadum's chart, the beauty of these charts is, if you have a uniformly loaded area which is rectangle of length L width B . And please understand that these methods have limitations or their particular methods.

So you can find out the state of stress only at the edges at a depth of z point p alright. Now let me first interpret this graph is a very longish expression.

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The influence factor would be $\frac{1}{4\pi} \frac{2mn(m^2+n^2+1)^{1/2}}{m^2+n^2+1} + \frac{I_A}{4\pi} \frac{2mn(m^2+n^2+1)^{1/2}}{m^2+n^2+1}$ upon m^2+n^2+1 multiplied by m^2+n^2+2 over $m^2+n^2+1 + \tan^{-1} \frac{2mn(m^2+n^2+1)^{1/2}}{m^2+n^2+1}$. So please do not remember ever, where $m = B$ by z and $n = L$ by z , it does not matter if you interchange B or L .

So it does not matter whether m or n get replaced or interchanged, conceptually these are nothing but the graphs which look like this, the Fadum's chart, this is the influence factor I_B , this is m and then this is how the n varies alright. This is a peculiar solution, one of the solutions I would say, if you have a informally loaded area of B and L . And if you have to find out the state of stress exactly at the edge or the corner point of the loaded area, alright.

Manipulations can be done in several ways. I hope you will realize and you may enjoy that if I have a uniformly loaded plate oh let me not segmentize it first. Suppose this is the plate or this is an area where q is known. So I hope you remember that $\sigma_z = I_B$ into Q by z^2 clear. So the moment you get I_B from this charts of this equation, just substitute over here you get the value of σ_z .

Now suppose if I ask you a to find out what is the state of stress just beneath the C g of this loaded area at a depth of let us say 10 meters, can you use Fadum's chart no is also correct

because look at this situation. So it has to be at the just beneath the corner ok, so what I can do is, I can discretize this whole thing in such a manner that I will create 4 units out of it clear. Now I think you can understand I can apply this concept fine, is this clear.

So 1 unit at the edge no issues I can find it out another unit at the edge I can find out another unit at the edge I can find out fourth and I will superimpose all of them. As if there were 4 sectors Q 1, Q 2, Q 3, Q 4 done, no issues, very good. Suppose if I say find out the, this is the loaded area let us say foundation and I want you to find out the state of z at this point. Please remember when I say state of a stress point this point means in the z direction at a certain depth clear, how will you use this concept now, do a bit of geometry, is this ok.

So I can do like this, I have made this section as a subset of a bigger square and I can solve it, apply your mind a bit and try to understand in what way the context of the application of Fadum's chart has changed, the loaded areas only this much agreed. So if I use method of superimposition, what I will have to do, if this is A, B, C, D and what I have done is I have projected on a bigger scale A prime, B prime, A double prime, A triple prime and D prime clear.

So if I have to find out the state of stress at this point using these charts at depth z , I will find out with the whole thing as the rectangle. So A double prime, A prime, B prime B, C, D prime, A triple prime, A double prime clear. So this becomes my rectangle first compute the state of stress through **this**, this place area does not exist clear. Now how would you use this super imposition principle you will have to subtract the effect of stress which is coming because of extended portions that is it.

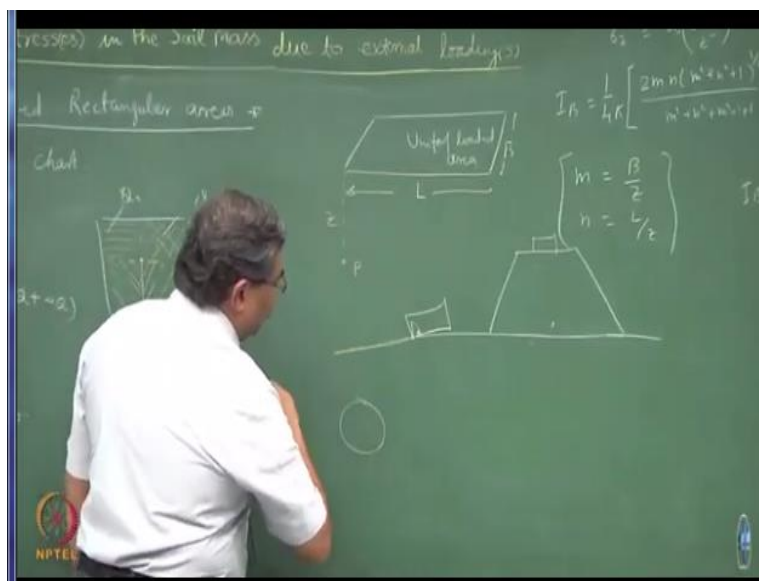
So method of super imposition can be done in 2 ways either you club the stresses make them summed up or you can use them as a subtraction also. So this portion is not contributing and hence I can subtract the stress which are going to come because of this strip and the process what has happened this strip has come 2 times. So add once clear and then you have the bigger one apply the correction.

You remember what we did, this was a loaded area and you were trying to find out what is the state of stress over here. So this is just a way to apply Fadum's charts for computing the state of stress is very simple as far as the mechanics is concerned, but the application part is going to be very interesting. So when you enter into the field, where you are a consultant and when you are applying these concepts how would you apply these concepts, very interesting situations.

Look at the modern day failures, I do not know how many of you are watching the YouTube's, where the major failures are occurring in the eastern part of the countries of the dams, are you aware of this, what the system is going to do over here. Suppose see where the litigation comes in the picture, are you getting this point, where you will earn money. I do not know whether you have ever realize this or not.

This is your house, you are living comfortably over here and then one day one builder comes and starts constructing something over here.

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Now what is going to happen, is this situation not similar to the one which you have analyzed just now, you were living over here and then some activity is going to happen over here. So this Q has become $Q + \Delta Q$ and because of this $Q + \Delta Q$ at this point the state of stress is changing. And we have done a lot of analysis, you know we have computed σ_{total} we have computed σ_{prime} , we have computed pore water pressures.

So that means all this is going to create a detrimental effect ok, so this was the example problem, second example problem. I might be having my house here and tomorrow railways decides to take a railway track close to my home, are you going to allow and do these computations in a clandestine manner and do not disclose that your house is going to be safe. So I can find out because of this extra loading what is going to happen at this place, vibrations is another issue as far as the loading is concerned.

I can find out how the state of stress is going to change and whether this is going to be acceptable to me or not fine. Because from next lecture onwards we are going to talk about the application of all these concepts in the form of consolidation. So if I am constructing something over here and if this system, you know gets distressed and god forbid, if this happens to be a heritage building. But then somebody has to safeguard that this operation goes on and we take guarantee that nothing is going to happen to the heritage building.

So you cannot just say like this, what you will be doing, you will be doing simple stress analysis and show here we are talking about loading now look at another situation. Your houses here and I am doing excavation inside, so I have created 3 situations. Now a person like me cannot sleep over here because I am an expert in my subject. Things which appear to be very simple on the blackboard and the paper, when they get delegated into the real life is the practice of the subject which is very, very interesting.