

In this figure, we are showing now the earthquake distribution during 1800 to 2007. This data is available in USGS website. So, reference is USGS. You can see the latitude and longitude wise, this distribution for entire India. They have grouped it in different magnitude ranges like this yellow dots, small yellow dots are for magnitude between 5 to 5.9. These green dots are for magnitude between 6 to 6.9. These orange dots are for magnitude between 7 to 7.9 and these red big dots are for magnitude greater than 8 and these stars are those earthquakes which are the significant earthquake in India where number of devastations and damages were maximum.

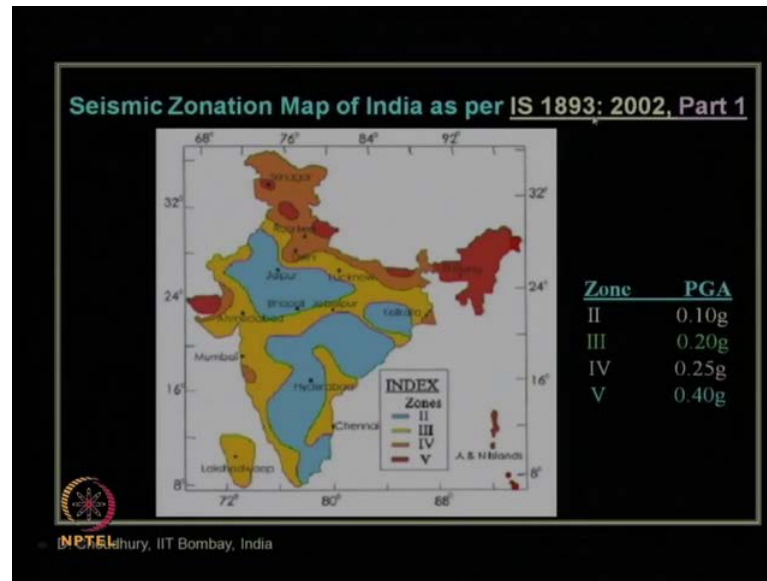
So, you can see over here like Bhuj earthquake, Latur earthquake, Koyna earthquake, Jabalpur earthquake, Bihar-Nepal earthquake, these are the several important earthquakes where when India experience severe disasters over the years and if you look at the intensity of this magnitude of earthquake points, mostly what you can say there highly located in this region, where our Andaman Nicobar island comes also in northeast region of India as well as this northern part of India that Himalaya and belt what we called.

So, the maximum number of occurrences of earthquake of magnitude 5, and above are mostly distributed among these area, but it does not mean that inside India or this peninsular region of central India had never had a chance to get experience of large earthquake, that we had already example of 1997, which was significant earthquake, 1993 Latur earthquake, 1967 Koyna earthquake and then Bhuj earthquake.

So, these are important earthquakes in the central region of the India are so-called we call peninsular India. So, here the number of occurrences is less, but it is present whereas, number of occurrences are very significant as well as very high magnitude of earthquake. There intensity of occurrences you can say is very well clustered in this northern north-eastern and this eastern part of India, in this island region inside the Bay of Bengal.

Now, let us see what our Indian seismic code refers to about the Seismic Zonation process for the India.

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So, if we look at here the seismic zonation map of India, as per our Indian standard code IS 1893 is code which we use for seismic zonation map. The part 1 of that IS 1893, the latest version is 2002 which divides entire India into four major zones. These zones are nomenclatured as zone 2, zone 3, zone 4, and zone 5.

So, zone 2 is this blue colour, zone 3 yellow colour, zone 4 orange colour. This one, this one and zone 5 is red colour region. So, you can see what are the different seismic zones as per our Indian seismic design code, and as per various seismic zones of India. Typical values of PGA EPI ground acceleration may be taken as these values as per the device one. So, 0.1 g, 0.2 g, 0.25 g and 0.4 g as per zone 2, 3, 4 and 5 respectively.

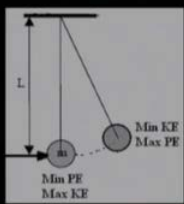
So, with this we have come to the end of our module number 1 that is introduction to geotechnical earthquake engineering. Next, let us do the recap of our module to which we have learnt in the previous lecture. Let us look at the slide here. In the previous lecture, we discussed about module 2 on basics of vibration theory and for that module, I have given the reference to another NPTEL video course on soil dynamics, the same module number that is in that video course of soil dynamics module 2 which was taken by me. So, one can go through that module 2 of soil dynamics NPTEL video course which discusses in detail about this basics of vibration theory.

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Dynamic loads :

1. Earthquake load,
2. Wind load,
3. Moving load,
4. Guide way unevenness,
5. Machine induced load,
6. Blast load,
7. Impact load etc.

Vibration



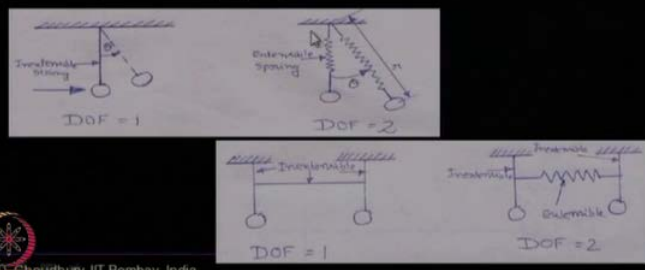
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So, what we learned in the previous lecture in this module 2 that various types of dynamic loads what is called dynamic loads, it has to be time variant load as well as it should create a vibration that is the exchange of kinetic energy to potential energy and vice versa. So, these are various types of dynamic loads and of course, our earthquake load for which we are studying this course is one of the major forms of dynamic load.

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Degrees of Freedom (DOF)

- No of independent co-ordinates (displacements) required to define the displaced position of all the masses relative to their all the position is defined as degrees of freedom.
- Generally in Dynamics, mass property dictates the DOF whereas in Statics, the stiffness property dictates the DOF



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Then we had learnt what is known as degrees of freedom. It is nothing but the number of independent coordinates required to define the displaced position of all the masses

related to their original position which is defined as the degrees of freedom. We have seen the simple examples of simple pendulum like this, where degrees of freedom is 1. If it is connected to a inextensible string, but if we change that string to an extensible string or extensible string, the degrees of freedom changes to 2. That means we require minimum two coordinates. This theta and this are to define the position of this system at any point of time in the space.

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Simple Vibrating System (SDOF system)

■ **Mass-Spring-Damper (MSD) System**

Friction-free surface

- m → Kinetic Energy
- k → Potential Energy
- c → Dissipation

■ **D'Alembert's principle**

➤ For any object in motion, the externally applied forces, inertial force and forces of resistance form a system of forces in equilibrium.

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Then, we had seen that how we represent a simple vibrating system through a single degree of freedom system. So, single degree of freedom system, the basic units we require to define a simple vibrating system are mass spring and dash port or damper which we call as MSD system. So, in this MSD system, mass represents the kinetic energy k or spring constant represents the potential energy of the system whereas, the damper or damping constant c represents the dissipation of energy or loss of energy. So, if there is no loss, we can represent a simple vibrating system only by using mass and spring that is m and k , but if we want to take care of the loss of energy also, then it will be m , k and c .

