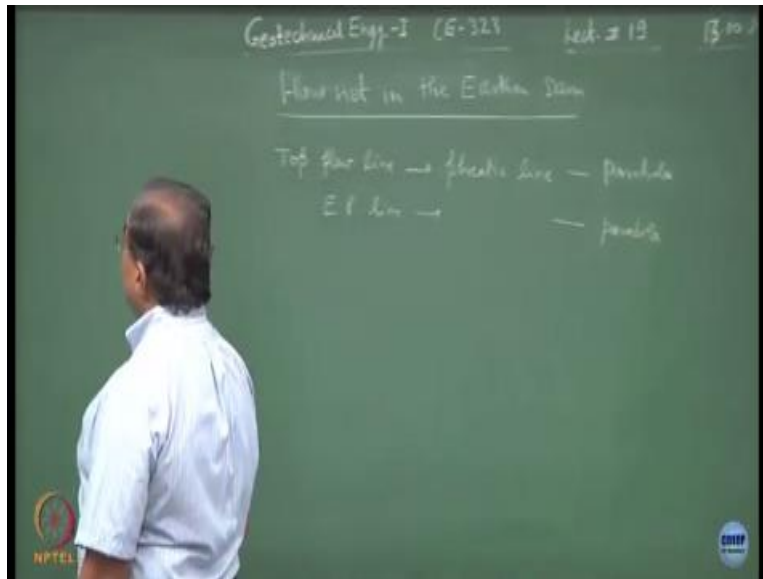


**Geotechnical Engineering I**  
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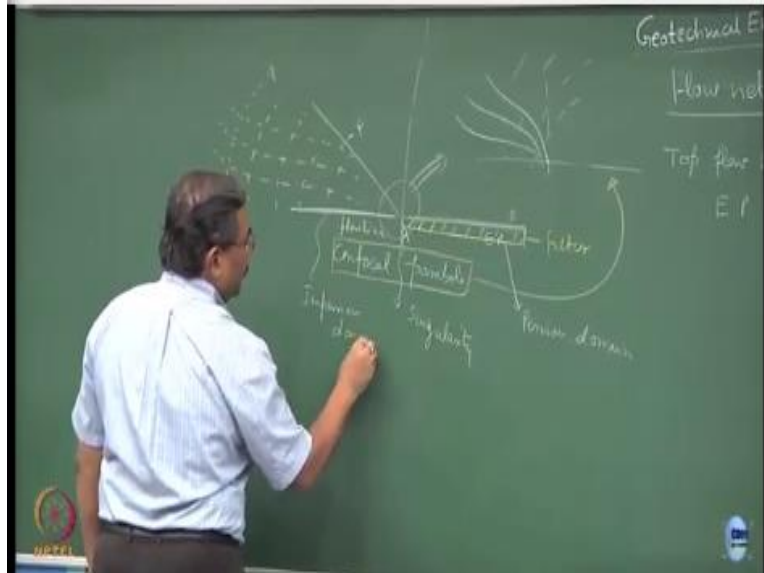
**Lecture-21**  
**Flow net in the Earthen Dam-I**

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So there is something known as the flow net in the earthen dam. So you must have realized that having proven that the top flow line or the phreatic line is a parabola. And hence the equipotential lines are also working to be a parabola.

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So the way we draw this is if I consider that this is the dam and if I have the top phreatic line like this. The next phreatic line like this and like this, the equipotential lines to this would be perpendicular to the flow lines. So this is how the fluid gets developed, so this is the psi function and this is the pi function, what you are observing here is that this is going to be a set of confocal parabolic alright.

