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**Geotechnical  
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**Lecture No – 18**

**Shear Strength**

Welcome, I am Professor J.N. Mandal, Department of Civil Engineering Indian Institute of Technology, Bombay. As I discussed earlier the triaxial test is performed under the different confining phasor, this table here is that under the confining phasor of 50 kPa and then you have to perform the test.

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**Soil Testing in Civil Engineering**

Deformation (mm)	Strain (%)	$A_c$ Corrected Area (mm <sup>2</sup> ) $\times 10^3$	$\sigma_3 = 50$ kPa		$\sigma_3 = 100$ kPa		$\sigma_3 = 150$ kPa	
			Load (kN)	$\sigma_1$ (kPa)	Load (kN)	$\sigma_1$ (kPa)	Load (kN)	$\sigma_1$
0	0	1.13	0	0	0	0	0	0
0.3	0.394	1.134	0.02	17.637	0.03	26.46	0.04	35.27
0.6	0.789	1.138	0.03	26.362	0.05	43.98	0.078	68.6
0.9	1.184	1.142	0.05	43.783	0.08	70.05	0.128	112.1
1.2	1.578	1.145	0.11	96.07	0.13	113.5	0.152	132.8
1.5	1.973	1.151	0.15	130.32	0.17	147.7	0.19	165.1
1.8	2.368	1.156	0.184	159.17	0.21	181.7	0.22	190.3
2.1	2.763	1.161	0.218	187.77	0.23	198.1	0.25	215.3
2.4	3.157	1.166	0.24	205.83	0.252	216.1	0.27	231.6
2.7	3.552	1.17	0.25	213.68	0.276	235.9	0.287	245.3
3	3.947	1.175	0.27	229.79	0.287	244.3	0.31	263.8
3.3	4.342	1.18	0.28	237.29	0.311	263.6	0.324	274.6
3.6	4.736	1.185	0.29	244.73	0.315	265.8	0.328	276.8

$$\text{strain (\%)} = \frac{\Delta L}{L_0} \times 100 = \frac{0.6}{76} \times 100 = 0.789\%$$

$$\text{Deviator Stress } (\sigma_d) = \frac{\text{Load}}{A_c} = \frac{0.03}{1.138 \times 10^3 \times 10^{-6}} = 26.362 \text{ kPa}$$

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Under the  $\sigma_3=100$  kPa and then under the confining phasor  $\sigma_3$  is 150 kPa, so under the three define confining phasor then we see that what will be the ultimate value.

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Soil Testing in Civil Engineering								
Deformation (mm)	Strain (%)	$A_c$ Corrected Area (mm <sup>2</sup> ) × 10 <sup>3</sup>	$\sigma_3 = 50 \text{ kPa}$		$\sigma_3 = 100 \text{ kPa}$		$\sigma_3 = 150 \text{ kPa}$	
			Load (kN)	$\sigma_d$ (kPa)	Load (kN)	$\sigma_d$ (kPa)	Load (kN)	$\sigma_d$ (kPa)
3.9	5.131	1.19	0.3	252.1	0.317	266.4	0.33	277.3
4.2	5.526	1.196	0.3	250.84	0.318	265.9	0.332	277.6
4.5	5.921	1.201	0.31	258.12	0.327	272.3	0.336	279.8
4.8	6.315	1.206	0.315	261.19	0.331	274.5	0.344	285.2
5.1	6.71	1.211	0.32	264.24	0.336	277.5	0.352	290.7
5.4	7.105	1.216	0.325	267.27	0.341	280.4	0.354	291.1
5.7	7.5	1.221	0.328	268.63	0.345	282.6	0.358	293.2
6	7.894	1.226	0.33	269.17	0.351	286.3	0.362	295.3
6.3	8.289	1.232	0.33	267.86	0.354	287.3	0.364	295.5
6.6	8.684	1.237	0.33	266.77	0.356	287.8	0.365	295.1
6.9	9.078	1.242	0.34	273.75	0.367	295.5	0.371	298.7
7.2	9.473	1.248	0.35	280.45	0.369	295.7	0.371	297.3
7.5	9.868	1.253	0.362	288.91	0.372	296.9	0.376	300.1
7.8	10.26	1.259	0.365	289	0.374	297.1	0.376	298.6
8.1	10.66	1.264	0.367	290	0.374	295.9	0.379	299.8
8.4	11.05	1.27					0.379	298.4
8.7	11.45	1.276					0.379	297

Ultimate value under the  $\sigma_3$  is 290, 290 that is  $\sigma_d$  means it is a deviator states, so we can write that when.