So, this is the picture of a simple vibrating system, single degree of freedom system with a mass, with a spring and with a damper. Now, it is degrees of freedom is single or 1, that is u . That is at any point of time, this u defines its position and applied dynamic load externally on the system is p of t which is a function of t . Then taking the free body

diagram of this mass, what we can write this weight is acting downwards $m g$. So, there will be a reaction from these rollers on this friction free surface that will balance this static weight.

So, the static equilibrium of the system is maintained by that way. Now, on the system what are the forces? This is external applied load p of t dynamic load, this is the force of resistance due to the damper which is the damper forces represented by that damping constant c times, the velocity. What is velocity? \dot{u} . What is \dot{u} ? That is the first differential of this displacement u with respect to time.

So, \dot{u} is nothing but du/dt and the spring force is k times u , that is spring constants time. The displacement and these displacements and this velocity is the relative velocity and relative displacement, that is if it is connected to a fixed end, then other end is 0 velocity. This end will be having a velocity of du/dt . So, no net velocity or relative velocities du/dt , but if both the ends are moving, then in that case, we have to take the relative or difference between the velocity between two ends to calculate the damper force, similarly for the spring force. We have to take the relative displacement and another force of resistance will come into picture that is $m \ddot{u}$ which is nothing but called inertial force. That inertial force is nothing but mass times acceleration. So, acceleration as we know that is $d^2 u/dt^2$, the second derivative of this displacement with respect to time.

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- Linear Model for Equation of Motion

$$m \cdot \frac{d^2 u}{dt^2} + c \cdot \frac{du}{dt} + k \cdot u = p(t)$$

- Governing Equation of Motion

$$m \ddot{u} + c \dot{u} + k u = p(t)$$

Units	MLT system	FLT system	SI unit
m	M	F/LT ²	kg
k	MT ⁻²	F/L	N/m
$\frac{c}{h}$	MT ⁻¹	F/LT ¹	N-s/m

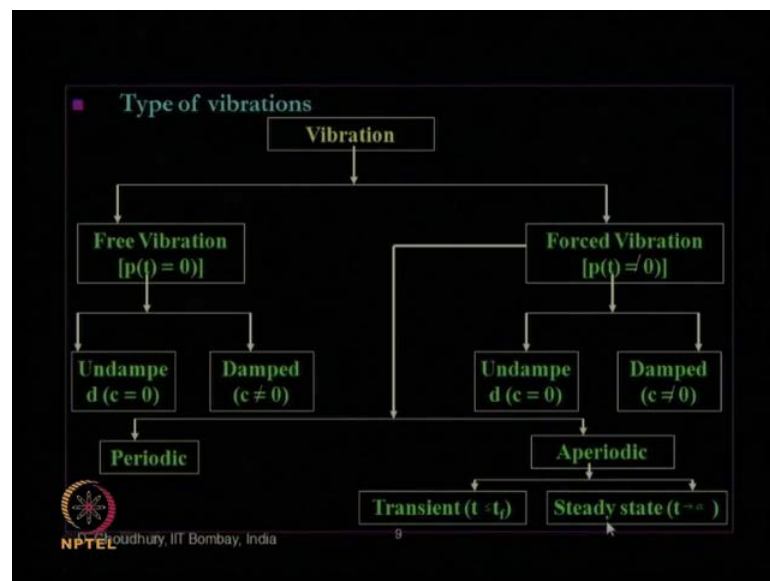

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So, using that if we consider a linear model for the equation of motion, we have seen that mass times acceleration is the inertial force damping. Constant times velocity is the damper force and spring constant time displacement is the spring force which will be equal to this externally applied dynamic load p of t . This is why for linear model because the assumption is masses not changing with time

So, it is constant damping; constant remains as a constant. It is not changing with time and the spring constant is also not changing with time. So, this is for linear spring, this is for linear damper and this is for constant mass system. That is why we say this is the equation governing equation of motion for linear model which on another notation, we write it as $m \ddot{u} + c \dot{u} + k u = p(t)$ and these are various units commonly used for m , k and c mostly. In SI unit, we use in kg for mass, that is Newton per meter for spring constant and Newton's again per meter for damping constant.

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Then, in our previous lecture, we had also seen what are the various types of vibrations. We have classified vibration into two major categories, free and forced vibration. Free vibration, when there is no externally applied dynamic load acting on the system that is p of $t = 0$, but forced vibration, when that is non-zero, then again within forced free vibration, we had two categories. Undamped free vibration and damped free vibration. Undamped means when the damping constant is 0. No loss of energy and damped means when the damping constant is non-zero that is loss of energies. Similar way the forced

vibration also can be undamped forced vibration and damped forced vibration depending on whether c is 0 or non-zero. Another sub classification of this forced vibration is a periodic and aperiodic. What is that? Periodic is that is this applied dynamic load repeats with time where aperiodic is it does not repeat with time within aperiodic.

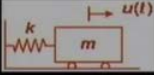
So, periodic example is we have seen the harmonic loading, any sinusoidal loading or co-sinusoidal loading. Those can be considered as periodic forced vibration, but when it is not repeating with time that is a periodic. We can also have to sub classification within that one is called transient, a periodic forced vibration. Another is called steady state, a periodic forced vibration.

So, transient means the time for which this force externally dynamic applied load is acting is for a finite duration. For example, earthquake load that acts for a finite duration. So, that is transient time, a periodic that is random whereas, steady state means that applied dynamic load acts for infinite times. For example, wind load can be considered as a steady state, a periodic forced vibration.

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- **SDOF system**
- **Free Vibration**
- *1. Undamped Free Vibration*

The structure is disturbed from its static equilibrium and then vibrates without any applied forces.




The equation of motion is: $m \ddot{u} + k u = 0$

The solution is: $u(t) = A \cos(\omega_n t) + B \sin(\omega_n t)$

$\omega_n = \sqrt{k/m}$ (rad/s) natural circular frequency

A and B are determined by the *initial conditions*


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Then, we have seen the solution for simplest case of free vibration with undamped condition. That means the equation of motion boils down to $m \ddot{u} + k u = 0$. We should know the initial condition that is at time $t = 0$. What is the displacement? What is the velocity for this free vibration to get the complete solution? The solution of this equation will be of this form, where these constants, a and b need to

be obtained from those known initial conditions and one of them must be non-zero at least and this ω_n is called natural circular frequency which is represented by root over k by m .