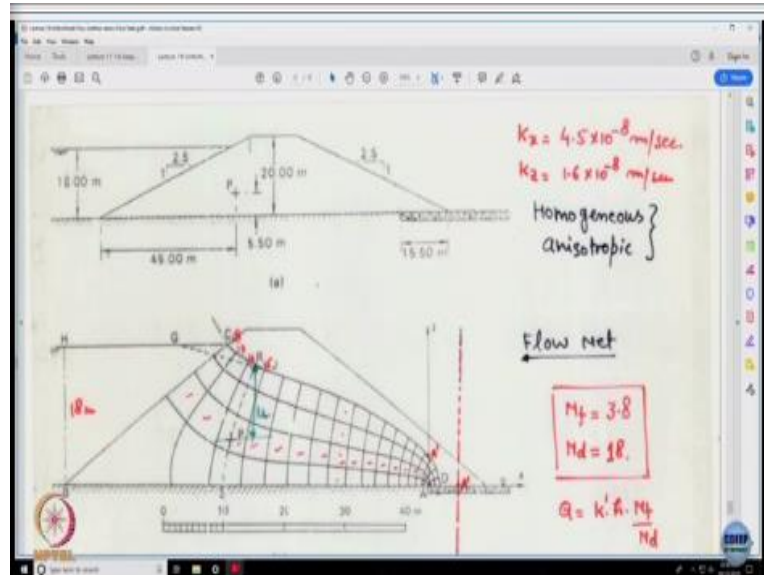
Now suppose, I consider this point now this is what is going to be the bottom flow line and this is connected to the atmosphere, this is going to be the equipotential line. So at point A I have the singularity, that means on the left hand side I have the flow line, on right hand side of the equipotential line and hence point A becomes singular. Now this type of a situation can be created by putting a filter layer.

Now the moment you put a filter layer, this becomes an equipotential, the atmospheric pressure prevails over here and hence the total head is going to be 0 at this point. The way we have plotted the flow lines, and we have assumed this to be the bottom most flow line, all the equipotential lines when you plot them in totality would extend up to this regime and this line would become the bottom most equipotential line clear.

So if at point A, if I enlarge things this is how it will look like. So I have the flow lines coming and meeting over here. And the equipotential lines starting from here, this is what the

representation of the confocal parabola would be. You can also assume that this is impervious surface and this is the pervious I will say this is the pervious domain, this is the impervious domain.

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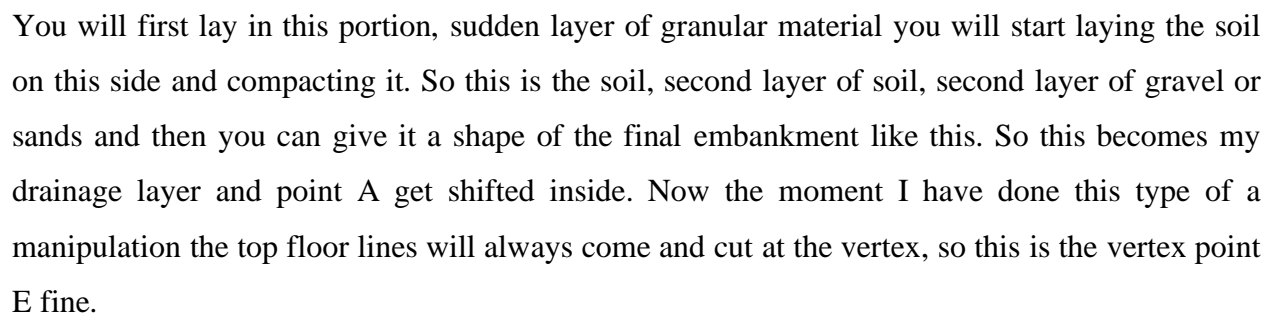


Now, we will take the situation where the 2 problems which I wanted to discuss related to the confined flow, I hope you must have realized it is very difficult to draw the flow nets in case of the embankment by hand. One has to be a very good artist, although is extremely difficult is a very complicated set of the flow nets. So suppose there is a situation like you have an embankment, this is the filter layer, what I have done is a close look at this figure will show that I have shifted the point A inside the body of the dam.

And that answers the question which he is asking that how would you control the seepage lines, so that they do not cut the downstream slope. So truly speaking this is the engineering solution which you are providing by shifting the entire layer of the drainage media inside the body of the dam. So what is going to happen, the moment I shift this filter layer somewhere here, point A becomes the focus of point A is the focus of the parabolic.

