Geotechnical Engineering I

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Lecture-17
Permeability of soil and Ground Water flow-II

We have been talking about the permeability and flow of ground water in porous media.

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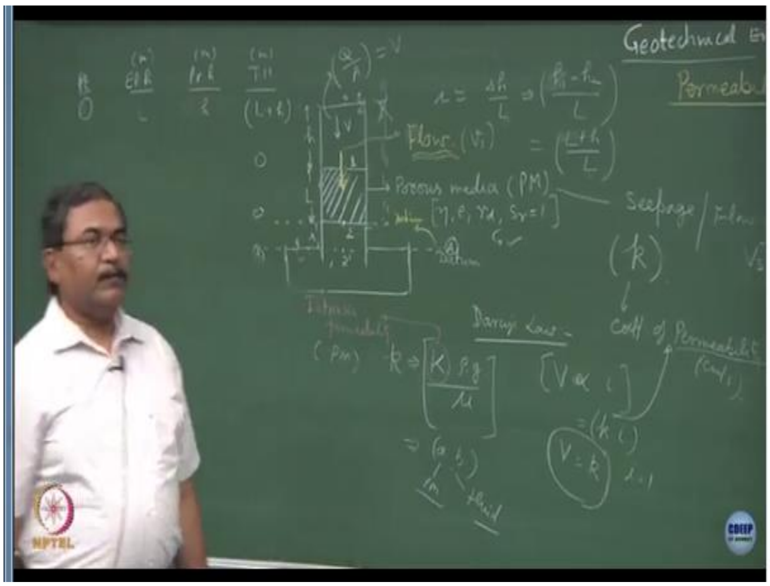

And we have been discussing about this problem that there is a porous media in case in a glass tube or a control volume. And then we were trying to analyze this problem as far as total head is concerned, pressure head is concerned and the elevation head is concerned. So as far as the elevation head is concerned, that is easy to obtain. So, if I say point elevation head, this is in meters, the pressure head is also in meters and the total head this is also in meters.

So point 1 was quite straightforward, we have already proven that between point 0 and 1 there is not going to be any flow because the total heads are same, point 1 the if I assume the elevation to be matching with the free water table. And suppose if I assume this as x I am sure you must be realizing that the most of the confusion is because of selecting the datum. So you have to add this x to this, a better way of doing this would be if I shift the datum at point 2 alright.

So the moment I shift datum to point 2 like becomes simple. At point number 2, the elevation head is 0. And what about the pressure head, this is exposed to the atmosphere. So the pressure head is 0, total head will be 0, I will tell you what is a fallacy which we are discussing last time. At point number 1 the elevation head is L, the pressure head is L + h, now sorry h and hence, the total head is L + h.

So the hydraulic gradient would be if I compute the hydraulic gradient this will be delta h upon L which is h 1 – h 2 over L this is nothing but L + h over L alright. The porous media is described by these parameters, that is porosity, void ratio, gamma d is known, S r is known, specific gravity is known and so on. Now the moment you shift point number 2 to let us say 2 prime, what is going to happen.

So this is where the trick is, I think you should be careful when you are writing these heads. If I shift to the point 2, what is going to happen, the elevation head at point number 2. So this is second situation, when I have shifted, when the datum is add let us say A A, at point number 2 the elevation head is x. Now what you have to understand is that this is directly exposed to the atmosphere because of it is connection with the free water table.

So if you put a piezometer over here, is not going to get you know, raise up to this point is not correct, why it is not correct. It cannot get lifted up to this point, what is the reason, because you have a situation where the boundary condition says that this line is exposed to the atmosphere. And hence the total head at this point is going to be 0, clear. Now those of you who are confusing this with the situation which we have been dealing with in the past.

