Geotechnical Engineering I

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Lecture-22
Flow net in the Earthen Dam-II

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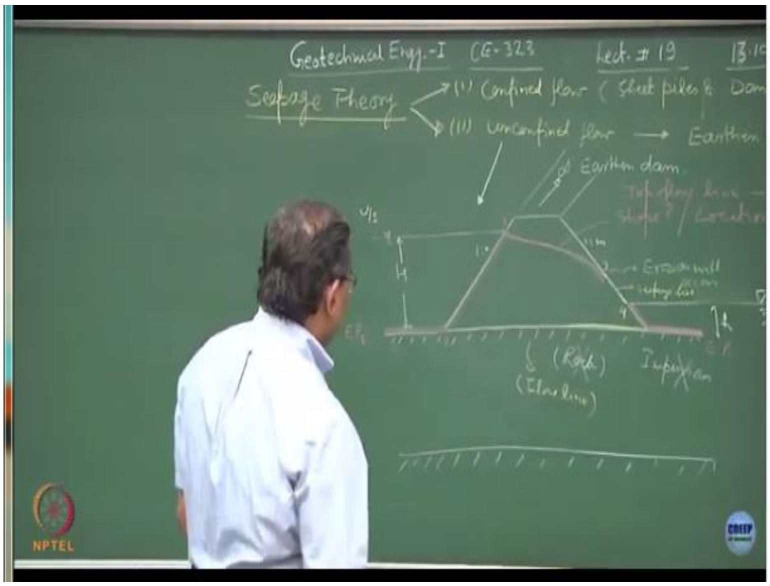

So we have applied seepage theory to 2 cases the one is the confined flow, mostly in case of the sheet piles and let say dam or weir. In today's class I will be discussing about the unconfined flow which is a characteristic of earthen dams and particularly homogeneous cross sections. And of course, to simplify the things I am assuming this to be isotropic that means k x = k z = k y.

By definition, the unconfined flow is occurring in a situation like this, that if I consider an earthen dam. So this is an earthen dam it is a cross section in fact, is a embankment, we can call this as an embankment also. This is a third dimension which is going up to the infinity into the plane perpendicular to the blackboard. Normally we define the slopes as 1 : m and 1 : n, where 1 is vertical and n is horizontal.

We might assume this to be impervious, it is rock and then suppose if I am retaining water over here of height H. I want to establish the domain of seepage which is occurring through the body of the earthen dam. Now based on what we discussed in the previous lecture, I hope you can realize that this is an equipotential, what about this surface line 1, 2 sorry this is also an equipotential, agreed.

So these are equipotential lines or equipotentials we will say, forget about lines and all, these are equipotentials. Can you tell me where the flow line would be, similarly on the downstream side if I say that this is the height of water which is being retained, so this is upstream side, this is a downstream side. I hope you can easily recognize with equipotential lines or the equipotentials this is also in equipotential alright.

Now, if you look at the bottom layer, what happens, this happens to the flow line ok, did you follow this. Intentionally what I am not trying to show here, the top flow line. Because the top flow line is going to be something like this ok. And we do not know what is the shape, what is location as a function of time itself, now this problem is becoming very complicated. So what we will do is we will take a steady state, so that we come out on the time facts.

So suppose the saturation takes place the top flow line sets in, now this is what the top flow line would be, ok. So this is the top flow line, we call this as the phreatic line also, where the pressures are atmospheric. I hope you can realize the moment h changes, what is going to happen, this line will drop down or go up. And that is the reason we call this thing as unconfined, because the porous media through which the discharge is going to take place happens to be the earthen dam.

The top flow line is not fixed and it is a function of so many parameters, a bit of variation in H is going to change it is location. The flow is confined between these equipotential lines and the flow line, the top flow line is not known. Now, what I am intentionally trying to show over here is, if you allow this type of a seepage pattern to occur, it goes and hits somewhere over here. And this is what we were discussing in the last lecture.

Meaning thereby if the line of seepage or the seepage flow hits the earthen dam at the downstream side from this point onwards the erosion will occur. What happens between let us say, if I define this as 3 and 4, now 3 and 4 becomes a seepage line. This is also a sort of a this is a discharge line better you call this as a seepage line only better let it be the seepage line alright.

Now we are interested in finding out what is the location of point 3 number 1, what is the velocity of the seepage which is taking place at this, what is the shape of the top flow line alright and what it is location. A comprehensive picture of this situation would be, if I consider the foundation soil also. So I will remove this impervious layer from here and I will shift the impervious layer somewhere over here.

Now what is happened, this portion we have already analyzed clear, so this is a sort of a confined flow in totality or in real life what happens is. We have a composite porous media domain through with the seepage is going to take place. So some part of the seepage is going to take place through the foundations, rest of the seepage is going to take place through the body of the dam.

So what I will do today is, I am not much eager to analyze the foundation system, I will go back to the unconfined flow problem through the body of the dam.