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Case-1 ①  
 $\sigma_3 = 50 \text{ kPa}$   
 $\sigma_d = \sigma_1 - \sigma_3 = 290 \text{ kPa}$   
 $\sigma_1 = (290 + 50) \text{ kPa} = 340 \text{ kPa}$

Case-2:  
 $\sigma_3 = 100 \text{ kPa}$   
 $\sigma_d = \sigma_1 - \sigma_3 = 297.1 \text{ kPa}$   
 $\sigma_1 = (297.1 + 100) \text{ kPa} = 397.1 \text{ kPa}$

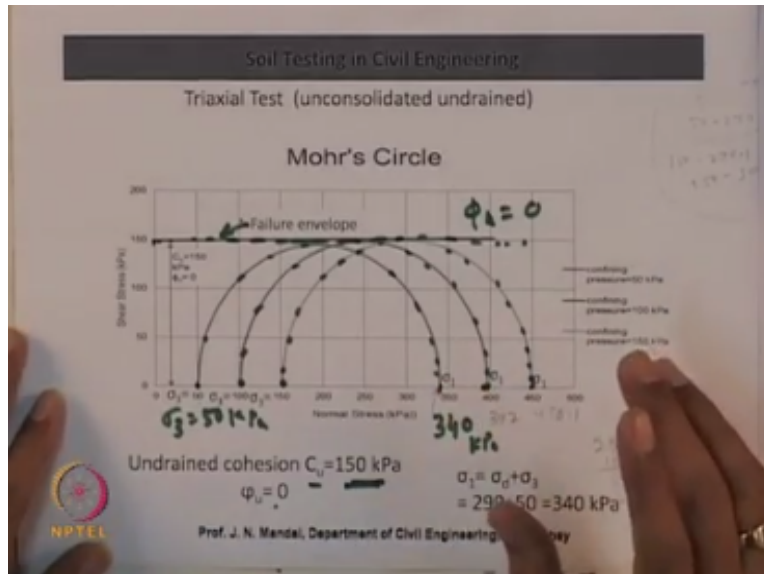
Case-3  
 $\sigma_3 = 150 \text{ kPa}$   
 $\sigma_d = \sigma_1 - \sigma_3 = 300.1 \text{ kPa}$   
 $\sigma_1 = (300.1 + 150) \text{ kPa} = 450.1 \text{ kPa}$

$\sigma_3$  case 1, when  $\sigma_3=50\text{kPa}$  then the deviator states that is  $\sigma_d$  and that is equal to  $\sigma_1-\sigma_3$  will be equal to 290 kPa. Similarly, for case 2 when the  $\sigma_3=100\text{kPa}$  then this is the maximum deviator states at here is 297.1, so case 2 when the  $\sigma_3=100\text{kPa}$  and we are having the deviator state that is  $\sigma_d=\sigma_1-\sigma_3=2197.1\text{kPa}$ . Similarly in case 3, when the is a case 3 here where the confining phasor is 150 kPa then the maximum deviator state above would 300.1 kPa.

So case 3, when  $\sigma_3=150\text{kPa}$  then deviator state  $\sigma_d=\sigma_1-\sigma_3=300.1\text{kPa}$  so from this test regional we can, we know what is  $\sigma_1$  that is  $\sigma_3$  that 50kPa, so for this confining phasor so  $\sigma_1$  will be 290+ that is 50 this kpa, so if you add this one 290+50 then it will give 340 kPa. Similarly,  $\sigma_1-\sigma_3$  297.1 under a confining phasor of 100kPa so we can write  $\sigma_1=297.1+\sigma_3=100$  so 100, so this kPa, so this will be about 397.1kPa.

