Geotechnical Engineering I
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Lecture-19
Seepage Theory

Welcome to lecture number 17 of geotechnical engineering 1, having done the seepage analysis. Now this is the right time to introduce you to the concept of seepage theory which is a quite a vast topic to discuss. So, until now we have talked about simple problems where the seepage is occurring through the porous media.

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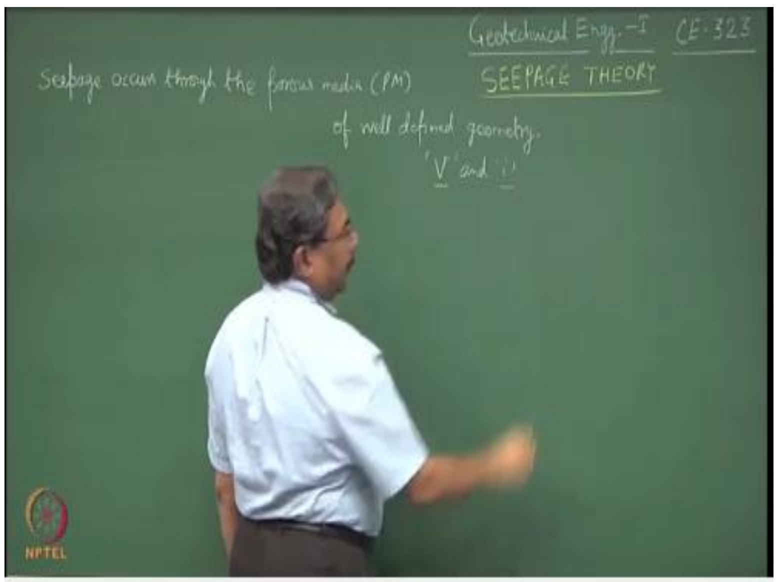

And it so happened that this porous media was you know of define geometry. Now, most of the time, when we deal with the porous media which is a well defined geometry. We assume the variation of n V and i you know easy to determine. But in most of the situations which we come across in the real life, the geometry of the porous media is not so well defined. And hence, it is becoming difficult or it becomes difficult to find out the variation of V and i in the real life situation.

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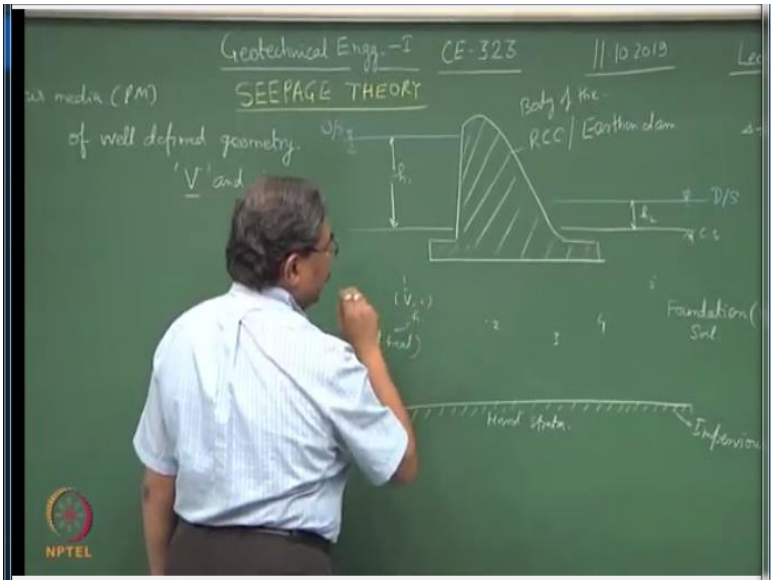

Suppose, if I say that there is a dam body now, this dam could be made up of RCC or this could be earthen dam alright. Now, this system is sitting on the ground in such a manner, so this happens to be the ground surface. And we have to show that there are 2 sites which are defined as upstream and downstream. So suppose this is the downstream water and this happens to be the upstream water.

And if I demonstrate the hard disk data by drawing this line, so this is the impervious layer, this is how we define the impervious layer. That means there is no flow which is going to take place across this line. If I define this as h 1 and if I say that the height the downstream water column is h 2. Now I hope you realize that it would be very difficult for me to find out the value of V and i and for that matter, even the h value where h happens to be the head at different points.

So this could be point number 1, point number 2, point number 3, point number 4, 5 and so on. Now, this is a typical cross section of a earthen dam or RCC dam. Now my interest is to find out how much seepage is taking place through the foundations of this system. So, this part is defined as the foundation soil or the foundation of the earthen dam, this is the body of the dam alright. Now very soon we will realize that the difference of head between h 1 and h 2.

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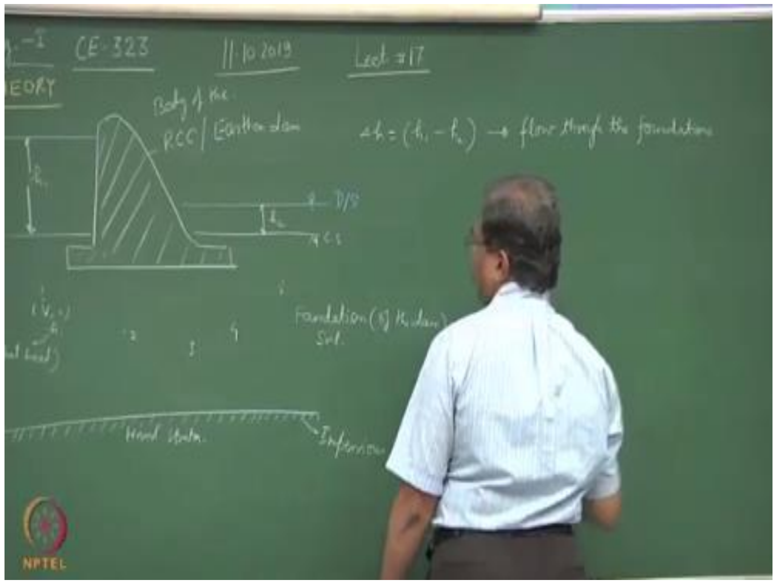

Now this is responsible for causing the flow through the foundations alright. So what I have to find out is I have to find out the values of 3 parameters v, i and h. Now as I said when you have complicated systems like this, it becomes very difficult to obtain the parameters as compared to the situations which we have already dealt with, so what we are supposed to do.