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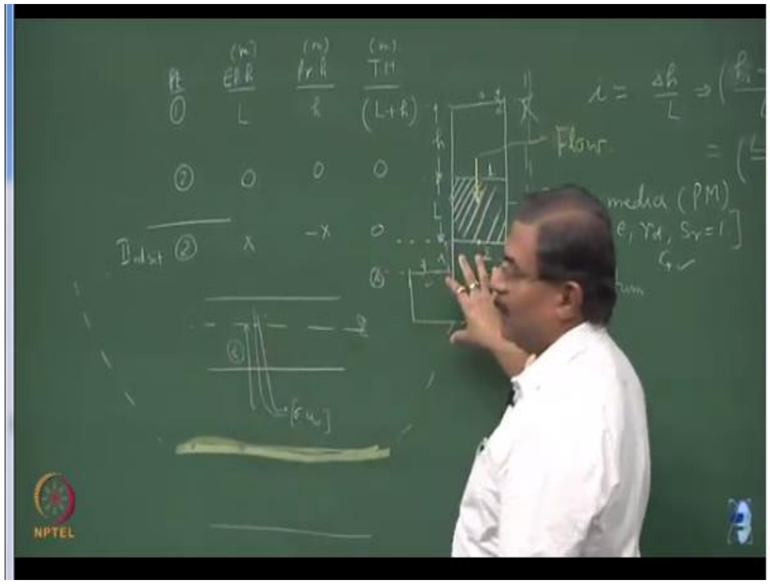

When we were considering different type of layered system and I had asked you to find out what is the state of stress at this point and find out the u w clear and suppose the water table is up here. In this case, now I can install a piezometric tube over here and this is what is going to rise up to this point and hence the head is going to be this much. So what is the difference between this situation and this situation anybody in the class and that is what you have to understand.

Here the boundary conditions are different as compared to this, is a continuum. I hope you are getting this point which is not exposed to the atmosphere. So if you install a piezometric tube over here, this will go and meet the phreatic surface. But in this case at this point and this point the atmospheric conditions prevail. So it is a good idea to shift the datum to the bottom most point of the sample. It becomes easy to go ahead with the analysis is this part clear.

This is the discrepancy, which I wanted to discuss in the last lecture is this part clear, another way of doing this would be if you say the elevation head here is x. If I install a piezometer here what is going to happen this is going to drop to - x and hence the total head will be equal to 0. I think the better way would be to shift the datum and do the analysis is this part okay. Now, what I will do is I will go ahead with the another situation, yeah.

So see, what we have done is we have established here that the pressure gradient across this sample is delta h upon L and delta h upon L is nothing but h 1 – h 2 upon L h 2 is 0. Once you switch the datum over here h 1 = L + h divided by L is the hydraulic gradient. Now this gradient is going to cause the flow through the porous media. So this yellow arrow shows the flow through the porous media, where point number 1 elevation head, see if you fix the datum over here.

So this is the elevation headed point 1 this is L, if you put the piezometric tube over here it goes meets up to here, so L h + clear. So pressure head is h at this point, elevation head is L, total head at this point is L + h fine, have you understood the discrepancy okay. Have you understood this part and this part the different between these 2, there is no boundary condition which is enforcing the atmospheric condition.

Now, if I do engineering how would you do surgery of the system, suppose if I include a layer of a porous media over here. Nowadays directional drilling is possible, you can drill laterally alright, a good example is our Santa Cruz airstrip, where the lateral drilling is done to take all the pipelines under the active run way. And then this I can connect to the atmosphere clear, so what I have done I have created the same situation as this over here.

Now at each and every point here, this is what is known as the filter. I have included a sand layer in case of your gas hydrates, you are doing hydro fracturing. And you are making this system more permeable, this is connected to the atmosphere and hence the pore water pressure at this point is going to be 0. Then the situation identical to this situation it is okay, any questions is this clear, so this is what the engineered system is I hope this point is clear.

Now let us do a bit more of extrapolation the situation with which we are dealing with here. So what we have done is we have established a flow condition over here and why flow is there because of delta h by L hydraulic gradient, whatever drop in column between point 0 to 1 is going to take place, this is the discharge clear, whatever flow is going to take place from point 1 to 2 because of the presence of porous media would be seepage or flow through porous media.

Now when the discharge or the flow is taking place through or the seepage is taking place through porous media, what we do is, we introduce the component V s, this is what is known as the seepage velocity. I hope you understand the difference between this, so this is your Q upon area of cross section which is nothing but the capital V. And what is going to enter into this system is because of seepage this will be the seepage velocity.

