Geotechnical Engineering I
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Lecture-18
Coefficient of Permeability

We have been discussing flow through porous media solved different examples and try to convey this message that in what way the flow modeling can be done. We have also talked about the hydraulic conductivity, the coefficient of permeability and I will discuss a bit more on coefficient of permeability in today's lecture, so basically k depends upon several factors.

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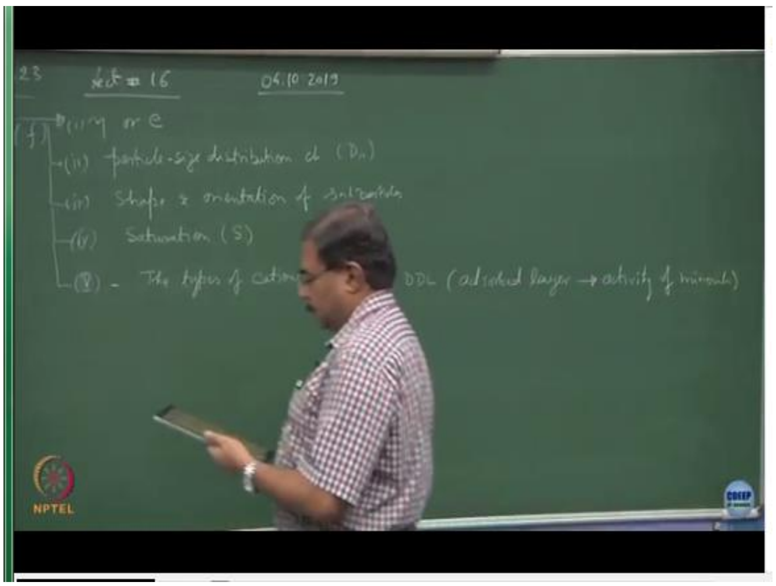

And these factors are the porosity or void ratio, the particle size distribution characteristics. We have used this concept D 10 and so on, the third parameter on which the k depends upon is the shape and orientation of the particles. The fourth parameter on which it depends upon is the saturation and normally we define this as s the another parameter on which it would depend is the type of cataions which are present in the soil mass and the thickness of diffuse double layer.

DDL stands for diffused double layer, we also call this as the adsorbed layer and which is an indication of the activity of the minerals, so if you remember the way we have defined k.

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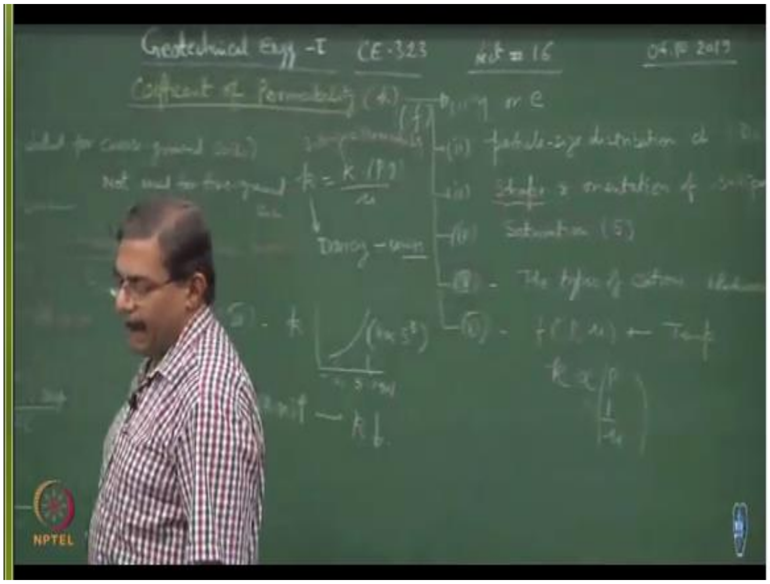

We said k is equal to capital K multiplied by rho upon mu into g and where capital K we have defined as the intrinsic permeability. So basically k depends upon the density of the fluid and the viscosity of the fluid and these 2 get influenced by the temperature alright. Now, this is the complete matrix, so how the environmental conditions influence the hydraulic conductivity or coefficient of permeability. Because they affect the density and viscosity of the fluid which is permeating and these are the other parameters.

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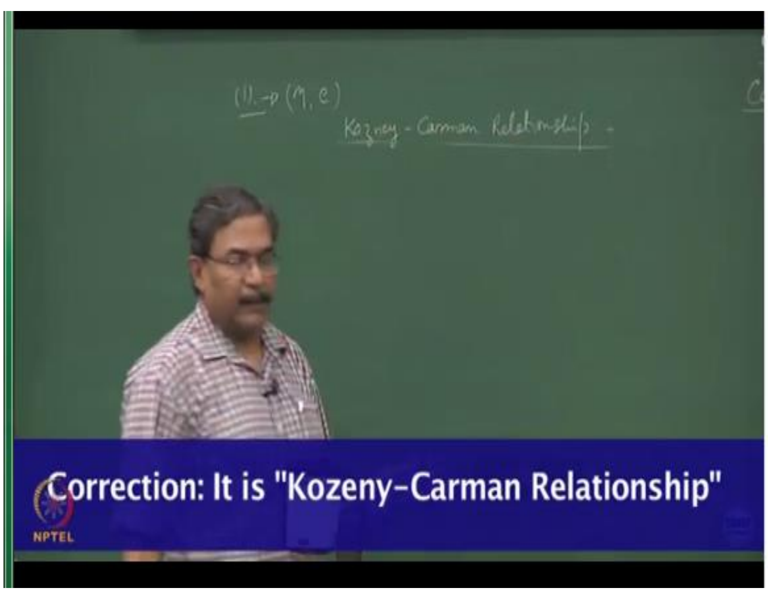

So the first parameter, if you want to understand what is the influence of the porosity or the void ratios, this is what is known as the Kozeny-Carman relationship.