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$u_{t=0} = u_0 \rightarrow u_0 = A$
 $\dot{u}_{t=0} = \dot{u}_0 \rightarrow \dot{u}_0 = B \omega_n$

$u(t) = u_0 \cos(\omega_n t) + \frac{\dot{u}_0}{\omega_n} \sin(\omega_n t)$

which can be written as $u(t) = C \sin(\omega_n t + \theta)$

$C = \sqrt{u_0^2 + (\dot{u}_0 / \omega_n)^2}$ $\cos \theta = \frac{\dot{u}_0 / \omega_n}{C}$ $\sin \theta = \frac{u_0}{C}$

natural period $T_n = \frac{2\pi}{\omega_n}$ (s) natural frequency $f_n = \frac{1}{T_n} = \frac{\omega_n}{2\pi}$ (Hz)

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We have seen this is the complete solution, where we have the C as the amplitude of motion and this T_n is the natural period of vibration. This is the initial displacement, this is the initial velocity and this is the behaviour of the displacement with respect to time. So, natural frequency can be expressed like this, natural period can be expressed like this. Then we have seen how to model the earthquake excitation as a forced vibration.

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■ Equation of motion: Earthquake excitation

(a) (b)

$$f_t + f_D + f_S = 0$$

$f_S = ku$
 $f_D = c\dot{u}$
 $f_t = m\ddot{u}'$

$$m\ddot{u}' + c\dot{u} + ku = 0$$
$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g(t)$$

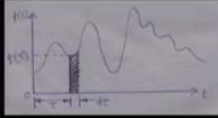
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So, this is the basic model where inertial force damper force and spring force equals to 0 where inertial force act's on the total displacement, because the acceleration acts on this mass for the total displacement, but damper and spring force acts further relative displacement compare to this ground and total displacement difference between that will give us the relative displacement.

So, by simplification, we got the basic equation of motion for earthquake excitation like this $m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g(t)$. So, \ddot{u}_g is nothing but the ground acceleration due to earthquake. So, that force will give us the dynamic force due to earthquake whereas, u is nothing but the relative displacement between ground and the top level, where masses concerned to be concentrated. Then we had also seen the forced vibration response to step excitation like this and the complete solution of that equation also we have discussed in our previous lecture.

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**Forced Vibration due to Arbitrary excitation
(Duhamel's Integral)**



$$dx(t) = f(\tau)d\tau h(t-\tau) = h(t-\tau).f(\tau)d\tau$$

So, $x(t) = \int_0^t h(t-\tau).f(\tau)d\tau$

$$x(t) = CF + PI$$

$$= e^{-\zeta\omega_p t} (A \cos \omega_p t + B \sin \omega_p t) + \int_0^t h(t-\tau).f(\tau)d\tau$$

Initial conditions, $x(0) = x_0, \dot{x}(0) = \dot{x}_0$

$$x(t) = e^{-\zeta\omega_p t} (x_0 \cos \omega_p t + \frac{\dot{x}_0 + \zeta\omega_p x_0}{\omega_p} \sin \omega_p t) + \int_0^t h(t-\tau).f(\tau)d\tau$$

where, $h(t) = \frac{1}{m\omega_p} e^{-\zeta\omega_p t} \sin \omega_p t$

If, $x(0) = 0, \dot{x}(0) = 0$

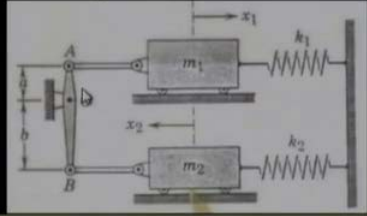
$$x(t) = \int_0^t h(t-\tau).f(\tau)d\tau \rightarrow \text{Duhamel's Integral}$$

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Then, for any arbitrary excitation, we have seen any random ocean like earthquake motion. We can take small infinite symbol portion and then, we can use Duhamel's integral to find out the particular integral part and complementary function part can be obtained by putting x equals to 0 in this basic equation of this one.

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For the system shown in Figure, mention (with reasoning) the number of degrees of freedom for the system for a small oscillation. Derive the governing equation of motion (from first principle). Consider, mass of the linkage AB and other connectors are negligible. Calculate the natural frequency and natural period of vibration for the system if $k_1 = k_2 = 90 \text{ N/m}$ and $m_1 = m_2 = 10 \text{ kg}$. And $c_1 = c_2 = 6 \text{ N-s/m}$. Estimate the damped frequency, damped period, damping ratio of the system.



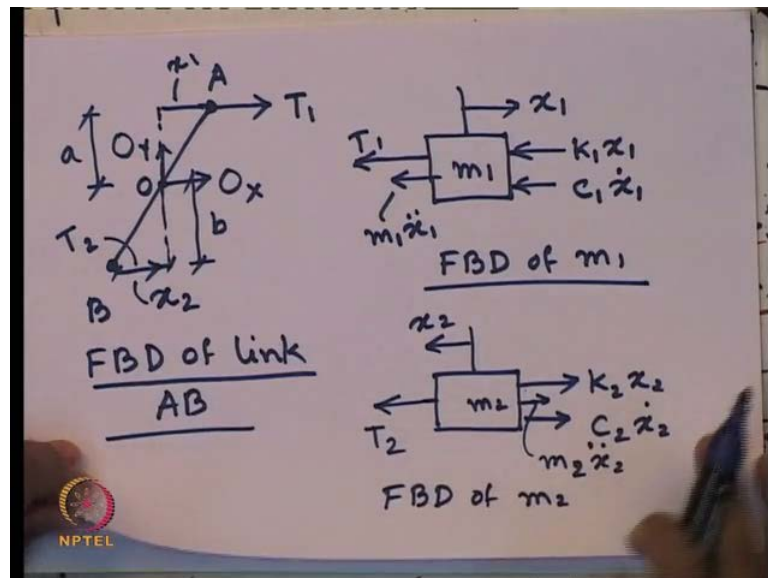
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So, we can now do a simple example problem on this topic of basics of vibration. Let us look at the problem over here. The problem statement is for the system shown in this figure here mentioned with reasoning the number of degrees of freedom for the system

for a small oscillation. So, for a small vibration, what is the degree of freedom for this entire system? Derive the governing equation of motion from the first principle for the system. Consider mass of the linkage AB and other connections like this connection, this connection, all are negligible. Compare to this mass m_1 and m_2 . They are connected to this roller on the friction free surface. We have displacement of mass m_1 x_1 T and for mass m_2 x_2 T in this direction; we have a hinge connection through this link A O B. At O, this hinge connection is there. So, the distance from A to O is a , and distance from O to B is b and it is connected through springs k_1 for mass 1 k_2 for mass 2. So, what it is asked that calculate the natural frequency and natural period of vibration for the system if this value of k_1 and k_2 are equal to this one and mass m_1 and m_2 are equal to this one?

Now, the next step. It is mentioned if suppose we add to dampers c_1 here and c_2 here with these values, then estimate the damped frequency, damped period and damping ratio of the system. So, let us see what will be the solution if we draw the free body diagram of this link.