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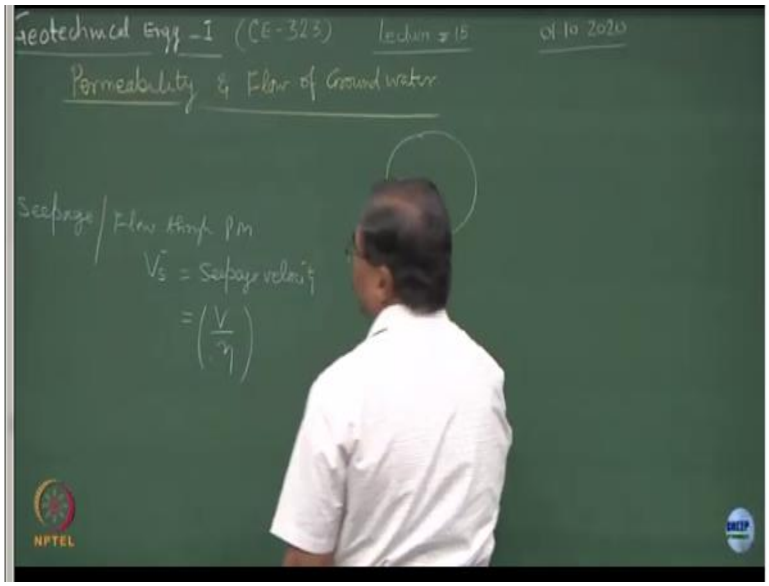

Now it so happens the seepage velocity is equal to discharge velocity upon porosity, how suppose if I consider a control volume and this is in the third dimension like this. These are the pores and rest of the portion is solid soil mass, so these are the pores. Suppose if I say area of the pores is A V air voice and area of the solids is A s, so the total area will be equal to A s + A v is this okay.

Now imagine if the discharge is taking place in this direction which is equal to Q, what is going to happen. The certain amount of water is going to enter through the pores not the entire flux because the solid portion is going to put lot of resistance in the flow and that is what we have defined as the hydraulic conductivity k, coefficient of permeability is this okay.

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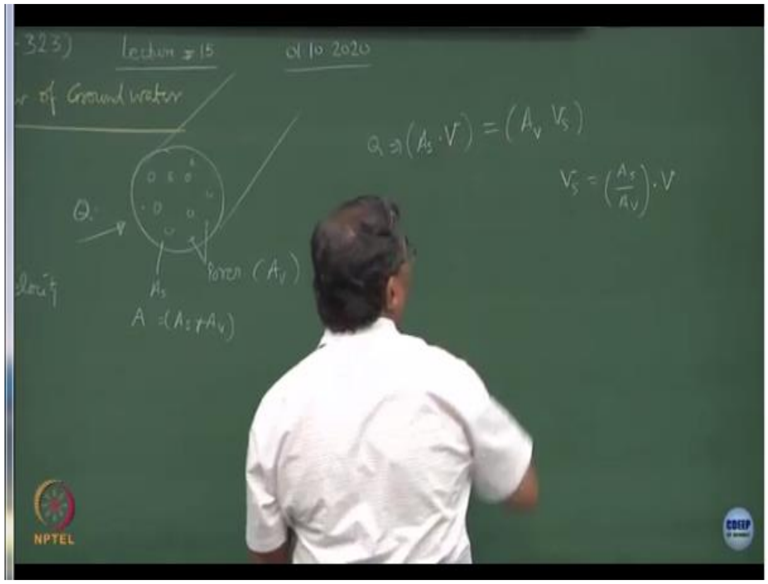

So if I say that the Q = A s and A s multiplied by V, is this okay and this will be equal to area of pores, multiplied by seepage velocity, this is the continuity. So that means, what I am saying is V s = A s upon A v multiplied by velocity V sorry which one on this side yeah there is the yes, you are right. So this is no I can deal with a solid also know okay, yeah you are right.

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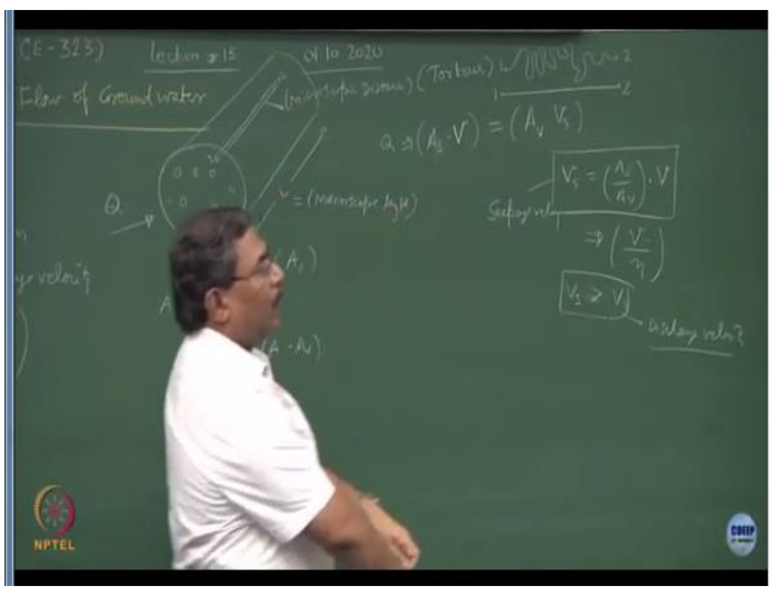

