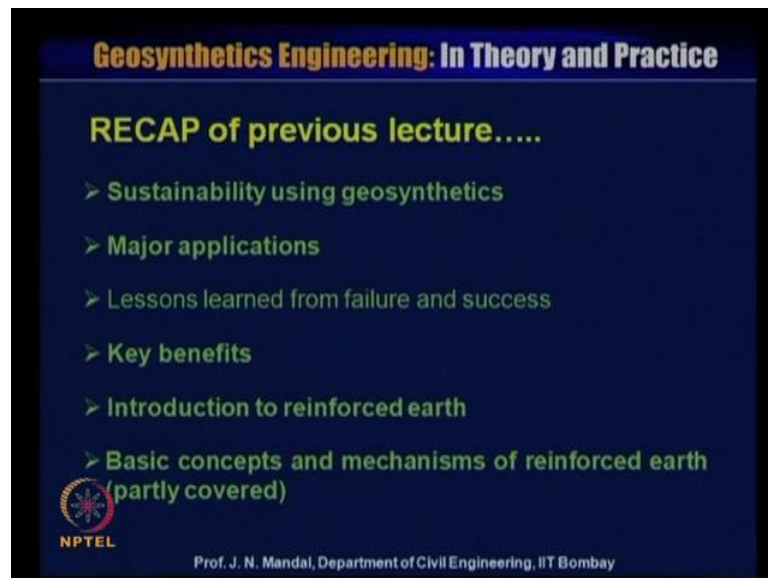


Geosynthetics Engineering: In Theory and Practices
Prof. J. N. Mandal
Department of Civil Engineering
Indian Institute of Technology, Bombay

Module - 1
Lecture - 3
Introduction to Reinforced Earth

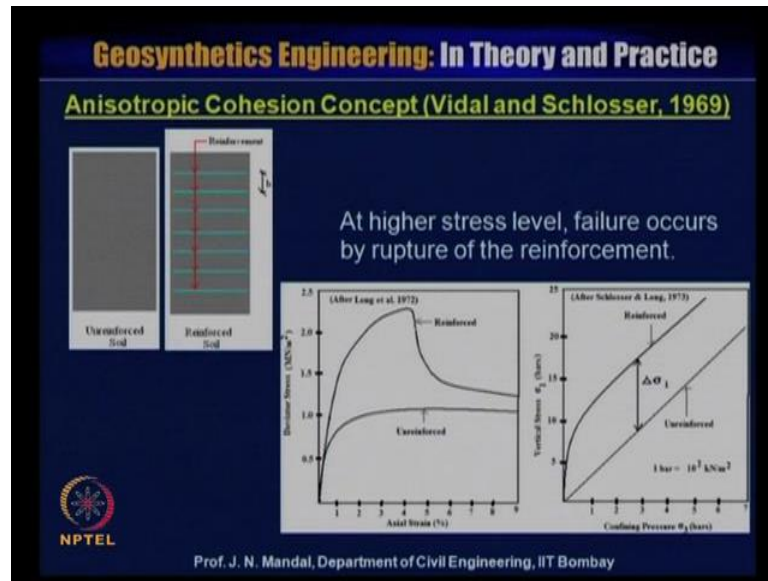
Dear student, warm welcome to NPTEL phase 2 program, video course on Geosynthetics Engineering in Theory and Practice. My name is Professor J. N. Mandal, Department of Civil Engineering, Indian Institute of Technology, Bombay, Mumbai, India. This module number 1, lecture number 3, Introduction to Reinforced Earth.

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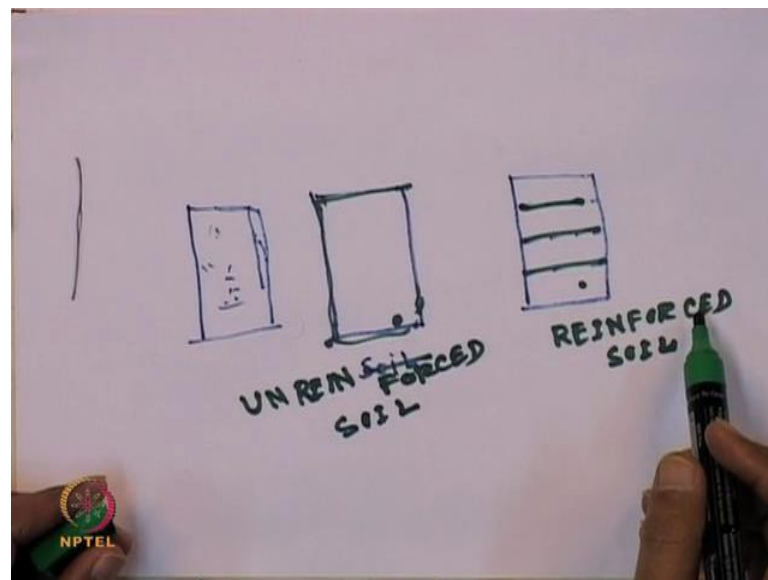
Now, I will address recap of the previous lecture, we covered the sustainability using the geosynthetics, major application and lesson learn from the failure and the success and what key benefit we can achieved. And next, the introduction of the reinforced earth and basic concept of mechanism of reinforced earth which we have partly covered.

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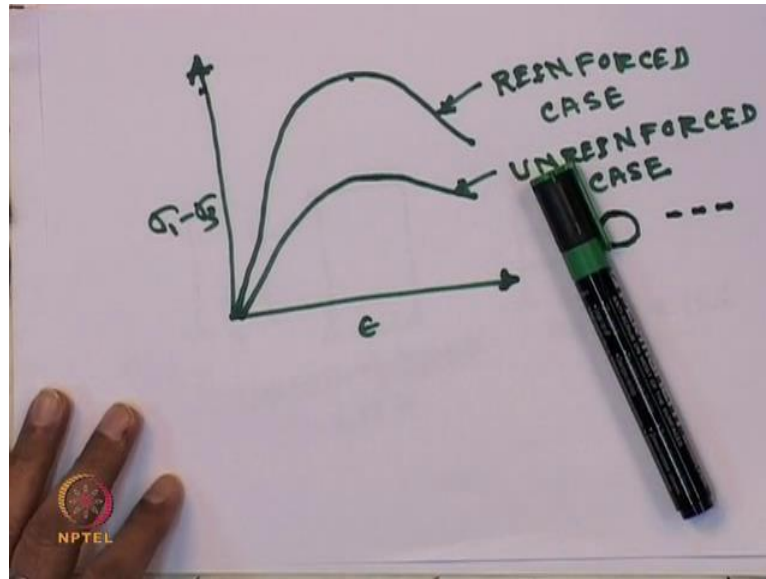
Next, I will explain the anisotropic cohesion concept given by Vidal and Schlosser in 1969.

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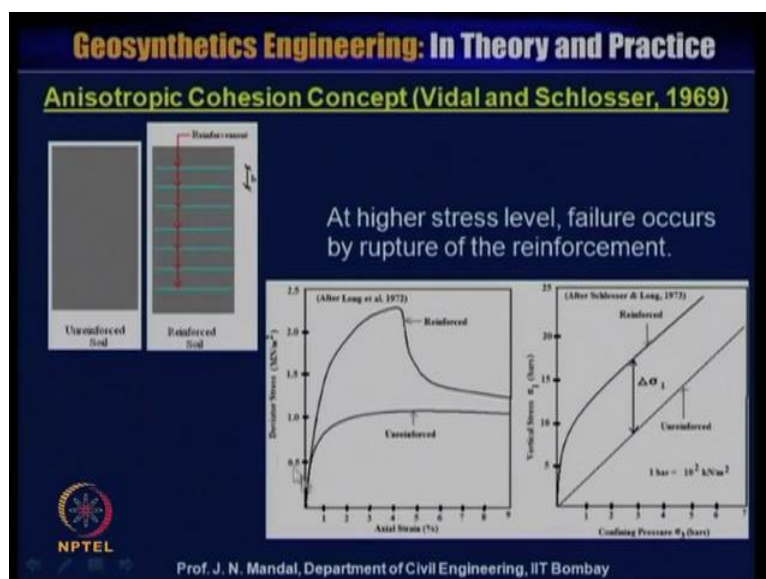
You can see here that, two triaxial compression specimen, this is unreinforced soil then, the other sample is the reinforced soil. Now, if you perform that axial compression test for both unreinforced soil you can say, this unreinforced soil and the reinforced soil. You can draw the relationship between the deviated stress and the axial stress.