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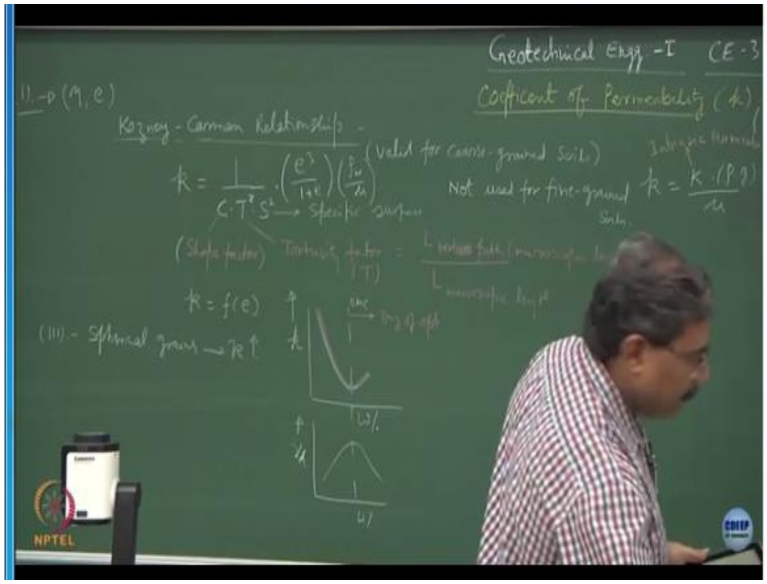

This relationship is valid for mostly coarse grain materials and it is written as the coefficient of permeability is equal to 1 upon C multiplied by T square into S square, C is the shape factor. It is understood that if the particles are a spherical in nature, coefficient of permeability is going to be higher as compared to the particles which are flaky or the particles which are angular or sub angular, T corresponds to the tortuosity.

If you remember we have talked about this in the previous lecture, tortuosity factor is the length of the tortuous path divided by the L is the physical or you may this is tortuous path is nothing but the microscopic path or microscopic length and this is the macroscopic length tortuous. So this is a square of T term and S corresponds to a specific surface. I hope now you can realize that is not going to be easy to determine these coefficients and these parameters can be obtained with the help of advanced instrumentation.

So, this is e cube over 1 + e multiplied by the density of water upon viscosity of water. The best way to find out shape factor would be laser microscopy. You can define the term that how the shape looks like tortuosity factor is a very difficult parameter to determine unless you go for some advanced instrumentation scheme like CT scanning all right or 4D X-rays these are the techniques which are being used nowadays.

This equation is normally not utilized for fine-grained soils alright, basically what it indicates is that k is a function of e. And you will find different variants in the books where people have tried e upon 1 + e also. But this is quite prevalent and this what is known as Kozeny-Carman relationship, shape and orientation of the soil particles. You remember, we have talked about the shape already, so the granular materials which are a spherical in nature.

So this is number 3 a spherical grains show very high hydraulic conductivity for the same density remember as compared to the fine grained materials. Now as far as the orientation of the particle is concern, I hope you remember this relationship which I had plotted sometime back when we were talking about the compaction curve and what I had shown is that if this happens to be the OMC you have the variation like this alright.

So this is how the variation of permeability or the coefficient of permeability would be with respect to moisture content. So at OMC the k value is minimum, on dry of optimum the hydraulic conductivity increases a bit with increase in the moisture content. And the best way to learn this would be the compaction curve, if you remember. So this is the moisture content and this is the gamma d, so these 2 are in combination to each other. Another thing where here we should remember is that on I am sorry, that is not dry of optimum.

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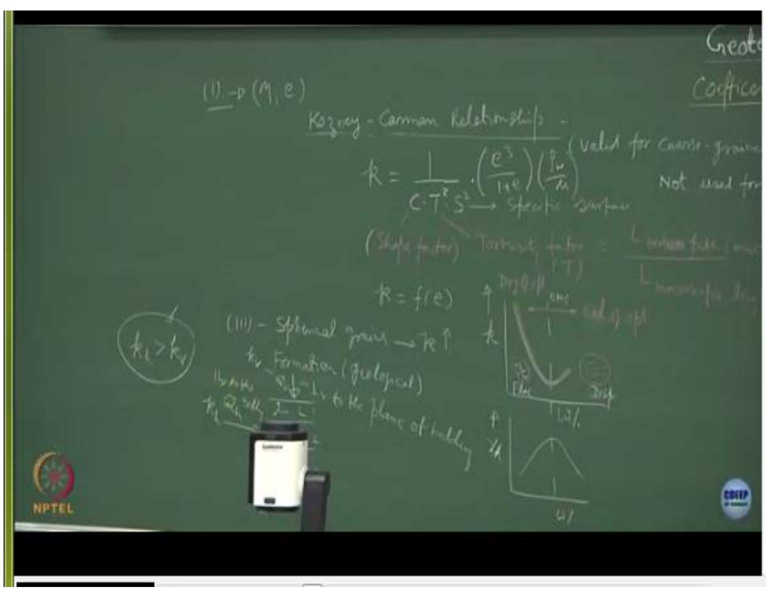