Then when case 3, when  $\sigma_3=150\text{kPa}$  then we can write the  $\sigma_1=300.1+150$  this is kPa, so that means this will give you about 450.1 this kPa. So what we are having that we want to draw the Mohr's circle with the, when  $\sigma_3=50$  then  $\sigma_1=340$ , when  $\sigma_3=100$  then  $\sigma_1=397.1$  kPa, when  $\sigma_3=150$  then  $\sigma_1=450.1$  kPa, so now with this data we can draw the Mohr's circle.

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Here is the Mohr's circle, this is for triaxial test unconsolidated undrained test and this is the shear stress and the normal stress, so as I said you that when  $\sigma_3$  is 50 that means here  $\sigma_3$  is 50 okay,  $\sigma_3$  is 50 then  $\sigma_1$  is 300 particular Pascal so this is  $\sigma_1$ , this is  $\sigma_1$  here is 340 so this is 340 kPa, here  $\sigma_3=50$ kPa and here 340 kPa so you know that this is  $\sigma_3$  this is  $\sigma_1$ , so you can draw a semi circle like this you can draw this line like this.

Now when the  $\sigma_3$  is 100 kPa then  $\sigma_1$  is 397.1 kPa, so here that is  $\sigma_3=100$  kPa and then  $\sigma_1$  is 397.1 so it is sometime here 397.1 kPa, so you can draw another that semi circle this circle is this, okay what  $\sigma_3=150$ ,  $\sigma_1=397$  kPa. Similarly case 3 when the  $\sigma_3=150$  then  $\sigma_1=450.1$  that means when  $\sigma_3$  is here 150 and  $\sigma_1$  is here about 450.1 kPa, so you can draw another that semi circle like this.

So then you can draw a line which is tangent to this chord and which is called the that failure envelope this is Mohr's circle failure envelope what triaxial test or unconsolidated undrained test from this unconsolidated undrained test under different confining phasor 50, 100 and 150 you can draw the failure envelope and then you can measure what should be the  $C_u$  value that means undrained cohesion value, here undrained cohesion value is 150.

So undrained cohesion  $C_u$  is 150 kPa whereas  $\phi_u=0$  this is 0, so from this triaxial test you can determine that what should be the undrained cohesion value  $C_u$ , because in case of the unconsolidated undrained test so this  $\phi$  value is equal to the 0, so only you can measure the cohesion intercept that  $C_u$  value. So from this test one can determine.

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## VANE SHEAR TEST

**Objective:**

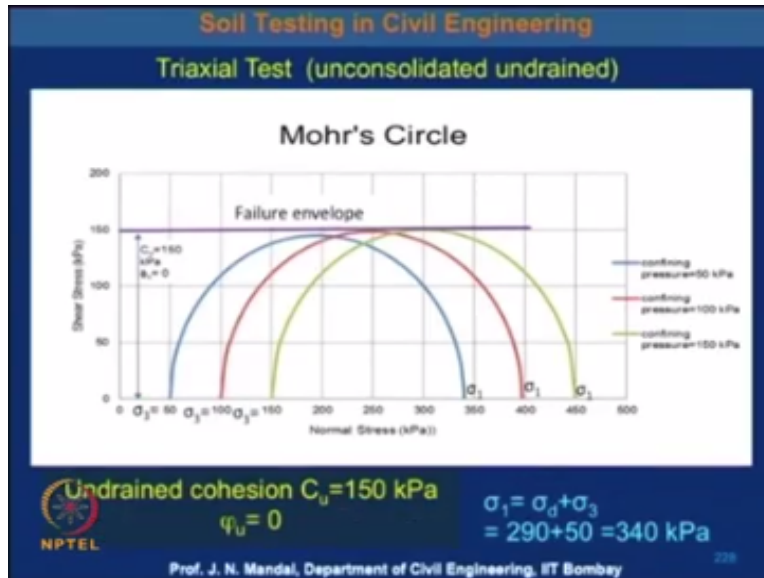
To determine the undrained shear strength of clays using laboratory vane shear test apparatus.

**Introduction:**

- Shear strength of soft clay deposits is difficult to obtain accurately in laboratory by conventional triaxial tests as getting undisturbed samples is very difficult.
- In such situations, the shear strength can be obtained by conducting vane shear test in the field.
- The vane is pushed into the soil up to the desired depth and a torque is then applied at the upper end.
- The torque is measured by noting the angle of twist.
- Shear failure occurs over a cylindrical surface (periphery and ends) having a diameter  $d$  equal to that of the vane.