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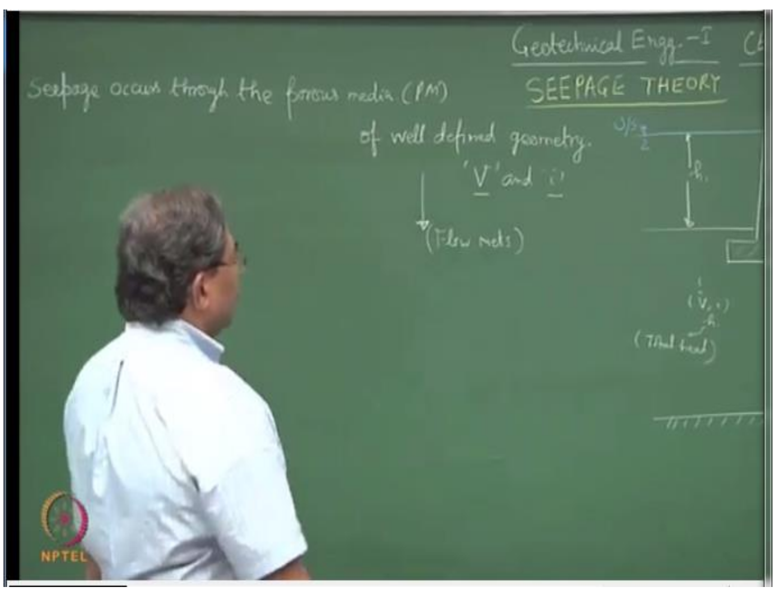

We take help of this seepage theory or the flow net.

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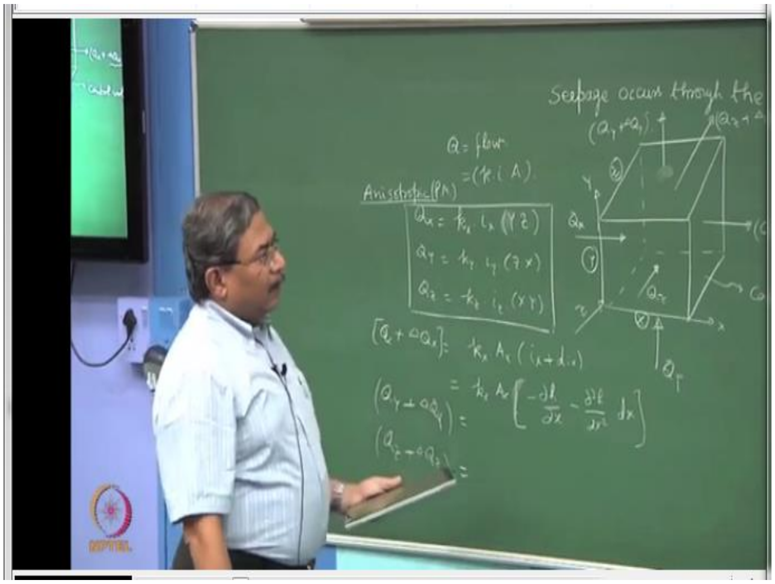

So if I consider a control volume alright, so this is the control volume and if I define y, x and z, these 3 directions, what I am interested in is finding out, how much flow is entering into all the 3 directions. So if I say that the flow is Q which is entering the control volume, so in the x direction the Q x enters and what comes out is Q x + delta Q x. I am just using delta x Q x to demonstrate that there is some change in the discharge.

In the z direction this is Q z and what comes out of the system is Q z + delta Q z. Similarly in the y direction, I can show that Q i or for the sake of science, you can say this is the Q y which enters the control volume and what comes out is Q y + delta Q i. Now I know that Q = k into i into A, where A is the **cross** area of cross section across which the flow is taking place. So for me it is easy to write, if I say Q x equal to and if I consider the soil mass to be anisotropic.

So let us generalize the situation anisotropic porous media. So the Q x will be equal to k x into I x into area through which the x is flowing, so x is flowing through let us say if I consider the dimensions as you know y and z. So this is the area of cross section through which the Q x is taking place. Similarly I can write Q i = k y, i y into z x and Q z = k z into i z x y alright. So I have the basic equations of the discharge or the flow which is entering into the control volume.

So this is the control volume and what I am assuming is that the dimension of this control volumes are x, y and z, so I can find out the incremental form also. The deltas of q x will be equal to, if I assume k x to be constant, you know what will be the function for this. I will skip these steps and you can say some time. We can write this as or what we can do is we can write as Q + delta x = k x into a x i x + d i x which will be equal to k x into a x i x + d ix which will be equal to k x into A x, what is the i x – del h upon del x, where del h is considered to be equal to h.

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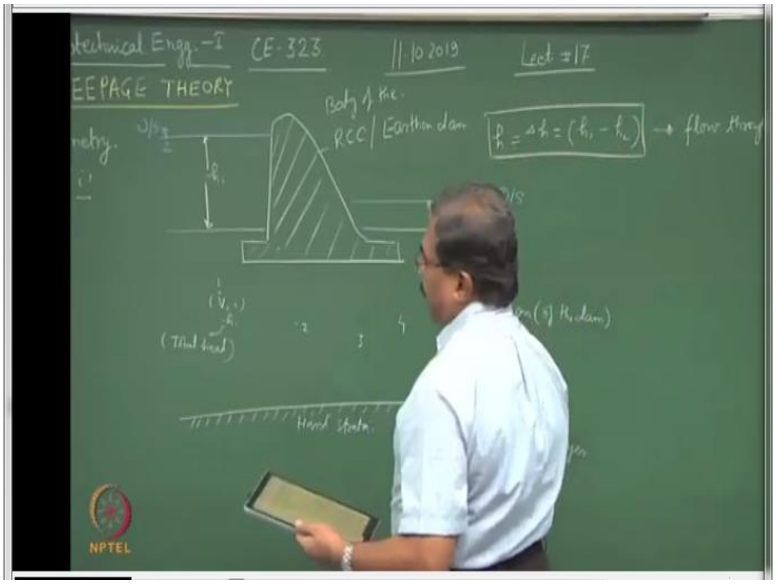

So if I consider this as h the hydraulic gradient is del h upon del x – del square h upon del x square into d x. Similarly, I can write for the other terms also that is Q y + delta Q y and Q z + delta Q z. Now if I use the continuity equation I can say that the total discharge which is passing through the system is flow ingress = the flow egress alright.