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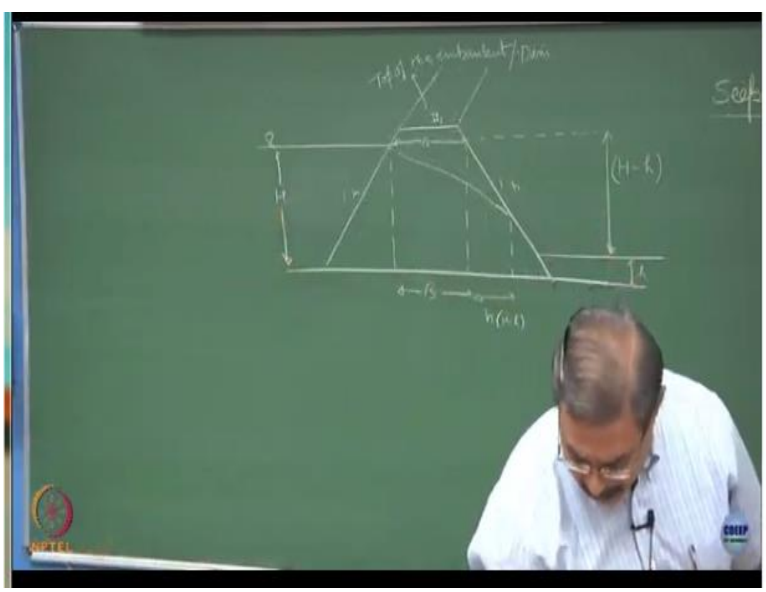

Now suppose if I consider the cross section of the dam as alright there is a water column over here there is a tail water here and this is the phreatic line which is developing over here. Now if I start doing the analysis, I will just give you few steps I might like to escape. If I consider this as 1 : m and 1 : n, if I draw this line further and if I treat this as H – h. If I shift the origin at this point, if I consider this as B, this will be the top of the embankment or the dam and let us say this is B 1.

So, this thing is going to be equal to n times H – h, see this is H and this is h. So this becomes H - h 1 : m, so this becomes n H – h. I think the first analysis we should be doing is with the I will have to change the phreatic line you are right.

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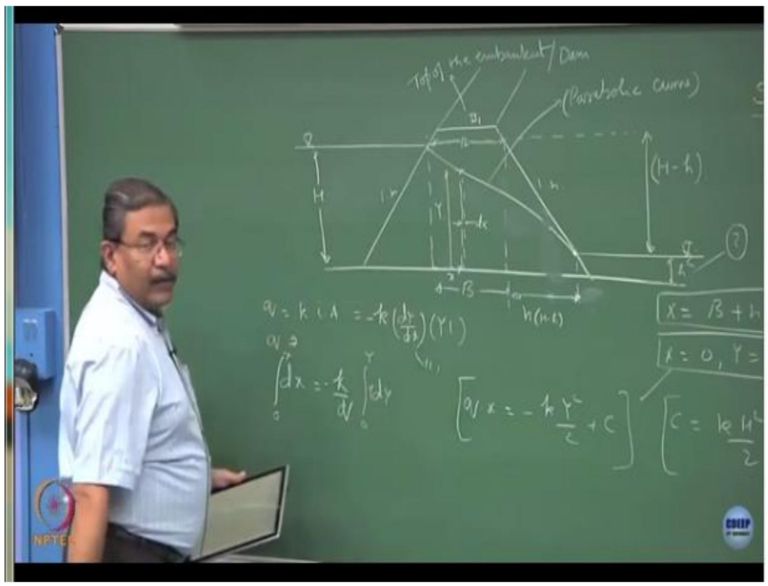

So I was not very careful, when I do this thank you. So in this case we are eliminating or we are putting the point 3 is known as the outcrop point. So in this case out crop point 3 happens to be meeting with the tail water, is this ok now. Now most of these analysis what we do is, we consider an element because the first task in hand is find out the shape and location of the top flow line, that means I have to establish what is the shape of this curve.

So if I assume at a certain x distance there is an element alright of thickness d x and at this point the height of this system is let us say y. I hope you can realize the d y by d x is going to be the velocity vector because this happens with a phreatic line. So let me explain to you the concept of the phreatic line first.

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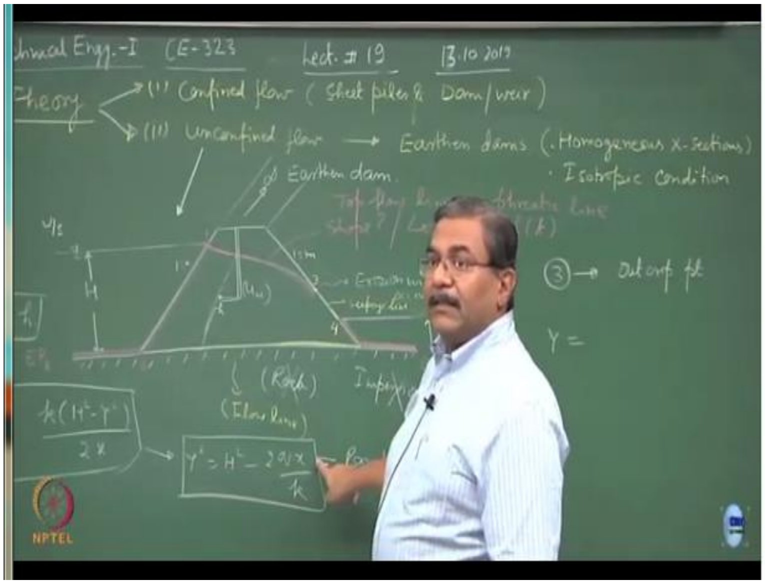

If I keep a piezometric tube at this point alright and if I know by somehow the flow net of the system. If I know the total head at this point, the total head is going to be equal to the equipotential line which is going through this point and cutting the top phreatic line. So just consider this for the timing when will draw the flow net will discuss this in details alright.