The moment it goes over here, all the flow lines will get diverted towards the filter. This is the engineering solution, which is given to make dams stable against seepage alright. And at the time

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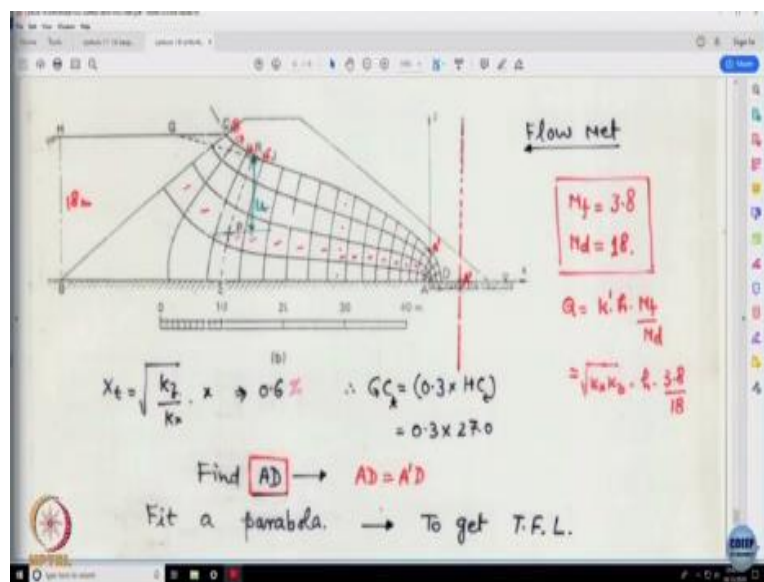


So the moment I have fixed the directrix I know the focus, I know the vertex E and I am assuming that  $AE = EF$ , total length of the filter is known  $y^2 + x^2$  under root will be equal to  $x + z$  0, is this ok, this is the property of the directrix. Any point sitting on the parabola would be having would has to satisfy this condition is this ok. So what I have done now, I know

this point I know the starting point as point 3 times L ok, where this is L this is a point of submergence.

For solving the parabola how many points we require 3, one is known, second one is fixed, third one I can obtain by using this geometry. So if I know the  $z=0$  value, I can consider a point over here, this is the equation which I get, fine. This discussion is mostly based on this construction. So if I know the  $k_x$  and  $k_z$  value, I think this is the problem which we have already discussed in the class, homogeneous soil mass anisotropic in the sense.

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Because hydraulic conductivities are different in  $x$  and  $z$  direction, what we will have to do is we will have to go for equivalent hydraulic conductivity which is root of  $k_x \times k_z$ , what is causing flow  $h$ ,  $h$  is known as 18 meters. So this 18 meter is going to cause the flow, after the flow net is constructed, I can count the NF and ND and this is 3.8 and 18. This function is transformation of  $x$  into  $x_d$  direction, so root of  $k_z$  by  $k_x$  would be 0.6 times  $x$ .

All the dimensions in the  $x$  direction will get reduced and hence you are seeing that the embankment has become steep,  $g_c$  is nothing but 0.3 times at  $h_c$ , if height is 18, this is 45, 45 into 0.6 would be 27, apply the correction again 0.3. So this becomes 0.3 times 27 if the flow net is done I can draw a line passing through  $a$  in the vertical direction wherever this cuts the top

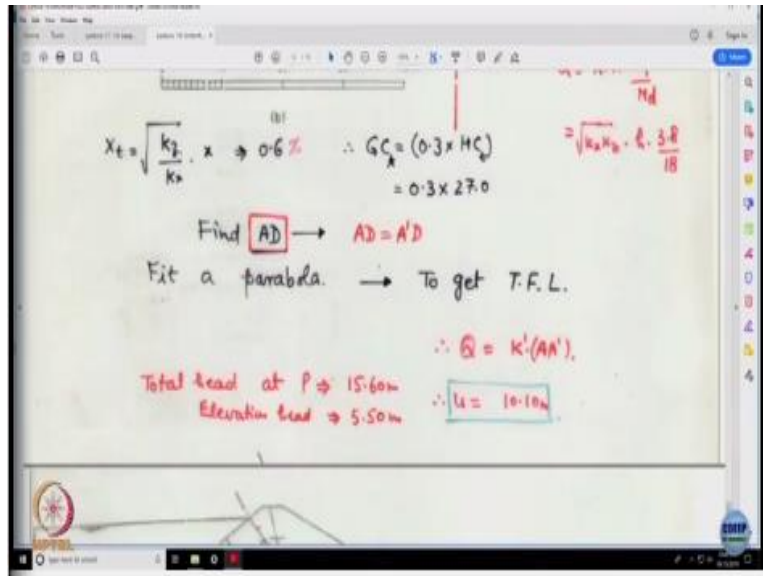
flow line this becomes my A prime, A A prime distance I can measure on the filter bed, I can draw a line passing through this, this will become directrix ok.