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So, if I draw this curve between the deviated stress and the axial strain, you can draw a curve like this for the reinforced case and you can draw the curve like unreinforced case. So, from this curve you can see that, there is a development of the stress in case of the reinforced soil compared to the unreinforced case. There are number of research work have been carried out in IIT, Bombay using this metallic reinforcement in the circular form as an enforcing material. Also in the form of the fiber, also geogrid material and different types of the other related material, so you will be knowing, what should be the behavior of the reinforced case.

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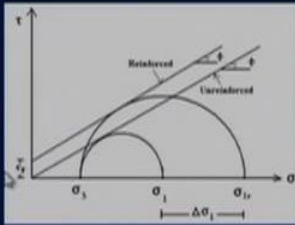
So here, we are now showing in this slide, that anisotropic cohesion concept, this is given by Vidal and Schlosser 1969. So, this curve shows between the axial strain in percentage and the deviators stress in kilo Newton per meter square. You can see that stress strain curve in case of the unreinforced soil, you can see that stress strain curve in case of the reinforced soil, that is Long 1973. So, you can see that, due to the interaction of reinforcement, there is a essential improvement of the deviated stress.

Now, we are talking that, how this anisotropic cohesion constant now, if you draw a relationship between the confining pressure on the x axis and the vertical stress in the y axis, you can draw a straight line for the unreinforced case. It is cohesionless soil, so this line will be straight line, which will be passing through the origin. When you are introducing the reinforcement, there will be a improvement of the vertical stress and that improvement is delta of sigma 1. It means that, due to the interaction of the reinforcement, there is a development or improvement of vertical stress of del of sigma 1.

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LCPC cohesion theory (Schlosser and Long, 1973)



For unreinforced soil, $\sigma_1 = \sigma_3 k_p$

The failure envelope for reinforced soil is defined by,

$$\sigma_{1r} = \sigma_1 + \Delta\sigma_1 = \sigma_3 k_p + \Delta\sigma_1$$

They compared the above equation with Rankine-Bell equation for c-phi soils,

$$\sigma_{1r} = \sigma_3 \tan^2 (45^\circ + \phi/2) + 2 c_r \tan (45^\circ + \phi/2) = \sigma_3 k_p + 2 c_r \sqrt{k_p}$$

Where, c_r = Reinforcement induced cohesion

Equating the above two equations, $c_r = \Delta\sigma_1 / 2\sqrt{k_p}$

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Now, LCPC cohesion theory, Schlosser and Long 1973 here, it shows between the tau and this stress, this is the unreinforced case, this is the sigma 1 major principle stress, this is sigma 3 minor principle stress. So, we can draw a Mohr circle and the failure envelope passes through the origin and tangent to the this circle and this unreinforced case, this angle of friction value is phi. But, in case of the reinforcement case and there is a

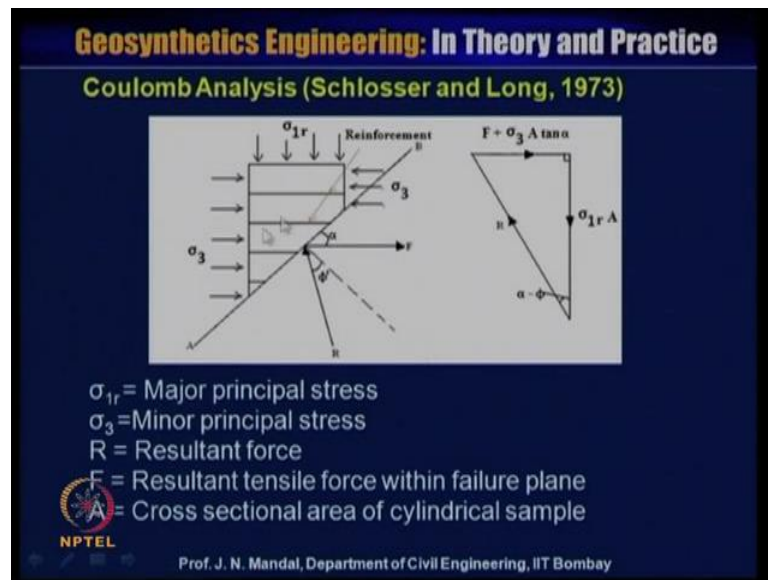
improvement of $\Delta \sigma_1$ and consequently, there will be the improvement of the adhesion value.

This concept is given by the Laboratory Central de Ponts et Chaussées in France by Schlosser and Long 1973. So, their concept so that for case of unreinforced soil then, σ_1 will be equal to $\sigma_3 \text{ into } k_p$. The failure envelope for reinforced soil is defined by σ_{1r} that is, this σ_{1r} will be equal to σ_1 plus $\Delta \sigma_1$ that is why, σ_{1r} is written is equal to σ_1 plus $\Delta \sigma_1$. Again σ_1 is equal to $\sigma_3 \text{ into } k_p$ so it would substitute the value of σ_1 is equal to $\sigma_3 k_p$.

So, this is $\sigma_3 k_p$, plus $\Delta \sigma_1$, they compare the above equation with Rankine-Bell equation for $c \phi$ soil. So, Rankine-Bell equation for $c \phi$ soils, σ_{1r} is equal to $\sigma_3 \tan^2 45^\circ + \phi$ by 2 plus $2 c_r \tan 45^\circ + \phi$ by 2. That means, this is $\sigma_3 \text{ into } k_p$ plus twice $c_r \text{ root of } k_p$ where, c_r is the reinforcement induced cohesion. Now, if we equate the above two equation, you can see this equation σ_{1r} is $\sigma_3 \text{ into } k_p$ plus $\Delta \sigma_1$ and this equation σ_{1r} is equal to $\sigma_3 k_p$ plus $2 c_r \text{ into root } k_p$.

So, if you equating this and with this equation so you can say that, $\Delta \sigma_1$ will be $2 c_r \text{ root over } k_p$ or c_r will be equal to $\Delta \sigma_1$ divided by $2 \text{ root } k_p$. So, from this, we can learn that, introduction of reinforcement how can improve the cohesion value, which we call the c_r , which we call the reinforced induced cohesion.

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Now, if you take the section of this triaxial compression test where, the horizontal layer of reinforcement are placed. If you take a cut this section A B where, the R is equal to resultant force, which is making at an angle of phi with the vertical of this plane A B. And there is a resultant tensile force within the failure plane that is, F which is making at an angle of alpha and sigma 1 r is the major principal stress, sigma 3 is the minor principal stress and A is the cross sectional area of this cylindrical sample.

Now, draw a free body diagram of this, this is the sigma 1 r into A which is acting vertically downward, this is the resultant force that is the R which is making at an angle of alpha minus phi and this is the F, which is the resultant tensile force within the failure plane. So, in that direction will be F plus sigma 3 into A tan alpha.

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From the force triangle,

$$F + \sigma_3 A \tan \alpha = \sigma_1 A \tan(\alpha - \phi)$$

Assume vertical spacing between reinforcements "h" and tensile strength of a reinforcement = T,


$$F = \frac{A \tan \alpha}{h} T$$

Combining the above equations,

$$\sigma_1 = \left(\sigma_3 + \frac{T}{h} \right) \tan \alpha \cot(\alpha - \phi)$$

By differentiating, σ_1 will be maximum when

$$\alpha = \left(45^\circ + \frac{\phi}{2} \right)$$

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Now, from this force triangle so we can write $F + \sigma_3 A \tan \alpha$ is equal to $\sigma_1 A \tan(\alpha - \phi)$. Assume, the vertical spacing between the reinforcement is h and tensile strength of the reinforcement is T . So, we can write F is equal to $\frac{A \tan \alpha}{h} T$, combining this above equation we can write σ_1 is equal to $\left(\sigma_3 + \frac{T}{h} \right) \tan \alpha \cot(\alpha - \phi)$ we are combining this earlier equation, this and this.