That means if I draw an equipotential line from passing through the point h and this is a top flow line, where the phreatic line is the point of intersection distance between the point of intersection of these 2 and this element is going to be the pore water pressure at point H. I will explain it again later alright. Now if you solve this expression, I will write quickly the steps, now what you can do is the, if you consider this element over here.

I can write the q = k into i into A and this will be equal to k into dy by dx into area cross section will be y into 1, this is the slope of the phreatic line. And if I integrate, let us say this function, I will be getting dx = k by q into dy y. So 0 to x and this is 0 to y, when I am writing dy by dx please be careful with a sign this will be minus. If you solve this expression, you will be getting q into x = - k y square upon 2 + C.

Substitute the boundary conditions x = 0, Y = H. And in second case, I can substitute this value over here and I get C = k into H square by 2. And hence, q will be equal to k H square - y square upon 2 x. I hope you can realize that this is the equation of the parabola, where I can write Y square = H square - 2 times q x upon k. This is the equation of the parabola, so what we have proven is that the top flow line happens to be a parabolic curve, is this ok.

The another equation will come when you substitute x equal to let us say B + n times H - h and at this point Y = h. I hope you can understand the basic objective is to obtain the value of h is the principle unknown why. Because I would like to find out what is the maximum discharge taking place through which the h becomes maximum. So I have to design the system in such a manner that the maximum discharge should not occur.

So I should be taking the value of h when it becomes maximum, so if I want to maximize h, what I will have to do is, I will have to write this expression in this form, a small q is not constant small q is a function of this, what do you mean, see that is what we have proven, y square is a function of x is a parabola. Your H is given constant is it not, q will remain constant for a configuration of H and a small h, k is constant and hence this is a parabolic function alright.

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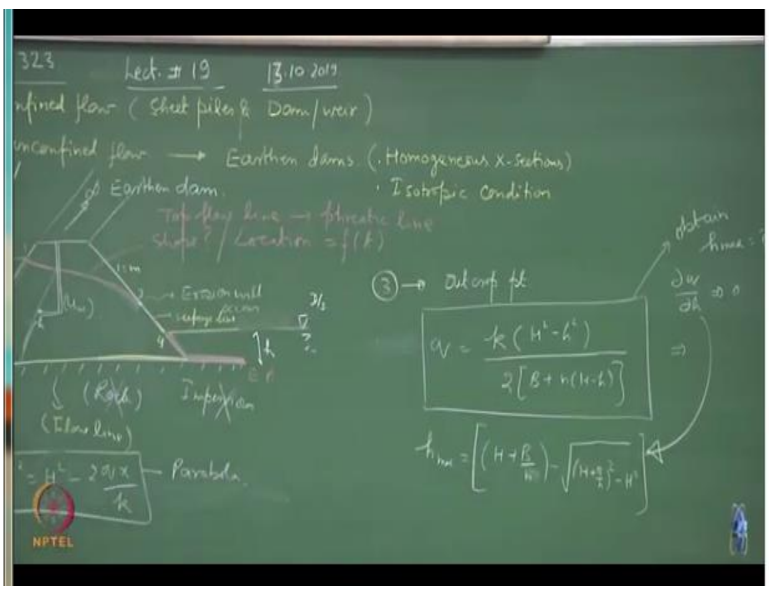

So what I can do is, I can write this expression as Q = k H square - h square over 2 times B + n H- h, this is a function which I get, is this part clear. So for a given H and a small h this is the flow regime which is getting developed. Now I want to optimize this function, so obtain h maximum. So if you differentiate this function and put it equals 0, what you will be obtaining is, you will be obtaining h which is maximum equal to H + B by n - under root H + B by n square - H square this expression.

In short what we have done is, we are finding out the nature of the top flow line for the condition. When capital H is causing the flow to occur in the homogeneous dam section under unconfined flow condition, tail height is h. And we want to see what is the maximum height of the tail water which is going to come for this system B 1, B in fact is the function of B 1 comma H because design head has to be fixed.

Now this is what is known as the free board, you must have been reading in newspapers why free bodies are important and why the cities are getting ground these days. There are so many cases including Maharashtra and so on, is it not m, n, so I can design the top width keeping in view the H the 2 types of slopes, how much water is to be retained, what is the free board. So I will include in this the free board let us say as z.