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So from this test you can determine that what will be the undrained cohesion value and from this triaxial test is very important so one can determine the shear strength and the shear strength parameter of the soil. Next we will discuss the Vane Shear test, so the main objective for the vane shear test to determine the undrained shear strength of clay using the laboratory vane shear test operators. Now shear strength of soft clay deposit is difficult.

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**Soil Testing in Civil Engineering**

**VANE SHEAR TEST**

**Objective:**  
To determine the undrained shear strength of clays using laboratory vane shear test apparatus.

**Introduction:**

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- In such situations, the shear strength can be obtained by conducting vane shear test in the field.
- The vane is pushed into the soil up to the desired depth and a torque is then applied at the upper end.
- The torque is measured by noting the angle of twist.
- Shear failure occurs over a cylindrical surface (periphery and ends) having a diameter  $d$  equal to that of the vane.



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To obtain accurately in the laboratory by conventional triaxial test at getting undisturbed sample is very difficult. So you cannot prepare a sample this is a saturated clay soil sample, it is very difficult to form the triaxial sample with the clay and it is a saturated clay in such case we can adopt the vane shear test. In such situation the shear strength can be obtained by conducting the vane shear test also in the field.

Now it is almost in the real test because we are performing this vane shear test in the field, the vane its push into the soil upto the desired depth and a torque is then applied at the upper end. The torques measure by noting the angle of twist, the shear failure occurs over a cylindrical surface periphery and the end having a diameter  $d$  equal to that of the vane.

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### Soil Testing in Civil Engineering

It is assumed that the shear strength ( $s$ ) of the soil is constant on the cylindrical sheared surface and at the top and bottom faces of the sheared cylinder. The torque applied  $T$  must be equal to the sum of the resisting torque at the sides ( $T_1$ ) and that at the top and bottom ( $T_2$ ). Thus,

$$T = T_1 + T_2 \text{ ----- (a)}$$

Where  $T$  is the torque

The resisting torque on the sides is equal to the resisting force developed on the cylindrical surface multiplied by the radial distance. Thus,

$$T_1 = (c_u 2\pi r H) r = (c_u \pi D H) \times \frac{D}{2} \text{ ----- (b)}$$

Where  $c_u$  is the shear strength

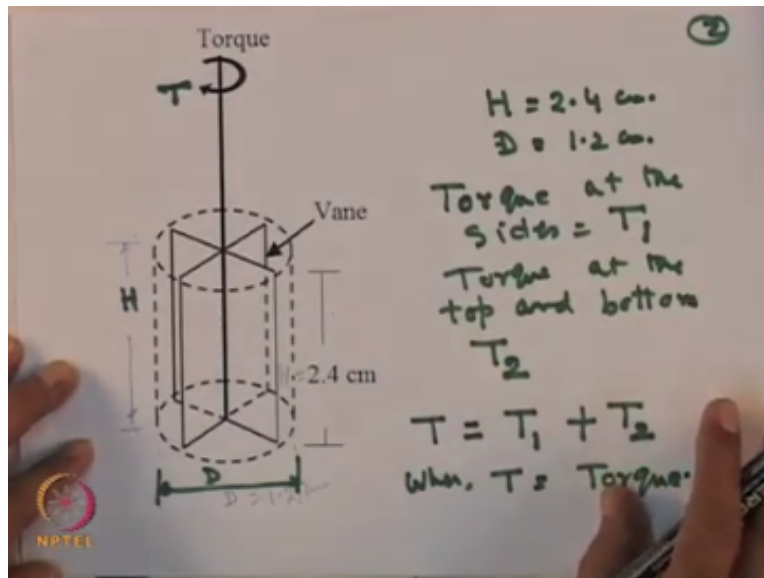
$D$  = diameter of vane

$H$  = height of vane



So, here we can discuss some about the vane shear test and this is the.

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This is the part of vane shear operators and this is the vane, and this is the height of the vane that is H, here is the H and h is generally 2.4 cm that means height of vane is 2.4 cm and it has a diameter this is the diameter which we can express as D and D is the diameter of the vane 1.2 cm, so you have to apply the torque this you put inserted into the soil and you can apply that torque this torque is designed as T.