This is wet of optimum you should correct me and this is dry of optimum alright. So dry of optimum we have particles which are highly disoriented alright, we call them as flocculated and on the wet of optimum we have the particles which are more dispersed. So by definition or by fundamental nature a dispersed structure shows less hydraulic conductivity as compared to the flocculated structure then we talk about the formation also.

And when I say formation this could be basically geological formation, there are 2 types of systems with come across, this is the layered soil mass. And once you say layer number 1, layer number 2, layer number 3 it so happens that this has become heterogeneous soil mass clear. But the assumption here is that each layer is isotropic in nature. So I will be discussing more about this isotropic nature or heterogeneity of the material, what isotropy indicates is.

If I take a point and if I measure it is properties in the x, y and z direction, all the properties are going to be same, this is the isotropy fine. But because of the multi layered system, it becomes inhomogeneous or heterogeneous. So a heterogeneous system could be isotropic layer wise, so isotropic nature means the properties are same at a given point in all the 3 dimensions alright. Now this type of formation may occur in nature by virtue of deposition.

You remember we had talked about all these things quite earlier when we are talking about the how soils are form. So in this case, we would be very eager to know what happens to the hydraulic conductivity when the flow is taking place perpendicular to the bedding direction. So this is the flow which is perpendicular to the plane of bedding, what is bedding. Sediments are being carried by the reverse and because of the density contrast or because of gravity they settle down. So, the formation of the entire soil mass is taking place layer wise.

Horizontal lines which I have shown are the bedding planes and this indicates the discharge in the vertical direction which is perpendicular to the plane of bedding. I will be more interested in also finding out what would be the Q h. So this is the horizontal flow of the fluid which is taking place through a layered system alright. Now I hope you can realize that this is going to be the parallel to the bedding.

Now as far as the saturation is concerned there is a relationship between K and S. So saturation of the soil is defined as I hope you understand the amount of water which is present in the pores to the volume of the pores ok. So normally this is a cubic relationship those of you who might get a chance to work in all these petroleum companies (()) (15:16) you will be using throughout your life this type of a relationship.

Because soils are not going to be saturated always why, because you have lot of methane, lot of gases trapped in the sediments clear. So this is a relationship which we are going to use normally we define saturation as something which is greater than 90% alright. So if soil sediments are approximately of greater than 90, 95%, we call them as saturated, if not they become unsaturated and about 30% less than we define them as dry alright.

So this is the relationship as far as saturation is concerned, now what is going to happen to the type of the cataions, the more presence of active minerals alright. So if active minerals their fraction increases, I have defined this as active minerals alright, what is your guess. So the more the active minerals present in the system, the double diffuse layer is going to be quite big, the resistance offered by the system to the fluid flow is going to be less.

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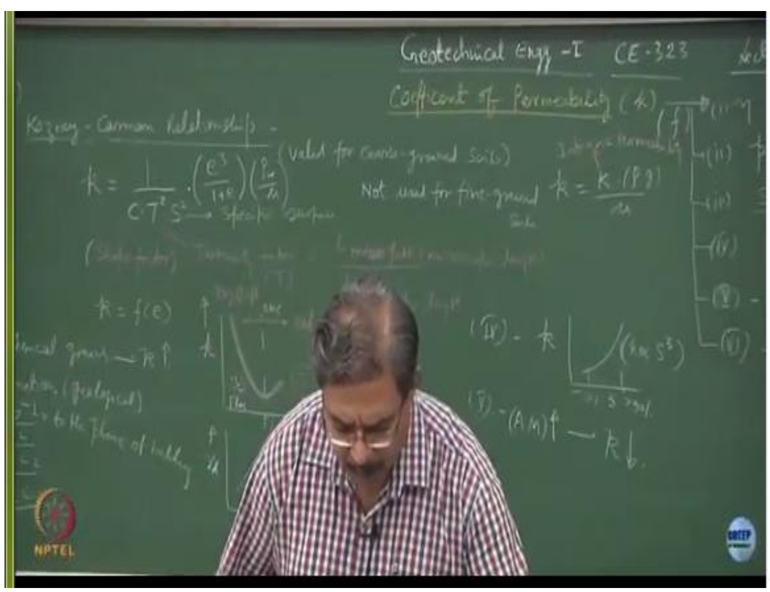

And hence your hydraulic conductivities are going to be less, the same thing is valid for the cation exchange capacity also alright, number 6 I think this we have already discussed. So k would be proportional to density of the fluid alright and inversely proportional to the viscosity of the fluid, is this ok. So a denser fluid is going to permeate easily provided is viscosity is less, is part clear or vice versa, any questions. We have also talked about the methodologies for determination of k.