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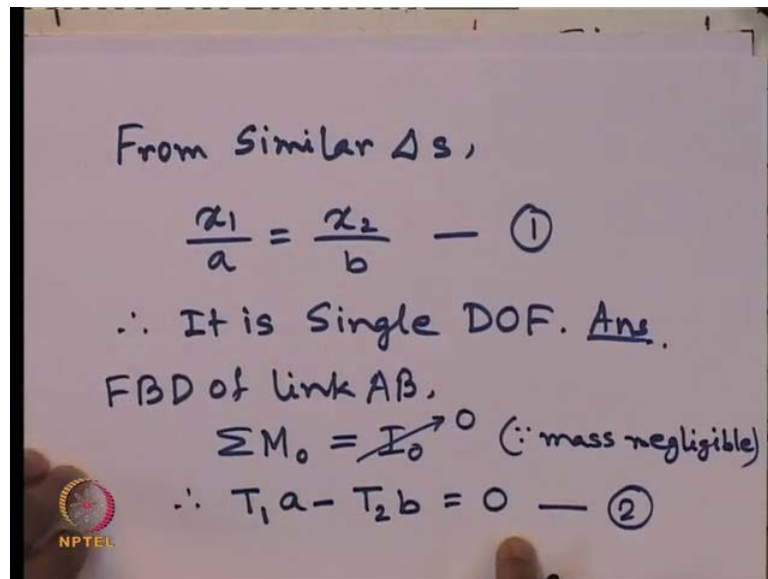


So, the link which is shown over here that AB link, AB we are drawing the free body diagram. So, this is moving by x_2 , this is moving by x_1 . So, this distance is x_2 , this distance is x_1 and corresponding forces are let us say, this is T_1 , let us say this is T_2 .

We have hinge over here O, where we can have reactions in the hinge in O X and O Y. These two directions and these dimensions are given A, and this distance is given as B.

So, if we draw the free body diagram of the masses, how it will look like? For m 1, we have x 1. So, force will be k 1 x 1. Let me draw for damper force. Also, for completeness c 1 x 1 dot and we have this inertial force m 1 x 1 double dot and the force in this link is T 1. So, this T 1 T 1 balances to that link. So, this is for the free body diagram of m 1, this is the free body diagram of link AB and for mass m 2, we will have k 2 x 2 c 2 x 2 dot. This is the direction of x 2, this one is T 2. So, this direction T 2 and this direction T 2 balances through that link and this one is m 2 x 2 double dot is the inertial force.

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So, this is free body diagram of m 2. Now, considering this similar triangles, these two from the free body diagram of link AB, what we can write that from similar triangles, we can write x 1 by a is equals to x 2 by b. Let us say this is equation number 1. We can have a look over here. See this is a, this is our x 1.

So, x 1 by a equals to this, x 2 by this b. So, that is what is shown over here. Therefore, what we can say about degrees of freedom? It is single degree of freedom. This is the answer of the first part of the question. Why? Because x1 and x2, they are not independent. They are dependent to each other. That means the entire system; this x 1 and x 2 are correlated. So, number of independent co-ordinative's either of them, hence the entire system is having a single degree of freedom system.

So, that is answer of first part. Now, from FBD of link AB, we can get if we take some of movement about that hinge point equals to $\dot{\theta}$ which is 0 because mass of that link is negligible, otherwise some inertial component will come. Therefore, we can write $T_1 \times a - T_2 \times b = 0$. So, this is our second equation. If we look at here, we are taking movement about this point. So, $T_1 \times a + T_2 \times b - T_1 \times a - T_2 \times b = 0$.

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The image shows a piece of paper with handwritten equations. At the top, it says 'FBD of m_1 ' followed by the equation $m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 + T_1 = 0$ labeled as equation (3). Below that, it says 'FBD of m_2 ' followed by the equation $m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2 - T_2 = 0$ labeled as equation (4). Then, it says 'From (3), using (2)' followed by the equation $m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 + \frac{T_2 \cdot b}{a} = 0$ labeled as equation (5). There is a small NPTEL logo in the bottom left corner of the paper.

So, with that, now from the free body diagram, so now using a FBD of m_1 , what we can write that $m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 + T_1 = 0$. This is say equation 3 and from FBD of mass m_2 , we can write $m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2 - T_2 = 0$. This is equation 4. Let us look here. So, $m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 + T_1 = 0$ and here, $m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2 - T_2 = 0$. That is what we had return to this equations.

Now, from this equation 3. So, from 3 using over equation 2, that means the relationship between this T_1 and $T_2 \times b$, what we can write that $m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 + T_2 \times \frac{b}{a} = 0$. That can be equation 5. Why? Because this T_1 is nothing but here equals to $T_2 \times \frac{b}{a}$. So, that is what I have written T_1 as $T_2 \times \frac{b}{a}$. So, using now equation 4 and 5, solving this 2 equations what we can write?

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Using (4),
 Eq. (5) becomes,

$$m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 + \frac{b}{a} (m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2) = 0$$

From, (1),

$$m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2 = 0$$

$$\rightarrow \left(m_1 + \frac{b^2}{a^2} m_2 \right) \ddot{x}_1 + \left(c_1 + \frac{b^2}{a^2} c_2 \right) \dot{x}_1 + \left(k_1 + \frac{b^2}{a^2} k_2 \right) x_1 = 0 \quad \text{--- (6)}$$

So, using four equations, 5 becomes $m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 + \frac{b}{a} (m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2)$. This equals to 0. How we got this? Look at here this equation 4, this one that is $m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2 = 0$. So, that is why we got this $\frac{b}{a}$ times $m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2$ which is this part. Hence, it becomes like this.

Now, from equation 1, now let us use what we can get from 1. From one we have a relationship between x_1 and x_2 through this a , and b . So, if we use that what we can write $m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 + \frac{b}{a} (m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2)$ into another $\frac{b}{a} (m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2)$. This will be x_1 because we have now used equation 1. So, $\frac{b}{a} (m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2) = 0$.

So, we have to be taking 1. Why? Let us go back again to equation 1. This x_2 is nothing but $\frac{b}{a} x_1$. That is what I am using here, $x_2 = \frac{b}{a} x_1$. That is why $\frac{b}{a}$, another $\frac{b}{a}$ came out and this become x_1 . Hence, on simplification, this equation we can write as $(m_1 + \frac{b^2}{a^2} m_2) \ddot{x}_1 + (c_1 + \frac{b^2}{a^2} c_2) \dot{x}_1 + (k_1 + \frac{b^2}{a^2} k_2) x_1 = 0$. Let us say this is equation 6. So, you can see everything is represented in terms of a single degree of freedom which is x_1 we have taken in this case.

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Using, $m_1 = m_2 = m$
 $k_1 = k_2 = k$
 $c_1 = c_2 = c$

Eq. (6), becomes,

$$\left(1 + \frac{b^2}{a^2}\right) (m\ddot{x}_1 + c\dot{x}_1 + kx_1) = 0$$

or, $m\ddot{x}_1 + c\dot{x}_1 + kx_1 = 0$ ($\because 1 + \frac{b^2}{a^2} \neq 0$)

Ans

So, now, what we can write further? Now, what is given condition using m_1 equals to m_2 equals to m . That is what is given to us, k_1 equals to k_2 equals to k and c_1 equals to c_2 equals to c . Using this equation 6 becomes $1 + \frac{b^2}{a^2}$. We can take common that becomes $m \ddot{x}_1 + c \dot{x}_1 + k x_1 = 0$ or $m \ddot{x}_1 + c \dot{x}_1 + k x_1 = 0$.