So it is assume that the shear strength is of the soil each constant on the cylindrical shear surface and at the top and the bottom phases of the shear cylinder. The torque T must be equal to the sum of the resisting torque at the side that you can say that torque at the side. Let us say that torque at the sides, let us say that  $T_1$  and the top and the bottom is  $T_2$  so torque at the top and the bottom that let us say that  $T_2$ .

So torque at the side is  $T_1$  torque at the top and bottom is  $T_2$  so that means the total the torque T, T can be expressed as  $T_1 + T_2$  so our T is the torque. Now you have to determine what will be the resisting force or resisting torque on the side is equal to the resisting force developed on the cylindrical surface multiplied by the radial distance.

So let us say that radial distance is equal to the R okay, radial distance is equal to the R because the diameter is equal to the D, so resisting torque on the side is equal to the resisting force developed on the cylindrical surface multiplied by the radial distance, so this is torque at the side so you can write that torque at the side that is what you call the  $T_1$ .

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(3)

$$T_1 = (C_u \cdot 2\pi r H) r$$

$$= (C_u \cdot \pi D H) \frac{D}{2}$$

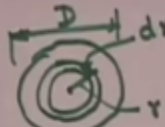
Where,  $C_u$  = Shear strength  
 $D$  = Diameter of Vane.  
 $H$  = Height of Vane

So this is  $T_1$  = what will be the shear strength that is let us say  $C_u \cdot 2\pi r H$  and this into  $r$  that means you can write the  $C_u \cdot \pi 2r$  is equal to the  $D$  and this is  $H$  so this into  $D/2$ , so where  $C_u$  is the shear strength and  $D$  this  $D$  = diameter of vane and  $H$  = height of vane, so you can see here the torque this torque  $T_1$  at the side that means torque  $T_1$  at the side if the shear strength is  $C_u$  and this is  $2\pi r h$ ,  $H$  is the height of the vane and  $D$  is the diameter.

So that means this  $r = D/2$  and this  $2r = D$  so that is why  $T_1 = (C_u \pi D H) D/2$  so you can write like this that this is  $D/2$ . And then the resisting torque that is  $T_2$  you have to calculate the resisting torque that is  $T_2$ , so resisting torque.

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Resisting Torque  $T_2$



$$\begin{aligned}
 T_2 &= 2 \int_0^{\frac{D}{2}} [C_u (2\pi r) dr] r \\
 &= 4\pi C_u \int_0^{\frac{D}{2}} r^2 \cdot dr \\
 &= 4\pi C_u \left[ \frac{r^3}{3} \right]_0^{\frac{D}{2}} \\
 &= 4\pi C_u \frac{1}{3} \cdot \frac{D^3}{8} = \pi C_u \frac{D^3}{6}
 \end{aligned}$$

Is  $T_2$  and this  $T_2$  due to the resisting force at the top and the bottom of the shear cylinder that means if this is the shear cylinder which is at the top and the bottom of the shear cylinder can be determine by integration of the torque developed on a circular ring of radius  $R$  and with  $D$ , let us say that if this is the circular and if you take as some small element like this and this distance is equal to  $r$  and this is  $dr$ , so this we can write the  $T_2$  will be equal to 2 because it is at the top and the bottom so 2 into integration 0 this is  $D/2$  because  $D$  is the diameter, diameter of the vane is  $D$  this is 0  $D/2$  is the radius and this into  $[C_u(2\pi r)dr]$  this is the torque, okay.

So we can write  $4\pi C_u$  and this is 0 to  $D/2$  this is  $r^2 \cdot dr$ , because  $2\pi r \cdot dr \cdot r$  because this is the torque so this you can write  $4\pi C_u$  and then this is  $r$  and this is  $r$ , so  $r^2 \cdot dr$ , so we can write that  $4\pi C_u$  and if you integrate it you can have  $r^3$  this divided by 3 and this whole to the power  $D/2$  this is 0 to  $D/2$ . Now if you can calculate this then you can have  $4\pi C_u$  and this is  $D/2$  cube that means  $D^3/H$  okay, so this is  $1/3$ th this will be  $1/3$ th, this is  $1/3$ th into this will be the  $D^3/8$  so this you can write  $D^3$  and this divided by 8,  $2^3=8$ .