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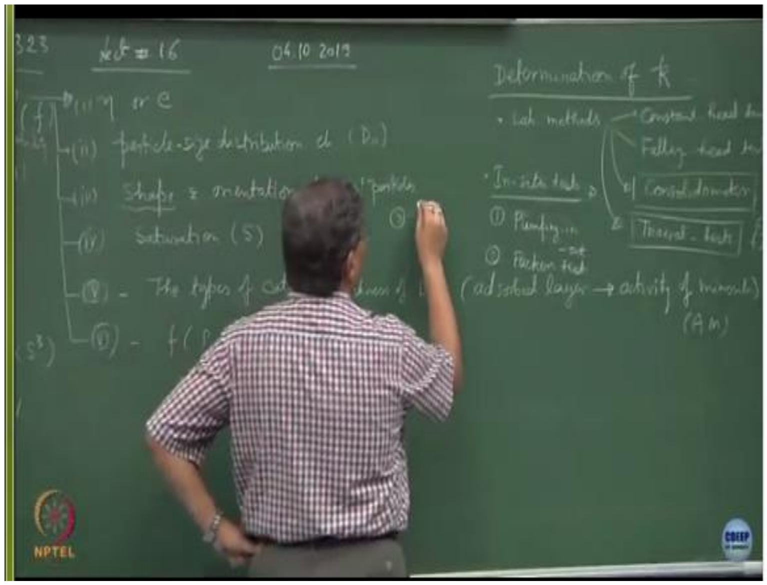

So until now we have discussed 2 methods which are mostly the laboratory methods, if you remember these are the constant head test and the falling head test. There is another test which we will be defining very shortly when we talk about the consolidation characteristics of soils. So I can use a consolidometer to obtain the k value. This has to be discussed slightly later when we discuss about the consolation characteristics. As I said, another method would be a triaxial method or triaxial test.

As far as the in-situ test are concerned, what we do is number 1 a pumping in test. So will discuss about this or it could be a pumping out test, what we do is we drill a well, and then pump out water or pump in water and we see how much the drop in the head is going to take place and unit time. There is another series of tests which is known as the packers test alright.

There is another interesting way of defining the permeability of the formations under in-situ conditions, particularly those of few who will get a chance to work in deep excavations in rock mass would be up Lugeon test. We call this as Lugeon index, so this is what is yours for you know fractured rock mass. Incidentally one of the ways to define the hydraulic conductivity would be in the units of Darcy also.

So k can also be defined in the form of Darcy alright depends upon in which continent you have working and in what type of problems you are working, any questions related to this, no, alright. So let me take up now some in-situ test.

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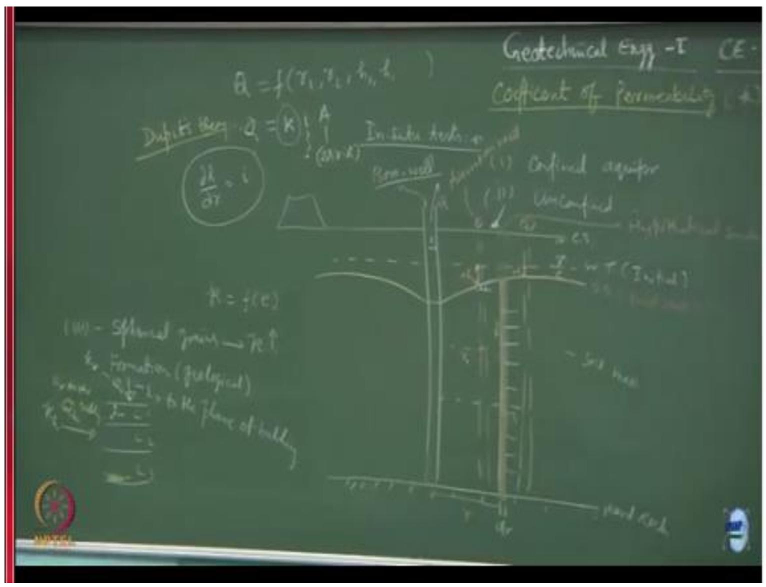

When we talk about the in-situ test we have to be very particular about in what type of aquifer, I am working this could be the confined aquifer. In your hydraulics scores I am sure you must have studied this and the second one is the unconfined aquifer. So by definition, suppose if this is the ground level and if I drill a well this is the bore well. Sometimes people also term this as a tube well, it is basically a bore well you are boring into the soil cutting the soil and putting down a casing.

And most of the time this casing is perforated alright, so this becomes a bore well that means your bored the soil and then you have lower down the strainer or a casing which is made up of steel to support the soil otherwise soil will collapse and the well is not going to function. So this is perforated and suppose we have a water layer somewhere here, initial water layer somewhere here. This is the water table not water layer sorry water table and somewhere here you have hard rock alright and this is the soil mass.

The system in which the boundaries are not defined clear and they are open this becomes unconfined aquifer ok. So you have to monitor closely the difference between 2 and 1 and I draw the figure for 1. I am just discussing now the unconfined aquifer case. Now suppose if I start pumping out water, what is going to happen. This water table is going to diminish, lower down clear, we had talked about effective stress analysis what when the ground water table decreases.

So imagine there is a building somewhere over here alright and without telling you, someone starts a industry and he or she is trying to suck out all the water for creating let us say utility items. Now what is going to happen, this water table is going to subside, is this ok, because of the discharge pumping out operations. This is the steady state situation I would say a final state because I have written steady state is this ok.