Once you have done this construction rest of the things are simple. So, if I know the NF and ND if I multiplied by the h value, I can find out the discharge taking place through this section. What he was asking, I have tried to answer over here. At a given point p if I want to know what is the total pore water pressure, the thumb rule is if the 2 equipotential lines are known, you interpolate in between draw equipotential which is passing through the point p draw a horizontal layer. If I put a piezometer here it will go and cut the top surface at this point where the equipotential is cutting the top phreatic line.

Point of intersection of the top phreatic line and this equipotential is going to give you the total head at this point. So once you have got the value of u you can compute whatever has been asked is this ok. I will repeat it again after drawing the flow net, if I want to find out pore water pressure at any given point, what I have to do is I have to interpolate the equipotential line passing through that point, find out where this equipotential cuts the top phreatic line.

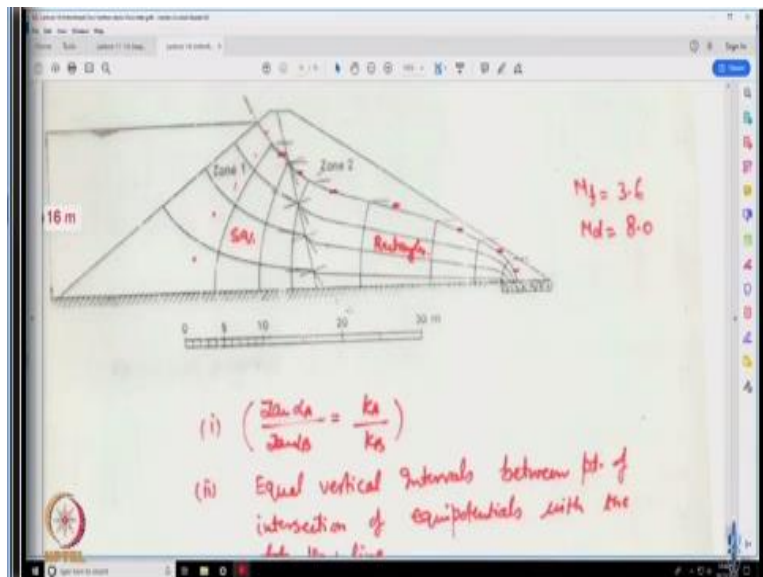
The vertical distance between this point and the horizontal plane which is passing through point p is nothing but the pore water pressure, this is a good example of unconfined flow under homogeneous conditions, anisotropic conditions.

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And you can solve the problems, practice it by following some book.

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The last thing which I want to discuss regarding the unconfined flow is most of time we come across a situation where we have different zones in the embankments. So suppose I want to reduce the you know seepage which is taking place through the body of the dam. So there are 2 ways, one I did is by maximizing the  $h$  value and getting the value of  $Q$ , which is the inverse process.

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Another better way would be if I design let us say, a core of impervious material in between. So suppose, this is the upstream water level and if I want to check the deepest taking place through this, what I have to do I have to put a impervious core. Now this becomes the situation that you have only come across in physics, when you are dealing with these Snell's law in case of light, you know.

If I consider this permeability as  $k_1$ , this permeability as  $k_2$  and if you realize how the flow lines are going to travel through them, what is going to happen. This flow line comes it is the second layer of a different permeability. The same thing is valid when we go from  $k_2$  to  $k_3$ , so the moment it comes out of the system. Depending upon  $k_3$ , there is going to be a deviation in the top flow line, so this becomes a case of the interface.