So this becomes a typical cross section of the earthen dam fairly simple the only point of interest you should remember is we are taking a hypothetical plane across which we are finding out the discharge. And just to make things clear, the slope of this line is nothing but the dy by dx. So which I am assuming to be equal to i rest is all simple mathematics yeah. So you check it for the global minima also by double differentiating it and get a solution that you can do.

Check whether this is going to be a global minima or not that is right. So q is going to be maximum for the h maximum that is right, correct. So the maximum discharge is going to get accumulated over here. And hence the h is also going to the maximum possible for the situation, so the both the things are interlinked, see another point here is the outcrop point.

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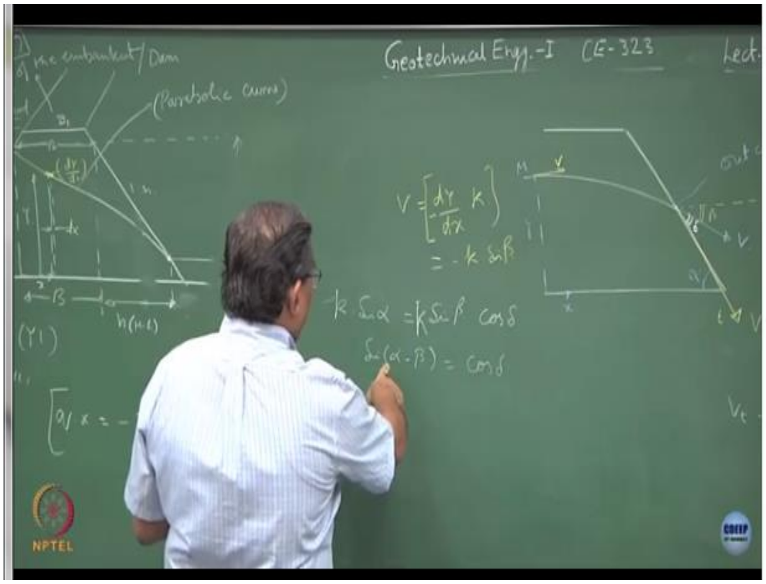

So if I consider a typical case, downstream, where the top flow line comes and cuts the embankment and if this is the alpha value. This is the velocity of the water, at this point m if I consider this is y and this as x, point o is the outcrop point this is inclined at an angle of let us say delta and with respect to horizontal this is beta. So if I say alpha = delta + beta, if I assume the component of V in this direction, let us say this is a t direction as V into cos of delta.

I can show that at this point the velocity is going to be equal to dy by dx and this will be equal to - k into sine beta. Now what we will do is, having done this   I will substitute the value of this V, which I have obtained from here. So this becomes V sine of beta into cos of delta, I can also obtain a term here v t as - k into dy by dt and this will be equal to - k into sine alpha.

So if I go for the equality of the 2, k into sine alpha = sine beta k into cos of delta, where I can write sine alpha - beta = cos delta, beta will be equal to alpha - delta. So I can say that this function no I have done a sorry.

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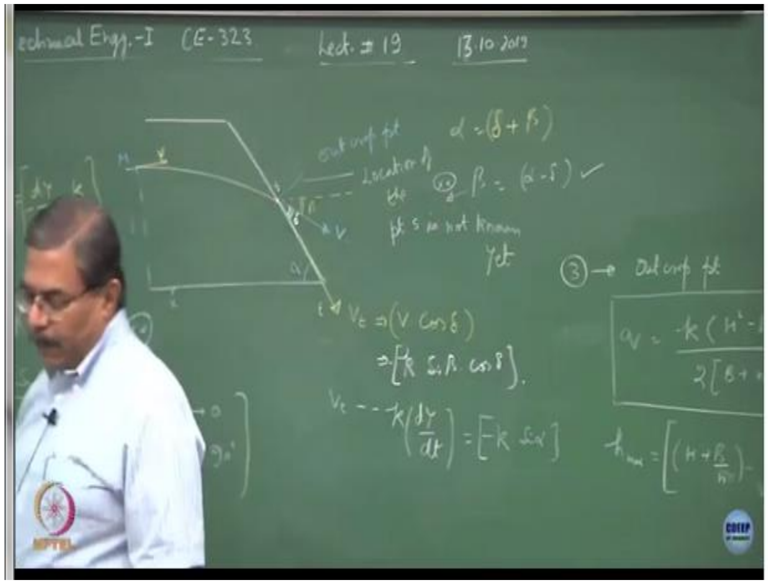

So if I substitute this over here I can show that sine alpha = sine beta cos delta. And if I substitute the terms I will be getting sine of alpha - delta = sine of alpha, what this indicates is that delta tends to 0, number 1. If delta tends to 0 that means, the discharge velocity at the free surface is going to be equal to whatever the V component is. So this always parallel to the downstream slope, this solution is valid when we say alpha is less than equal to 90 degree

So what we have done is, we have talked about the velocity of the outcrop point, we do not know what is the location of this point is still. So location of the point o or let me put it as s is not known yet, so we will try to find it out. Now there are different methods of finding out the location of the outcrop point.