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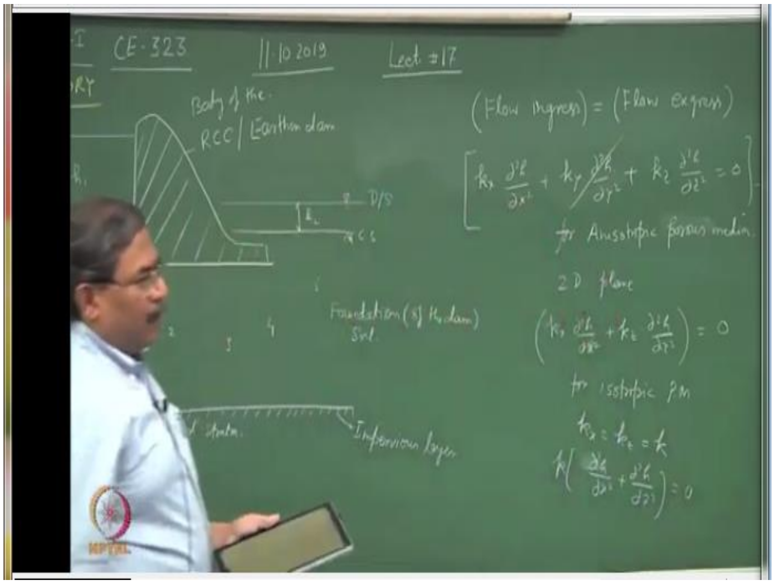

So that means, all these functions can be written as your Q terms are nothing but this where this is A x, this is A y, this is A z. So I can skip some steps to save time and what I will be getting is I will be getting as k x into del square h upon del x square + k y del square h upon del y square + k z del square h upon del z square = 0. Now this is what is known as the Laplace equation for anisotropic system, anisotropic porous media.

If I am talking about a 2 dimensional plane y can be eliminated and this becomes, we can eliminate this or we can assume this to be 0. And hence, we have k x del square h upon del x square + k z del square h upon del z square = 0. To make this system isotropic porous media I can assume that k x = k z = k. And hence, I will be getting this equation k del square at h upon del x square + del square h upon del z square = 0, what this indicates is del square h = 0 and this is again the Laplace equation.

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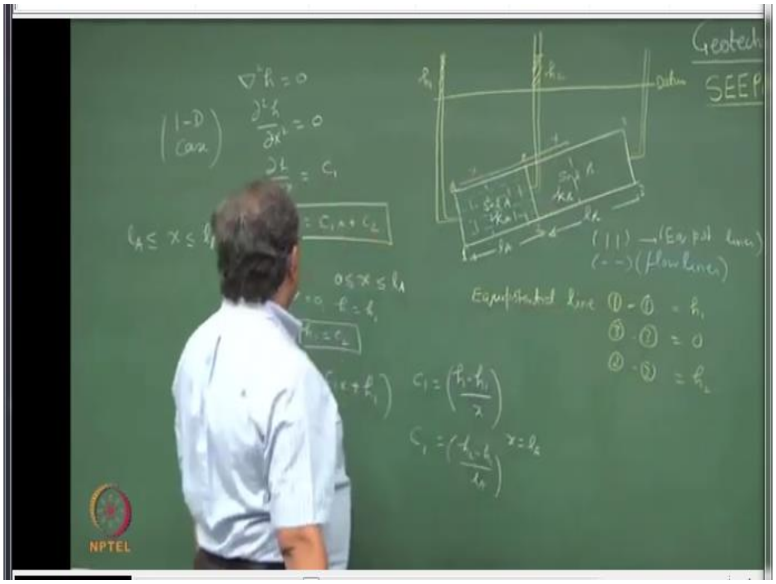

Now if I play with this equation there are 2 ways of interpreting this. Now h happens to be the total head which is causing flow to occur in this case. So suppose if I write it as, if I consider 1 dimensional case this will be equal to the del h upon del x = c 1 or if I further integrated I can say h = c 1 x + c 2 alright. So using the Laplace equation, I can also solve these problems let us take a simple case.

Suppose there is a system like this, there is a composite soil system soil A and soil B this is the length of the sample l a, l b. I am considering k a and k b as their hydraulic conductivities. Now if I connect this soil mass with let us say a water tub or a water bath alright. So these are the piezometric tubes somewhere here I can take the datum and with respect to this datum if I say that this is h 2 this is h 1 alright.

Now try to find out the discharge and the hydraulic gradients which are existing in the soil mass A and soil mass B of different lengths. So let me introduce the concept of equipotential lines here and flow lines. I can assume 1, 1 as the equipotential line because at each and every point on the line 1, 1 the potential is constant and this is equal to h 1, at 3 and 3 the total potential is 0. Now, somewhere in between that is a 2 and 2 we are assuming this to be h 2.

Now suppose if I ask you to solve this problem establish the discharge seepage velocity through this system by using the Laplace equation is fairly simple. So let me consider first a distance x starting from point 1 and x is such that, b this is less than l A and greater than 0. If I consider x to be, this should be my reference point, so what is going to happen, now at this point at x = 0, h = h 1 alright.

So h 1 = c 2 is this correct ok, what will be the discharge, so discharge can be obtained ,the moment h 1 is known, what I should be doing is this h is corresponding to somewhere in between alright. So all these lines are going to be parallel to each other, that means all the equipotential lines where the h is acting is going to be parallel to 11, 22 and 33. The lines which are going to cut them perpendicular to each other would be the flow lines.

So the blue ones are the flow lines and the white ones which I have drawn are the equipotential lines. Now suppose if I say that h, if this function is done I can substitute the value of h 1 c 2.