Now the way these tests are done is you create a well of certain dimension start pumping out and then you create 2 observation wells. So observation wells could be located in the vicinity. This is observation well number 1, this is observation number 2 and what we are going to observe how much the groundwater has depleted, is this ok delta h in both the cases. So this is in ground this is in the well number 1 observation well number 1, this is observation well number 2.

I hope you can understand that this delta x is nothing but again the discharge which is taking place. Now sorry this has been created because of discharge Q is this ok, Q has caused this delta h1 and delta h2 to appear. Now suppose if I define the well radius as capital R from the radial distance to this is let us say r 1, this is r 2 alright. And suppose if I take at a distance of r another hypothetical well which represents a hypothetical surface ok.

So this is a hypothetical surface, I hope you can realize when you are pumping out water the discharge is going to take place perpendicular to the surface is this correct, have you understood this. So this is a sort of a cylindrical curved surface perpendicular to this, the discharge is going to occur. So if this is r and thickness of this element is d r clear, height of this surface is let us say h fine, can I write a relationship between Q as a function of r 1, r 2, h 1, h 2 and so on.

Because Q is nothing but k into i into A is this ok, so k is the principal unknown remember, we are doing the field test to obtain this coefficient of permeability, what is i, area is not an issue area I can obtain, what is the area. The curved surface will be 2 pi r into h, is this ok, so I need not to bother about this thing. So that means area will be 2 pi r into h, what is i hydraulic gradient now how will you find out a hydraulic gradient.

Now, this is where we have to take help of a theory which is known as Dupit's theory, this is valid only very close to the wells. So as you go away from the well what will observe is Dupit's theory is not valid because this curve flattens. Now, as per Dupit's theory the slope of this line corresponds to the i value, that means del h upon del r would be equal to i, is this ok. Now, what we know, we know the value of Q.

Because the moment I start pumping out test, I know how much volume of water I have taken out per unit time. So Q is known, I have 2 observation wells why, because if you substitute these values over here, you will realize that this is going to be a quadratic equation.

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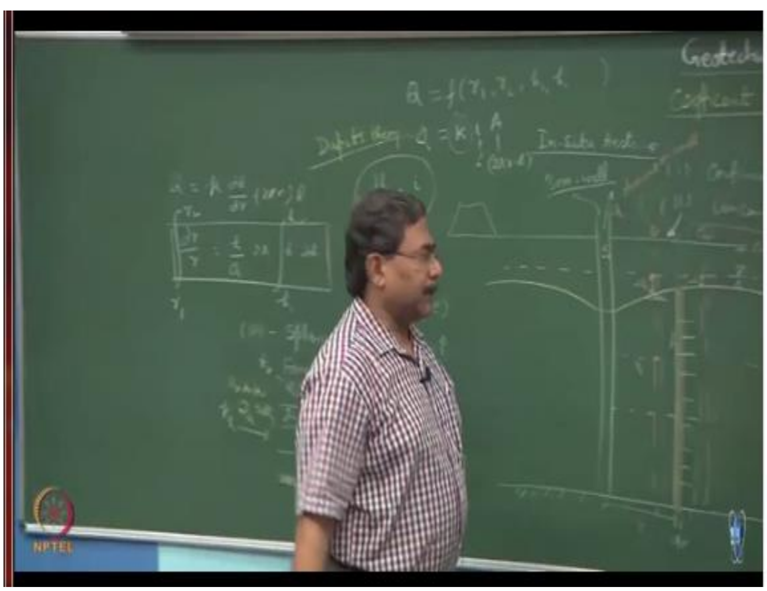

So, k into del h upon del r multiplied by 2 pi r into h ok, so k i into A. So I can write this as del r upon r = k by Q and 2 pi into h into del h is this ok. So, del r upon r = k by Q del h 2 pi h. Now this becomes my governing equation, I can solve this easily what I have to do. The whole idea of creating 2 observation wells was I know the r 1 I know the r 2 clear. I know the total values of heads and I can solve this expression, is this ok.

So what I can do, I can use a sort of a magnetic tape not magnetic tape I can use a electronic device, I will lower it down the moment it gives me a beep. It is understood that I have touch the water table, I know this value. Similarly I know this value, I can substitute over here and I can get the values of k the difference between this situation which is. Now so, basically what is the difference between unconfined and confined aquifers, the more you take out water.

The more water table keeps on drowning there is no constraint on this, so these type of aquifers are known as unconfined aquifers. Now suppose if I introduce a layer of aquifer in this manner.