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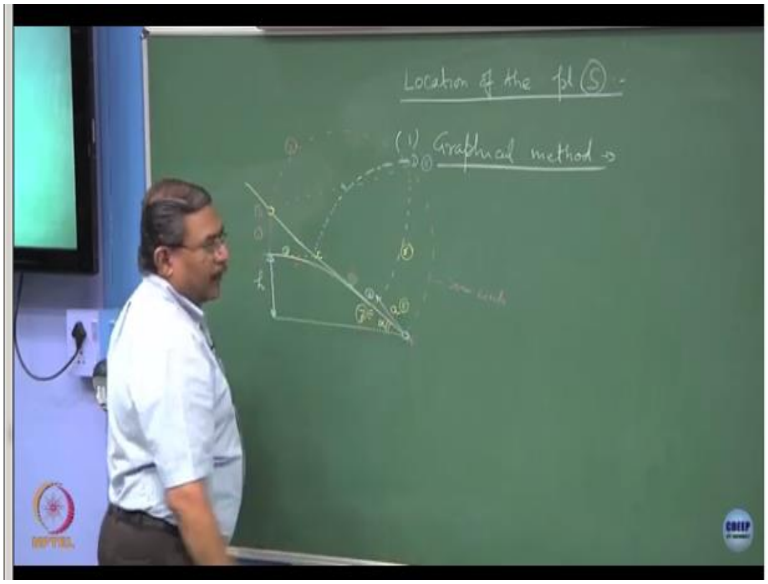

So this is the point s, one of the simplest method is you adopt the graphical method, please follow the steps which I am to going talk about. If this is a downstream side of the slope, I have taken this as alpha, what we are interested in finding out is the location of this point which is s and this is the height. This is the height of the water column let us say h or whatever the first step is extend this in the vertical direction and let it cut the downstream slope.

So extend this line vertically up, now wherever this cuts suppose if say this is a, this point is B on a B draw a semicircle ok. So the first step is this, number 2 step is this, draw a tangent to the phreatic line and let it cut the downstream slope. Let us say this point is C, so this is the step number 3 take a C as the radius and keeping a as the centre draw an arc. So a is the centre and a C is the radius get the point number D.

So this is the step number 3, this is step number 4, get point number D is 5, keeping centre as B and take BD as the radius and draw another arc let it cut over here. So this becomes your point number E, so it so happens that this is the point s with the way I have drawn has to be corrected alright. So this is the point s which matches with E and normally we define this distance as a. So having done point number 5, this becomes step number 6 and obtain 7 a is 8, is this ok I will repeat it.

Basically we wanted to find out the location of the outcrop point. So take the downstream side of the dam extend the height of the water column, let it cut the dam surface downstream stream get the point B take a as the centre this as the radius sorry this whole thing as the diameter and then you complete the semicircle. Extend the tangent drawn to the top phreatic line, wherever if this cuts the inclined surface point c, take a as the centre aC as the radius get point D.

B as a centre DB as the radius get point E, aE corresponds to A and A is the location of the out crop point. So until now we have done 3 things, we have defined the shape of the top flow line, we have defined the location of the point s and we have also proven that the velocity vector at point s outcrop point is going to be parallel to the phase of the slope it is seepage or simple discharge alright clear.

So and this surface is not going to be an equipotential which I showed you in the earlier case. So it is a freely discharging surface, is this point ok. Now the same thing we will prove I drew this, yeah because that was one of the situations yeah.

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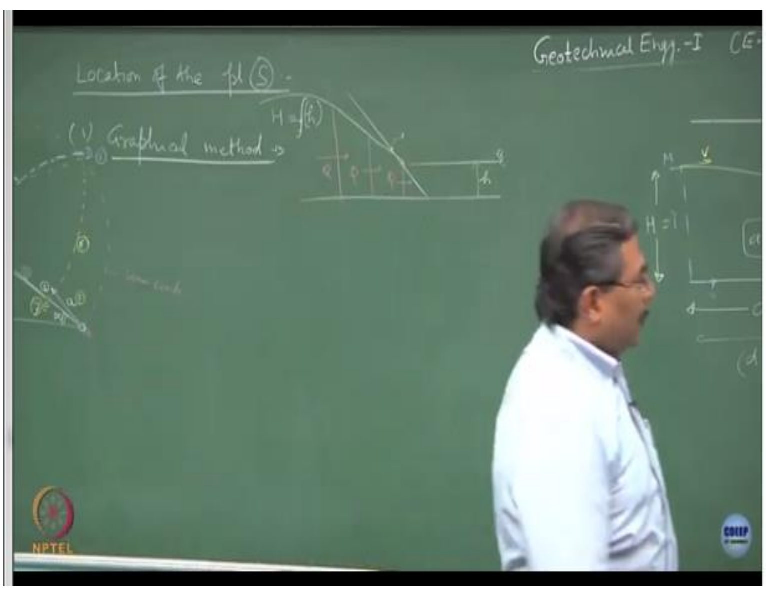

So basically in that case what I did is, I have located intentionally the point s meeting with h. I hope you can understand this is one of the a specific cases, what is the reason for doing this. I will come back and explain to you, if this point s is lying somewhere over here, this is going to be more damaging to the body of the dam, you understand. Because the seepage line is coming and hitting it over here, so the erosion will start here.