So, if you combine then, σ_1 is equal to here, it will be the $\sigma_3 A \tan \alpha$ this is $\sigma_3 A \tan \alpha$, A is cancelled from both side and this is $\tan \alpha$ by h into T . So, this is $\tan \alpha$ and this is T by h and this is $\tan \alpha \cot(\alpha - \phi)$, this will be the $\cot(\alpha - \phi)$. So, if we combine this and this equation, you can form this equation, we show σ_1 is equal to $\sigma_3 + \frac{T}{h} \tan \alpha \cot(\alpha - \phi)$ and A is cancel from both side. Now, if you differentiate this equation, σ_1 will be the maximum when the α value will be equal to $45^\circ + \frac{\phi}{2}$.

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$$\sigma_{1r} = \left(\sigma_3 + \frac{T}{h} \right) \tan \left(45^\circ + \frac{\phi}{2} \right) \cot \left(45^\circ + \frac{\phi}{2} - \phi \right)$$

$$\sigma_{1r} = \left(\sigma_3 + \frac{T}{h} \right) \tan \left(45^\circ + \frac{\phi}{2} \right) \tan \left(45^\circ + \frac{\phi}{2} \right)$$

$$\sigma_{1r} = k_p \sigma_3 + k_p \frac{T}{h}$$

Again, $\sigma_{1r} = \sigma_1 + \Delta\sigma_1 = k_p \sigma_3 + \Delta\sigma_1$

Therefore, $\Delta\sigma_1 = k_p \frac{T}{h}$

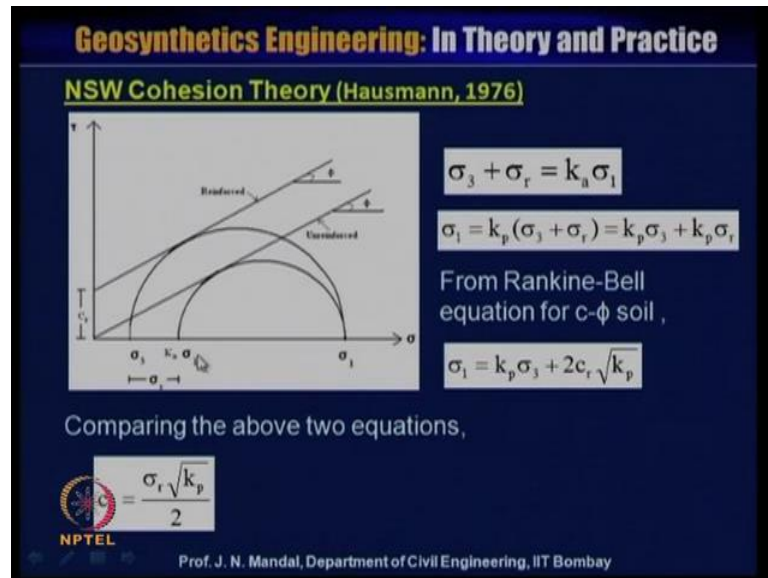
$$\frac{\Delta\sigma_1}{2\sqrt{k_p}} = \frac{k_p \frac{T}{h}}{2\sqrt{k_p}} = \frac{\sqrt{k_p} T}{2h}$$

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Now, we can write then, σ_{1r} is equal to σ_3 plus T by h into $\tan 45$ degree plus ϕ by 2 into $\cot 45$ degree plus ϕ by 2 minus ϕ σ_3 h is equal to σ_3 plus 3 by h $\tan 45$ degree plus ϕ by 2 into $\tan 45$ degree plus ϕ by 2 σ_{1r} is equal to, this is k_p into σ_3 plus k_p into T by h . Again we know the earlier, the σ_{1r} is equal to σ_1 plus $\Delta\sigma_1$, σ_1 again is equal to k_p into σ_3 . So, σ_1 is equal to k_p into σ_3 plus $\Delta\sigma_1$.

Therefore, we can write that, $\Delta\sigma_1$ is equal to k_p into T by h now, if you correlate this equation and this equation then, you can have $\Delta\sigma_1$ will be k_p into T by h because $k_p \sigma_3$ $k_p \sigma_3$ is cancelled. So, we know that earlier, that cohesion or the reinforced induced shear value or the adhesion value, the shear is equal to $\Delta\sigma_1$ by $2\sqrt{k_p}$. Now, if you substitute the value of $\Delta\sigma_1$ that means, $\Delta\sigma_1$ is equal to k_p into T by h by $2\sqrt{k_p}$. So, shear is equal to $\sqrt{k_p}$ divided by 2 into T by h with this theory, if you know what will be the tensile strength of the metallic reinforcement and what will be the spacing between the reinforcement then, you can calculate the shear value of course, you know what will be the value of k of p .

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Now, NSW New South Wales cohesion theory is given by Hausmann 1976 in this slide, this is the sigma of 1, this k sigma 3, this is sigma 3 there is a development of sigma 1. So, this is Mohr circle for unreinforced case with making at an angle of phi, this is the reinforced case and this angle of friction is equal to the same and the phi, there is a development of the shear value here. So, from here, we can write that, sigma 3 plus sigma r is equal to k into sigma 1, so we write sigma 3 plus sigma r is equal to k into sigma 1.

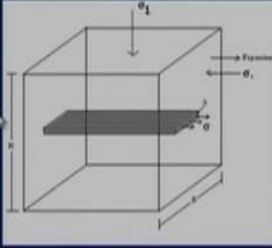
Again sigma 1 is equal to 6 sigma 1 is equal to k p into sigma 3 plus sigma of 1 so this you can write, k p into sigma 3 plus k p into sigma r. We know from the Rankine and Bell equation for c phi soil, sigma 1 is equal to k p into sigma 3 plus twice shear into root of k p. Now, if you compare these above two equation, so we can write that, c r will be equal to sigma 1 root k p, this divided by this divided by 2 because if you compare this and this then, this is k p into sigma r.

So, this you can write that, c r will be equal to sigma r root k p by 2 so you can calculate that, what will be the cohesion theory, which is given by Hausmann in 1976. So, whatever it is in the French or in Australia, this value is almost the same if we put in some of the problem you can find that, cohesion some say is working well.

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Geosynthetic Engineering: In Theory and Practice

Considering failure by rupture of reinforcement (Hausmann, 1976).



σ = Ultimate tensile strength or yield strength of reinforcement

Additional Stress in the soil due to inclusion of reinforcement = σ_r

Therefore, $\sigma_r \cdot B \cdot H = \sigma \cdot A$

$$\sigma_r = \frac{\sigma A}{BH}$$

A = Cross sectional area of the strip

$$c_r = \frac{\sigma_r \sqrt{k_p}}{2} = \frac{\sigma A}{2BH} \sqrt{k_p}$$

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Now, considering the failure by rupture of the reinforcement, which is given by Hausmann. Now here, this is the ultimate tensile strength or the yield strength of the reinforcement, this is the reinforcing chief element which is giving the ultimate tensile strength or yield strengths of the reinforcement, which is designated at sigma. Now, additional stress in the soil due to the inclusion of the reinforcement is sigma of r, so this give you the additional stress in the soil for the inclusion of the reinforcing material.

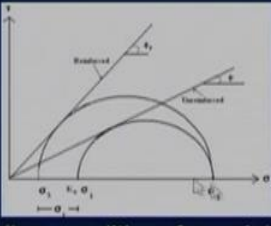
Now, this sigma r into this B and this is height H and this is B so this sigma r into B into H is equal to sigma into A because this tensile strength of the strip is sigma and the A is equal to cross sectional area of the strip. So, this sigma into A will be balanced by this sigma r into B into H, this area so sigma r into B into H, this area will be equal to sigma into cross sectional area of the strip A. So, that is why, it has been written that sigma r into B into H is equal to sigma into A.

Now, you can write that, sigma r is equal to sigma A divided by B into H and we know that, c r is equal to sigma r root over k p by 2, which already we have shown earlier. Then, you substitute the value of sigma r here so sigma r is equal to sigma A by B H so sigma r is equal to sigma A by B H, this 2 is remain. So, 2 B H and root over k p. So, considering the failure by the rupture of the reinforcement also, you can calculate that what will be the cohesion intercept for this soil.