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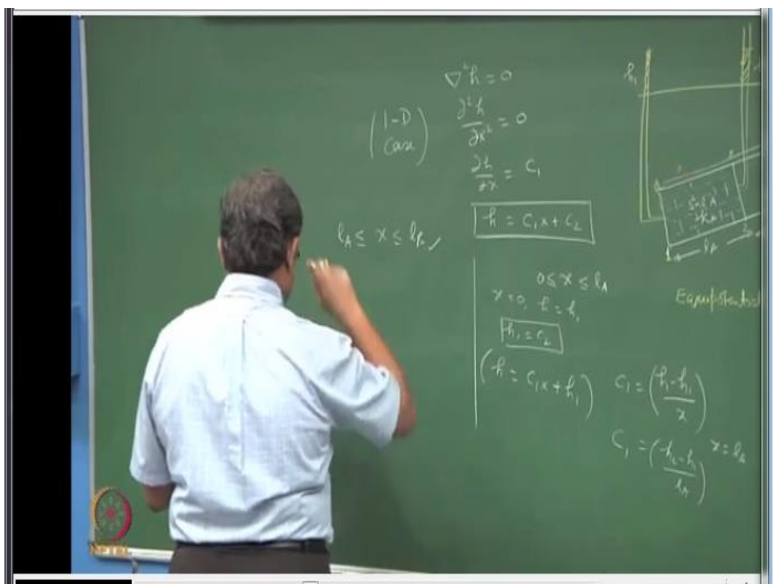

So h = c 1 x + h 1, so basically c1 comes out to be h – h 1 over x, x = l A, this will be equal to h is h 2 – h 1 upon l A. So I have the composite function now, I can substitute the value of this over here and I can solve this expression. Now similarly, what you should be doing is try to find out the values of h in the second domain that is when the value of x is less than equal to l B and l A. Suppose this is the point which I have considered here, let us say this is the x value alright.

Now I will give you the final expressions and please try it yourself to save some time you can show that. In this case the h will be equal to – h 2 upon l B x + h 2 upon l B l A + l B, so c 1 I can compute as – h 2 upon l B and c 2 can be computed as h 2 over l B l A + l B. So this is the head distribution which we are trying to find out. Now another thing which I would like to do is, I will like to find out the discharge which is taking place to the system.

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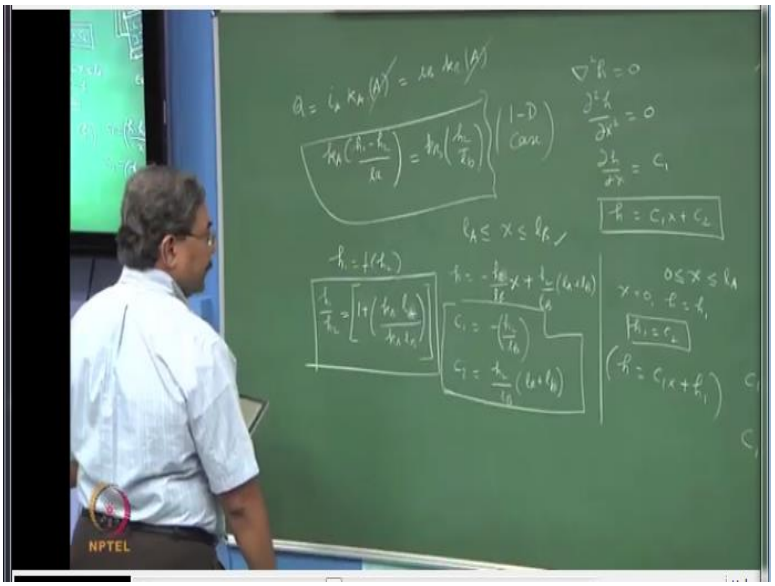

So, the Q value will be equal to i in let us say soil A multiplied by k A into area of cross section, in this case remains same of the sample. So this is equal to i B multiplied by k B into A, **A** can be eliminated alright. So this is going to be now k A into h 1 – h 2 upon l A = k B into h 2 over l B. So I have got another relationship between h 1 and h 2. So the principle unknown is h because I do not know what is the variation of h along the flow path.

So from here I can get h 1 as a function of h 2 and if you solve this expression you will be getting this as h 1 upon h 2 will be equal to 1 + k B into l B l A upon k A into l B A alright. So I can use this expression, I can use these expressions which we have derived and we can solve h 1 and h 2. So this is the application of the Laplace equation for analyzing the 1 dimensional flow these type of problems we have done earlier also.

I just wanted to site an example of how to use the generalized seepage theory to obtain the solutions to the problems which are going to be more complicated.

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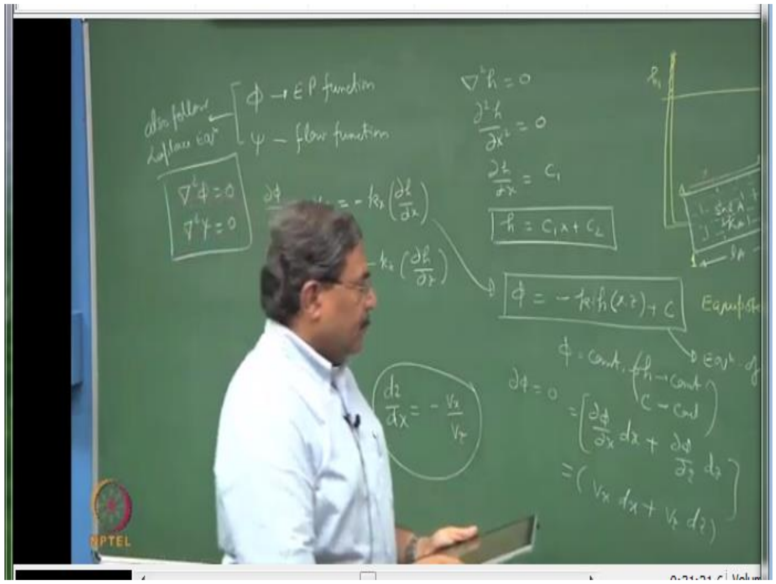

If I use the 2 terms as pi and psi, so the flow lines will be defined with or designated with psi.