So the better way would be let this point remain submerged into the water table clear, this is one of the ways and I can create a reservoir here, where I can allow people to do some amusement or whatever yeah. So I am sure you must have realize, what we are trying to do is, we are trying to relate H and h alright, is this ok. So I can always create a situation where the top phreatic line comes and meets this point, I can say that s corresponds to or a value tends to 0 that also I can do.

So this is one of the specific situations which we talked about, just to derive the parabolic equation. Otherwise what is going to happen please realize this very conveniently what I did is by assuming this point over here, I have imposed this condition of h at this point. At this point, unfortunately you do not have h valid, are you getting this point. Because this happens to be a discharge point this is not a seepage point that is what we have proven.

So in order to get a mathematical solution, we have force this point to be lying with the top tail water, so that we can obtain a solution fine. So let us do now the analytical solution to obtain the outcrop point.

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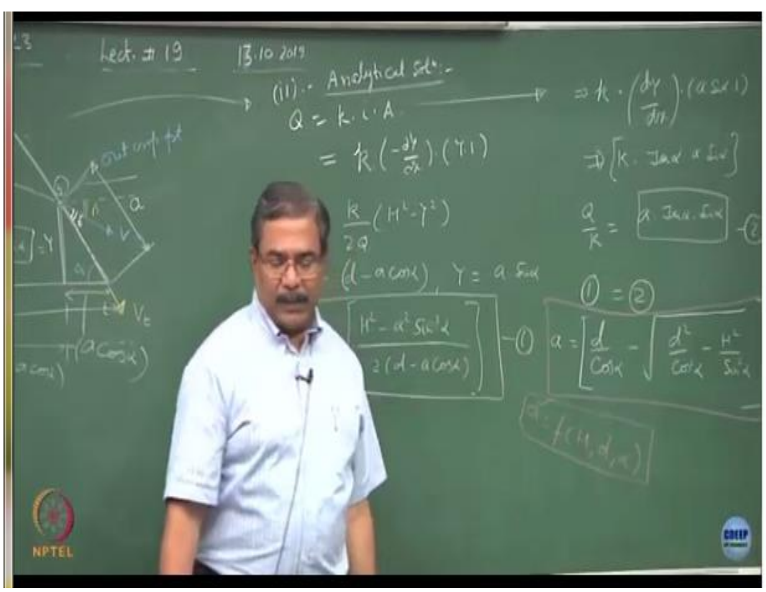

In this expression, suppose if I in this figure, if I write that Q = k into i into A and this will be equal to let us say, k del y by del x into area Y into 1, solve this expression. In such a manner that if this y = H capital H and say this distance is d and this outcrop distance is A this Y will become a sine alpha can I show starting from this function if I come to, let us say k upon 2 Q H square - Y square substitute the values x = d - a cos alpha and at this point Y = a sine alpha is this ok.

So this much is a cos alpha, so this distance would be d - a cos alpha. If you solve this expression you will be getting Q by k = h square – a square cos square alpha sine square alpha upon 2 d - a cos alpha this is the equation number 1. Another equation I can obtain by equating Q to be equal to k into, can you tell me what will be the other equation. Suppose if I use this function a sine alpha the area of cross section through which the flow is taking place. So Y term becomes A sine alpha into 1, what will be the hydraulic gradient here.

If I assume this as dy by dx, is this ok, so this I can write as tan alpha into a sine alpha. In other words, Q by k will be equal to a tan alpha into sine alpha, so I am defining the discharge in 2 ways. If I equate these 2, I will be getting an expression, the final expression would be a = d by cos alpha – under root d square by cos square alpha - 2 square h square by sine square alpha. So this is the expression which gives you the value of a.

In other words, a depends upon what, a would depend upon capital H number 1 alright, distance of the section at which H is acting, so d and the alpha value. Coming back to your partial answer, what is the peculiarity of this equation which is linked with the question which you are asking. At this plane, the Q is Q, at this plane, also Q is Q at this plane also Q is Q agreed continuity, what do you observe here that the outcrop point is independent of Q, is this ok.

So one of the questions have been answered it does not matter where I put the tail and the outcrop point clear. Because the location of the outcrop point is only the geometry of the downstream side of the earthen embankment. So what we have shown is a point and now please sit down and try to solve this and prove that a what you get from here is same as the a what you are getting from here.

So these are 2 ways to solve the location of the outcrop point, so this is the analytical solution fine. But I am sure you must have realized the derivations are very, very informative, they give you a lot of information about how to design the cross sections of the dams homogeneous cross section of dam.