So, this is our governing equation of motion. Thus, it is the second part of the answer because this $1 + \frac{b^2}{a^2}$ term is never 0. That is the reason. So, this is governing equation of motion which was asked in the question. So, if we look at the slide here, first it says identify the number of degrees of freedom for a small oscillation that we have identified the answer. Single degree of freedom system then was derived the governing equation of motion from first principle, so from basics first principle now we have arrived. This is the equation of motion.

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Natural circular frequency,
$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{90}{10}} = 3 \text{ rad/s}$$

Natural period,
$$T_n = \frac{2\pi}{\omega_n} = 2.1 \text{ sec. } \underline{\text{Ans.}}$$

$\therefore f_n = \text{Natural frequency,}$
$$= \frac{1}{T_n} = 0.478 \text{ Hz.}$$

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Now, let us calculate the values. Whatever has been asked in this question, we have to find out. Natural circular frequency which is ω_n is nothing but root over K by m . What is the values of K , given that is 90 m is 10 kg which comes out to be 3 radian per second. That is another answer what is asked that calculate the natural frequency. What is natural period?

So, natural period is T_n which is 2π by ω_n which will be after putting this value of ω_n here about 2.1 second. So, this is answer of another part. Let us look here what was asked in the question. Calculate the natural frequency? Natural circular frequency is three radiant per second and natural period of vibration is 2.1 second. So, if somebody is interested to calculate, they can calculate f_n in terms of hertz also. That will be 1 by 2.1 So, therefore, f_n which is natural frequency will be equal to 1 by T_n that is 0.478 hertz. Next, it is asked what is the, let us see here. What is asked here estimate the damped frequency, damped period and damping ratio.

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Critical Damping,
 $C_c = 2m\omega_n$
 $= 2(10)(3) \text{ N.s/m}$
 $= 60 \text{ N.s/m.}$
Damping Ratio $= \eta = \frac{C}{C_c}$
 $= \frac{6}{60} = 0.1 \text{ or } 10\%$

The image shows a whiteboard with handwritten mathematical derivations. It starts with 'Critical Damping,' followed by the formula $C_c = 2m\omega_n$. The values are substituted as $2(10)(3)$ N.s/m, resulting in 60 N.s/m. Then, the 'Damping Ratio' is defined as $\eta = \frac{C}{C_c}$, which is calculated as $\frac{6}{60} = 0.1$ or 10% . An NPTEL logo is visible in the bottom left corner.

So, to find out these things, we need to compute first critical damping. Critical damping C_c . C_c is nothing but $2m\omega_n$ which is 2 into m is 10 kg ω_n . We obtain 3 radian per seconds so much Newton second per meter unit, so 60 Newton second per meter. So, damping ratio is nothing but let us say C by C_c . How much is c given? 6 . How much C_c we computed? 60 . So, it comes out to be 0.1 or if we represent in percent, it is 10 percent. Damping ratio is 10 percent. What is the damped period?

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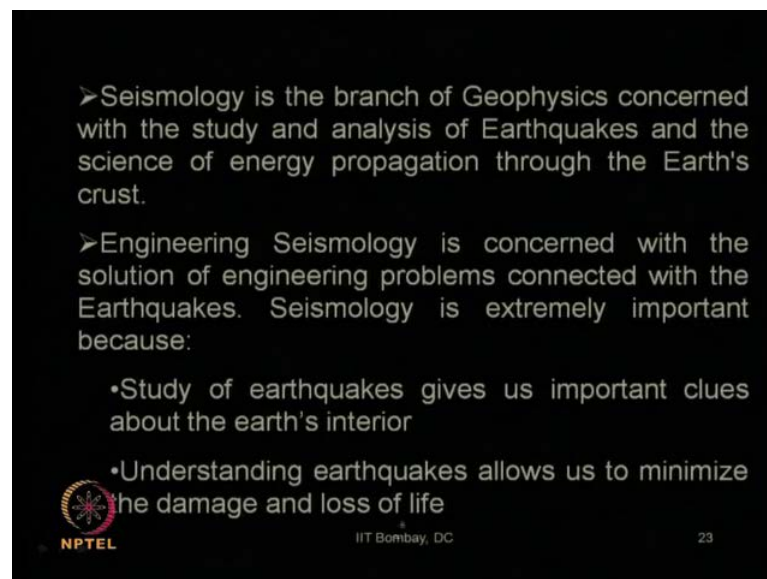
Damped circular frequency
 $\omega_d = \omega_n \sqrt{1 - \eta^2}$
 $= 3 \sqrt{1 - (0.1)^2} \text{ rad/s}$
 $= 2.985 \text{ rad/s}$
 \therefore Damped period $T_d = \frac{2\pi}{\omega_d} = 2.104 \text{ sec.}$
Damped frequency $f_d = \frac{1}{T_d} = 0.475 \text{ Hz}$
Ans

The image shows a whiteboard with handwritten mathematical derivations. It starts with 'Damped circular frequency' and the formula $\omega_d = \omega_n \sqrt{1 - \eta^2}$. The values are substituted as $3 \sqrt{1 - (0.1)^2}$ rad/s, resulting in 2.985 rad/s. Then, the 'Damped period' is calculated as $T_d = \frac{2\pi}{\omega_d} = 2.104$ sec. Finally, the 'Damped frequency' is calculated as $f_d = \frac{1}{T_d} = 0.475$ Hz. The word 'Ans' is written at the bottom right. An NPTEL logo is visible in the bottom left corner.

Let us first find out damped circular frequency ω_d , which is ω_n natural circular frequency times $1 - \eta^2$. Why this one? Because it is under damped because damping ratio is less than 1, which will be $\sqrt{1 - 0.1^2}$. So, ω_d will be 2.985 radian per second therefore, damped period T_d will be $T_d = \frac{2\pi}{\omega_d}$ if we put this value over here we will get 2.104 second and therefore, damped frequency will be $f_d = \frac{1}{T_d}$. Putting this value, we will get 0.475 hertz.

So, that is the answer of the total problem. So, with this recap and this problem which we have solved completely, now we ended the module number 2. Let us start our next module for this course on geotechnical earthquake engineering which is module 3. This module 3, we will be discussing on the topic of engineering seismology. Let us learn what is engineering seismology?


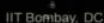
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> Seismology is the branch of Geophysics concerned with the study and analysis of Earthquakes and the science of energy propagation through the Earth's crust.