So, let it be like this, so now, what it indicates is that the seepage velocity is always greater than the discharge velocity which one is faster. The water molecule has to travel from one point to another point either in the microscopic length or in the macroscopic length. So L will be the macroscopic length to maintain the continuity, now this path is going to be tortuous, that means this path is not going to be straight line, this path is going to be something of this sort alright.

So, this is 1 and 2, which I have represented here as 1 and 2, however the linear dimension is straight. So imagine, if the discharge is taking place through point 1 and 2, the seepage is going to take place through 1 and 2 but to maintain the equity or the continuity, the time is same. The discharge in the form of seepage is going to be same as the discharge in the form of the discharge.

And hence, this is what comes out to be from this analysis, see, when the water is entering from 1 to 2, there are 2 possibilities either the water moves through the macroscopic length which is the physical dimension of the sample or through the (()) (15:05) of the soil sample which are quite tortuous in nature. That means, in the same time if the same distance is to be covered, the velocity through this is going to be more than velocity through this, what it indicates is that the velocity of sorry flow of water is going to be lesser.

When you have macroscopic length, but when the discharge is taking place through the microscopic length, which is the tortousity, the seepage velocity is going to be higher than the discharge velocity. Normally, how would you maintain the continuity, suppose if I give you a cylindrical sample and if I say that the Q is moving from here and Q is coming out, there is no loss of fluid clear, imagine like this.

So this is what actually we are trying to analyze, this is another attribute of the porous media which we are going to use over here. So when I say the flow has initiated because of the hydraulic gradient, there is going to be a velocity component V s. So V is known capital V a porosity is known, V s is known and you can show how V s is changing with respect to V. Now because of the virtue of this porosity, as I said V s is going to be more than capital V the discharge.

So this is what we have to include over here, is this part okay. Now if you go back to this equation, where the Darcy's law says Darcy's law is valid for discharge velocity. So we say that discharge velocity is proportional to hydraulic gradient correct or this is equal to k into i, what is k coefficient of permeability, what is coefficient of permeability. This is the resistance offered by the porous media to the flow of fluid, is this okay.

Units are in centimeter per second, so there are 2 definitions which are prevalent in this direction and this context not direction. The one definition is how do we define permeability, the permeability is defined as the rate of flow of water under laminar conditions through a unit cross section, cross sectional area of a porous medium under a unit hydraulic gradient, hydraulic gradient is i and the standard temperature conditions.

So basically permeability is the rate of flow of water under laminar flow condition through a unit cross sectional area of a porous media under a unit hydraulic gradient and standard temperature conditions. That means, if I put i = 1, the discharge velocity is equal to k. Now most of the time what is done is, that there is a relationship between coefficient of hydraulic conductivity and I think last time I told you to stick to small k.

Because I wanted to preserve capital K for some other notation, so this is alright this capital K is known as intrinsic permeability alright. Just concentrate on the board for a minute, try to understand what is this concept which I am introducing over here. The hydraulic conductivity K is let us say 2 variables a multiplied by b, capital K depicts the porous media, this is a fundamental characteristic of the porous media.

And that is why we call this as intrinsic permeability intrinsic means fundamental nature fundamental behavior. It is not going to change, however the term, so this is the pm and what is b, b is the fluid clear. In other words, hydraulic conductivity or coefficient of permeability is a sort of a interaction of porous media with a fluid. Mathematical definition would be, this is how you define the interaction between solid porous media and the fluid clear.

So the mapping is between the material porous media and the fluid, what is rho into g density unit weight clear divided by mew viscosity of the fluid. So attributes of the fluid attribute of the porous media conductivity clear. I hope now you can understand easily what I have to determine not capital K. I should be determining sorry not small k. I should be determining capital K, because this is what is corresponding to a porous system, fluid can be anything.