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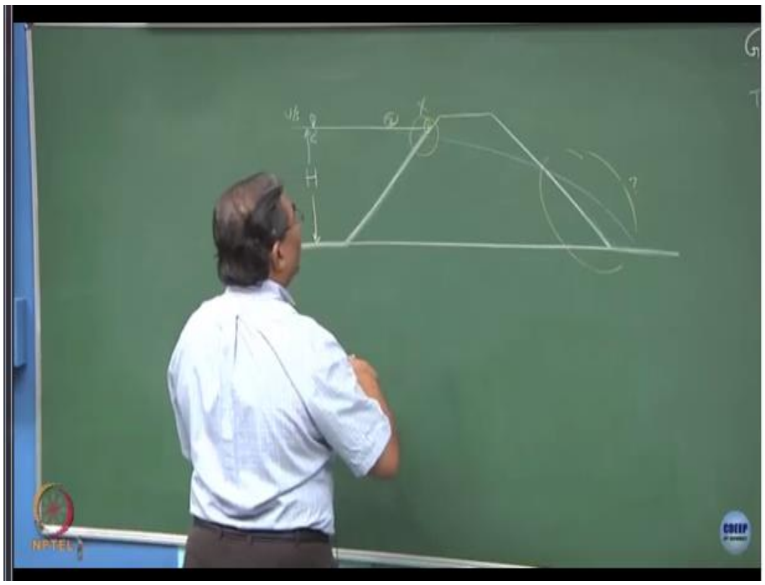

This is the upstream side of the water of height H ok. This is the equipotential line and this line is also an equipotential from the end point to the point where I have taken Y = H alright, that is how I think I created boundary conditions yeah. So from this point, this is a cos alpha and then I am substituting the value of a as this thing a sine alpha Y truly speaking, like this is an equipotential line the top phreatic line should have been perpendicular to this.

But what we have proven until now is, that the top phreatic line is a parabolic curve alright how to fit the parabola on this. Now please stop writing and see what I am doing, this will give you a better understanding of what we are going to discuss subsequently. The only possibility to fix a parabola on this figure or superimpose on this figure would be if the parabola goes like this is, is this correct agreed. Because what we did is, we have shown we have derived the expression.

Now if you put this equation and superimpose on this cross section, this is how the situation would be. Now tell me, what are the geometrical irregularities I have induced in the process let us start counting, is this possible very good, why not possible you are right answer is correct. It is not possible, why it is not possible, look at the flow line and the equipotential line, they are not perpendicular to each other number 1 fallacy or I would say this is violating del square pi = 0 and del square psi = 0 case.

It is not, because the way you will plot it you will normally know that is going to cut, so you can differentiate that parabolic function over here you will find this not alright. The second what is the second loophole look at the figure a started is included in that yeah, you are right. So this is the number 1, discrepancy number 2 discrepancy is this, number 3 yeah that has come already.

So perpendicular to the very good that also no downstream will come later, first you finish this part, what is next, come out of that discussion that I have no said that there is some problem over here. Now what is the other problem, major problem what I have drawn here is, a top flow line, is the some problem with that. Look at, where the top flow of the phreatic line is going. It should have been in the porous media.

Imagine you had trying a top flow line which is going outside the porous media that means this is also there is a problem, agreed. So what should be done, create the correct phreatic line in the dam body and that answers your question, is this part ok. Even then what is going to happen, I am sure if you even if you shifted whatever you do, there is going to be a discrepancy like this, try this now clear and there is an answer for that, so let us assume the situation.

Now the question is, how are you going to come out of this situation, what we have to do is, we have to reshape the equation of parabola which we have derived. So let us start applying corrections from point number 1, is this part ok. The discrepancy part, have you understood, what we should be doing.

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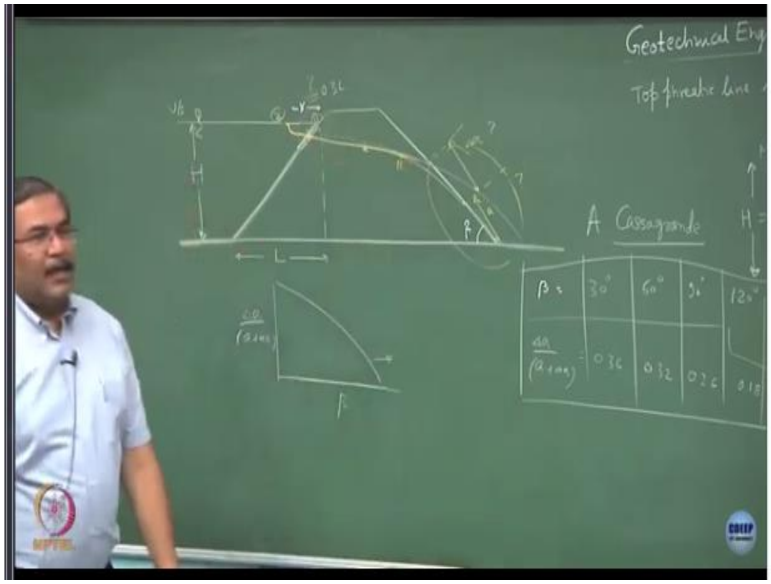

As you rightly said, the flow line is going to be perpendicular to the equipotential and not starting from this point, where the water is matching with the body of the dam it will be at a certain distance x. So the top phreatic line would start like this ok. Now this is near perpendicular then what will happen, this will go further down at this point where is the outcrop point now, outcrop point should have been somewhere here.