Another type of interface might occur if I consider a dam where I have provided 2 filter here ok. In this case  $k_2$  is tending to 0 because I wanted to make system impervious, so I have use clay to make a core of the dam through which nothing should permeate, very small value of  $k_2$ . But here I am sure you will realizing that  $k_2$  is tending to infinity because this is made up of rock cores, gravels, sands, there is a specific purpose.

Now, this is the filter clear, what is going to happen, the top flow line will come hit across this and then it enters into  $k_2$  whether it will enter into  $k_2$  or not is the big question. So when the



flow is taking place across the discontinuities or the boundaries, what you have to do is, you have to solve this problem by assuming a simple model. And let this model be like this that this is the interface where the 2 materials are located.

This is let us say A material, this is B material if I draw a channel of flow, clear. So this is the material A, permeability is  $k_A$ , this is material B permeability is  $k_B$ . Now the question is you have to find out what is going to happen to these flow lines, when the interface is there. So draw the perpendicular to these interfaces and then clear. If I ask you to draw the flow net it can be done, this angle is  $\alpha_A$  angle of incidence of the flow line in the media A.

This angle is  $\alpha_B$ , the angle of deviation with respect to perpendicular drawn on the interface I will show interfaces a thick line. I can rather flow nets, this is the first equipotential line, second equipotential line, third equipotential line, I can assume this to be let us say  $x$  y A,  $L_A$ . So  $y_A$  is the breadth of the channel,  $L_A$  is the length of the channel, same thing can be done over here also.

So this is  $y_B$  and  $L_B$  can you draw a relationship between  $\alpha_A$   $\alpha_B$ . So derive a relationship between this oh sorry  $y_A$ ,  $L_A$ ,  $y_B$ ,  $L_B$  and what is causing the flow to take place  $\Delta H$ . So I can have a situation where  $\Delta H_A$  and  $\Delta H_B$  are provided, the continuity says that discharge will remain same, fine. So if discharge remain same  $Q_A$  will be equal to  $Q_B$ , that means, this will be  $k_A \Delta H_A \text{ upon } L_A \text{ into } y_A \text{ into } 1$ .

And this will be equal to  $k_B \Delta H_B \text{ upon } L_B \text{ } y_B \text{ into } 1$  fine continuity equation. Try to solve this expression and hope you will realize that what you will be getting is  $\tan \alpha_A \text{ upon } \tan \alpha_B$  equal to  $k_A \text{ upon } k_B$ . Of course you have to assume some relationship between  $y_A$  and  $y_B$   $L_A$  and  $L_B$ . So if I assume both sides the squares,  $y_A L_A$  cancels out  $y_B L_B$  cancels out, this is one of the solutions, these are perfect the squares clear.

So I have simplified the whole thing and I am getting this expression, now what is the interpretation of this expression, that is very important. So the whole story started from the interface problems like this, this  $k_1$  is a finite value, the value of hydraulic conductivity for the

filter layer is tending to infinity clear. So what is the relationship between  $\alpha_1$  and  $\alpha_2$ ,  $\alpha_2$  is going to be 0 why, what is the meaning of this.

The seepage line will never enter in a media which is highly permeable and hence what is going to happen, the flow is going to take place parallel to the plane of the filter, it bypasses the media. Now this concept is used for designing most of the filters in industry as well as in various civil engineering projects. So coming back to this situation, what we have been talking about.

I think I have given you the basics and you can extend them to this situation  $k_1$ ,  $k_2$  permittivity contrast. We have zone 1 zone 2, what I have done is zone 2 you have to go for equivalent permeability transformation, flow channels will remain same 123.6 the same thing is going to enter into this also. Number of drops will not get affected because of the hydraulic conductivity, I hope you agree with this.

Because number of drops have nothing to do with the permeability of the material they follow the geometry. So here you have equipotential of 100% and here you have equipotential of 0% because this is a filter layer. So 16 head value is getting dissipated from this point to this point, how many head drops 1, 2, 3, 4, 5, 6, 7, 8 clear. So I have got NF by ND, NF by ND is a constant term we discussed in the previous lecture.