It could be water, it could be contaminated water, it could be hydrocarbon, it could be gases, it could be bacterial suspension, whatever, clear. So this is what please remember (()) (22:20) okay, yeah, so you are talking about this portion, that means there is no, you are right. So there is no porous media over here, this is the discharge, this is the seepage, this is again the discharge you are right, with portion, in this portion.

In this portion viscosity is not so important, because you are not talking about the seepage taking place through the porous media, this is the discharge only. So suppose if you remember last time, we had derived some expression falling head test. So as the time passes by, what is going to happen this column is going to decrease, is it not. So h is decreasing and then we are taken is – del h 1 del t, so this is a free discharge, this is the seepage, there is a difference. And seepage is 1 minute, seepage is attributed to all these characteristics.

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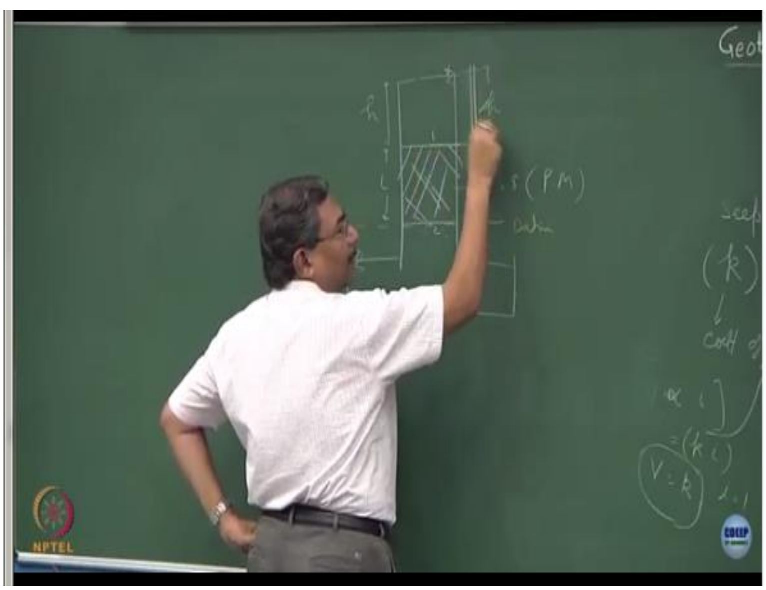

This is a glass tube and I am filling up the sample over here, this is the soil sample porous media. This is your point number 1, this is your point number 2 clear and this is kept in the water bath come out of all confusions simplify things take your this is h water table, this is the water level here. And the length of the sample is L, shift the datum to this point number 1 to get rid of all these confusions.

So this becomes my datum, elevation head at point 1 is known, what you want is, pressure head, how would you obtain the hydrostatic pressure head over. If you put the piezometer here this will go up to this height clear. So that means the piezometric head at point 1 is h, the piezometric head at point 1 is going to be rising up to the height h to equilibrate with the atmospheric conditions.

At this point, now, the elevation head as become 0, what is the pressure head because this itself is atmosphere 0 finished, total head at this point is 0 total head at this point is going to be L + h. Coming back to your question, head across this and this is L + h, the gradient is L + h upon L which is going to cause the flow into the sample. Otherwise this is free discharge, this is also free discharge agreed, is this part clear.

By virtue of being a porous media the void ratio, density, soil properties, specific gravity all are define, what you are ensuring is. This is in steady state and hence the sample is fully saturated, if sample is not saturated then the problems will occur. We always talk in terms of meters of the pressure head, if you remember the pressure head at this point is the height of the piezometric raise in water, so this is going to be h clear.

Now suppose, now I am going to play with the boundary conditions. So please have a close look at what I am going to discuss.