> Engineering Seismology is concerned with the solution of engineering problems connected with the Earthquakes. Seismology is extremely important because:

- Study of earthquakes gives us important clues about the earth's interior
- Understanding earthquakes allows us to minimize the damage and loss of life

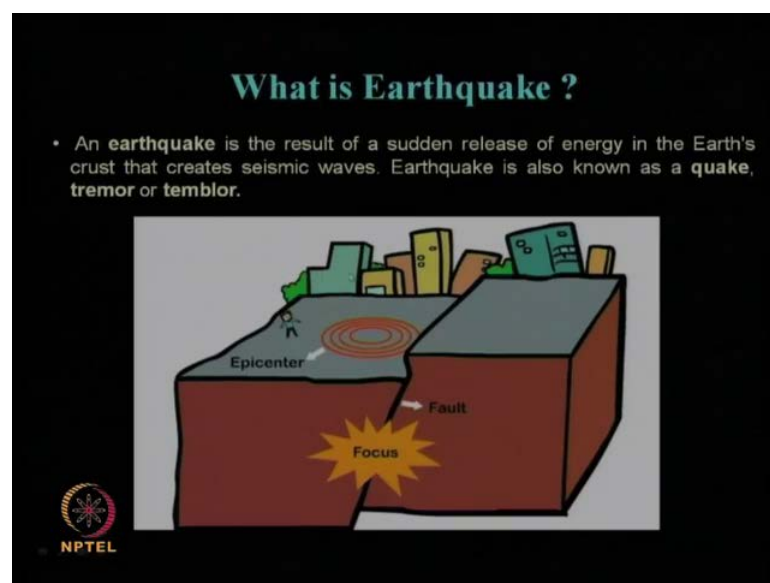
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If you look at here, the definition says seismology is the branch of geophysics which is concerned with the study and analysis of earthquakes and the science of energy propagation through the earth's crust. So, that is called seismology which is a branch, a subdivision of geophysics whereas, engineering seismology, it is concerned with the solution of engineering problems connected with the earthquakes. So, seismology is extremely important. Why? Because of the study of earthquakes give us important clues

about the earth's interior and understanding of that earthquake will allow us to minimize its damage and loss of life.

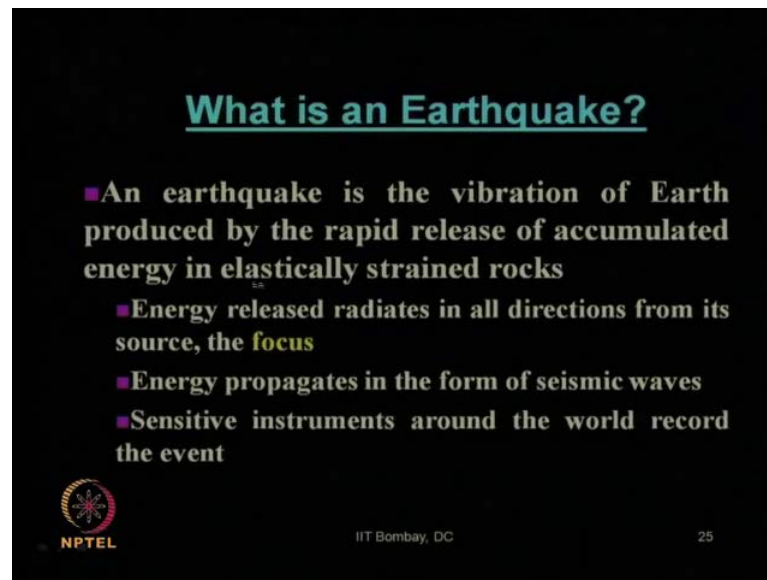
So, that is the reason we should learn seismology and in particular, engineering seismology, so that we can provide a better solution to handle this earthquake related damages to reduce or stop the loss of life and property. So, if you go to the basic definition of earthquake as per the seismology or engineering seismology is concerned, let us look at this picture.

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So, what is earthquake? Earthquake is the result of a sudden release of energy in the earth's crust that creates seismic waves. So, earthquake is also known as quake tremor or temblor. So, these are the various other alternative terms which are used for earthquake like only quake, a tremor or temblor. So, what it says? There is a sudden release of energy in the earth's crust. So, when that release of energy occurs, it creates the waves which are called seismic waves and that finally comes up to the ground and which damages all the buildings and structures whatever is located on the top of this ground. So, that is the basic concept of earthquake.

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What is an Earthquake?

- **An earthquake is the vibration of Earth produced by the rapid release of accumulated energy in elastically strained rocks**
 - Energy released radiates in all directions from its source, the **focus**
 - Energy propagates in the form of seismic waves
 - Sensitive instruments around the world record the event

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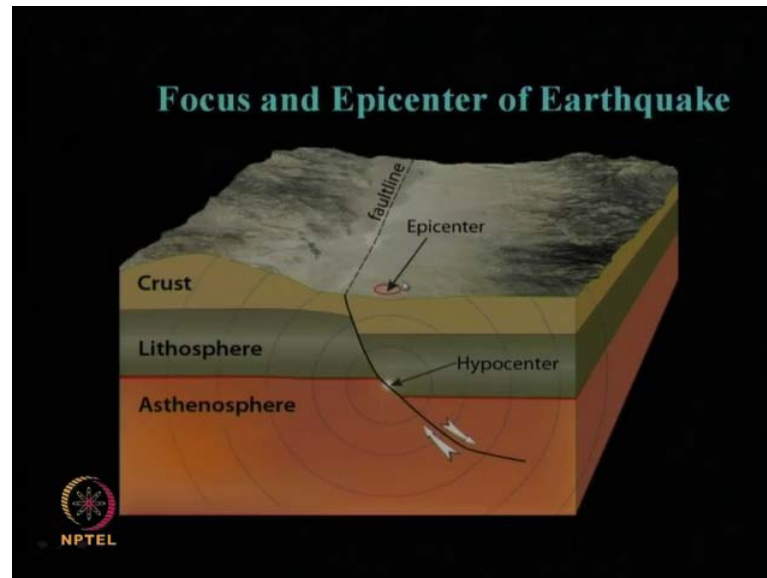
So, let us see the further definition of earthquake. An earthquake is the vibration of earth produced by the rapid release of accumulated energy in elastically strained rocks. So, this energy released, it radiates in all the directions from its source that is source means where the earthquake gets generated in the earth's crust when that energy gets released due to the vibration process in the strained rocks. So, energy gets released. Automatically those radiation of that energy occurs in all the directions and that point where that release of energy occurs is called focus of earthquake.

So, energy propagates in the form of seismic waves and sensitive instruments all around the world can record the event. We will see later that earthquake occurring at a particular place on the world map can be recorded by various other corners of the world like suppose an earthquake has occurred in Japan, in India. It can be recorded in US. It can be recorded, but there are few limitations etcetera which we will see later on from the seismology or geophysics point of view, because of propagation of the waves and their different zone concept, blind zone, etcetera, but typically we can say it is not that an earthquake occurring at a particular place has to be recorded in the close vicinity of that place only. Otherwise, suppose if any earthquake is occurring in deep sea, we could have never ever measured that earthquake if that would have been the reason.

So, as the earthquake releases energy have which travels in the form of seismic waves, so that travels for a much longer distance which can be recorded in all this sensitive

instruments all over the world and that is the reason, even an earthquake occurring in a mid-desert or in a mid-sea or in a mid-ocean can also get recorded in this sensitive instruments placed all over the world.