That means you can write the  $T_2$  the resisting torque is equal to  $\pi C_u$  and  $D^3/6$ , so you are having that one resisting torque  $T_2 = \pi C_u D^3/6$  and earlier also you have obtained that  $T_1$  that is  $T_1$  is equal to this value, you have got this one  $T_1$  value you have got this value. So  $T_1$  also will be the  $C_u \pi D^2 H/2$  okay. So now the combination of  $T_1$  and  $T_2$  will give you that what will be the  $T$  value okay, and I will show you that from the earlier equation for  $T_1$  and  $T_2$ .

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$$T = T_1 + T_2 \quad (5)$$

$$T = \pi C_u H \cdot \frac{D^2}{2} + \pi C_u \frac{D^3}{6}$$

$$= \pi C_u \left[ \frac{D^2 H}{2} + \frac{D^3}{6} \right]$$

$$C_u = \frac{T}{\pi \left[ \frac{D^2 H}{2} + \frac{D^3}{6} \right]}$$

$T$  = Torque.  
 $D$  = Diameter of vane, and  
 $H$  = Height of vane.

So you know that  $T=T_1+T_2$ , so  $T$  will be equal to  $T_1=\pi C_u$  it will be this one  $T_1 (\pi C_u H)D^2/2$  so you can write  $\pi C_u H \cdot D^2/2 +$  that  $T_2$ ,  $T_2$  we have determine here so  $T_2$  will be equal to  $\pi C_u D^3/6$  so you can write  $\pi C_u D^3$  this divided by 6, so this torque will be this, so we can write this way  $t=$  you can take common  $\pi C_u$  so this will be equal to  $D^2 H/2 + D^3$  this divided by 6 okay, so from this equation you can determine what will be the  $C_u$  value.

$C_u$  value will be equal to torque  $T/\pi D^2 H/2 + D^3$  this divided by 6 so you will have this equation that means what is  $C_u$  and its shear strength of the soil that you can determine using this equation. Where here you know that  $T$  is equal to the torque and  $D$  is equal to the diameter of vane and  $H$  is equal to height of vane, this is height of vane okay,  $C_u$  is equal to this. Now you can perform the test and you know what will be the diameter of the vane id known you know what will be the height of the vane is known and you have to calculate the torque that mean  $T$ .

So if you know the torque and  $D$  and the  $H$  then you can calculate what will be the  $C_u$  value, since  $C_u$  can be determine from the vane shear test, now let us when you are rotating this when you are applying the torque that means.

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(5)

$$T = \frac{(\theta_{\text{final}} - \theta_{\text{initial}})}{180} K \pi$$

Where,  $K$  = Spring Constant.  
 $\theta_{\text{final}}$  = final angle of twist, and.  
 $\theta_{\text{initial}}$  = Initial angle of twist

Torque is equal to the  $T$ , so you can have some final angle of the twist okay, for example that here rotating this when it is inserted into the very soft clay and then is rotating okay, so this is the  $T$  so you have something  $\theta_{\text{initial}}$  you can have something  $\theta_{\text{final}}$  so this torque can be determine within this equation that means  $\theta = \theta_{\text{final}} - \theta_{\text{initial}}$  this divided by 180 this into  $K\pi$  where,  $K$ = spring constant and  $\theta_{\text{final}}$ =final angle of twist and  $\theta_{\text{initial}}$ = initial angle of twist.

So from the vane shear test if you know that what angle of twist that initial angle of twist and what should be the final angle of the twist and then you can calculate the  $T$  you know that what will be the  $K$  the spring of constant, so if you can determine this  $K$  value knowing this  $T$  value knowing this spring constant and  $\theta_{\text{initial}}$  and  $\theta_{\text{final}}$  then you can determine the what will be the torque, so if the torque is known then you can determine what will be the  $C_u$  value, you know the equation  $C_u$  is related with the  $T$  and also related with what is the diameter of the vane and what will be the height of the vane.

So if you know height, if you know diameter and if you know the torque then you can determine that what should be the  $C_u$  value by the vane shear test, and this vane shear test is appropriate for the saturated clay soil sample and it is much more realistic because this test is performed in the field, thank you.

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