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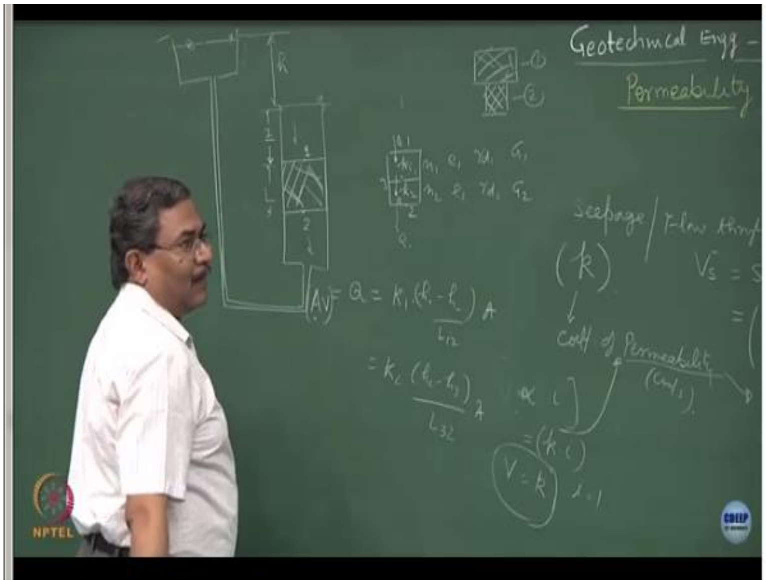

This is my Q and this is the porous media fine, now what I will do is, I will connect it to a water bath, is this okay. So all those who are asking this question of the surface being exposed to the atmosphere, I think now you are realizing something what I have done, have I done something substantially different or is the same thing what I have been discussing until now. Then you have understood what I am going to talk about.

It does not matter where I connect this point 2, so what I am basically doing is, this is the head difference between the 2 water levels. And the logic says that the hydraulic gradient is the 1 between the difference of the energy state between the 2 surfaces and that is what is going to cause the flow. Suppose if I lower it down a bit, what is going to happen, the discharge is going to place from some take place from 1 to 2.

The moment I have lifted it up, what is going to happen discharge amount take place from 2 to 1 is this point clear, simple mechanics, have you understood this. I can change the flow direction through the porous media just by manipulating the h, h could be positive, h could be negative. So if I bring it down and if I keep this cistern or what you call it as a reservoir somewhere here, what is going to happen.

Now the delta of the free water surface and this free water surface is going to cause the discharge to take place, let us prove it. You can fix the datum somewhere work it out now, when you go back today please do this with different datums you know the total energy here donor total energy here and you can prove that the discharge going to take place from 2 to 1, is this part okay, have you understood this.

Now suppose if I asked you to draw the free body diagram of this element, there is another manipulation, first understand the manipulations, different type of manipulation which I can do. I can replace this porous media by a multi layered system agree, that means I can say this system has permeability k 1, this has k 2. In earlier case you must have realize that the dissipation of the hydraulic gradient is going to take place between 1 and 2 and that was linear.

Now because of introduction of k 2 and k 1 what I have done, I have manipulated the system much more, I am tending towards more natural processes. And let me complicate it further k 1 with porosity e 1 porosity e 2 e 1 e 2 into gamma d 1 gamma d 2 G 1, G 2, what does a nature offers you, same thing, what I have created, I have created a multilayered system, 2 layer system, where now you are going to be in a difficulty, but this simple.

Now what I can do is, this 1 and 2 I can replace with point number 3 and I can still go ahead with the same thing, what I require is one equation of continuity, that means this Q is going to be the same as this Q is going to be the same as the Q which comes out that is it. In other words, if my Q remains same I will say k 1 multiplied by h 1 – h 2 over L 1 2. This will be equal to k 2 h 2 – h 3 is this correct, L 3 2 and area of cross section.

So area cross section you multiply, fortunately the area of cross section remains same, so that does not come the picture, another manipulation what I can do, I can reduce the area cross sections. So I can create a sample like this the first layer and then second sample is sitting like this okay. So this is your first sample, this is your second sample or a multilayered sample whatever, is the concept clear.

As far as the total head pressure heads are concerned they remain same. Only thing is now you are introducing a point at which the state of energy is not known which can be solved by using the continuity principle, yeah. So this Q is nothing but your A into V alright, so this V is here and this V is here also that is it, is this okay concept clear. Look at the pressure gradient that is important, so at this point and this point, the pressure gradient was straight line earlier.

Now what has happened, the slope has got changed, why, because you introduce another point another property. So earlier this was like this, now who knows it might have become like this and you simply create a datum over here. Now you will not be confuse at all, I hope all this is gone is the same thing. In this previous case, what did I do, I kept, how does it matter whether you are keeping it in a tube connected to this or whether this whole thing is inside a water bath it does not matter.