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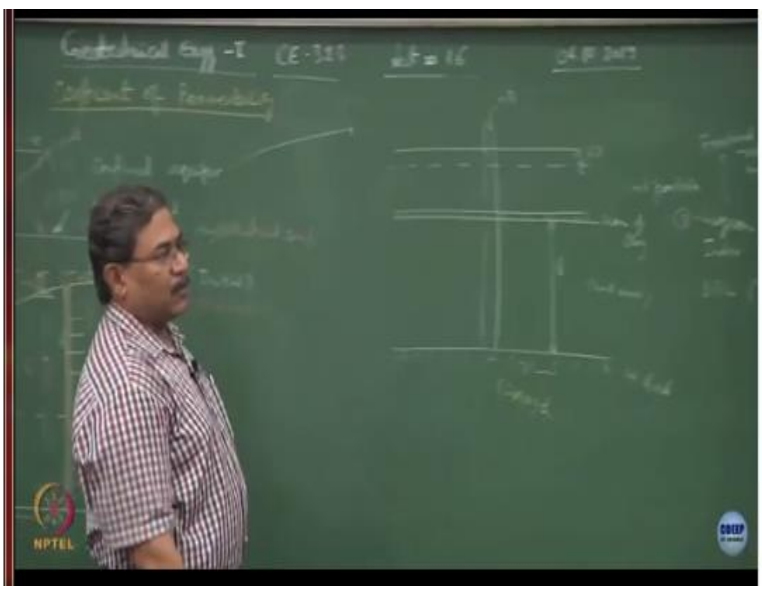

This is the ground surface and this is the layer of the aquifer, this is the hard rock, this is the ground surface and this happens to be let us say is clay of seam of clay. And I am pumping out water, I hope now you realize that this is what is going to be the soil mass. And what has happened the soil mass is now constrained between the 2 impermeable boundaries 1 and 2 clear. So what is going to happen, this surface is always going to be bound by this, is this ok.

So that means when you have put a constraint on the porous media through which the discharge is going to take place, this becomes a confined aquifer case. Here what is happening here the surface area is changing with respect to time the more you pump out, this value of h keeps on dropping down, here this is going to be constant. And then what about the water table, the water table would be somewhere here, this is the initial water table ok.

And the more and more I stuck it out what is going to happen, it has to be below this, that is it. So if you solve this expression was going to happen here, this your area term will be if I take it as a distance of r, this will be 2 pi r into h, h remains constant. Now we are going to use these concepts quite a lot in our analysis. So what we have talked about until now is coefficient of permeability, what are the parameters on which it depends, how to obtain permeability, coefficient of permeability in the laboratory in the field.

Then these 2 situations and I hope this helps you ok. So one more thing I would like to discuss that is the unsaturated hydraulic conductivity. So, until now what we have discussed is about the saturated states of the material present day geomechanics is heading towards unsaturate state of the material. So sometimes people also want to know the unsaturated soil hydraulic conductivity k u.

So we are now differentiating you must have noticed very cleverly the difference between k sat this, all k saturated alright and k u. There are many schools including the Cardiff where most of the guys are working only on k u why, because most of the geotechnical engineering applications are related to unsaturated state of the material, nuclear waste disposal buried cables, buried pipelines, buried hydrocarbon facilities, nuclear explosions, municipal solid waste alright.

Chemical reactions going on in the soil mass as the is there in find that the moisture front moves out and soil becomes unsaturated. There is a very interesting way of obtaining it though it is not a very accurate solution but to begin with I think you can use the concepts what we have discussed in the classroom.

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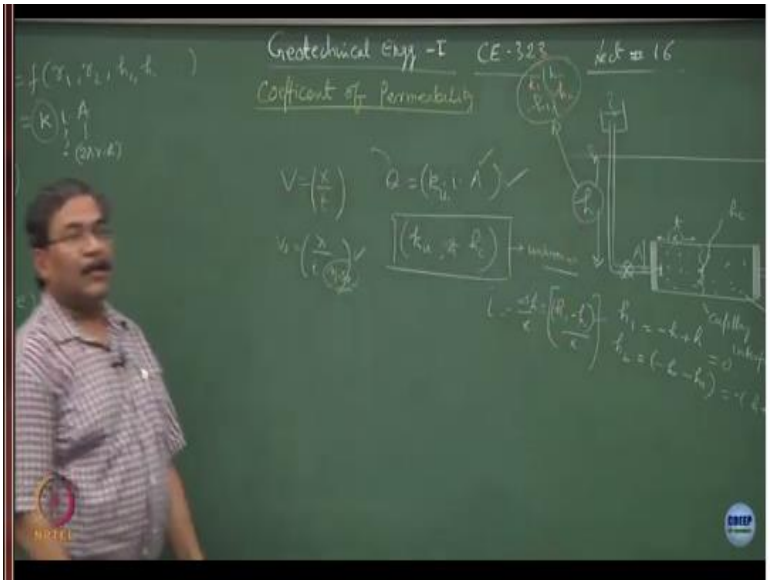

So what normally is done is, you take a glass tube and seal it from both ends by trapping some soil mass into it. So this soil mass is unsaturated, or let us say dry soil mass which is compacted at some gamma d alright, submerge the entire thing into a water bath. So what I have shown here is the water bath is this ok and let the moisture front move in the, I can create different type of boundary conditions.

Now suppose if I shift I think coming back to our discussion which we are having in earlier lectures. If I create at point 1 and point 2, where point 2 is the capillary interface, so the moment I opened the valve, what is going to happen. This water is going to enter into the dry soil mass and imagine you can do this experiment also, you fill up the dry soil and then open the valve, let the water move in.

So this is the distance x in a given time t of the waterfront, which is moving from left to right, is this ok you can imagine, compute the total head at point 1 and compute the total head at point 2. Suppose if I say the total head at point 1 is 0, suppose if I shift my datum somewhere here, at the height of h, the elevation head is - h and the piezometric surface would be h. So total energy at this point is 0, what about this point.