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Let us see the next terms which are focus and epicentre of earthquake. So, what is this basic terminology? If you see this point, what just now we have mentioned the point where that release of energy occurs inside the earth's crust. That is the point call focus of earthquake. The other name of that focus is also called hypocenter of the earthquake. That hypocenter and focus is the same term. So, that hypocenter from that point if you project it vertically on the ground surface from this point, where it meets the ground surface; that is called epicentre of earthquake.

So, what does it mean? Epicentre means it is always has to be on the ground surface. So, epicentre does not mean that it is the point where earthquake energy gets released. It is the point which is nothing but the vertical projection of the energy release point of the earthquake on ground. So, from this point, energy gets realised and then, seismic waves travels in all the directions and if you project it vertically, that will give you the epicentre. Now, you can of course ask me that earth if we take like this, you can take vertical projection like this in all the directions because it is a kind of a circular arc shape from one point inside the circle. You can have several projections, but whatever projection on the ground surface will give you the least distance that signifies your

epicentre. Is that clear? So, the least vertical projected point on the ground will give us the location of epicentre of an earthquake on ground surface. Now, we want to see what are the basic reasons for earthquake?

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What causes an Earthquake?

Movement of Tectonic Plates

Earth is divided into sections called Tectonic plates that float on the fluid-like interior of the Earth. Earthquakes are usually caused by sudden movement of earth plates

Rupture of rocks along a fault

Faults are localized areas of weakness in the surface of the Earth, sometimes the plate boundary itself

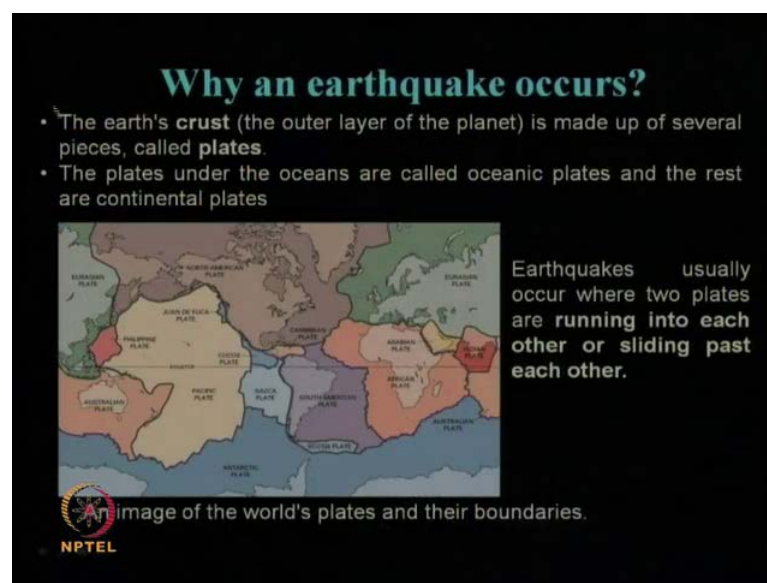
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So, what causes an earthquake? There are two basic causes of earthquake. One can be called as movement of the tectonic plates and another can be called as rupture of rocks along a fault. So, movement of tectonic plates what occurs here like earth, it is divided into sections, various sections or various plates which are called as tectonic plates and this tectonic plates, they float on the fluid like interior of the earth. As we know earth is having three major different portions like core of earth, then mantle of earth and then, crust of earth.

So, these crust plates or tectonic plates, these typically move on this fluid like interior of the earth and these earthquakes are usually caused by sudden movements of these plates. These tectonic plates, they move and when they move suddenly with respect to each other that cause an earthquake, this is called plate tectonic earthquake. The another cause of earthquake can be rupture of rocks along fault. Now, what are the faults? Faults are localized areas of weakness in the surface of the earth like on the earth surface. So, this is the earth surface which is composed of tectonic plates. Those plates suddenly move with respect to each, creates earthquake.

Now, within the tectonic plates wherever we have weakness in the rock, in the earth crust, those are called as faults. So, faults are localized areas of weakness in the surface of earth. Sometimes, this two plate boundary itself can be considered as a fault because that is a weakness that is open point, where that energy inside the earth can get released through that weak point or those faults. So, whenever there is a rupture when this rock breaks out along this weak point or these faults, those are called fault related earthquake. So, one is plate tectonic earthquake, another is fault rupture earthquake or fault related earthquake. So, these are the two major causes of earthquake.

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Now, why an earthquake occurs, it can be another question. So, let us see like the earth's crust, there is the outer most layer of the earth planet which is made up of several pieces as we have mentioned just now which are called plates or tectonic plates. So, you can see various plates. So, over here in this picture, the plates under the oceans are called oceanic plates whereas, the rest of the plates which are below these continents are called continental plates.

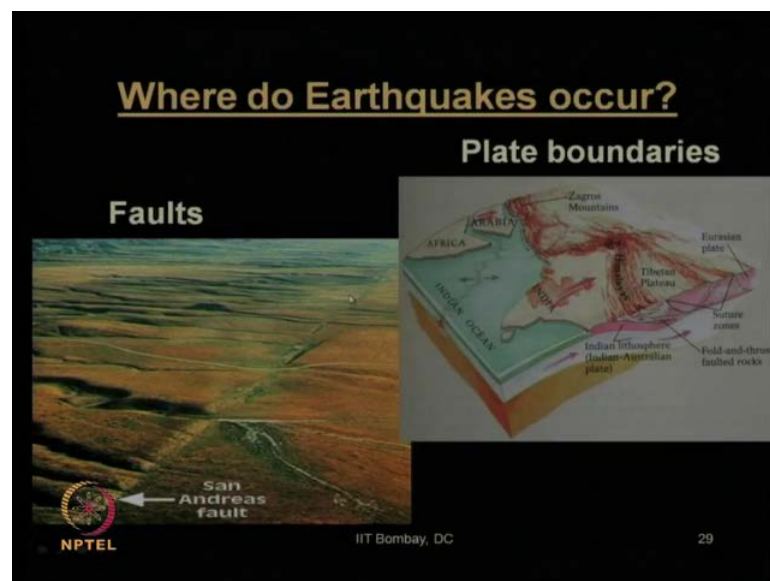
So, if you divide these plates tectonic plates in two major category we can say one can be oceanic plate another can be continental plates. So, you can see over here several plates, nomenclature like earths usually. Earthquakes usually occur where these two plates are running into each other or sliding past each other. What does it mean? As I am showing

over here, when these two plates move with respect to each other like this or it can move like this or one plate can get entered into another plate like this.

So, several combinations or individual movement of this plate can create the earthquake. So, that is why it is return over here. Earthquakes usually occur when these two plates are running into each other, that is one is entering into the other one or subducting we call. Later on we will use these technical terms or when these two plates slide past, that is move with respect to each other by the process of sliding, it can be horizontally vertically inclined anything and this image shows the worlds plates and their corresponding typical boundaries like as you can see the entire world map. It is now placed in the two dimension as you can see this part continues to this portion. So, you can see over here. This is Eurasian plate. This plate is called Eurasian plate Europe cum Asia portion of Asia and Europe. That is why the name Eurasian plate this continuous over here. So, that is why the other portion on this side. This is Indian plate, this part is Australian plate. You can see over here, so this continuous over here to up to this. So, this is Australian plate.