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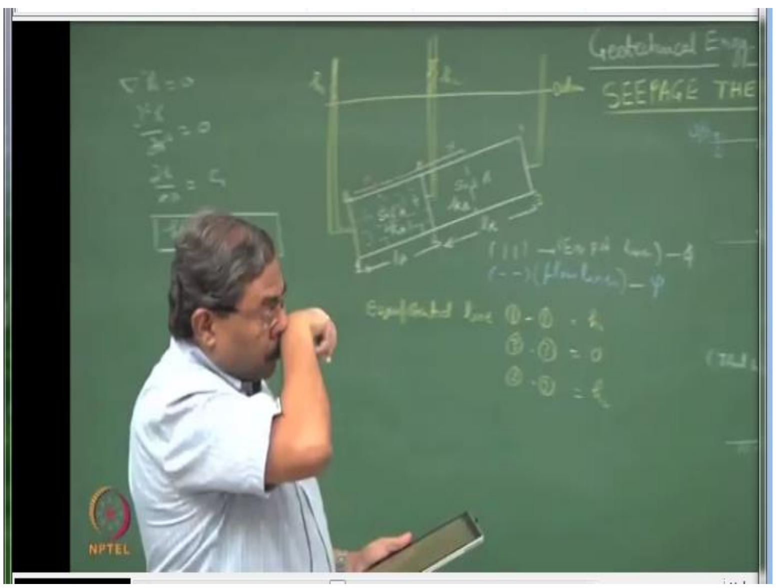

And the flow lines will be designated with pi alright, so this is the equipotential function and this is the flow function the characteristics or the properties of these functions are that if I said del pi by del x this will be equal to V x alright. And this is what is equal to - k x del h upon del x and del pi over del z will be equal to V z discharge velocity - k z del h upon del z the –sign indicates that x increases the h decreases alright.

So this is what the interpretation is, I can show that del square pi = 0 and del square psi = 0. Now what this indicates is that the functions pi and psi also follow Laplace equation. Now, if I solve this expression this will give me pi = - k h as a function of x and z + c. Now this could be an equation of a curve or this could be a straight line also. So what this indicates is that the equipotential function or equipotential line could be either a curve or a straight line.

Now suppose if I take pi as constant clear, so h becomes constant and c also becomes constant. If this is a situation, I can say that del of pi will be equal to 0 and del of pi can be written as del pi by del x into dx + del pi by del z into del z dz sorry. Now just now we have define del pi by del x as V x + del pi by del z as V z. If you solve this expression, what will be getting is dz upon dx will be equal to - V x upon V z, so this becomes your function number A. Now the same thing I can do for the flow function also.

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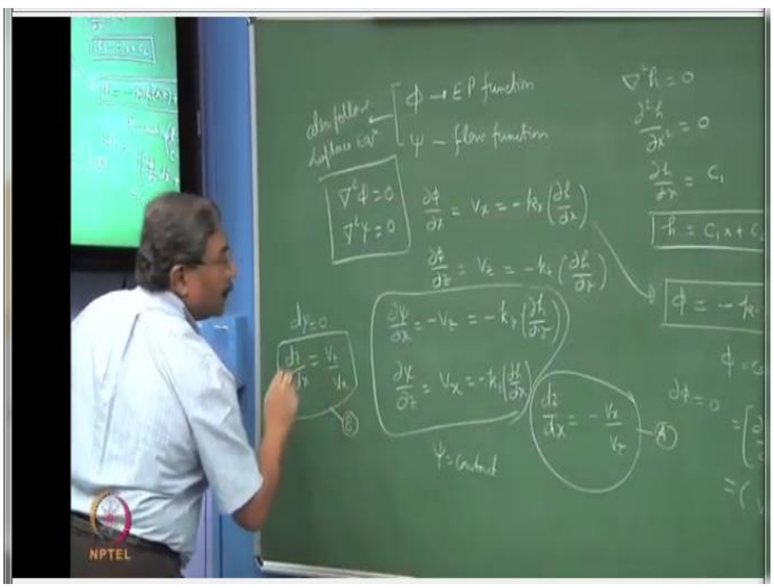

If I define del psi by del x = – V z and del pi by del z = V x. So this function can be written as - k x into del h upon del z and this can be written as - k z into the del h upon del x. I can use this function again to show that if psi is constant d psi will be 0 and this will yield dz by dx as V z upon V x. Now if I designate this as B you can make out that A is perpendicular to B.

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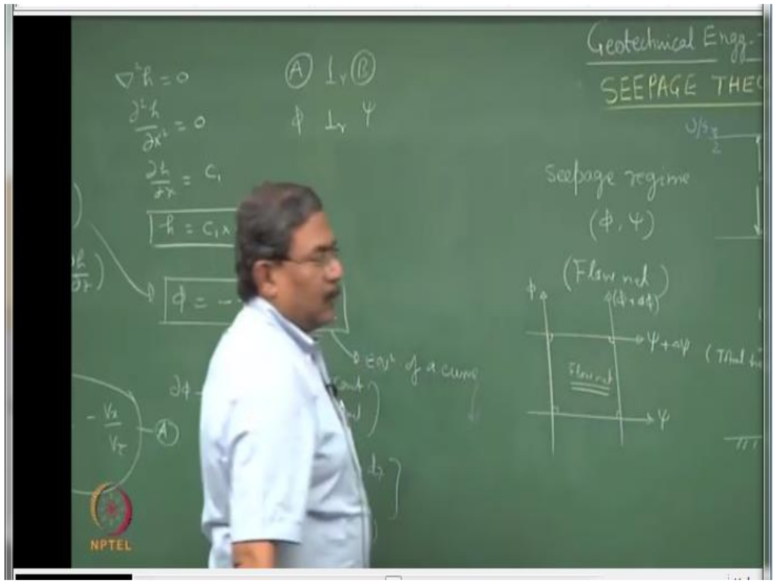

So A is perpendicular to B is this part ok, now A happens to be a equiponential line. So equipotential line is always perpendicular to the flow line, this is what we have derived. So when we talk about the seepage theory where all this is going to be used, this whole thing can be applied to solve or to establish the seepage regime. We call this as seepage regime and seepage regime is defined by pi and psi functions and this is also known as the flow net clear.

That means, if I maintain this condition and if I define this as a psi function which is the flow function this could be pi + delta pi psi and this is an equipotential function which is pi and another function here I can take as delta pi as long as the perpendicular or you know what to call it as a normality criteria is established that means pi is perpendicular to psi function, this is a flow net.

Those first requirement of the flow nets is that pi function should be perpendicular to the psi function. So what we have done until now is we started from a 3 dimensional flow which is entering into a control volume of the soil we are assuming that because of the seepage the volume of the control volume does not change, soil is incompressible flow takes place goes inside it comes out, we have used the continuity equation for flow.

And then we have derived the functions for equipotential lines and the flow lines. And we have shown that these 2 functions are always perpendicular to each other. And this type of arrangement of pi and psi function is known as flow nets, now I will discuss about how to utilize flow nets for solving different problems. So let us play with this flow nets a bit more let us generalize this.