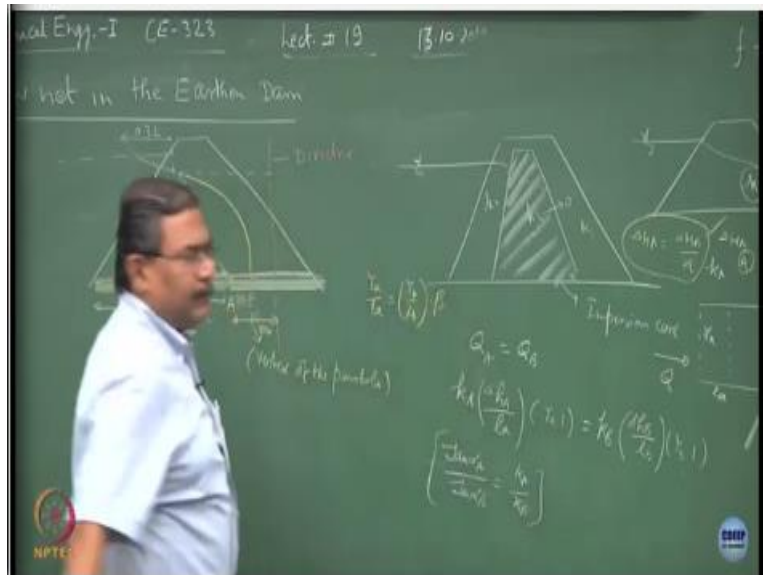
And if I multiplied by the  $A A'$  value, you remember  $A A'$  value, this is nothing but the distance of the focus from the directrix, so this can be obtained very easily. So though these problems look very complicated, they are very easy to solve. The moment you have got NF upon ND the moment you know the focus point which is intentionally done by u clear. You know the top flow line, get the value of  $A A'$  directrix is known, do a little bit of computation by using that expression.

And hence the discharge across the plane can be opt obtain very easily. So NF upon ND multiplied by  $A A'$  the discharge through the body of the dam clear. So in this case, it is a beautiful example of how 2 zones have been club together to do the simple analysis. So I am

using the concepts of isotropic systems, homogeneous individually but I have 2 zones which are heterogeneous in nature.

And I am showing here the concept what we have used over here, that there are 2 ways of solving this. Either I should be having a relationship between  $\Delta H_A$  and  $\Delta H_B$ . Suppose this is  $x$  not  $x$ , suppose some  $\alpha$  value alright. So what I am doing is, I am defining the relationship between the head drop across first layer and head drop across second layer and how they are related, I will get one solution clear. The second way of doing this would be the geometry of the system.

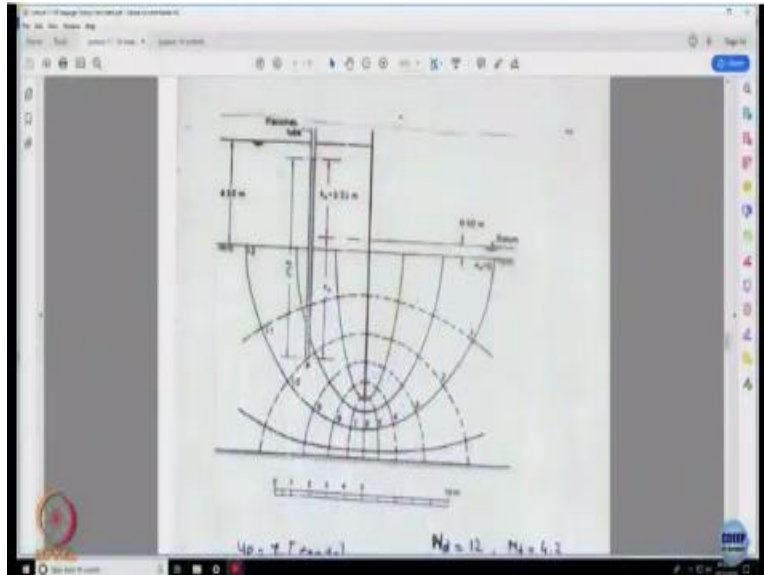
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So I may say that  $y_A$  upon  $L_A$  is equal to  $y_B$  upon  $L_B$  multiplied by some  $\beta$ , this is my choice. Because this is a geometry, I can control the geometry of the flow or I can assume the distribution of the heads across this point. Ideal situation would be if you keep the head constant between this and this, there is no problem, that is what actually we have done in this case. In the previous lecture, I had stopped after discussing the case of the sheet piles.