Now, South American plate. This is Caribbean plate, this is Nazca plate, this is Arabian plate, this is African plate, this one is North American plate and this is Juan de Fuca plate. This is Philippine plate, this is cocos plate, this is pacific plate, this is Antarctic plate and this is Scotia plate.

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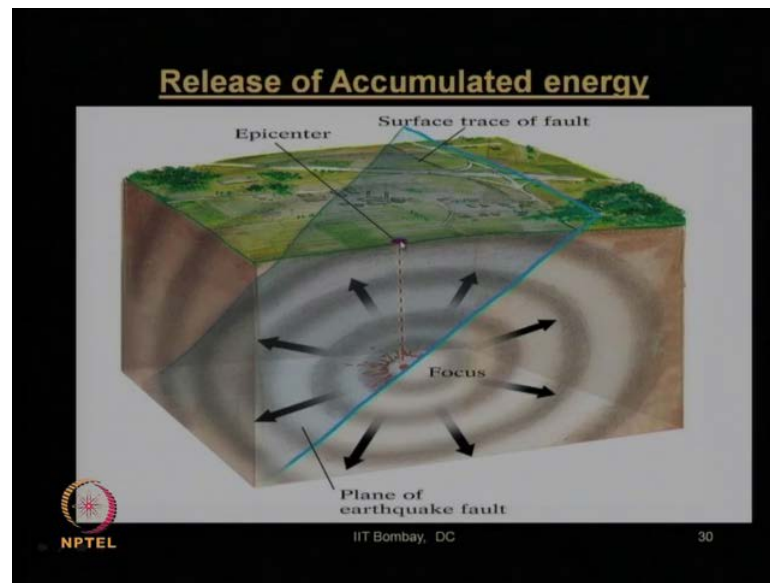


So, these are the various major plates all over the world and they are various plate boundaries. Now, where do earthquake occurs, we have seen already. What causes an earthquake? There are two major causes. As we have mentioned, one is plate tectonic earthquake and another is called fault rapture earthquake. So, these are the two causes of earthquake that is where do earthquake occurs. It can occur at the plate boundary that is between the boundary of two plates. It can occur like Eurasian plate is here, Indian plate and sometimes, they are combined and together they are called Indian-Australian plate as I have shown in the previous figure, just now Indian plate and Australian plate.

So, you can see over here, Indian and Australian plate are trying to go below this Eurasian plate. As this Indian and Australian plate tries to go below this Eurasian plate, in between we have Himalaya, Mount Himalaya that is still moving up. That is why the geologists, they say the height of Himalaya is increasing day by day even though it is couple of millimetres or centimetre, but still the height of that Himalaya is increasing because there is always a pressure or a movement of this plate and from this side another plate. So, automatically what happens if you see over here? Indian plate and Eurasian plate, Indian plate is subducting over here and Eurasian plate is here. In between we have already Himalayan Mountain as the boundary. So, in that boundary, automatically it moves up.

So, in plate boundary, these are the reason or these are the locations where earthquake can easily occur because of this plate movement. That is the reason why this Himalayan belt is having more occurrence of earthquake which had occurred. As I have shown in one of the figure earlier that most of our Indian and Indian sub-continental region earthquake are mostly clustered in this region of Himalayan region, whether northern part of India or north eastern part of India. Major reason goes to this plate boundary movement or plate tectonic movement of this Indian plate and Eurasian plate whereas, if we talk about the fault rapture earthquake, another type of earthquake classic example as can be given is the San Andrea's fault which is the fault located in California state of USA. So, along that fault, whenever there is a because fault means it is already have location of weakness in the earth crust. So, obviously energy when it tries to get released, they will prefer this fault location or these locations where from the earthquake energy gets released. That is the reason why this along this fault number of earthquakes can occur again and again.

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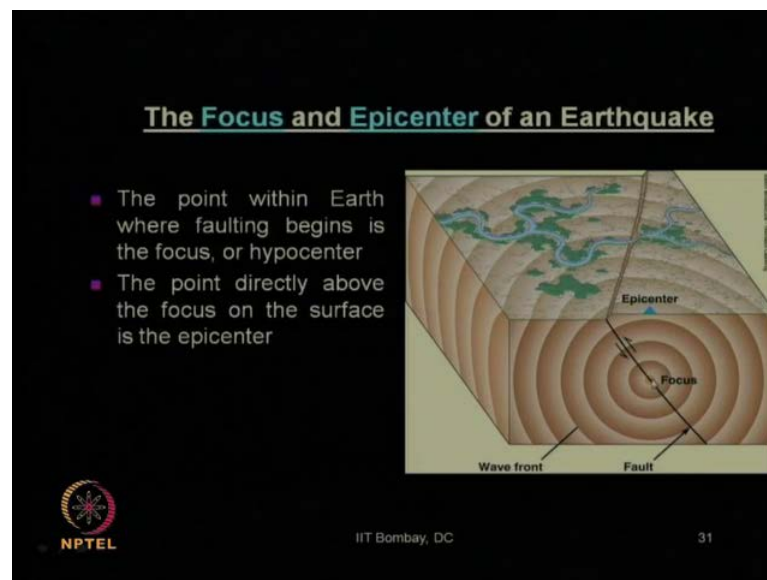
So, this picture shows how the release of accumulated energy can occur from the focus of the earthquake or hypocenter of the earthquake. So, if you see this point, the earthquake energy gets released and finally, the waves are travelling in all the directions. So, if we say this is the plane of earthquake fault and that plane of earthquake fault which creates a trace on the ground surfaces called surface trace of the fault, so on the ground whatever fault we see, suppose we see this fault. Just now I gave the example of San Andrea's fault say this is the San Andrea's fault. It does not mean that the fault location is limited within this length and only a few meters of depth or area can be limited. It can extend in any direction like this.

So, geologists or seismologists, they gave us, they generally give us the length of the fault, depth of the fault and this area of the cross sectional portion of the fault as a fault characteristics or fault data base information which are necessary because otherwise suppose, if you think that fault is located in this direction, the depth if you considered in this direction, obviously your calculation of fault, characteristics, everything will go wrong.

So, through this picture, it shows that we have to identify this plane of earthquake fault, their depth, and their projected or cross sectional area and which traces on the ground surface is nothing but the surface trace of the fault. Even it is possible sometimes this surface tracing may not be seen on the ground surface. What I want to mean may be this

fault may criss-cross another fault. In that case, its surface trace on the ground is not visible. In that case, it is called a hidden fault. We will come to that later. So, from this focus as I said earlier, if you draw a vertical projection on the ground surface, on the ground surface wherever it meets that point is called the Epicentre of the earthquake.

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So, this is another picture which shows the focus and Epicentre of the earthquake. This is the focus, this is the fault, these are seismic wave fronts and the vertical projection on the ground, this will be the epicentre, the point within the earth where faulting begins is called focus or hypocenter and the point directly above the focus on the surface is called as epicentre. So, with this, we will stop our lecture today. We will continue our lecture in the next class.