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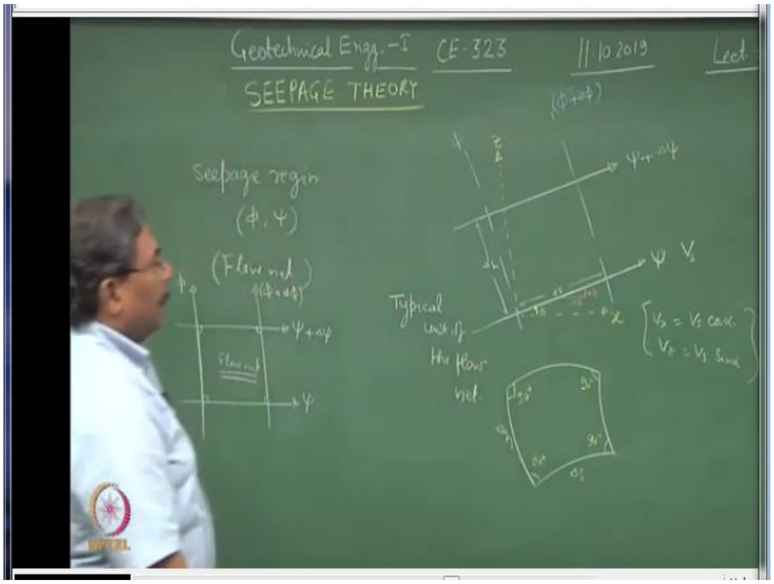

So suppose, if I assume that this is the psi + delta psi and there is another flow line, which is depicted as psi. Now perpendicular to this would be the equipotential functions depicted as pi + del pi. In such a manner that this is delta s distance and this is the delta n. So there is a specific reason of assuming this as delta s and delta n, this is the direction of the flow. So that means, the discharge velocity is in the s direction.

Now if I take the components of this vector in such a manner this is x axis and this is the z axis what is known as rotation of the plane. So I am just rotating the axis from sn to xz with this s alpha fine. I can define V x as V s cos alpha and V z as v s sine alpha. I can also define the term cos alpha s, If I take the slope of this line and if I define this as delta x delta z. So cos of alpha will be equal to del x upon del s.

This can also return as dx upon ds and sine of alpha will be equal to delta z upon delta s, which will be equal to the dz upon ds right. Now suppose if I say what is del pi by del s.

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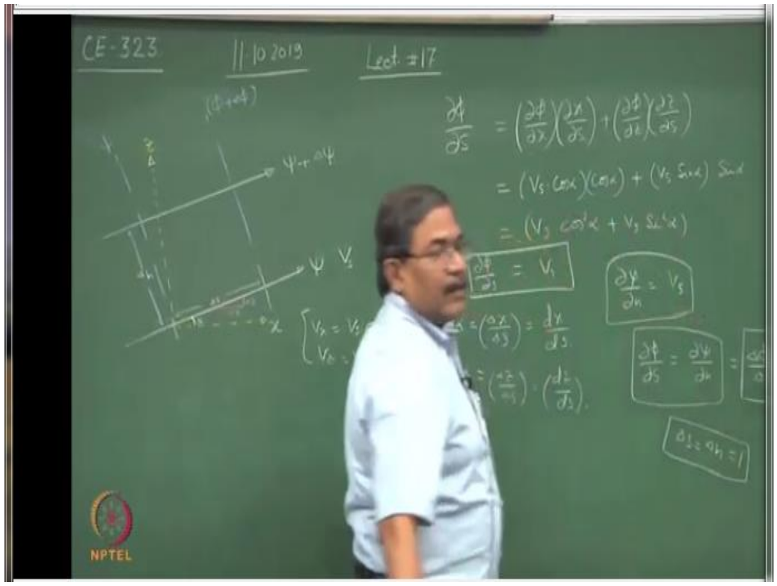

Incidentally del pi by del s is nothing but rate of change of Equipotential in the s direction alright. So how pi is changing along the direction of the flow is what is being depicted as del pi by the del s. So this will be equal to del pi del x into del x upon del s + del pi by the del z into del z upon the del s. So this is nothing but what is del pi by del x, del pi by del x is V x, so V x is V x into cos alpha and dx by ds is cos alpha +, this will be V s sine alpha into sine alpha.

So this is equal to V s cos square alpha + V s sine square alpha this will be equal to V s alright. Similarly I can also show that del psi by del n will also be equal to V s. Now what this indicates is that del pi by del = del psi by del n, which can be written as delta pi over delta s = delta psi over delta n and that is what I had said. That there is an intention of choosing this delta x and delta n as the steps or the a spacing between the equipotential lines as delta s.

And delta n as these you know spacing between the flow lines. Now if I put a condition that delta s = delta n and suppose to simplify things, if I make it equal to unity. So this is the typical property of a square, that means the flow net are going to be geometrically a square units either they will be curve or they will be linear. So there is no harm, if I assume delta s = delta n = 1, what I m saying is delta pi = delta psi alright.

So the flow net by definition is a square entity made up of a linear system or this could be nonlinear system also provided the delta s ok. So this becomes the delta n the curved path and this becomes the delta s the condition of normality cannot be sacrificed. So this is 90 degree and this is 90 degree, so this is a typical unit of the flow net.

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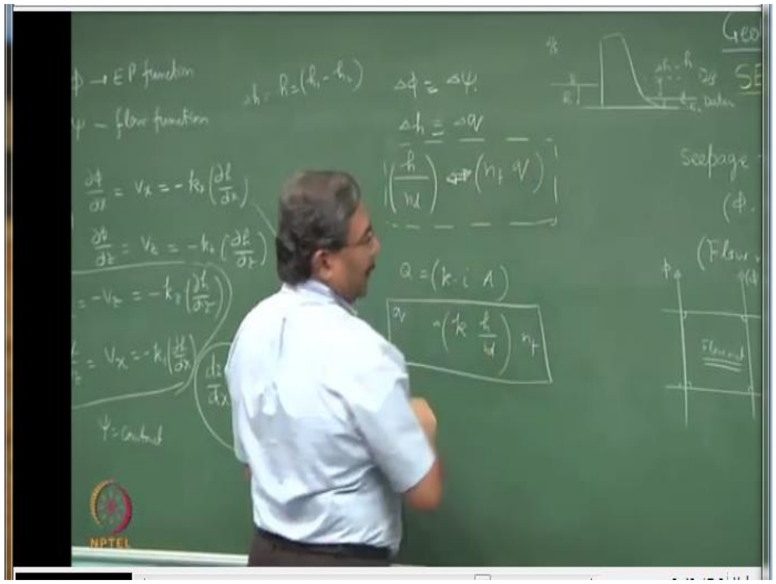

Suppose if I start from this function that is delta pi = delta psi, this is nothing but delta Q and this will be delta h. Now if I introduce 2 terms that is the total head you remember h = h 1 – h 2 which is also equal to delta h. So this head is causing the flow to occur, a quick review of the discussion which we had earlier, if this is the body of the dam this is h 2 this is h 1 head downstream upstream.