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➤ At low stress level, failure in a reinforced earth tends to occur by slippage. If friction along the reinforcement is proportional to vertical stress and cohesion remains zero,



$$\sigma_r = F \sigma_1$$

$$\sigma_3 + \sigma_r = K_a \sigma_1$$

$$\sigma_3 + F \sigma_1 = K_a \sigma_1$$

$$\frac{\sigma_3}{\sigma_1} + F = K_a$$

$$\frac{\sigma_3}{\sigma_1} = K_a - F = \frac{(1 - \sin \phi_r)}{(1 + \sin \phi_r)}$$

$$\sin \phi_r = \frac{K_a - F - 1}{F - K_a - 1}$$

Failure conditions for variable σ_r (Hausmann, 1976)

F = friction factor characterizing reinforcement interaction with the cohesionless soil. If $F = K_a$, $\phi_r = 90^\circ$, failure occurs by rupture rather than slippage.

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Now, at low stress level, failure in the reinforced earth tends to occur by slippage, if friction along the reinforcement is proportional to the vertical stress and cohesion remains 0. In look at here, this is the unreinforced Mohr circle and this is the failure envelope and which is making at an angle of phi and this is the particle stress for the reinforced case, which is making at an angle of phi of r. This is the failure envelope for the reinforced soil case and sigma of r is proportional to sigma of 1.

So, we can write that, sigma r is equal to F into sigma 1 for cohesion remain 0 so we can write here that, sigma of 3 plus sigma of r is equal to k a into sigma 1. This is sigma 3, this is sigma r, this is k a sigma 1 so we can write this sigma 3 plus sigma r is equal to k a into sigma 1. Now, sigma 3 plus sigma r again equal to the F into sigma 1 so you substitute this value of sigma r here. So, you can have sigma 3 plus F into sigma 1 is equal to k a into sigma 1.

Now, you divided by sigma of 1 so you can we write sigma 3 by sigma 1 plus F will be equal to k a. We know sigma 3 by sigma 1 is equal to k r that is, is equal to 1 minus sin phi r by 1 plus sin phi r so sigma 3 by sigma 1 is equal to k a minus F, this is k a minus F so this will be 1 minus sin phi r by 1 plus sin phi r. So now, if you solve this equation, you can have sin phi r is equal to k a minus F minus 1 divided by F minus k a minus 1. So, if you solve this equation, you can have sin phi in this form. For F is equal to the friction factor, charactering the reinforcement interaction with the cohesionless soil.

Now, in this case, when this F will be equal to k of σ_1 so ϕ_r have a limiting value of 90 degree. So, we can conclude that failure occur by the rupture, rather than the slippage.

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Diagram illustrating the forces on a reinforcement strip in soil. The strip has width B and length L . The soil height is H . The vertical stress is σ_1 , and the shear stress is τ . The force F is the horizontal force exerted by the strip.

Equations shown:

$$BH \sigma_r = 2 \tau B L e$$

$$\sigma_r = \frac{\sigma_1 \tan \delta \times 2 B L e}{BH}$$

$$\tau = \sigma_1 \tan \delta \quad \sigma_r = F \sigma_1$$

$$F = \frac{2 \tan \delta B L e}{BH}$$

Slippage of reinforcement (Hausmann, 1976)

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Now here, it is showing that, slippage of the reinforcement which is given by the Hausmann 1976 you can see, this is the reinforcement strip and there is a shear stress, which is acting along the plane of this reinforcement and this reinforcement with these B dash and this length is equal to L dash. So, there will be the shear stress acting along the plane of the reinforcement in this direction and this is expanding and this is the force of σ_r and this is acting on this area that is, this is the B and this is the H .

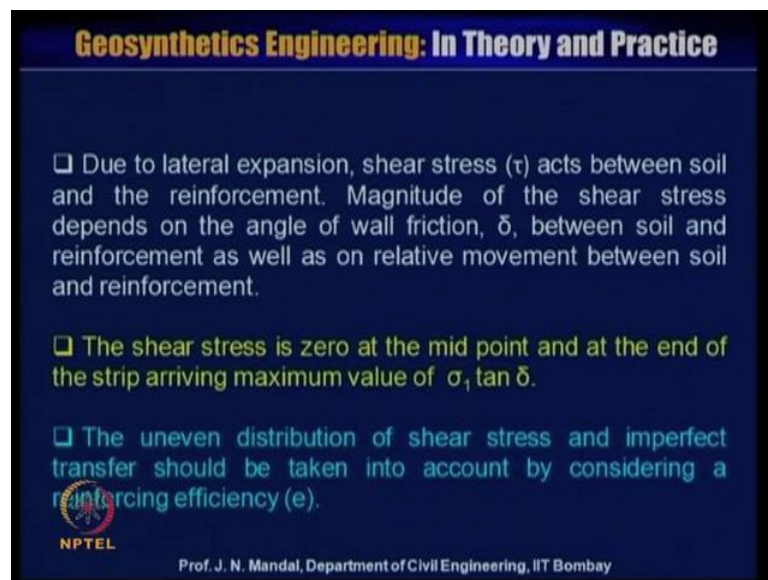
So, we can write the σ_r into B , B into H that mean area, will be equal to 2τ into B dash into L dash. Now, this area is B dash into L dash and τ is acting on the both the side of the reinforcing strip that is why, 2τ and this area is equal to B dash into L dash. So, again we know, this τ is equal to $\sigma_1 \tan \delta$, so because there will be the vertical stress is σ_1 and there is the development of friction between the soil and the reinforcement, which is designated at δ .

So, τ also can be expressed as $\sigma_1 \tan \delta$ so we can write that, σ_r will be equal to, this is 2 , 2 and τ is equal to $\sigma_1 \tan \delta$, $\sigma_1 \tan \delta$ and B dash into L dash, this is B dash and L dash, this divided by B into H , this is B into H . There is another term which is e , I will explain it later about this efficiency e and again that σ_r is equal to F into σ_1 is known to you. So, F will be equal to σ_r by σ_1 so

you can write, F is equal to $2 \tan \delta B L \sigma_1$ divided by B of H because σ_1 σ_1 's will be cancelled.

So, you can write this F so this is the figure shows the slippage of the reinforcement now, when there will be development of the stress shear, you can see that initially, the shearing stress is almost will be the 0 at the middle of the reinforcing strip and gradually, it will be the maximum almost at the end.

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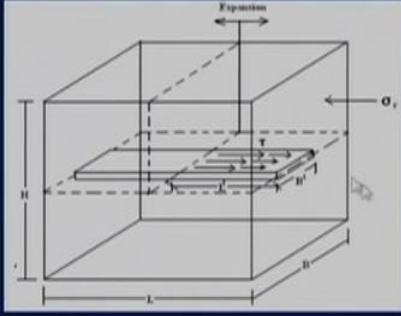
- Due to lateral expansion, shear stress (τ) acts between soil and the reinforcement. Magnitude of the shear stress depends on the angle of wall friction, δ , between soil and reinforcement as well as on relative movement between soil and reinforcement.
- The shear stress is zero at the mid point and at the end of the strip arriving maximum value of $\sigma_1 \tan \delta$.
- The uneven distribution of shear stress and imperfect transfer should be taken into account by considering a reinforcing efficiency (e).

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So, I am just saying from this that, due to the lateral expansion, shear stress is τ which is acting between the soil and the reinforcement. The magnitude of the shear stress depend on the angle of wall friction δ between the soil and the reinforcement as well as relative movement between the soil and the reinforcement. Shear stress, as I said is acting 0 at the midpoint and at the end strip arriving the maximum value that is, $\sigma_1 \tan \delta$. That is, uneven distribution of the shear stress and imperfect transfer should be taken into account by considering a reinforcing efficiency e .

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$BH \sigma_y = 2 \tau B' L'$

$$\sigma_y = \frac{\sigma_1 \tan \delta \times 2 B' L' e}{BH}$$
$$\tau = \sigma_1 \tan \delta \quad \sigma_y = F \sigma_1$$
$$F = \frac{2 \tan \delta B' L' e}{BH}$$

Slippage of reinforcement (Hausmann, 1976)

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What I am trying to say here because that there is a shear stress is less near to the middle of the strip and gradually, it is increasing and you can have a maximum stress here. So, these are uneven stress distribution, which we cannot consider in any kind of the design. So, what you have to be consider, some efficiency and that efficiency which is designated as e.