And I had emphasized on the concepts of determining the discharge through the opening below the sheet pile and the impervious strata. And this is there I had talked about how to compute the number of drops and number of flows, which you can utilize to obtain the discharge.

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There is another clause of problem, which normally is discussed in the realm of seepage analysis and that is what is depicted over here. So there is a sheet pile, which is retaining certain height of water 4.5 meter and there is a tail water or the downstream water and the datum has been considered over here with 0.5. And I am interested in finding out what is the pore water pressure at a given point in the porous media.

So once you have obtain the flow net, it should not be difficult for you to compute the pore water pressure at a given point. Because remember the entire analysis we have done until now is by assuming the  $\Delta H$  which is the total head and that causes a seepage regime to get developed into the porous media. So  $\Delta H$  is  $H_1 - H_2$  that is 4 and this head is getting distributed in the porous media.

So at this point p if I know the equipotential I know that total head, if the datum is known I know the position head of point p which will be equal to  $-z_p$ . And if I insert a piezometric tube over here, the total height of the water column in this tube would be this much that is  $H_p + -z_p$  alright. So this is how we can compute the pore water pressure at a given point, rest remain same and you can compute the discharge which is taking place through this orifice or the open space.

I will move on to another interesting problem, there is a sheet pile and I would like to find out what is the pressure which is getting exerted on the sheet pile along it is depth, so this pressure is

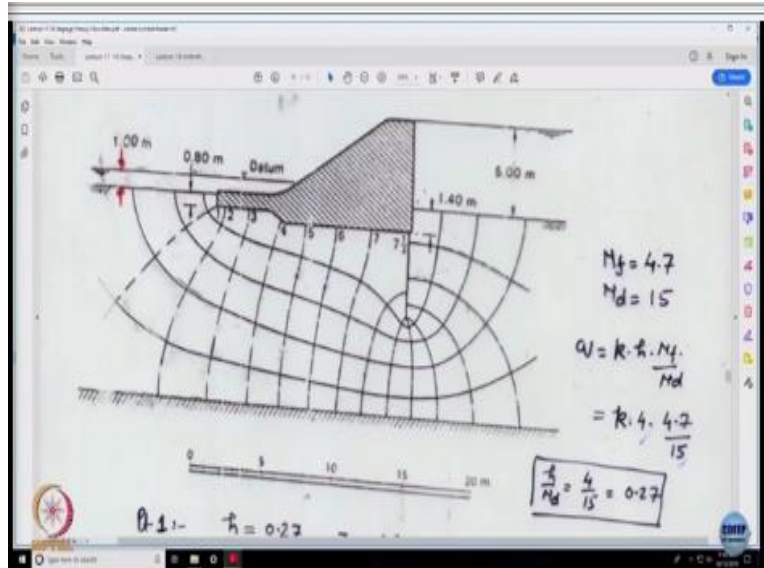
because of the seepage. In the second course, when we are dealing with earth pressure theory, we will find out the earth pressure which is coming on the sheet pile. And then the combination of the 2 will give you the total pressure which is acting on the system.

The concepts remains same these are sheet pile, this is the downstream ground surface, this is upstream ground surface, there is a water table layer in the backfill of the sheet pile. At steady state condition the flow regime develops in the porous media and the height which is attained by the downstream water table is 4.0 above the ground surface. If the flow net is known, you can work out the number of drops at point 1, 2, 3, 4, 5, 6, 7 along the sheet pile.