If you are putting vacuum over there, so this does not matter, keep the datum as it is. Pressure head this point will become minus of your value of vacuum which you are applying that is it, is this okay. So if this point is being sucked with a suction pump, what will be the pressure at this point minus that value. So what happens to the hydraulic gradient, this is a good question. So if you suck something what happens, now is getting the realization of what we are discussing, the discharge increases.

So minus of this thing and this minus of minus of this thing that means a hydraulic gradient is going to become higher clear. So V is proportional to i, so that means, the moment of the fluid through this porous system will get enhanced. Now let us manipulate this a bit more, direction of flow is clear to all of you. This head is causing everything to happen.

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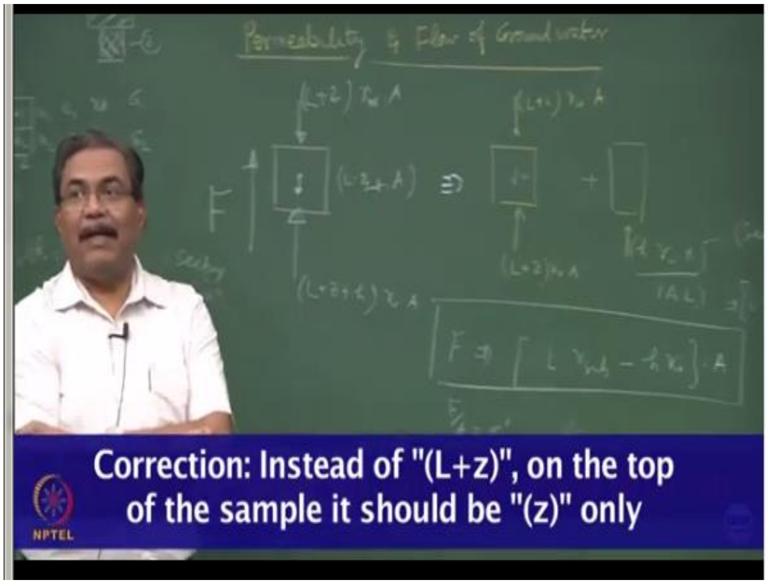

Now suppose if I draw the free body diagram draw it, how many forces are acting okay you let us do it together. So is there something acting from a top, how much is that, there is a sample, what is the total head at point 1, h total head. This h is not directly connected to point 1, there is a dampener in between which is soil mass clear. So that means, at this point elevation head is known L, if you are putting the datum head over here, at this point the pressure head is z. So total is L + z into gamma w into A, is this part clear, what is happening here.

This point, what is the elevation head 0, what is the pressure head, if you connect this to the piezometric column what is going to happen L + z + h this surface is connected to the atmosphere here. So this is L + z + h gamma w into A what is the other force which is acting on this element very good, excellent. So what is the weight component sorry, yes. So weight will be equal to L, L very good L, what gamma what type of gamma sorry gamma dry, whole thing is put in a steady state condition, how can this be dry yes, do not do these mistakes these are basics alright.

So gamma saturated and A nice are you happy with this, what is the other force which is going to act on this element. I said something if I raise this and if I bring it down, I can modulate something yes you are right not gradient force. So that means yes he is right, have you understood, have you ever seen a dashpot okay. If you go to the hospitals, you must have seen these saline. So what does they they put it on a stand.

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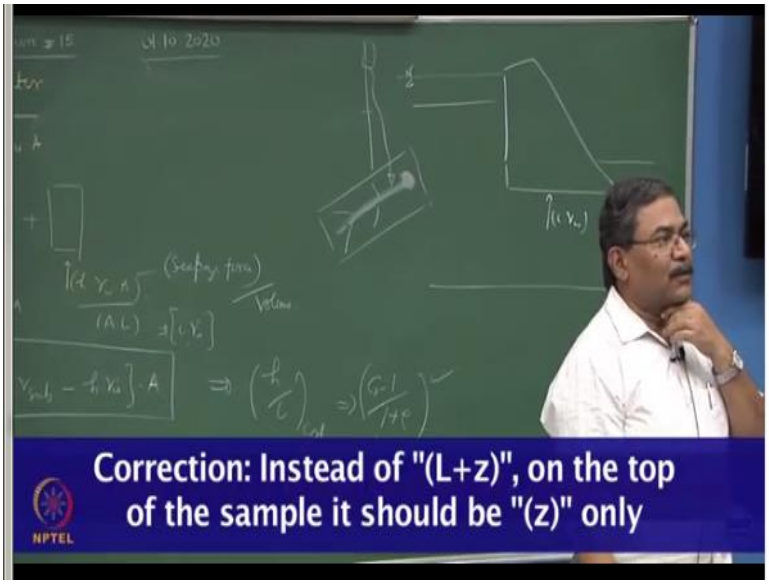