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- Due to lateral expansion, shear stress (τ) acts between soil and the reinforcement. Magnitude of the shear stress depends on the angle of wall friction, δ , between soil and reinforcement as well as on relative movement between soil and reinforcement.
- The shear stress is zero at the mid point and at the end of the strip arriving maximum value of $\sigma_1 \tan \delta$.
- The uneven distribution of shear stress and imperfect transfer should be taken into account by considering a reinforcing efficiency (e).

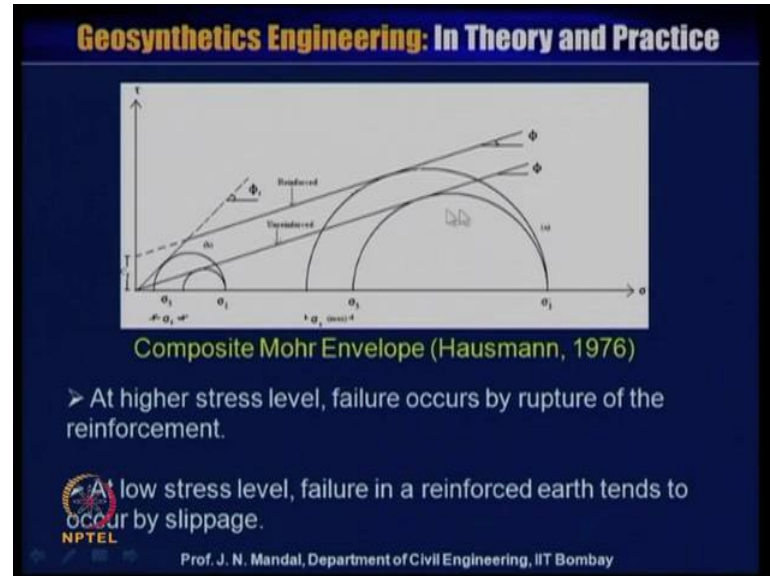
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So, we have to consider this designation e, which we call the efficiency e so that is why, when you can see that there is a uneven distribution of the shear stresses and the

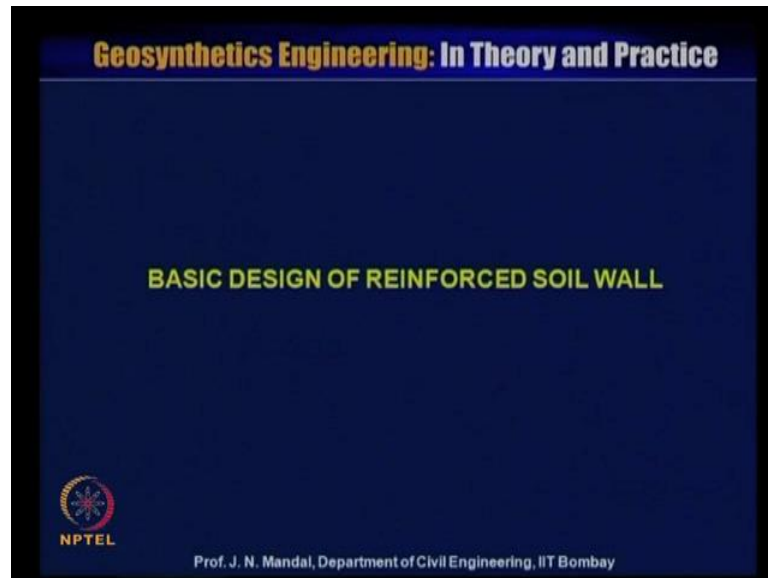
imperfect transfer should be taken into account by considering this reinforcement efficiency e .

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Now, this is composite more envelope even by Hausmann 1976, these are the one is the unreinforced case, this angle of friction is ϕ , this is the Mohr circle, this is another Mohr circle for the inputs case. You can see, there is a angle of ϕ_r and ϕ , it is the same as in case of the unreinforced case but there is a development of the adhesion value. The other case you can see in terms of the angle of friction, this is the ϕ in case of unenforced whereas, in case of the enforced case and this value is ϕ_r , this is the higher value. So, at the higher stress level failure occur by the rupture of the reinforcement, at low stress level failure in a reinforced earth tends to occur by the slippage.

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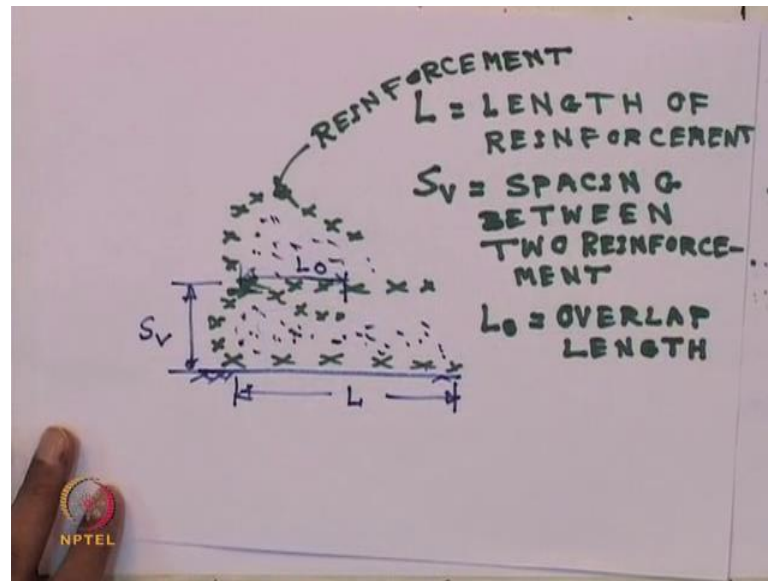


Next, we will now address the design of geosynthetics reinforced soil system, a number of triaxial compression test of cohesionless soil with reinforced soil have been performed. And we have covered from anisotropic cohesion concept and then, the Laboratory Central de Panss Chaussces Paris the cohesion theory. You have also covered the New South Wales cohesion theory and failure by rupture of the reinforcement, failure by slippage of the reinforcement and composite more envelope without and with reinforcement case.

Now, I will address the basic design of reinforced soil wall, the reinforcement material is flexible. Let us look at this paper, this is a kind of flexible material so I wanted to show that, how we can form a reinforced soil retaining wall with the aid of this paper. We can place this paper on the ground surface, then fill up with the some granular material. And then, we can wrap it, we can wrap it like this and this is called the overlap and this is called the length of the reinforcing material.

And next, we can use another layer of the reinforcing material and then, you fill up with the soil and then, you can wrap it, like that you can construct a reinforced soil retaining wall. So here, this is called the facing element, which is called wrap facing element so in this wall, I am saying in sketch like this.

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If this is a ground surface and I am using this reinforcing material like this and then, this material it can be wrapped and can be placed like this and this is filled up with the backfill material. Next, we place the another layer of reinforcing material and you can wrap it like this and you can fill up this material. Like that, you can continue and can construct a reinforced soil wall with the paper and the backfill material.

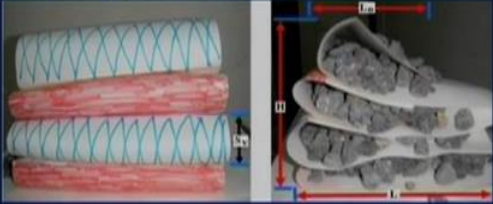
The length of the reinforcing material which can be defined as L and this is the spacing of the material which can be expressed as S of V and this is the overlapped length and this can be expressed as L of O . So, we can say that, L is defined as a length of reinforcement and $S V$ is spacing between two reinforcement and $L O$ we can define as a overlap length.

So, this we call the backfill material and this we say that reinforcement, this we call the reinforcement. So, even then, we can form a reinforce soil retaining wall with the help of the paper and the backfill material, it may be like a sand or the granular material or even then, with the stone.

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Geosynthetics Engineering: In Theory and Practice

The reinforced earth concept can be used for retaining walls. For basic understanding, a model reinforced soil wall is constructed using paper and gravel/sand.



➤ For internal stability, we have to determine the length of reinforcement (L), overlap length (L_0) and spacing of the reinforcement (S_v).

➤ For external stability, we have to check for sliding, overturning and bearing capacity failures.