I can work out the pore water pressure and just now I have discussed how to obtain the pore water pressure at a given point, remember each equipotential line defines the total head. So if you know the elevation head of the point, total head minus elevation head will give you the pore water pressure it is a simple example which I thought I will discuss over here. These problems I have taken from the Craig book.

So my suggestion is please follow either Craig book or any other book whatever you might be having access off. Another interesting problem and this has something to do with what you were asking in the last lecture. There is a body of the dam or a wear in hydraulics course, you must have studied the design of wears.

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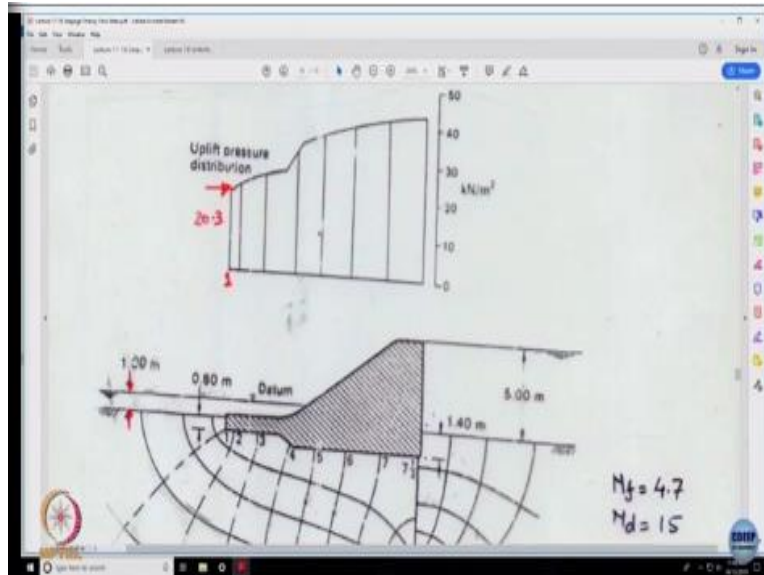


So this is the cross section of the dam which is retaining water with a water head of 5 meter and downstream side you have a water tail of 1 meter. As a thumb rule is that tail water level is considered as datum, draw the flow net number 1, find out the NF ND. So I think now you can realize that drop number 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15. So I have drawn ND as 15, the flow lines would be 1, 2, 3, 4 and then some fraction of the squares which I am considering as 0.7.

So number of flow lines I have taken as 4.7 you can compute Q value as usual. And this is a discharge is taking place through the bottom end of the you know sheet pile you may say from the opening, I can find out this discharge across any element. So suppose if this is the element across which I want to find out the discharge, what I should be knowing is what is the head difference across any element and then NF upon ND I will be using.

And then I can compute the Q value, what I want to highlight over here is these type of analysis are done to find out the uplift due to the seepage.

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If you remember, when we were talking about the seepage force computation, I had drawn a figure where I had created a earthen dam. And then at the base of the earthen dam I had shown you that is a seepage pressure which is acting. Suppose you have to find out the pore water pressure at point 1, 2, 3 and so on. Now, this is a pore water pressure which is acting at the base of the earthen dam or any system which is sitting on the soil media.

Point 1, the elevation head is 0.8 alright datum is at 1, so the elevation head at point 1 is - 1.8. If the equipotential line which is passing through point 1 is known, I know the total head at this point, I can compute the pore water pressure  $H - \text{elevation head}$ . It might so happen suppose if I want to find out the pore water pressure between 3 and 4, let us at some point 3 prime, what I will have to, I will have to refine my flow net in such a manner that one of the equipotentials passes through 3 prime.

So in that case the whole flow net will get changed, this is what somebody was discussing yesterday that and I was talking about the precision of the flow nets alright. So please practice these situations follow some book and I think they should not be any problem. Once you know the pore water pressures, you can obtain the distribution of the pore water pressure which is acting at the base of the system.

And then I can impose this pressure for finding out the stability of the dam. Now with this I finish my discussion on confined flows. As I said in most of the cases here whatever we have discussed until now, the porous media is confined by the boundary conditions alright. Now I am going to take the situations where unconfined flow takes place.