If this is a datum at the tail water or downstream water this is delta h which is equal to h and this head is causing the flow to occur. So if I introduce a term delta h as number of drops this is going to equal to delta h and what about the q. The q is taking place through number of channels, so what will happen to number of channels. It gets multiplied by number of channels is it not, so what will be that term this will q, the total discharge.

So the delta h is the total head is getting divided by number of drops and the total discharge is getting multiplied by total number of flow lines. Now this is what we are going to utilize in further analysis is a equivalent sign we will be using these terms. So if I say that the total discharge is equal to k into i into A normally what we do is, we consider this as 1 into 1 which we have already done and area of cross section is defined always as per unit third dimension of the element.

As if the flow is entering into the system clear, so that means A will become 1 and then this q is nothing but q into n f and then k into i h upon nd is this ok I was just trying to show the equivalence over here. So let it be this is equivalence this is ok, this is equivalence. So basically what I am trying to show is, when we talk about the total q value, if I am dealing with the delta q function. So this function will get multiplied by n f.

So that means the total q will be equal to this multiplied by n f is this ok because there are so many flow channels which are contributing to the discharge. So this concept I have used over, let us say, I think I will eliminate this term to not complicate things then it is alright.

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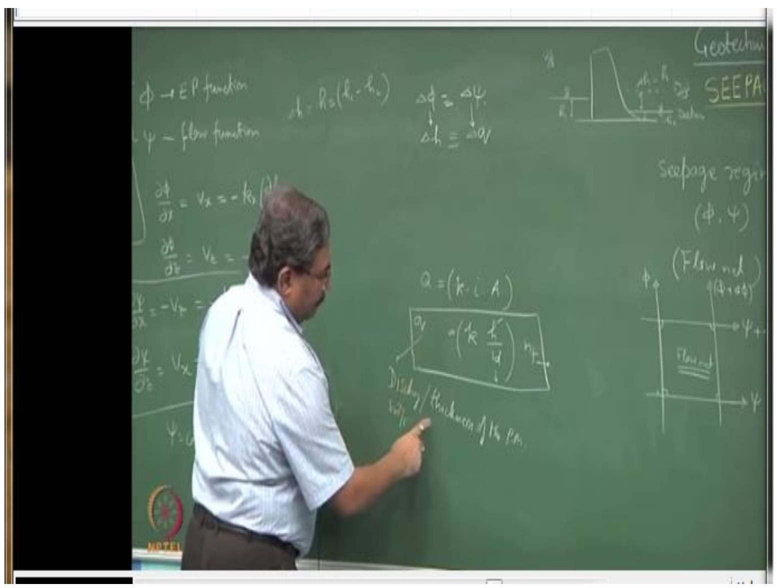

No, no my idea was that delta pi is nothing but delta h function and delta psi is delta q function this is what I am trying to show. So basically if you use this function q which is the total discharge taking place through a system, what I have to do is I have to plot the flow nets. I have to compute what is h which is causing flow to occur, I should be knowing what are the number of drops of the potential which are occurring and what are the number of flow channels.

This is what is going to give me the discharge per unit length per unit thickness of the element of the porous media. So remember the discharge units would be meter cube per second, area cross section we have taken here as 1 into 1. So this is your discharge will be meter cube per second per unit length in the third dimension, now if you have anisotropic soils.

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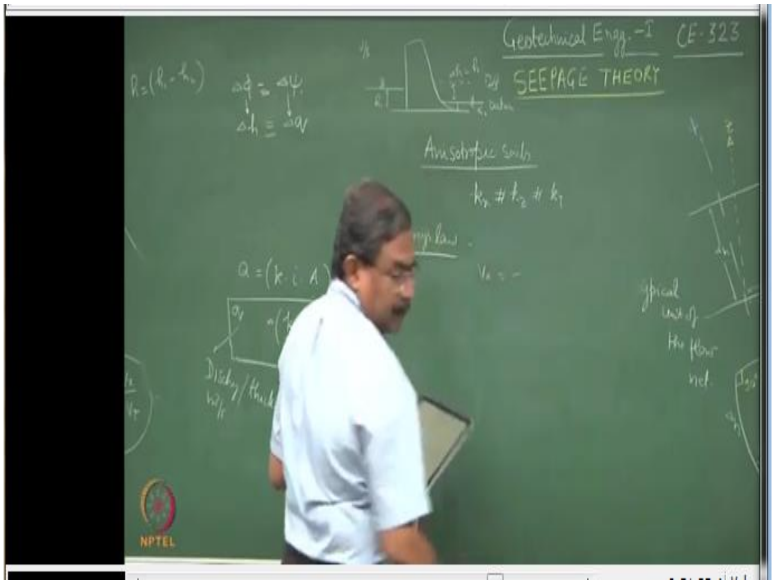

This was for the isotropic situation you know whatever we had done, so here we are assuming that k x is not equal to k z is not equal to k y alright. But suppose if I still consider the 2 dimensional flow, I can use the Darcy's law and I can say V x = - k x yeah this is already written there, so I can use this function.