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Next, I am showing you the reinforced earth concept, how it can be used in case of the reinforced soil wall. So, you should understand that, what should be the basic concept and mechanism of reinforced earth. So, this is the model and we have used paper as a reinforcing material and you have is the sand and the gravel. You can see here, this is the paper and then, it has been wrapped, like this paper and these wrapped and here, is the sand.

And this the length of the reinforcement and this is the vertical distance between the two reinforcement and the height of the wall is capital H and here, L_0 is the overlapped length and this is the facing element. So, when we will design this reinforced wall system, if required for the internal stability and also the external stability. So, from the internal stability, we can determine that, what will be the length of the reinforcement, what will be the vertical spacing between the two reinforcing element and also, what will be the overlap length.

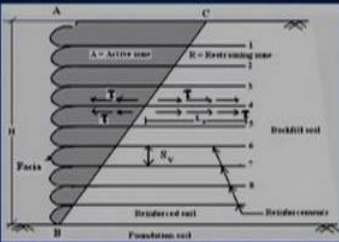
For external stability, we have to check the sliding, the structure may collapse due to the sliding or structure may collapse due to the over turning or structure may fail due to the bearing capacity. If the ground is soft then, you require proper ground modification system or it can be filled by global stability. So, for external stability, you have to determine, what will be the factor of safety against the sliding, what will be the factor safety against the over turning, what will be the factor of safety against the bearing

capacity as well as what will be the factor of safety for Gibbs heated failure or global slope stability failure.

You have to be very careful about the backfill material so when you design you require for the internal stability as I mentioned that, you require the what will be the length of the reinforcing material, what would be the spacing between the reinforcement and also, the overlap. So, we require certain theory, what is happening how the reinforcement fail.

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The diagram shows a cross-section of a reinforced soil wall. On the left is the 'Facing' element. Horizontal reinforcement layers are shown extending into the 'Backfill soil'. A diagonal failure line, labeled 'Failure line (BC)', divides the soil into two zones: 'A = Active zone' on the left and 'B = Restraining zone' on the right. The wall height is denoted as 'H'. The failure line starts at point 'B' at the base of the wall and goes to point 'C' at the top of the soil. The diagram also shows 'Foundation soil' at the bottom and 'Reinforcement' layers. Arrows indicate the direction of forces and stresses within the soil.

Failure line (BC) divides the reinforced fill into two zones:

- (A) Active zone, and
- (B) Restraining zone

Cross section of a reinforced soil wall

- > In active zone, the shearing stresses from the soil to the reinforcement act towards the facing element. This means that reinforcement is pulled towards the facing element.
- > In restraining zone, the shear stresses from the soil to the reinforcement act away from the facing, thereby tend to hold the reinforcement in soil.

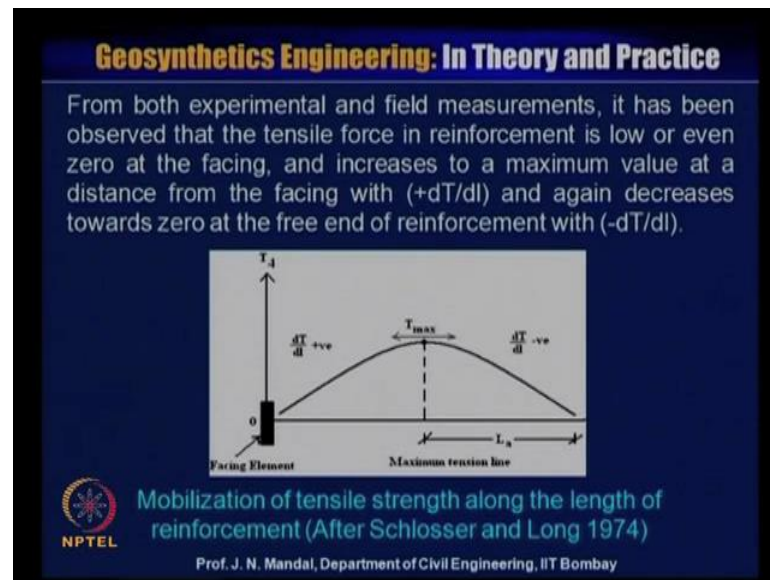
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What system we should follow a path, you can see this figure, these are the number of the reinforcement layer and starting from 1 2 3 4 like this. And this is height of the wall is H and this is the facing element and the failure line is BC, which divide the reinforcement fill into the two zone, this left hand side is the active zone and right hand side is the restraining zone. So, this is the cross section of a reinforced soil wall and spacing between the reinforcement is S V.

And in the active zone, the shearing stresses from soil to the reinforcement acts towards the facing element this means, that reinforcement is pulled towards this facing element But, in the case of restraining zone, the shear stresses from the soil to the reinforcement act away from the facing element thereby, tend to hold the reinforcement in soil.

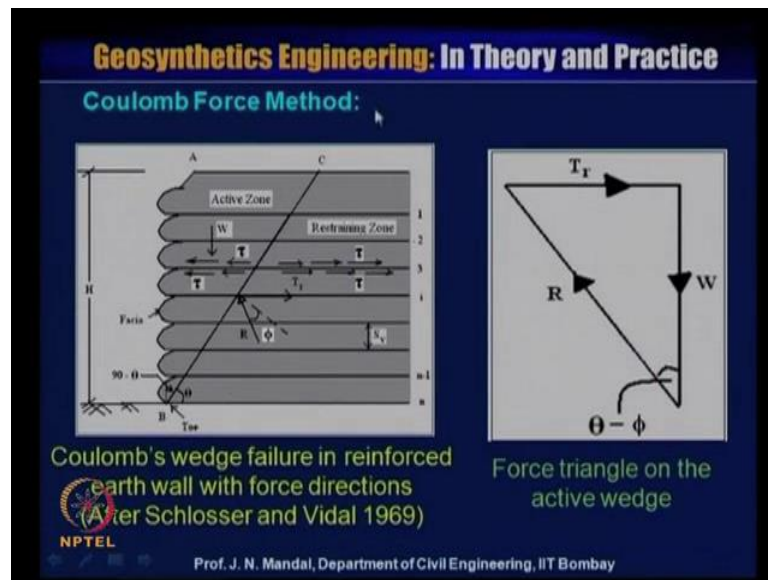
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Now, what is happening, how we are drawing the failure envelope, you can look at this figure for mobilization of the tensile strength along the length of the reinforcing element after Schlosser and Long 1974. It has been observed from both experimental and field measurement that, tensile force in the reinforcement is low even then, 0 and the facing element. And then, increases to a maximum value that is, T_{maximum} at a distance from the facing element and again decreases towards the 0, as the free end of the reinforcement.

You can see that, maximum tensile force does not occur near to the facing element, it is always occur at the some high from the facing element. So, locus of the maximum tensile force element will give a point of the failure envelope so it means that, there is no tensile force developed near to the facing of the element. It may be like this or it may go up and then, go horizontally and then, go down, also different types of the failure system has been observed. So, if this is the kind of the failure system, also has been compared with the analytical method and it found that, it matching well. So next, we will discuss about the Coulomb force method.

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So, in this method, that coulomb's wedge failure in reinforced earth wall with force direction, this is after Schlosser and Vidal, 1969. And you can see here, this BC line, this is the wedge ACB and this AC line making at an angle of θ with the horizontal and this is the toe of the wall and this angle is θ . So, this angle will be 90 minus θ , this is the facing element and here, there is a tensile forces which is acting, which is defined as T of r and the resultant force r , which is acting at an angle ϕ with the normal to the plane BC .

And here is this ACB weight, here W is the active zone, weight of the active zone and this side is the restraining zone. And you can see that, τ shearing stresses which is acting towards the facing element and this is the τ which is acting as a straining zone τ , which is acting away from the facing element. Now, if you draw a force triangle on the active zone you can see, this T_r which is acting horizontally and this is weight of the active zone, which is acting vertically downwards. And this is the reactional force, which is passing through this and which is making at an angle of θ and ϕ so this is the force triangle on the active wedge.