If I consider this point just on the left hand side of the waterfront, what will be the head on this point capillarity. If the dry phase which is just being negotiated by the water, so at point h 2 to the elevation is - h and the capillary head is – h c. So the total head would be - h + h c, so you know the hydraulic gradient, what is the length of the path x. I can do this experiment I can measure the x over a period of time and I know how much x is moved in with t, h c is unknown alright.

So the moment I know h upon x which is the hydraulic gradient, what is the value of discharge and here I want to find out k unsaturated. So this k will become unsaturated truly speaking you may debate that Darcy’s law is not is not valid for unsaturated hydraulic conductivity. But still we are trying to use it because there is no harm in getting something which is approximate as compared to very precise, is this ok.

So if you know the area of cross section of the system i is known, what are the principle unknowns, k is unknown and what else h c, what about the velocity term x by t, is this ok. And if I want to find out seepage velocity, this will be x upon t upon porosity, I am not very sure about what is the saturation state soil I put a penalty on this porosity, I will multiply by s r. So what I have done I have penalize the porosity, you understand what is the meaning of penalizing some parameter uncertainty.

If S r = 1 porosity is eta is fine, if s r is not equal to 1 this will be some fraction of porosity which is contributing to the whole process. So now if I say that this Q is what k i into A and I know the V s term is alright, what I can do A into V s is going to be the discharge, can I equate this function with this function. And then what is going to be unknown x I can measure t I can measure k u and s c are unknown.

So in this equation, you are going to have 2 unknowns, is this fine. So what I should be doing, if I play with this h, so what did I tell you I can raise and lower down the column of the saline solution or the water column. So if I play with h, what is going to happen, I will be having different values of h let us say if I create h 1 and h 2 corresponding to time t 1 and t 2, are you getting this point. So I have 2 equations, I have 2 unknowns, I can solve them.

So those of you who might pursue your carrier and unsaturate state of solid mechanics, your studies will start from this point onwards. So, let us move on now to the layer systems of soils, until now we have been considering all homogeneous isotropic situations clear. Now let us introduce into it inhomogeneity, non homogeneous sometimes people call this, inhomogeneity is a better word.

So how inhomogeneity will be introduced by considering layers what we were discussing there. Now the mechanics part is coming in, the hydrostatics we did then we started with the hydrodynamics of the porous media clear. And now we have related the hydrodynamics to the different field applications and now we are going to come to the extremely real life situation, so that is what we discussed over there, the way the formation of the deposits occurs.

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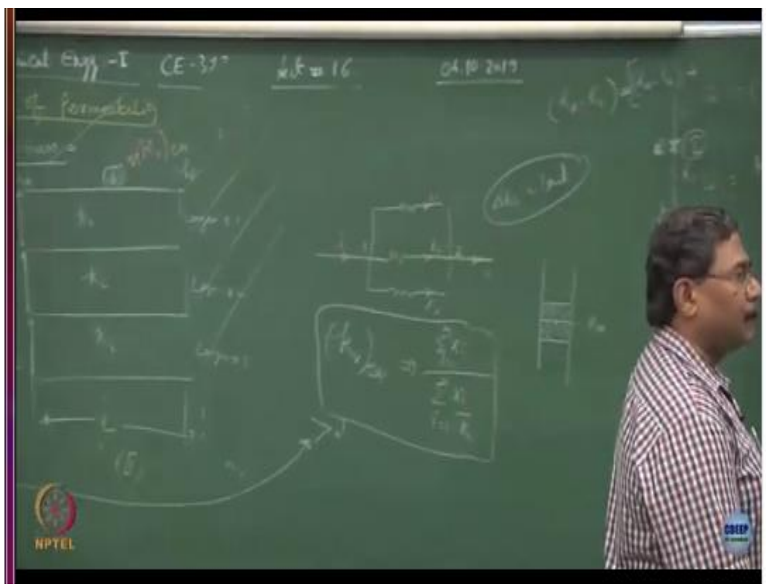

So if I consider is layered system of soil mass, so this is layer number 1, layer number 2, layer number 3 and so on. We might be having layer number n as well alright, I repeat is a inhomogeneous system heterogeneous system of soil mass. Because you have different layers, but what we are assuming is that each layer is isotropic in nature that means at a given point the properties, density, void ratios, hydraulic conductivity are going to be constant.

In most of the pumping out tests what happens, fine particles from the soil mass get washed out clear. So, why pumping out tests are detrimental to the safety of structures, when you are pumping out water excessively, the chances are all the fines will also move out. And the moment fines move out the density of the system is going to decrease because remember the fine fraction only contributes to the density of the system, why you have a system like this.

And all these fine particles were making the system more compact dense, now when you were doing pumping out operation all these particles have washed out because of what i gamma w previous lecture. See the beauty of geo mechanic says that you cannot study this in discreteness, are you getting the application all the concepts, which I have discussed in last 2, 3 lectures. All these fine particles are getting lifted up by i into gamma w per unit volume.

So the more and more pumping you do the more and more hydraulic gradient develops clear and these particles get lifted up which results in a loss of void ratio and gamma d. The system becomes more porous, a more porous system, the chances are that it will collapse it will show more volumetric deformations, it will show more subsidence is the story clear fine. So, I have knit lot of concepts together in today's discussion.