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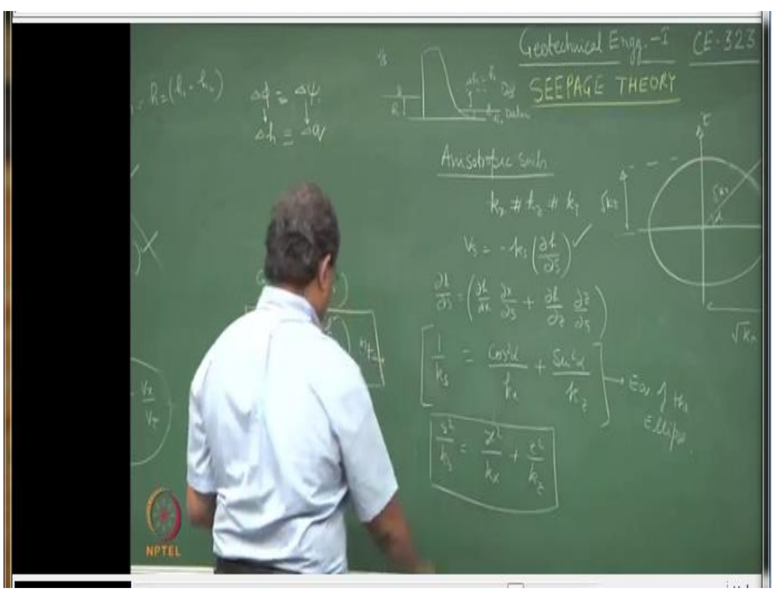

I can say V s this V s = - k s into del h upon del s, now if I solve this function the del h upon del s this will be equal to del h upon del x, del x upon del s + del h upon del z into del z upon del s. This can be shown to be equal to 1 upon k s, if you substitute V s term V x term and V z term you can show this to be equal to cos square alpha upon k x + sine square alpha upon k z, I hope you can realize that this is the equation of the ellipse alright.

And if I ask you to draw the ellipse, if this is the x direction, z direction, this is the s direction, this is the alpha term, this is under root of k s, this is under root of k z and this is under root of k x. This can be also written as s square upon k s = x square upon k x + z square upon k z. Now there is a interesting way of depicting the anisotropy by using the Laplace equation which we heard earlier assume to be one dimensional flow and for isotropic conditions.

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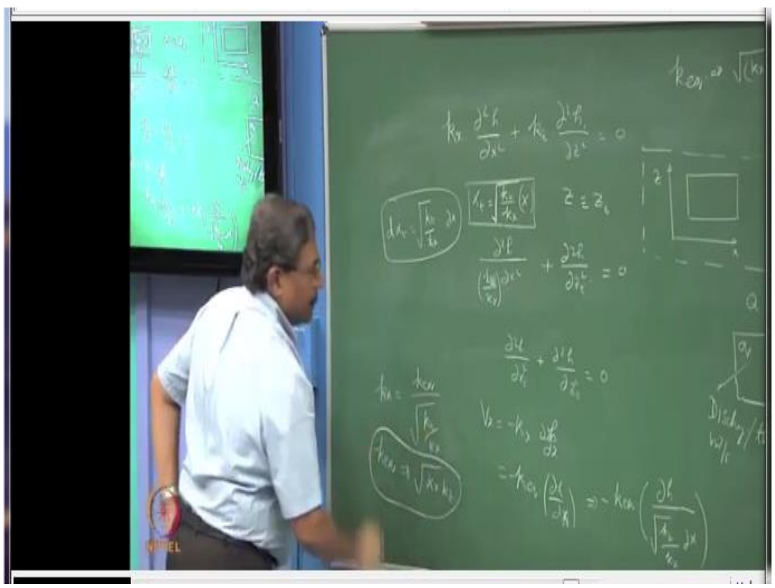

So **sa** if I say that k x del square h upon del x square + k z del square h upon delta z square = 0. This is the equation for anisotropic condition, I can create isotropy by assuming z, z is equivalent z let us say t alright. In that case I can write del square h upon this plus if I say this is del square h upon del x square and this is k z comes over here divided by k x is this ok. So this is equal to 0, if I assume that x t = k z over k x into x, can I replace this term with del square h upon del x t a square + del square h upon del z square.

So what I have done is a anisotropic situation has been converted into isotropic situation by transformation. So the flow net which we have been talking about on x z plane, suppose sorry which was a rectangle why, because we have k z and k x, this can be transformed to a scale x and z t. Now the moment I transform it over here, what is going happen, this is going to be a square is this fine.

Now this type of transformation is used normally to deal with the anisotropic situation, here see what we have done, here we have manipulated with the permeability coefficient. So k z upon k x is treated as del x t upon del x = k z upon k x. So this is a anisotropic coefficient, I can also write x t upon. So there is a mistake I have done. So this part would be under root because when you are doing square term over here.

This should be the correct function, is it not it is ok only this will be outside the under root because this whole thing is the x square. So if you say del x t will be equal to under root of k z upon k x into del x.

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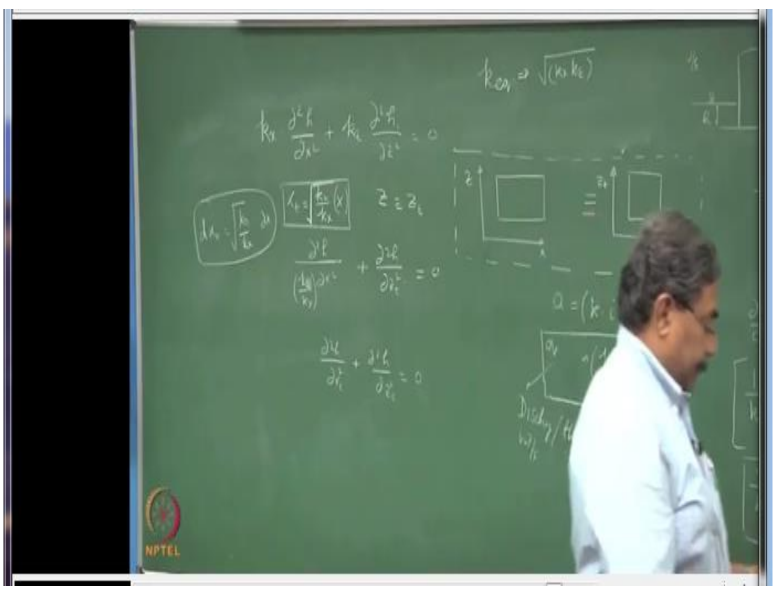

I can define another term as k equivalent of the anisotropic system and that should be equal to, suppose if I write that k equivalent would be under root k x into k z can you prove this, again you go back to the basics. So V x = – k x into **x** del h upon del x, now if I replace this term by k effective or equivalent this will become del h upon del x t. Now this will be equal to - k equivalent, what will be del h upon del x t.

This will be under root k z upon k x into del x, that means k x will be equal to k equivalent divided by under root k z by k x and this would give you the expression that k equivalent would be equal to under root k x into k z.