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From the force triangle, $T_r = W \tan(\theta - \phi)$

Now, $W = \frac{1}{2} \gamma H^2 \tan^2(90^\circ - \theta)$

Therefore,

$$T_r = \frac{1}{2} \gamma H^2 \tan^2(90^\circ - \theta) \tan(\theta - \phi) = \frac{1}{2} \gamma H^2 \cot^2 \theta \tan(\theta - \phi)$$

$$\frac{dT_r}{d\theta} = 0 \quad T_r \text{ is maximum when } \theta = (45^\circ + \phi/2)$$

$$T_{r(\max)} = \frac{1}{2} \gamma H^2 \tan^2\left(45^\circ - \frac{\phi}{2}\right) = \frac{1}{2} K_a \gamma H^2$$

resultant tensile force is distributed in all the reinforcement

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From this force triangle of the active wedge, we can determine that, what is T_r that means, T_r is equal to...

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Coulomb Force Method:

Coulomb's wedge failure in reinforced earth wall with force directions (After Schlosser and Vidal 1969)

Force triangle on the active wedge

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This is T_r is equal to $W \tan \theta - \phi$ so here, you can write that, T_r is equal to $\tan \theta - \phi$.

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From the force triangle, $T_r = W \tan(\theta - \phi)$

Now, $W = \frac{1}{2} \gamma H^2 \tan(90^\circ - \theta)$

Therefore,

$$T_r = \frac{1}{2} \gamma H^2 \tan(90^\circ - \theta) \tan(\theta - \phi) = \frac{1}{2} \gamma H^2 \cot \theta \tan(\theta - \phi)$$
$$\frac{dT_r}{d\theta} = 0 \quad T_r \text{ is maximum when } \theta = (45^\circ + \phi/2)$$
$$T_{r(\max)} = \frac{1}{2} \gamma H^2 \tan^2 \left(45^\circ - \frac{\phi}{2} \right) = \frac{1}{2} K_a \gamma H^2$$

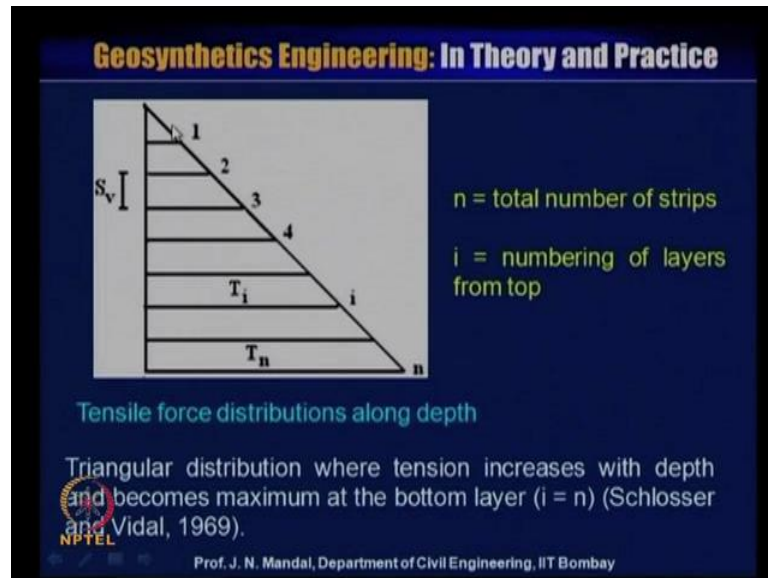
NPTEL resultant tensile force is distributed in all the reinforcement

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Whereas, W is equal to half into gamma H square tan 90 minus theta so here, W this is theta, this is 90 minus theta and this is height H. So, this A C will be equal to H 90 minus theta and this height is H so this area will be the half into H into H 90 minus theta half into base into altitude so half into H into H 90 minus theta. So, we can write that, weight is equal to half into gamma H square tan 90 minus theta. Therefore, so this T r can be written that, if you substitute this value of W here that means, W is equal to half gamma H square tan 90 minus theta and then, tan theta minus phi.

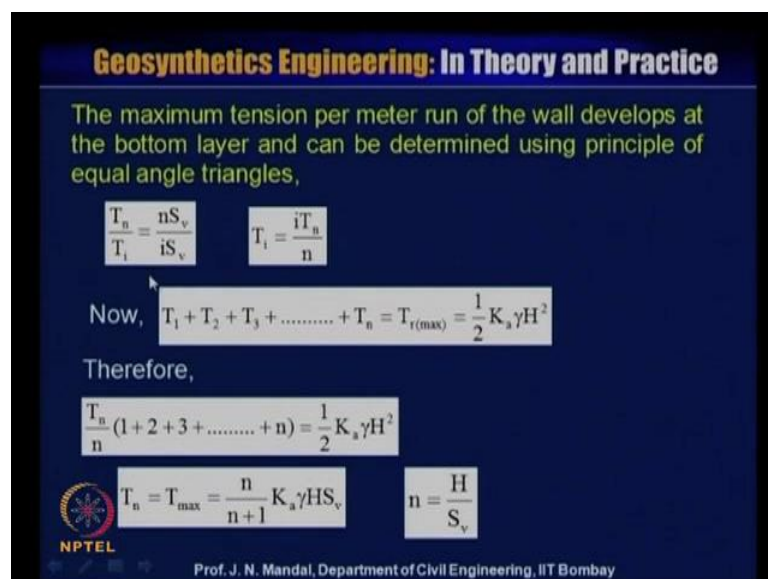
You can have half gamma H square, this is cot theta into tan theta minus phi now, we have to check that, what will be the T r maximum value. When T r will be the maximum when theta will be 45 degree plus phi by 2 that mean, d T r by d theta will be 0. So, T r maximum will be half gamma H square tan square 45 degree minus phi by 2 so this is equal to half into k of a gamma into H square. We know that, k a coefficient of active work pressure, that is equal to tan square 45 degree minus phi by 2. So, we can write that, T r maximum is equal to half k a gamma H square, this resultant tensile force is distributed in all the reinforcement layer.

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So, you can see here, the number of layer of the reinforcement 1 2 3 4 5 like this where, n is equal to total number of the strip and i is equal to number of the layer from the top and this is the spacing between the two reinforcement, which is designated at S of V . Then, you can see from this figure, the how the tensile force distribution along the depth so it is a triangular distribution for tension is increasing with depth and becomes the maximum at the bottom layer when i is equal to n this also is specified by given Schlosser and Vidal in 1969. So, we can conclude that, tension is increasing and the maximum tension developed at the bottom of the reinforcing layer.

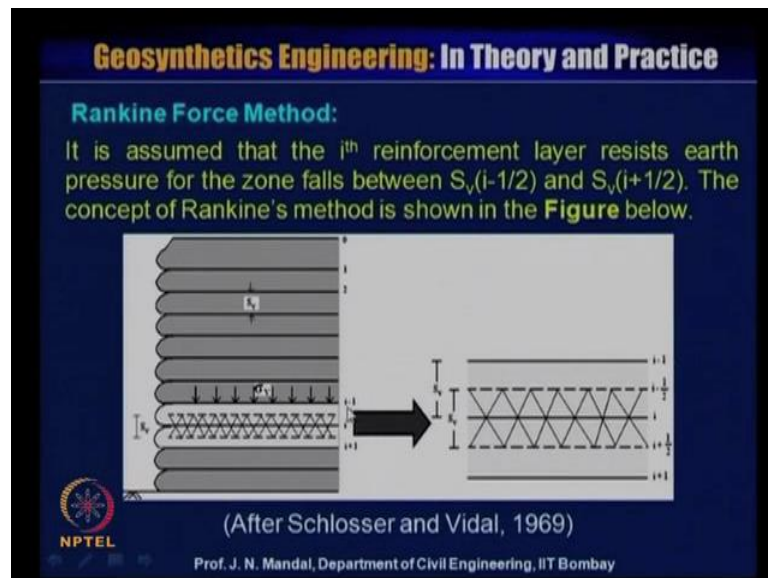
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This maximum tension per meter run of the wall developed and the bottom layer, and can be determined using the principle equal angle triangle. So, we can write T_n by T_i , $n S V$ by $i S V$, T_i will be equal i into T_n divided by n . Now, T_1 plus T_2 plus T_3 plus dot dot dot plus T of n is equal to T at maximum that is, half $k a \gamma H^2$. Therefore, we can write T_n by n , 1 plus 2 plus 3 plus dot dot dot plus n is equal to half $k \gamma H^2$. T_n is equal to, T maximum is equal to n divided by n plus 1 $k a \gamma H S V$ or we can write that, n is equal to H by $S V$.