If you are lying on a bed over here clear, so what you do normally they put a saline here correct this is a same situation what dot normally sisters will do they will module the height of the saline. So somewhere they will fix it, so the small kit they cannot put it 1 meter high, why imagine the hydraulic head. Now we are going to see what is going to happen, we are going to model that thing is this okay, so if I lower it down and bring it up.

I can create a force which is going to act on the system. So this is your seepage force take where this seepage force is acting, what is the plane on which this seepage force is acting. You have to look at the figure, where were your datum is, at this point, why. Because the entire pressure head and elevation head is acting at this point clear. So truly speaking, what is happening, you have 1, 2, 3 forces and they are being balanced by F and this is what is known as seepage force.

Now, so this is the seepage force that video clipping was regarding what, quick sand, we are going to create quicksand condition, how. So little bit more of understanding for all of you, you know let us put it in components. Let us filter out the seepage force in the form of hydraulic gradient. So, what I can do is I can say L + z into gamma w A, of course, weight is acting and this becomes L + z gamma w A +, if I write like this, is there any problem, what I have done.

I have filtered out the effect of the hydraulic gradient h was the driving force which is causing seepage to occur from point number 2 to 1. Because this is how the flow is going to take place, so the flow is now taking place from 2 to 1, positive gradient is here, less gradient is here alright. So suppose if I write this expression, F is equal to, now this is what is known as seepage force, normally we define seepage force per unit volume.

Now, if I do per unit volume, what is going to happen this will become A into L and this can be written as i into gamma w. So i into gamma w is the seepage force which is acting on the system clear. The whole situation could be something like, you know this is a dam made up of concrete or earthen earth and for me, there is no difference between this system, this system and this system, is this okay.

The moment I have created h over here, reigns might do it in the due course of time, what has happened. There is the h which got created over here, look at the base what is going to happen i into gamma w per unit volume is acting here, have you understood the consequence of this, I will come to that, do not worry. First, let us talk about the seepage there is the big culprit know, seepage is listen.

But what I should be worrying about is the seepage forces, so this is what I am trying to explain right now we will take care of that part do not worry. So what I am trying to do is this situation is a simplified model to do the analysis. This is out of the way I discussed something and this is the real life situation which forces us to study, what is the influence of hydraulic gradient on the stability of the structures, is this part clear and what we have done systematically is.

We have shown that this is the hydraulic gradient which is the seepage force per unit volume. Now rest is all simple mechanics you can solve this I will skip few steps and try to solve this function. If you can write it quickly it saves some of my time. So this will be equal to L gamma submerged - h gamma w into A, try to show this. This is the seepage force, what is F upon A stress clear and suppose if I say that this is effective stress why, h gamma w is what pore water pressures.

So L gamma submerge minus pore water pressure is nothing but the effective stress and if I put a condition that this tends to 0, this is nothing but your sand boiling. Now if you simplify this, what will be getting, can you do it quickly, i comes out to be, what is the i h upon L. We define this as critical this will be equal to G - 1 over 1 + e, try to prove this, is this correct Jasmine, is this equation correct, you must remembering, is this okay Mini.

So you try to prove that i h upon L critical will be G - 1 over 1 + e, what it indicates is, a soil is going to boil under hydraulic gradients. And at that time hydraulic gradient is this which is equal to the specific gravity - 1 over 1 + void ratio. Have you understood the whole Khakani or not. So you were talking of the weight, what you have to do is F will be equal to what, F + this equal to this plus this.

A better way of defining this would be this minus, this minus this is equal to F, mathematically they are same is same as the, h correct h is causing the entire a story. So what is h, this h, this is the incremental pressure, now do an exercise. Suppose h tends to 0, what is the meaning of this, water column matches with this water column and try to see what happens. So what is our intuitive feeling, if h tends to 0, what is going to happen, hydraulic gradient is going to be 0 that is it. So in other words, this is the response of the porous media to the external gradients hydraulic gradients.