So, I have to match it with this outcrop point which we have obtained as a, the distance between this point and the point where the parabola is cutting is the error delta a. We have added 2 more uncertainties now, in the process of solving the situation, what are these 2 unknowns. One is x, another one is delta a, x we can sort out by assuming that if the length of the submergence of the upstream slope is L, x is going to be equal to 0.3 times L.

This I have taken care of, that means the initiation of the flow line is going to be from a distance of point 3 times L. If L happens to be the submergence length of the slope in the x axis, number 2 this line itself is an equipotential line. So the way I have drawn is still wrong, what should have been done yes, you are right it will be perpendicular comes like this cuts perpendicular and goes like this ok, the another uncertainty is empirical in nature delta a.

So what people have done is, they have come out with design charts, and in design charts if beta is known, where beta is the downstream of angle if this is 30 degree, 60 degree, 90 degree, 120 degree is it possible, 120 degree it is possible.

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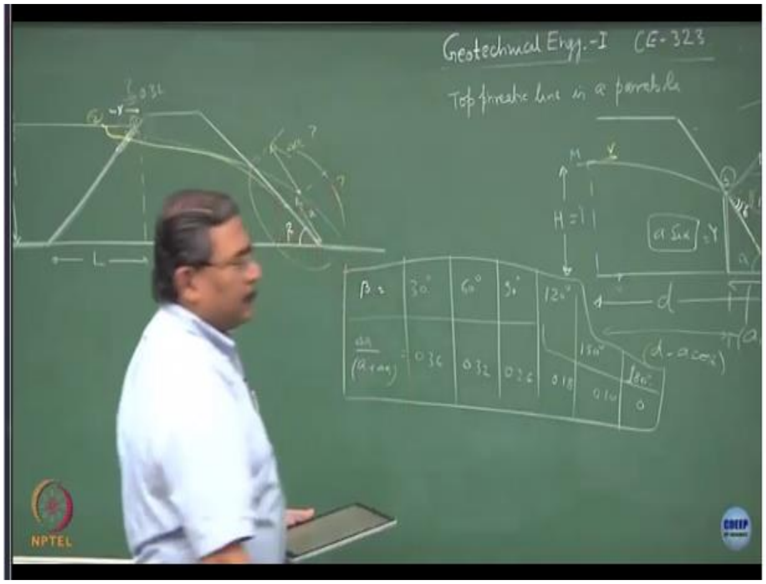

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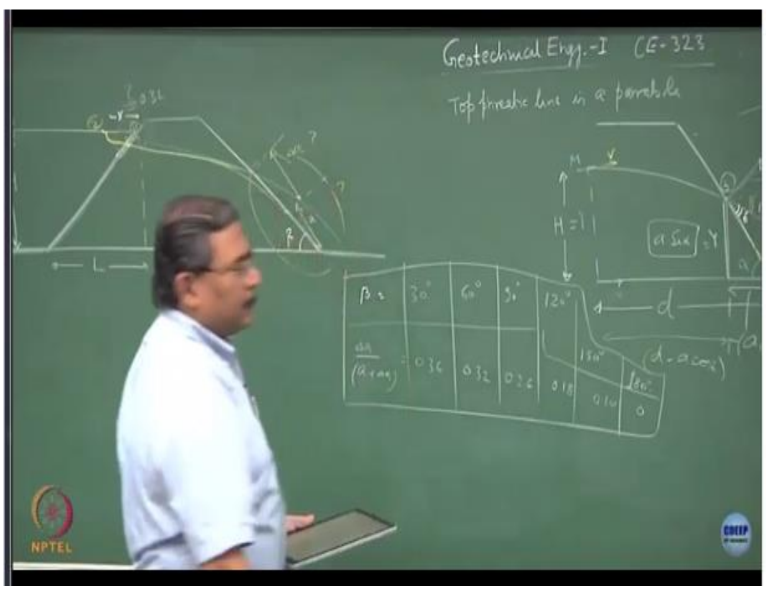

We will discuss such cases 150 degree, 180 degree the value of delta a upon a + delta a is 0.36, 0.32, 0.26, 0.18, 0.10 and 0. This was given by call this Arthur Casagrande, we call this as Casagrande graph, where delta a upon a plus delta a is inversely proportional to the beta, delta a is the error term truly speaking what you are doing, you are pushing this whole thing inside clear.

So this can be pushed only when this point is a fictitious point and it goes inside, so that the graph matches with the outcrop point that is it, is a fictitious point.

So this we have taken care of alright, so normally these type of situations are drawn graphically. First you apply correction over here, number 2 correction over here, come up to a certain point over here, let the graphic discontinuous, a start from the downstream side, fix value of a obtain the value of delta a, a + delta is known, fit this portion of the graph again and then let it be discontinuous up to point B and in between, you can match the 2 graphs.

Nowadays, you have FEM packages which can do these things for you quite easily. But I thought it is important to discuss in the class, the conventional way of doing the things.