If n value is equal to the 1 then, you can have $k a H$ into $S V$, if n is equal to H by $S V$, if you know what will be the height of the reinforced soil wall and if we know that, what should be the spacing between the two reinforcement then, you can calculate what will be the number of the reinforcement is required. Or other way, if you know the number of the reinforcement and the height of the wall, you can also calculate what will be the spacing between the two reinforcement. So, we can calculate the number of the reinforcement, we can calculate also the spacing of the reinforcement next, we will discuss about the Rankine force method, what rankine say.

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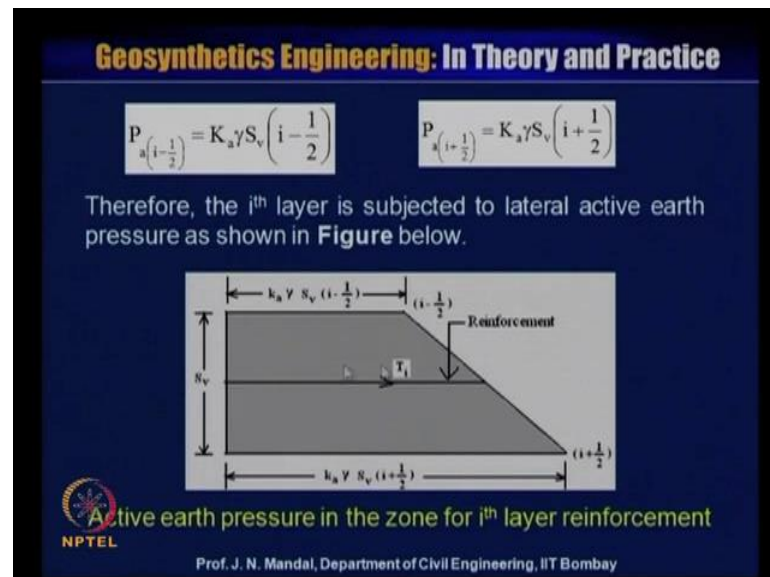


It is assumed that, in the i th reinforcement, this is i th reinforcement, layer resist the earth pressure for the zone fall between that S of V into i minus half and S of V into i plus half. This concept of ranking method is shown in this figure let us say that, this is the reinforced soil wall and this is the number of the reinforcement 0 1 2 3 like this in

this layer, i th layer. You are considering i th layer and this is the spacing between the two reinforcement and this is the vertical stress σ_v is acting on this reinforcement.

And this is one of the reinforcement in i th layer is i and top layer will be i minus 1 and this bottom layer will be i plus 1. Now, if you take the middle of this layer, I am showing here, the middle of this layer then, in this to this is half S V and this to this is half S V . So, total will be the S of V and this we consider as a i layer and this top will be equal to i minus half and this bottom will be equal to i plus half so this is given by Schlosser and Vidal 1969.

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So, how we can calculate the pressure so this is the active earth pressure in the zone for i th layer reinforcement. If the i th layer reinforcement is T of i and its spacing is S of V and in the top is, this is i minus half and the bottom is i plus half. So, this will be K_a into γ into S V into i minus half in this layer, at the bottom it will be K_a into γ into S V into i plus half. So now, we can take as this as a trapezoidal so we have to calculate that, what should be the area. So, area will be the half of this distance plus this distance and then, multiplied by the S V so this will give you the area of this trapezoid.

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T_i = Developed tension in reinforcement = Area of active earth pressure diagram

$$T_i = \frac{1}{2} K_a \gamma S_v \left[\left(i - \frac{1}{2} \right) + \left(i + \frac{1}{2} \right) \right] S_v = i K_a \gamma S_v^2$$

Vertical stress at i^{th} layer, $\sigma_v = \gamma i S_v$

Therefore,

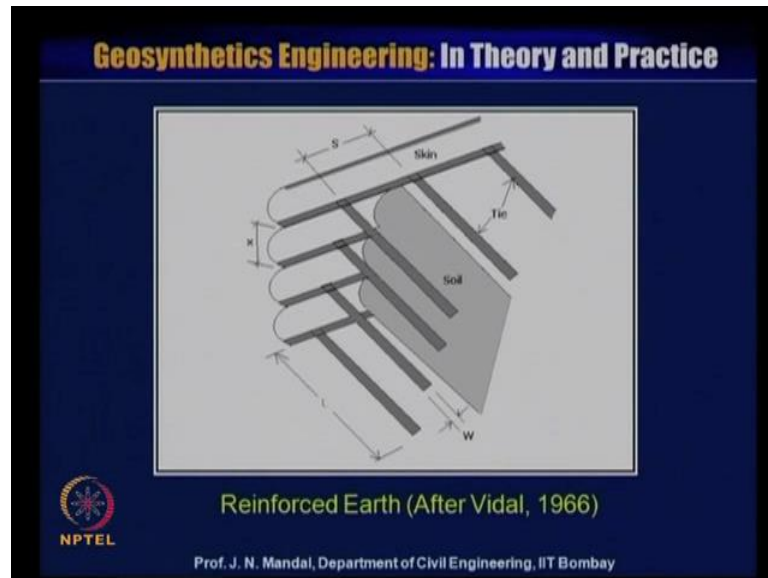
$$T_i = K_a S_v \sigma_v$$

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So, we can show you that, next slide that, how we can write this, that area of the active earth pressure diagram or what that T_i development of the tension in the reinforcement. So T_i , you can write is equal to half into $k_a \gamma S_v$ into $i - \frac{1}{2}$ plus $i + \frac{1}{2}$ into S_v , this is the area of the trapezoid. That means, half into summation of top and the bottom layer and this height that is, S_v . So, we can calculate and we can have T_i is equal to $i k_a \gamma S_v^2$.

So, vertical stress at the i^{th} layer, it can be expressed σ_v is equal to $\gamma i S_v$ therefore, T_i can be written as $k_a S_v$ into σ_v . So, we can calculate, what will be the developed tension in the reinforcement and this developed tension in reinforcement is nothing but the area of the active earth pressure of diagram, we can calculate that tension in the reinforcement.

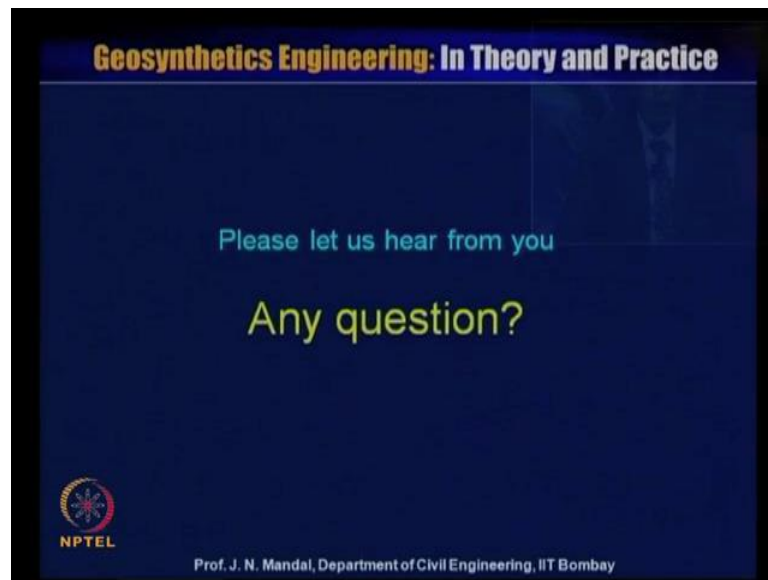
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Now with this, that Rankine earth pressure and coulomb's earth pressure theory, this Vidal in 1966 developed the first reinforced earth retaining wall and you can see that, he has used as a metallic reinforcement. Here, strip in the form of very thin sheet, it is galvanized metallic reinforcement whose length is equal to L and width of the reinforcement is W . So, when we will design then, we have to be consider what will be the width of the metallic strip, what will be thickness of the metallic strip.

And also, what would be the length of the metallic strip and then, what will be the horizontal spacing and S and what will be vertical spacing. And this is as a skin element, which we call the facing element, which is made of the prefabricated concrete. It may be in the semicircular or it may be in the ((Refer Time: 59:10)) shape form, so with this, I conclude this lecture.

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Please, let us hear from you any question.

Thank you for listening.