So let me attribute some dimensions to this. let us say this is of length L and X 1 is the width X 2, X 3, this is let us say I will simply say k 1, alright. Life is already very complicated, so I am not changing other parameters k 2, k 3 and so on. I can create 2 situations here if you remember flow taking place perpendicular to the bedding and flow parallel to the bedding clear. Now all of you have done equivalent circuits resistances in series and parallel and batteries in parallel and series 10 + 2 physics hope sure.

Can this system not be represented like, now suppose if I asked you to fill in the blanks, what is flowing through this on the right hand side, current. Current gets distributed, is this correct, so you said here that i get distributed in the form of i 1, i 2, i 3 and it comes out from the system, what remains constant, very good voltage, where is the voltage getting applied, across the resistances clear.

So if you consider this as point number 1 and this as point number 2, what you said is the delta V 1 2 remains constant, is this ok. Suppose, if I allow flow of water to take place through this, what is going to happen, is this situation not similar to the one which I have drawn, have you understood clear. As if multiple pipes are connected to this phase through with the discharge is going to take place.

So, that means Q h = Q 1 + Q 2 + Q 3 Q n and the analogy is this, water remains constant. Water remains constant is the voltage across not in this case voltage hydraulic gradient. So if this is h A on this plane and on this plane if it is h B what is the hydraulic gradient would be delta sorry delta h = h A - h B over length this becomes i is this ok. Now you can complete the whole exercise.

So, your Q h will be equal to k equivalent horizontal, is this ok multiplied by i, what is i h A- h B over L into area of cross section, what is the area of cross section x 1 + x 2 + x 3 clear. And most of these aquifers are the soils, beddings are going to be in the third dimension like this, is this ok. So this is going to be let us say w width, I can always take perpendicular unit section of one meter length.

So as the sheets of the soils are sitting over each other, so that means the area of cross section is going to be now x 1 + x 2 + x 3 it is multiplied by w, is this part clear, no issues, analogy is clear. And then now you know what is Q 1, will be equal to k 1 i x 1 into w, is this ok. So you know all the expressions, tell me what will be be k h equivalent quick yes, very nice. So this would be sigma k i into x i nice sigma very good.

So you have understood the whole thing i could be 1 to n and this i will be also 1 to n, is this part clear no confusions. Now if I revert the situation, so we have already got for h equivalent, now suppose if I change the direction of the flow or if I am interested in finding out what is going to happen to let us say k V equivalent. Draw on your notebooks equivalent circuit first and try to understand what is getting distributed now.

In this case the current got distributed agreed, now was going to get distributed sorry voltage true. So as if this system, is current entering into a system of r 1, r 2, r 3 and so on. And about point number let us say that was A B like in the lateral direction, here I can say x y no I will not use x y, I will use let us say what should I use 1 and 2 ok. So this is your point number 1 and this is your point number 2.

As if multiple layers of the resistances are as arranged in a series manner work out k equivalent, I think you must be remembering the equation also. So how do you solve this function, so if I say the Q is entering into the layers and it remains Q always, is this ok. So this Q is equal to we know this or we have to measure this one will be equal to k 1 into i 1 what is the i 1. So now this is going to be between 1 no.

Let us say this is a and b h a – h b over x 1 into area of cross section what is area of cross section w into l like this you can complete the whole thing, what you are observing here is the next time is going to be h b – h c, is this ok. We did this problem in the last lecture also, remember this case I was talking about a situation where starting from P m let us the P m material, I will create a multi layer system and then we were debating upon that at this point I do not know the head.

I do not know the head at point b, c, d intermediate points clear. So how would you solve this equation or how would you solve the situation, are you getting this point. The question here is I do not know what is the head at point b, c, d and so on. I hope you understand this, instead piezometer and then do something. Otherwise this a interesting question, otherwise you have another equation in this form this Q remains constant use this equation correct.

So this is equal to this and this will be equal to the next one and so on. So ultimately you are getting a relationship between a h a, h b, h b, h c and the total head across this is going to be let us say h a and z. So this will be equal to k vertical equivalent multiplied by i, so i will be h a – h z upon l, what is the value of l x i sigma is this ok fine, multiplied by w into l. This is nothing but the total length of the system the total length of the flow and your area of cross section will be l multiplied by w fine.

So this is going to be same as each one of them and what is the relationship between h a - h z, this will be equal to h a – h b the hydraulic gradient is getting dissipated or getting added up. So this total head will be equal to this plus, this plus, this plus this. So you write here, this will be equal to this plus another term and so on. So can you guess, what will be the k vertical equivalent would be any guess 1 upon r = 1 upon r 1 + 1 upon r 2 that equation you remember, what happens in case of capacitance c 1 + c 2.

Now this concept nowadays being used in geomechanics to create equivalent circuits of soils and on which many of them are working. So we do impedance spectroscopy at very high frequencies and we try to get the response of the material and when we do modeling and all. So yeah tell me what this would be sigma x i very good sigma very nice. Please try to prove this, what is your intuitive feeling which one equal to more k h equivalent vertical or k h equivalent horizontal are you sure 100%.

So this proves that k h is greater than this one and hence mostly the flow of ground water is in the lateral direction and so on, it is ok.